Electricity & Magnetism
Lecture 3: Electric Flux and Field Lines

Today's Concepts:

A) Electric Flux

B) Field Lines

→ Gauss' Law

Your Comments

What the heck is epsilon 0?

IT'S JUST A CONSTANT

$$\vec{E} = k\frac{q}{r^2}\hat{r} \qquad \vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$$

$$k \equiv \frac{1}{4\pi\varepsilon_o}$$

$k = 9 \text{ x } 10^9 \text{ N m}^2 / \text{C}^2$

$\varepsilon_o = 8.85\, x\, 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2$

I don't understand electric flux, how it's derived and the formula. I also need someone to explain ε_o or ε not. Cause I don't know what that is.

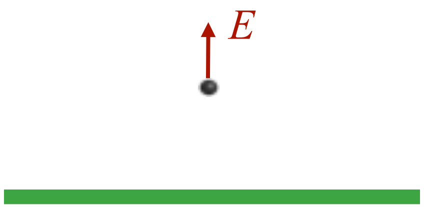
"Calculating Electric Field from Arc of Charge!" [hint]

"Why is gauss' law so important? Why is flux a useful value?"

My Comments

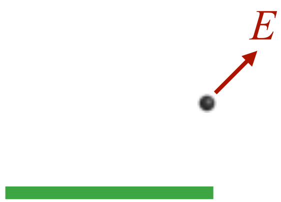
- You will need to understand integrals in this course!!!
- Forces and Fields are Vectors
- Always Draw a Picture First. What do the Forces/Fields Look Like?

Prelecture



Infinite line of charge

Activity Guide



Finite line of charge
(constant λ)

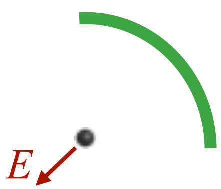
Exam?

Finite line of charge
(non-constant λ)

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

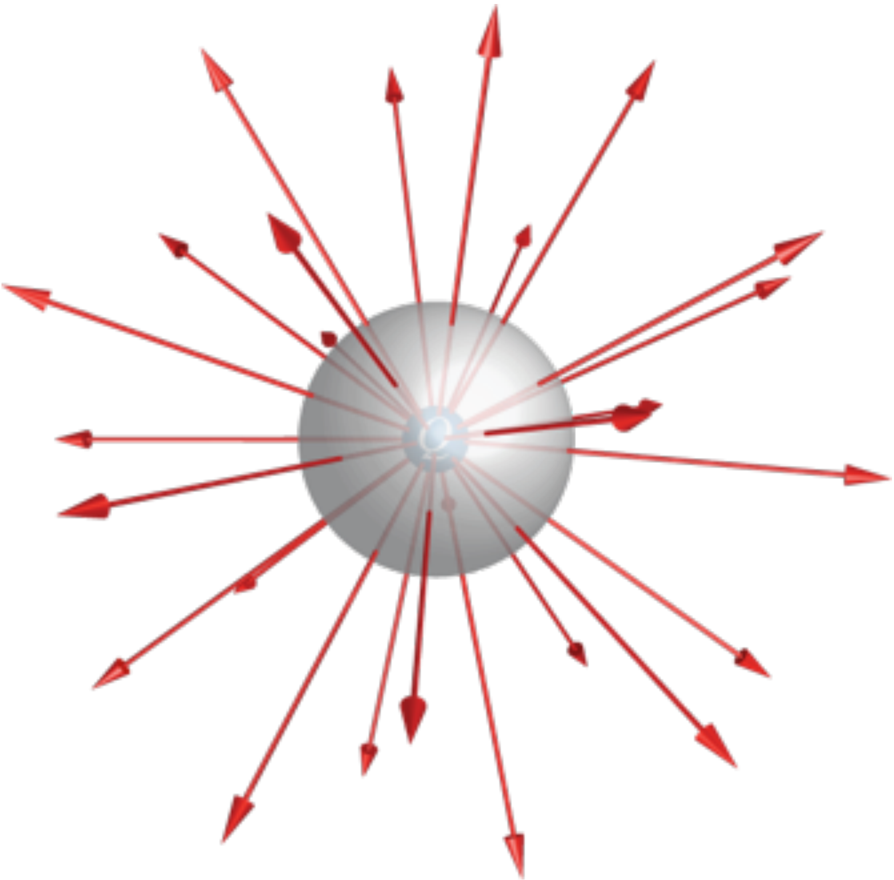
WORKS FOR ALL!

