Electricity & Magnetism
Lecture 2: Electric Fields

Today's Concepts:

A) The Electric Field

B) Continuous Charge Distributions

Your Comments

Suddenly, terrible haiku:
Positive test charge Repelled by optimists
Cup half electric

What if there are two non parallel infinity line and a point A,
what will the angel between the two lines affect the electric field at point A?

I could not understand the concept of the continues or the infinite line of charge. Can Yooooooou please go over it on class

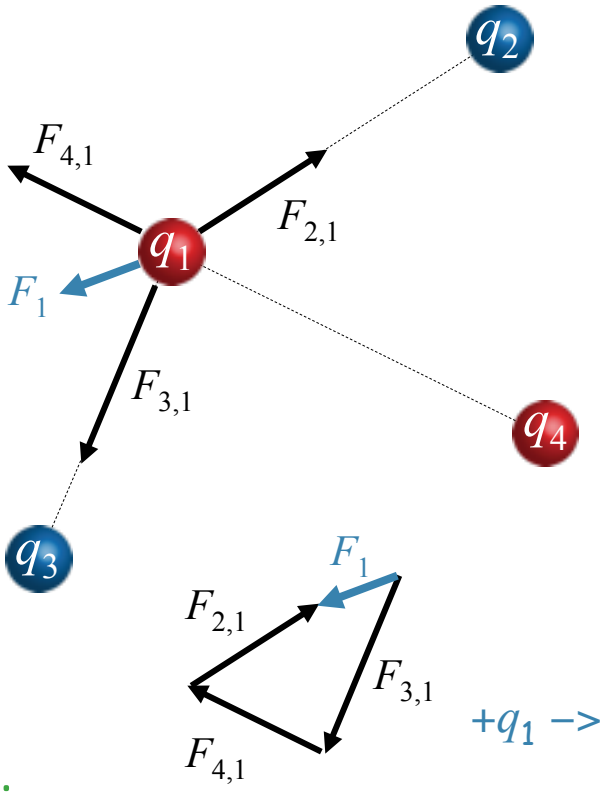
??Infinite lines of chargee??? it would help alot to go over those, everything else is simmplle

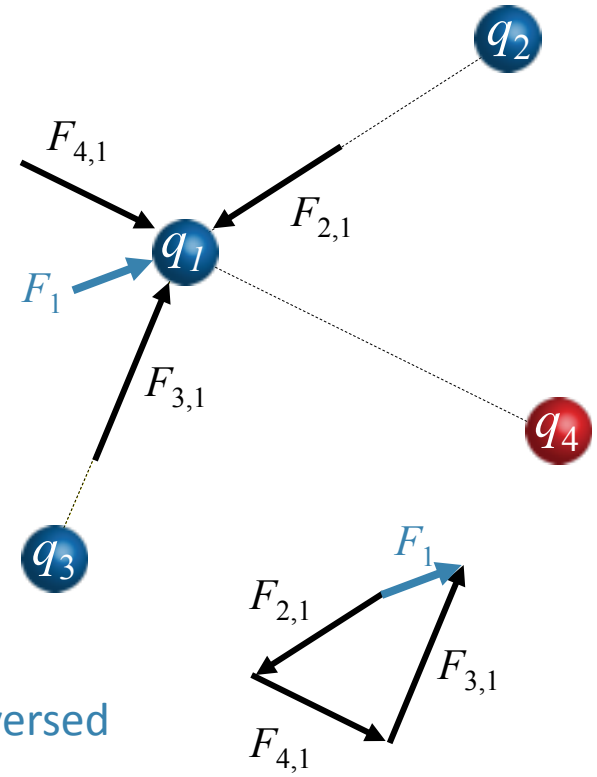
Positives and negatives, what do they really mean? It seems like we, as humans, just decided to name them this.

I get the feeling that I still don't entirely understand fields, so if we could work on that, it'd be super awesome.

Coulomb's Law (from last time)

If there are more than two charges present, the total force on any given charge is just the vector sum of the forces due to each of the other charges:





$+q_1$ –> $-q_1$ Þ direction reversed

MATH:

$$\vec{F}_1 = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} + \frac{kq_1q_3}{r_{13}^2}\hat{r}_{13} + \frac{kq_1q_4}{r_{14}^2}\hat{r}_{14} \rightarrow \vec{E} = \frac{\vec{F}_1}{q_1} = \frac{kq_2}{r_{12}^2}\hat{r}_{12} + \frac{kq_3}{r_{13}^2}\hat{r}_{13} + \frac{kq_4}{r_{14}^2}\hat{r}_{14}$$

Electric Field

"What exactly does the electric field that we calculate mean/represent? "

"What is the essence of an electric field? "

The electric field E at a point in space is simply the force per unit charge at that point.

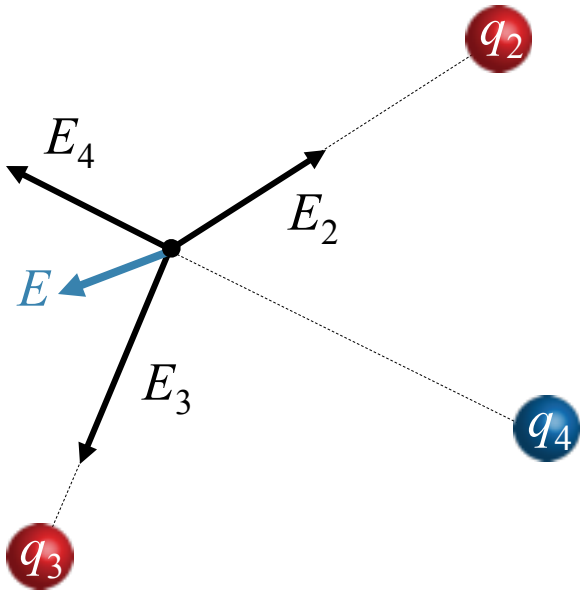
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric field due to a point charged particle

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

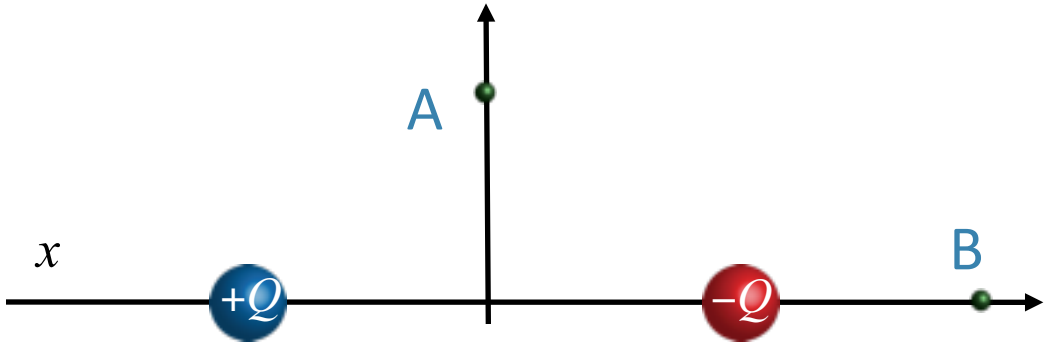
Superposition

$$\vec{E} = \sum_i k\frac{Q_i}{r_i^2}\hat{r}_i$$



Field points toward negative and Away from positive charges.

CheckPoint: Electric Fields1

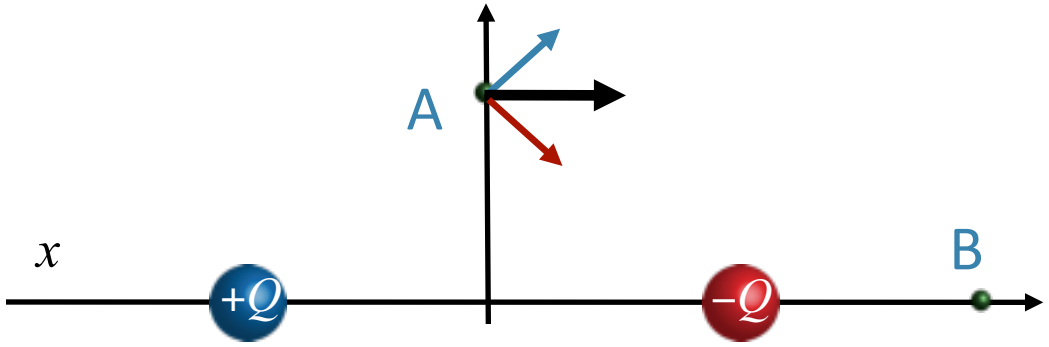


Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is the direction of the electric field at point A?