Homework



Arc of charge (constant λ)

Electric Field Lines



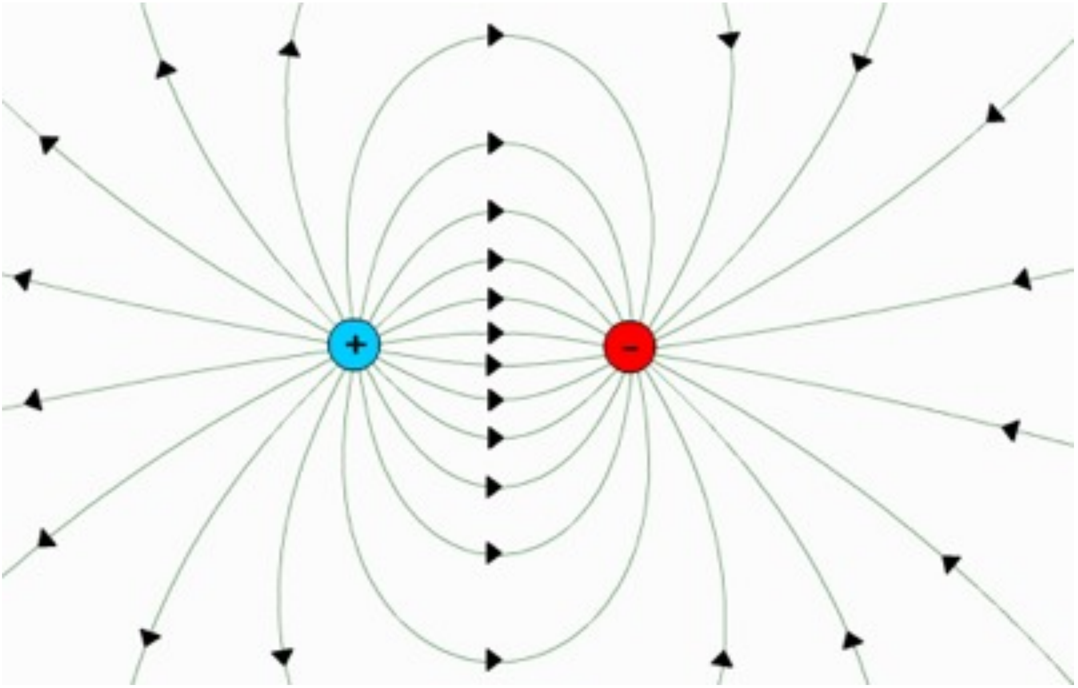
Direction & Density of Lines
represent
Direction & Magnitude of E

Point Charge:

Direction is radial

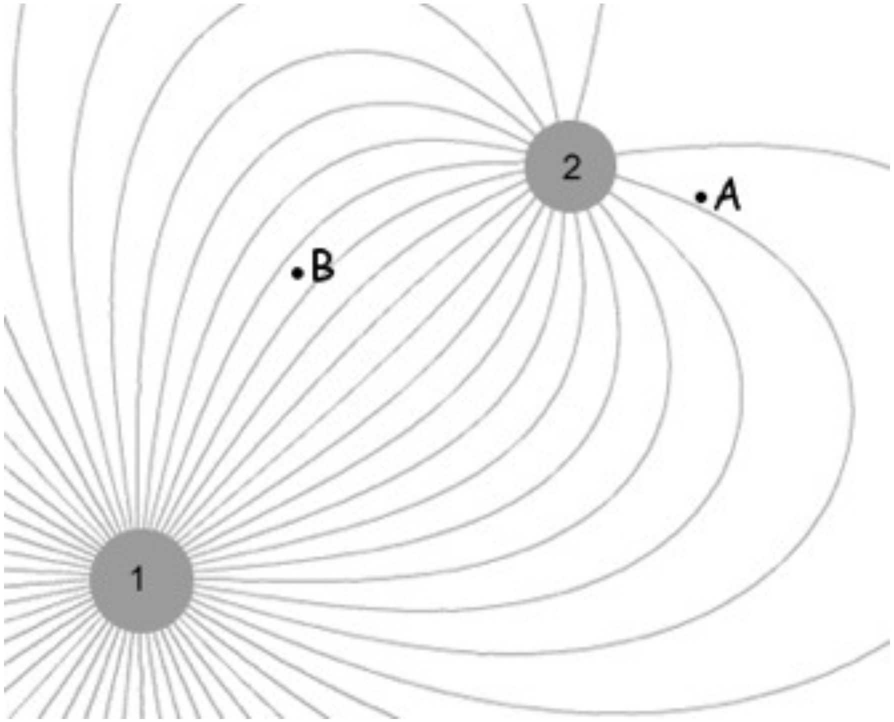
Density $\propto 1/R^2$

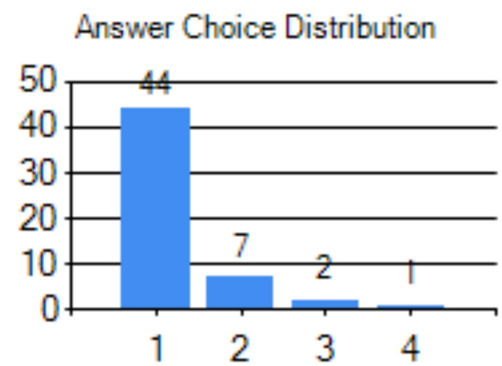
Electric Field Lines



Dipole Charge Distribution:
Direction & Density
much more interesting.