- ○ Up
- ○ Down
- ○ Left
- ○ Right
- ○ Zero

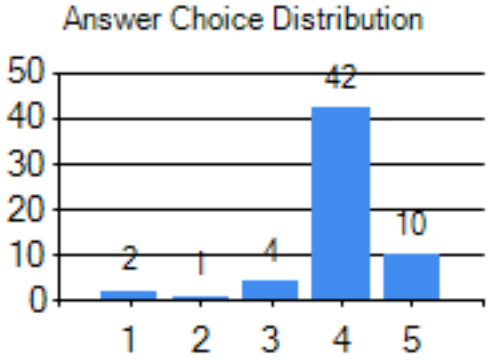
CheckPoint Results: Electric Fields1



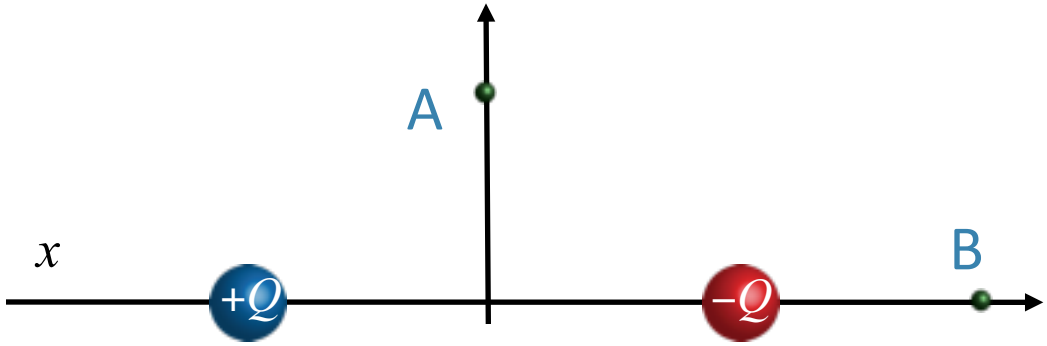
Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is the direction of the electric field at point A?

- ○ Up
- ○ Down
- ○ Left
- ○ Right
- ○ Zero



CheckPoint: Electric Fields2

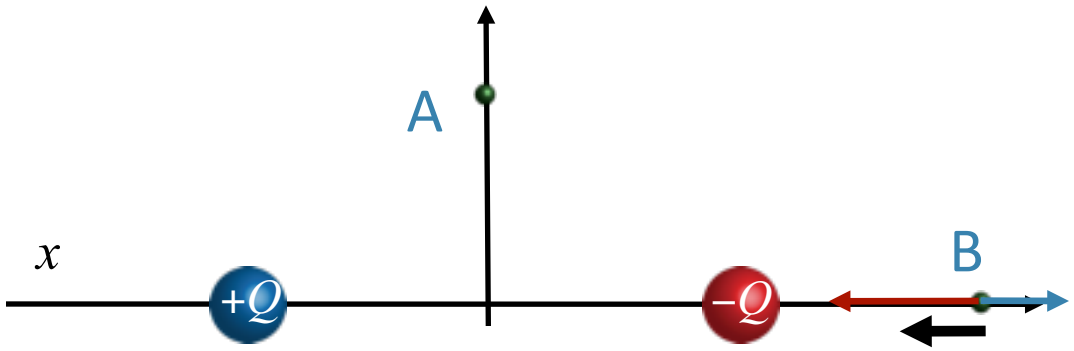


Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is the direction of the electric field at point B?

- ○ Up
- ○ Down
- ○ Left
- ○ Right
- ○ Zero

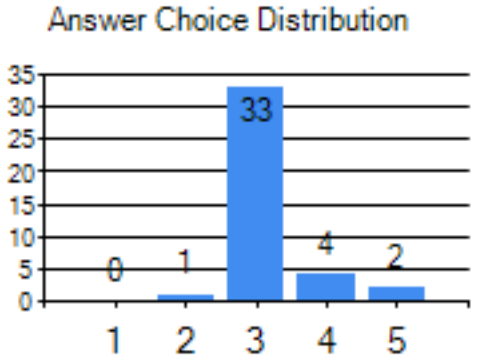
CheckPoint Results: Electric Fields2



Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

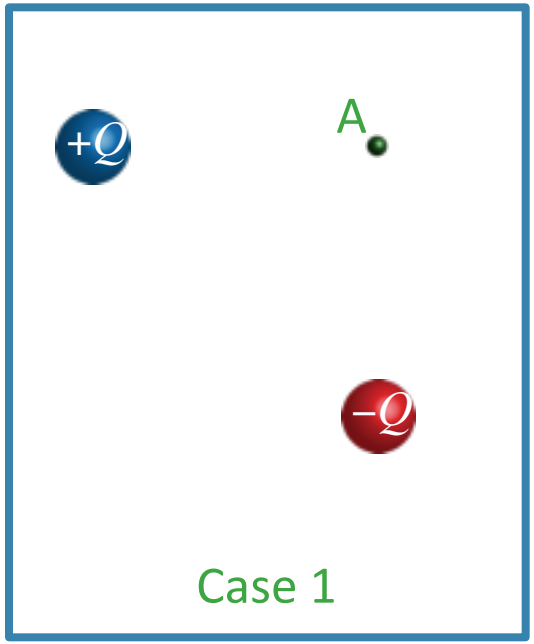
What is the direction of the electric field at point B?