CheckPoint: Field Lines - Two Point Charges 1

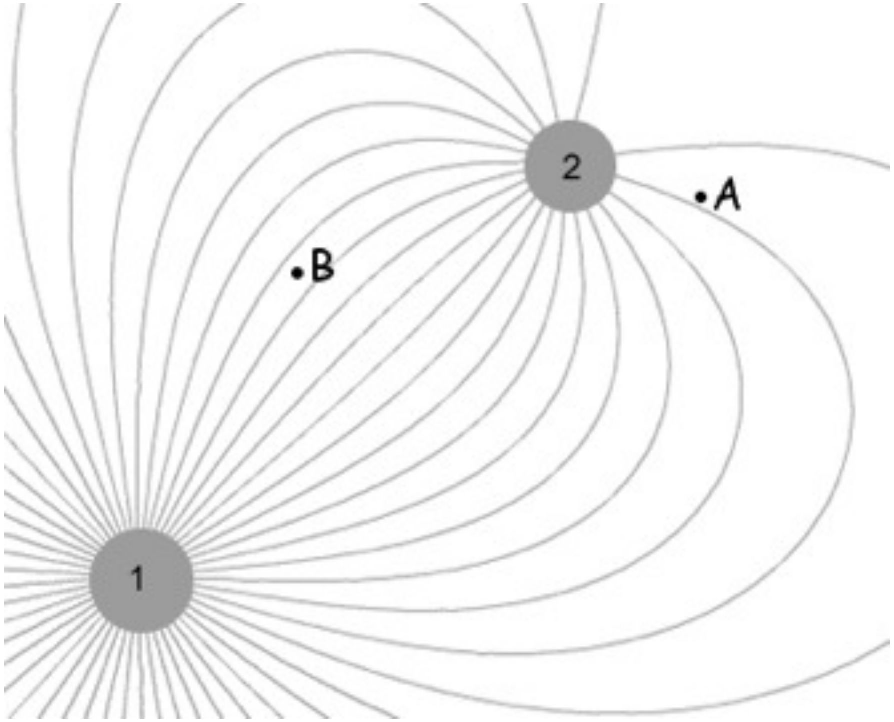


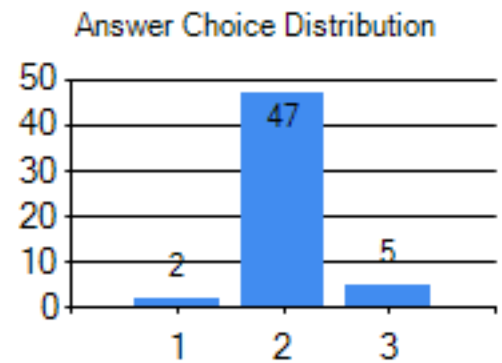


Compare the magnitudes of the two charges

- ○ $|Q_1| > |Q_2|$
- ○ $|Q_1| = |Q_2|$
- ○ $|Q_1| < |Q_2|$
- ○ There is not enough information to determine the relative magnitudes of the charges.

CheckPoint: Field Lines - Two Point Charges 2

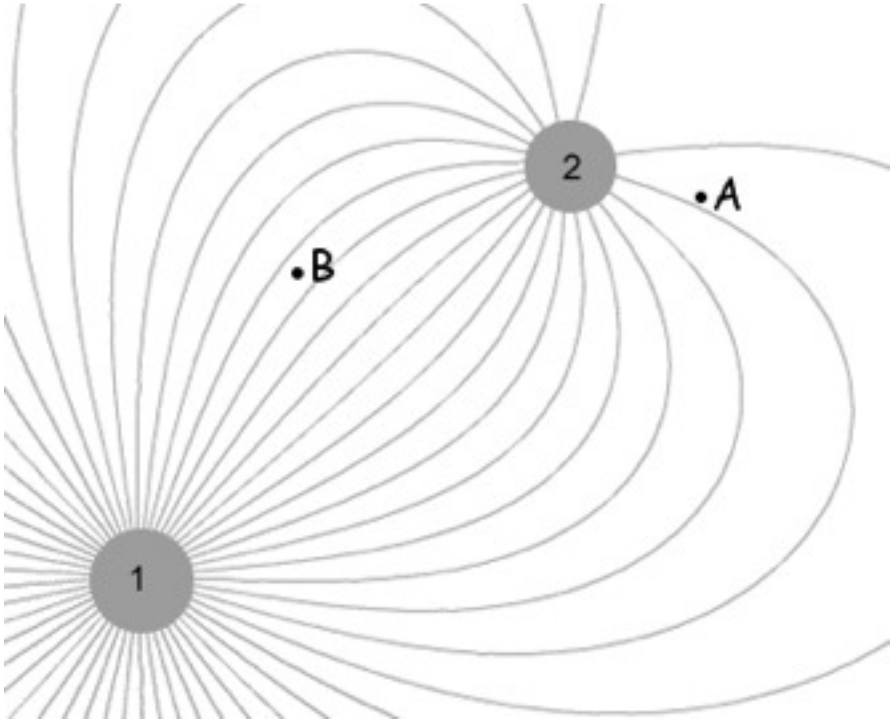


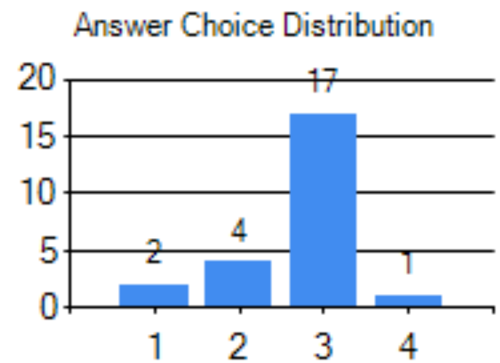


What do we know about the signs of the charges from looking at the picture?

- ○ Q_1 and Q_2 have the same sign
- ○ Q_1 and Q_2 have opposite signs
- ○ There is not enough information in the picture to determine the relative signs of the charges

CheckPoint: Field Lines - Two Point Charges 3





Compare the magnitudes of the electric fields at points A and B.

○ $|E_A| > |E_B|$

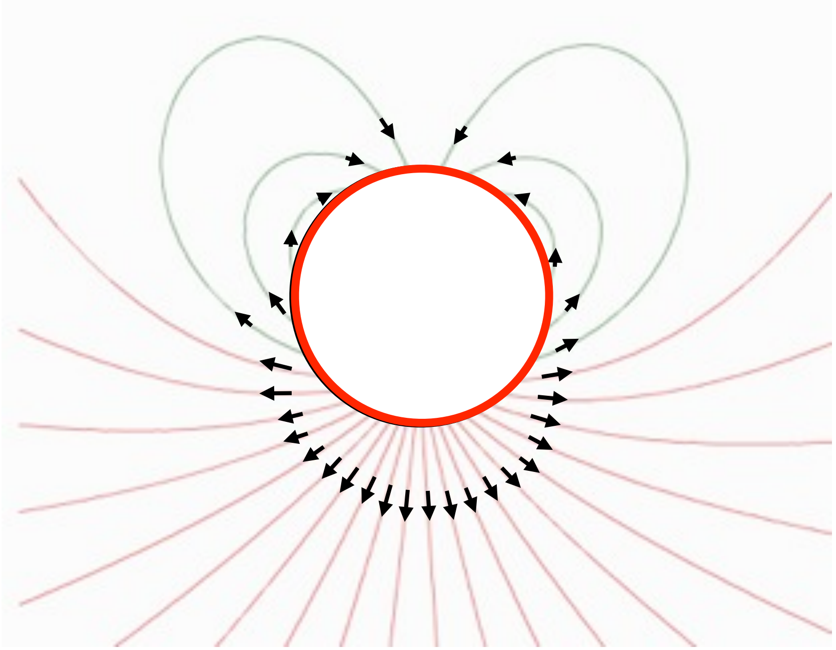
○ $|E_A| = |E_B|$

○ $|E_A| < |E_B|$

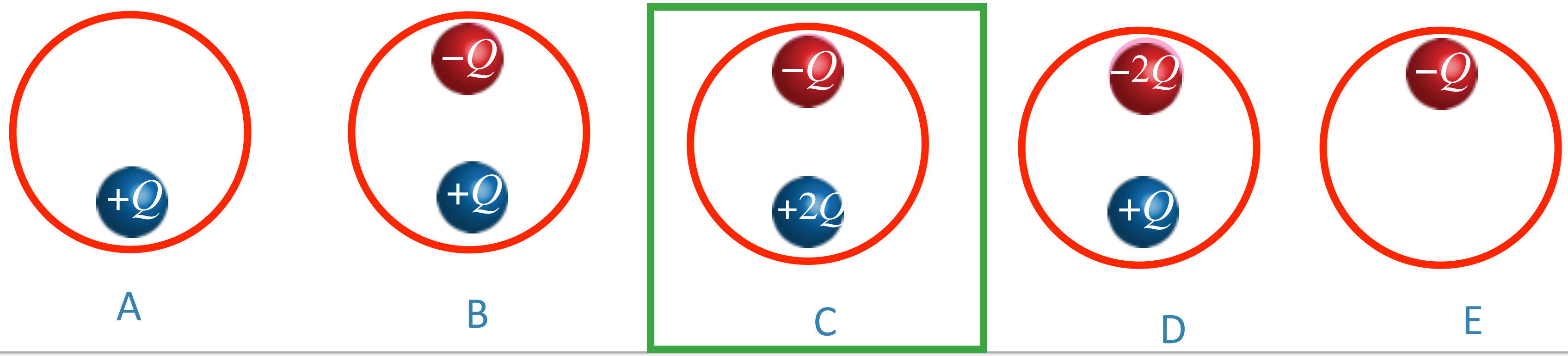
○ There is not enough information to determine the relative magnitudes of the fields at A and B.