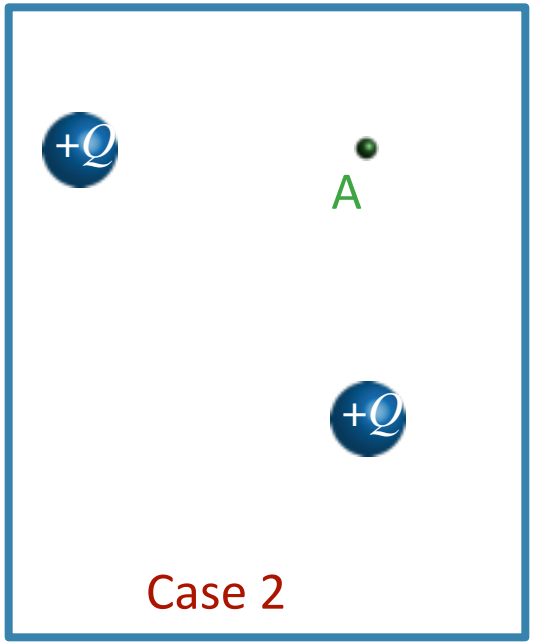
- ○ Up
- ○ Down
- ○ Left
- ○ Right
- ○ Zero



CheckPoint: Magnitude of Field (2 Charges)

In which of the two cases shown below is the magnitude of the electric field at the point labeled A the largest?

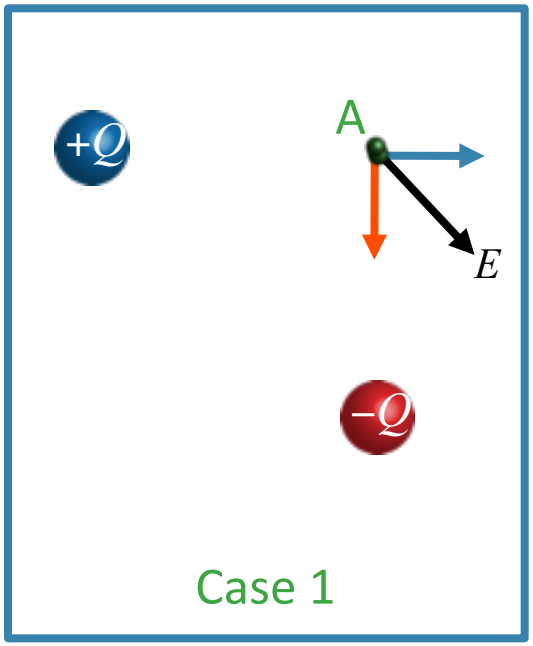


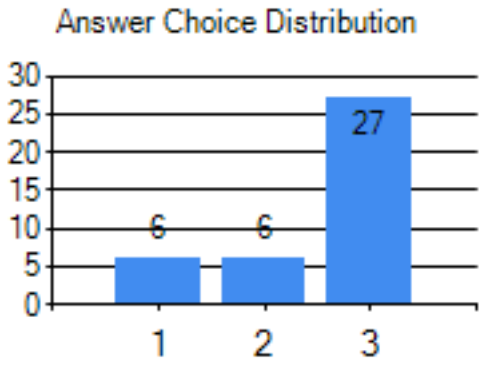


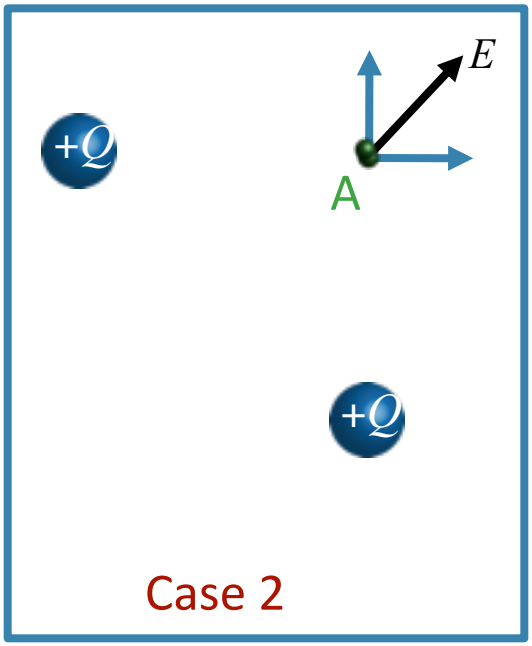
- ○ Case 1
- ○ Case 2
- ○ Equal

CheckPoint Results: Magnitude of Field (2 Chrg)

In which of the two cases shown below is the magnitude of the electric field at the point labeled A the largest?