Point Charges

"Telling the difference between positive and negative charges while looking at field lines. Does field line density from a certain charge give information about the sign of the charge?"

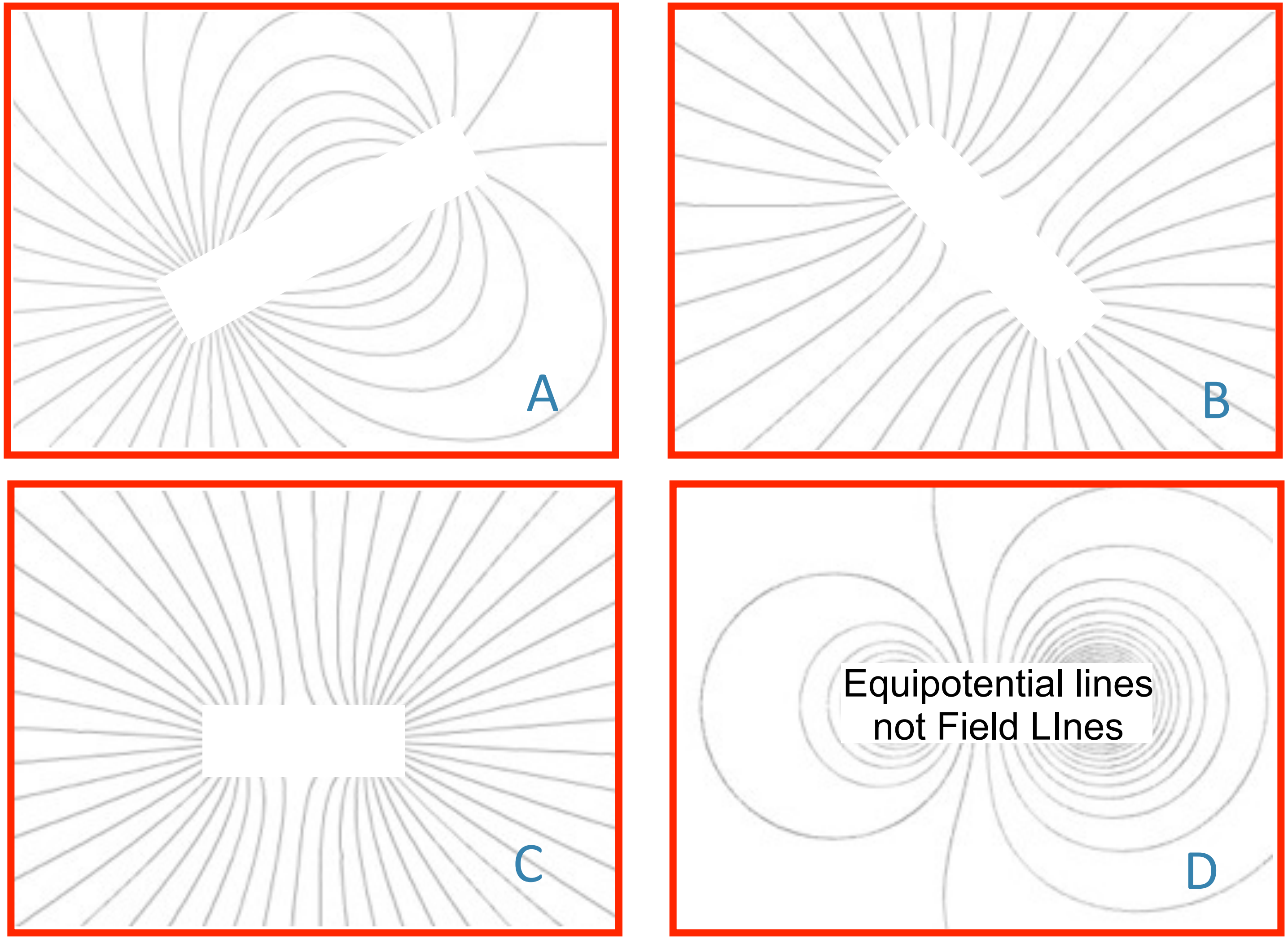


What charges are inside the red circle?



Clicker Question: Same Sign, Diff. Magnitudes

Which of the following field line pictures best represents the electric field from two charges that have the same sign but different magnitudes?



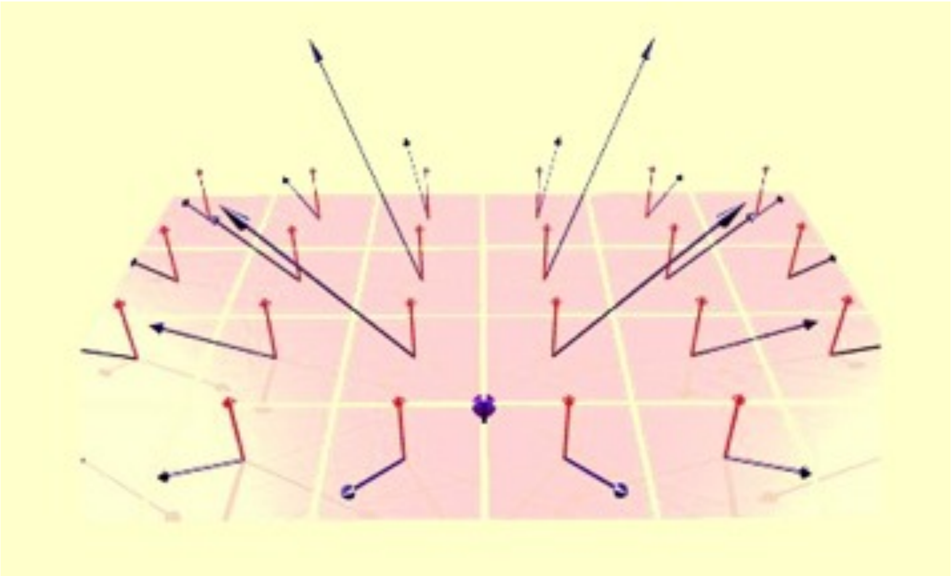
Electric Flux "Counts Field Lines"

"I'm very confused by the general concepts of flux through surface areas. please help"