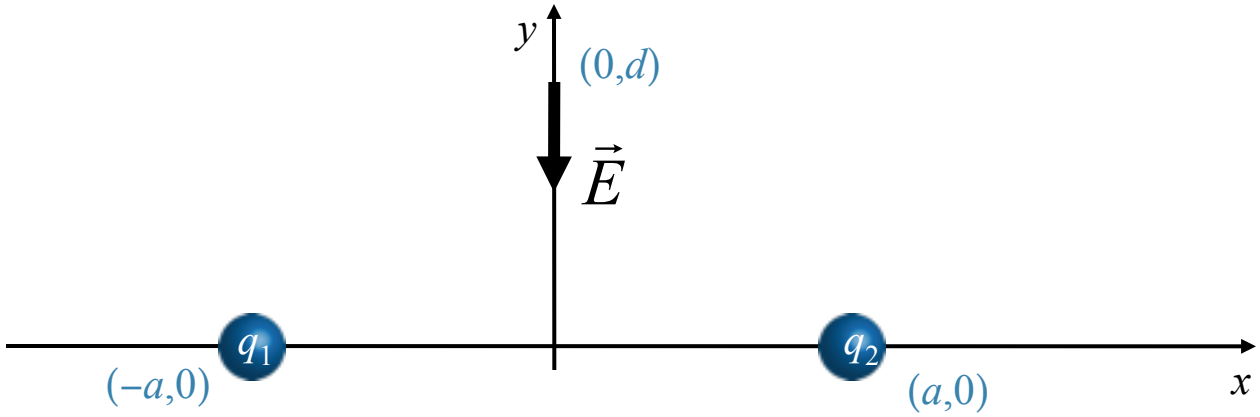


- ○ Case 1
- ○ Case 2
- ○ Equal

"The upper left +Q only affects the x direction in both and the lower right (+/-)Q only affects the y direction so in both, nothing cancels out, so they'll have the same magnitude. "

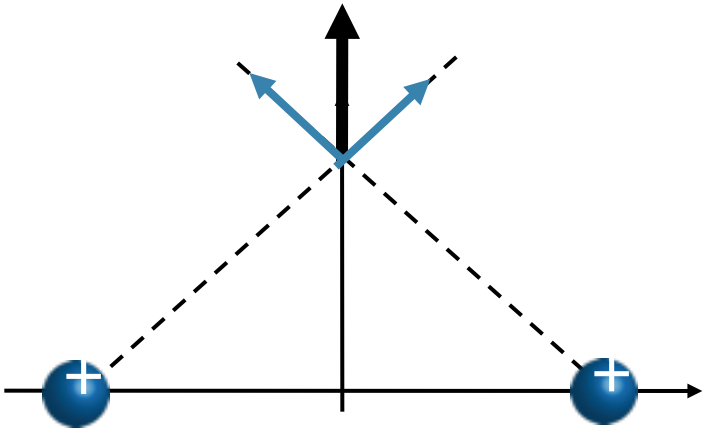
Clicker Question: Two Charges

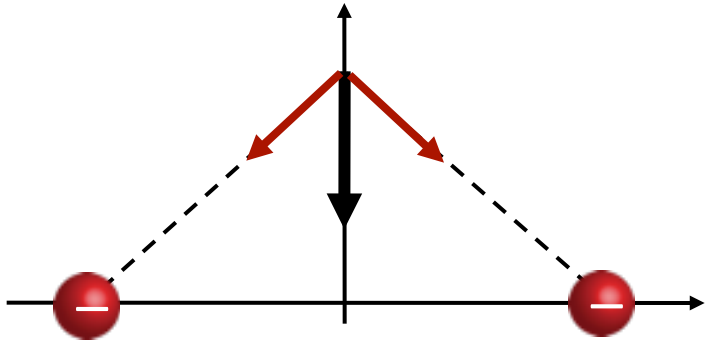
Two charges q_1 and q_2 are fixed at points $(-a,0)$ and $(a,0)$ as shown. Together they produce an electric field at point $(0,d)$ which is directed along the negative *y*-axis.

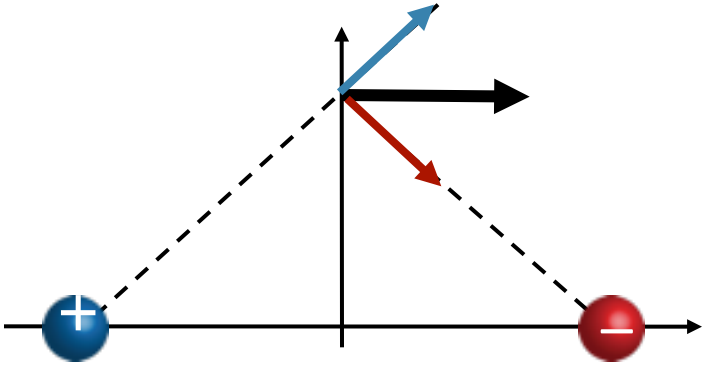


Which of the following statements is true:

A) Both charges are negative
B) Both charges are positive
C) The charges are opposite
D) There is not enough information to tell how the charges are related



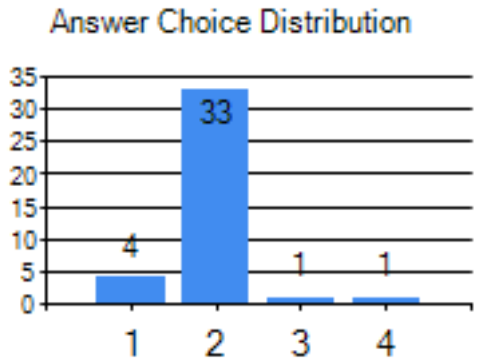




CheckPoint Results: Motion of Test Charge

A positive test charge q is released from rest at distance r away from a charge of +Q and a distance 2r away from a charge of +2Q. How will the test charge move immediately after being released?

- ○ To the left
- ○ To the right
- ○ Stay still
- ○ Other



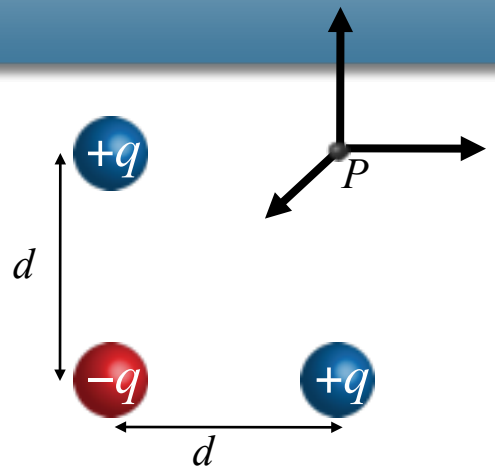
"The force is proportional to the charge divided by the square of the distance. Therefore, the force of the 2Q charge is 1/2 as much as the force of the Q charge. "

"Even though the charge on the right is larger, it is twice as far away, which makes the force it exherts on the test charge half that as the charge on the left, causing the charge to move to the right."

The ratio between the R and Q on both sides is 1:1 meaning they will result in the same magnitude of electric field acting in opposite directions, causing q to remain still.

Electric Field Example

What is the direction of the electric field at point P, the unoccupied corner of the square?



A) 

B) 

C) $E = 0$

D) Need to know d

E) Need to know d & q

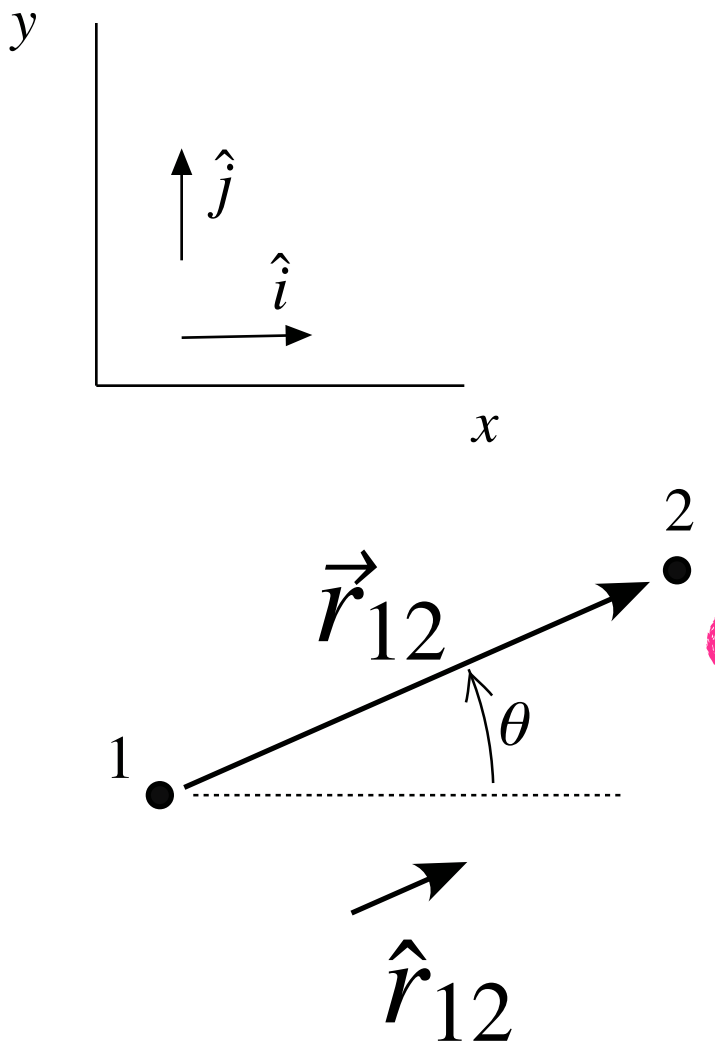
Calculate E at point P.

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

$$E_x = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2}\cos\frac{\pi}{4}\right)$$

$$E_y = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2}\sin\frac{\pi}{4}\right)$$

About $\hat{r}$

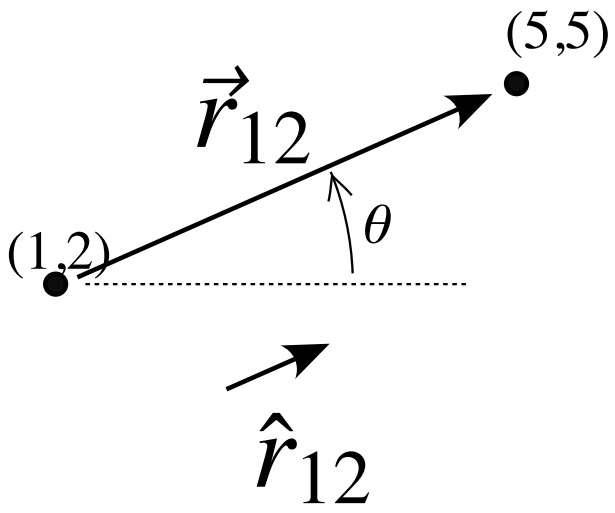


$$\vec{r}_{12} = r_{12}\cos\theta\,\hat{i} + r_{12}\sin\theta\,\hat{j}$$

$$\hat{r}_{12} = \frac{r_{12}\cos\theta\,\hat{i} + r_{12}\sin\theta\,\hat{j}}{r_{12}}$$

$$\hat{r}_{12} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

For example



$$\vec{r}_{12} = 4\hat{i} + 3\,\hat{j}$$

$$r_{12} = \sqrt{4^2 + 3^2} = 5$$

$$\cos\theta = \frac{4}{5}$$

$$\sin\theta = \frac{3}{5}$$

$$\hat{r}_{12} = \frac{r_{12}\cos\theta\,\hat{i} + r_{12}\sin\theta\,\hat{j}}{r_{12}}$$

$$\hat{r}_{12} = \frac{4}{5}\,\hat{i} + \frac{3}{5}\,\hat{j}$$

Continuous Charge Distributions

"I don't understand the whole dq thing and lambda."

Summation becomes an integral (be careful with vector nature)

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i \quad \rightarrow \quad \vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

WHAT DOES THIS MEAN ?

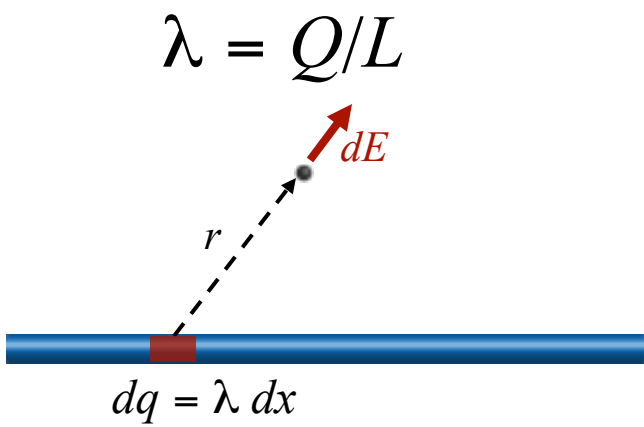
Integrate over all charges (dq)

r is vector from dq to the point at which E is defined

Linear Example:

pt for E •

charges ● ● ● ● ● ● ●



Clicker Question: Charge Density

"I would like to know more about the charge density."

Linear $(\lambda = Q/L)$ Coulombs/meter

Surface $(\sigma = Q/A)$ Coulombs/meter^2

Volume $(\rho = Q/V)$ Coulombs/meter^3

Some Geometry

$$A_{sphere} = 4\pi R^2 \qquad A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \tfrac{4}{3}\pi R^3 \qquad V_{cylinder} = \pi R^2 L$$

What has more net charge?.

A) A sphere w/ radius 2 meters and volume charge density ρ = 2 C/m^3

B) A sphere w/ radius 2 meters and surface charge density σ = 2 C/m^2

C) Both A) and B) have the same net charge.

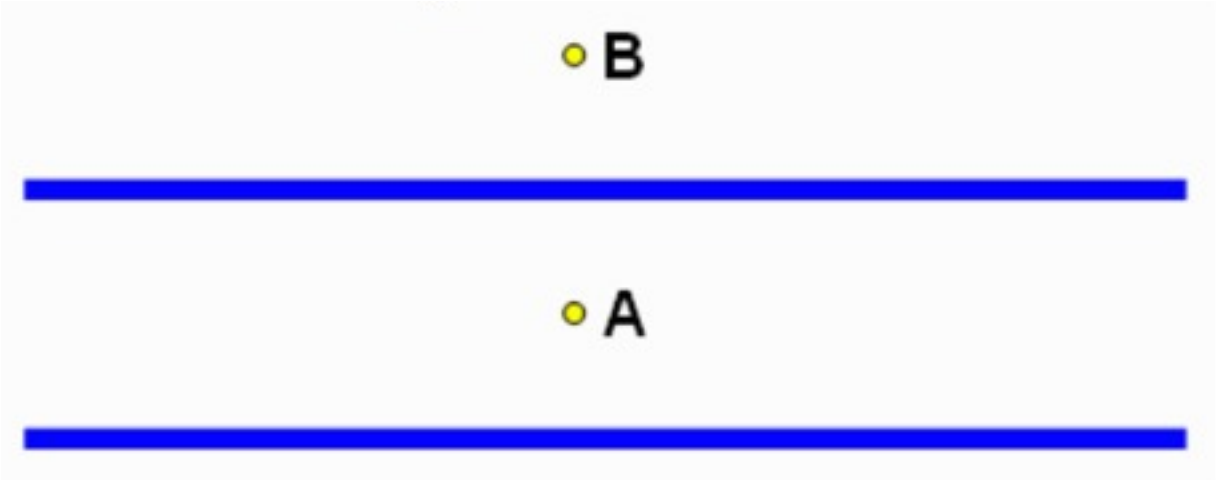
$$Q_A = \rho V = \rho \tfrac{4}{3}\pi R^3$$

$$Q_B = \sigma A = \sigma 4\pi R^2$$

$$\frac{Q_A}{Q_B} = \frac{\rho \frac{4}{3}\pi R^3}{\sigma 4\pi R^2} = \frac{1}{3}\frac{\rho}{\sigma} R$$

CheckPoint: Two Lines of Charge

Two infinite lines of charge are shown below.



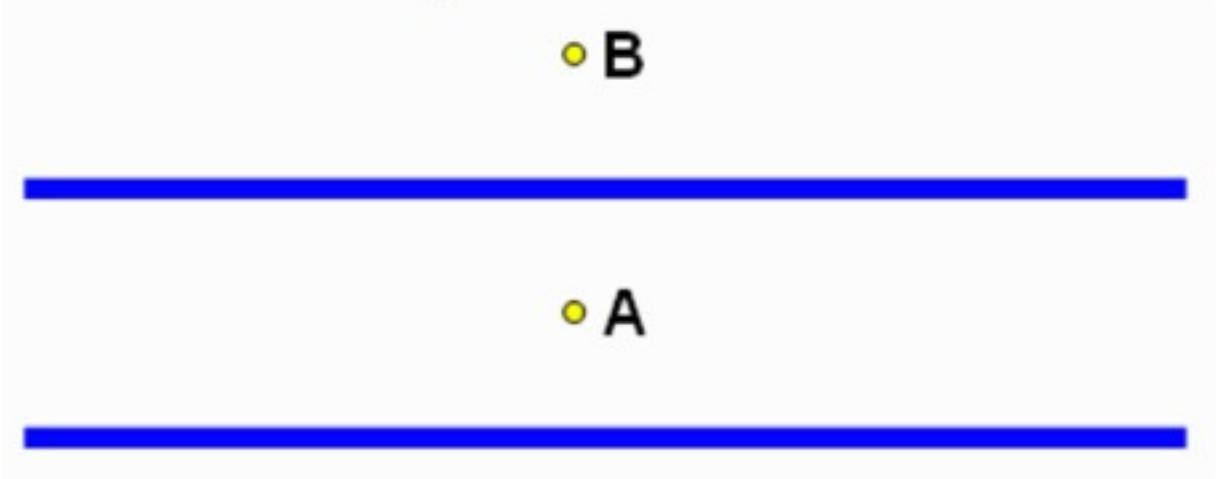
Both lines have identical charge densities $+\lambda$ C/m. Point A is equidistant from both lines and Point B is located a above the top line as shown.

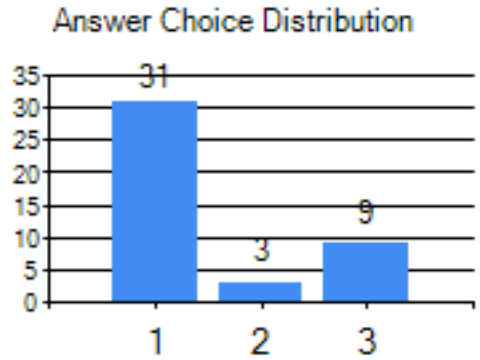
How does E_A, the magnitude of the electric field at point A, compare to E_B, the magnitude of the electric field at point B?

- ○ $E_A < E_B$
- ○ $E_A = E_B$
- ○ $E_A > E_B$

CheckPoint: Two Lines of Charge

Two infinite lines of charge are shown below.





Both lines have identical charge densities $+\lambda$ C/m. Point A is equidistant from both lines and Point B is located a above the top line as shown.

How does E_A, the magnitude of the electric field at point A, compare to E_B, the magnitude of the electric field at point B?

- ○ $E_A < E_B$
- ○ $E_A = E_B$
- ○ $E_A > E_B$