$$\Phi_S = \underbrace{\int_S \vec{E} \cdot d\vec{A}}$$

Flux through surface S

Integral of $\vec{E} \cdot d\vec{A}$ on surface S



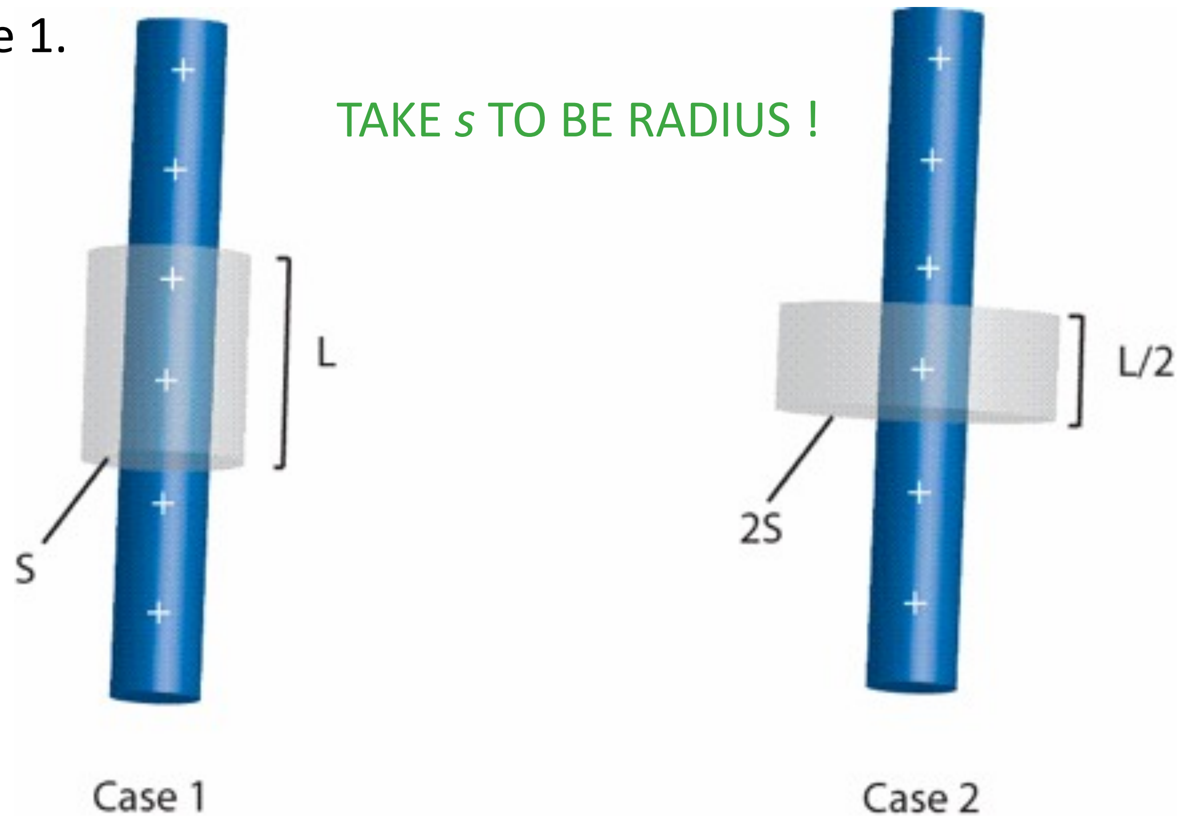
CheckPoint: Flux from Uniformly Charged Rod

An infinitely long charged rod has uniform charge density of λ, and passes through a cylinder (gray). The cylinder in case 2 has twice the radius and half the length compared to the cylinder in case 1.

Compare the magnitude of the flux through the surface of the cylinder in both cases.

A. $\Phi_1 = 2\ \Phi_2$
B. $\Phi_1 = \Phi_2$
C. $\Phi_1 = 1/2\ \Phi_2$
D. None of these

TAKE *s* TO BE RADIUS !



CheckPoint Results: Flux Unif. Charged Rod

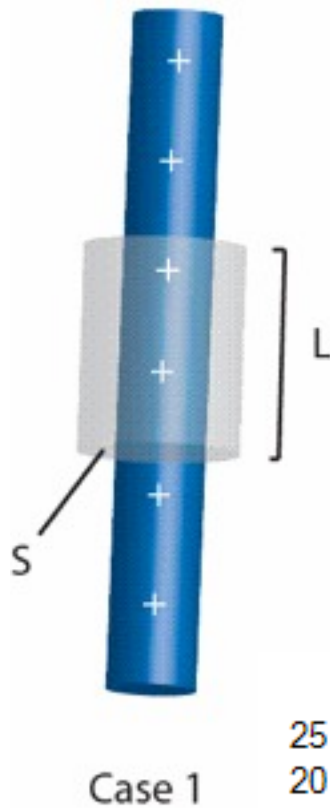
Compare the magnitude of the flux through the surface of the cylinder in both cases.

A. $\boxed{\Phi_1 = 2\,\Phi_2}$

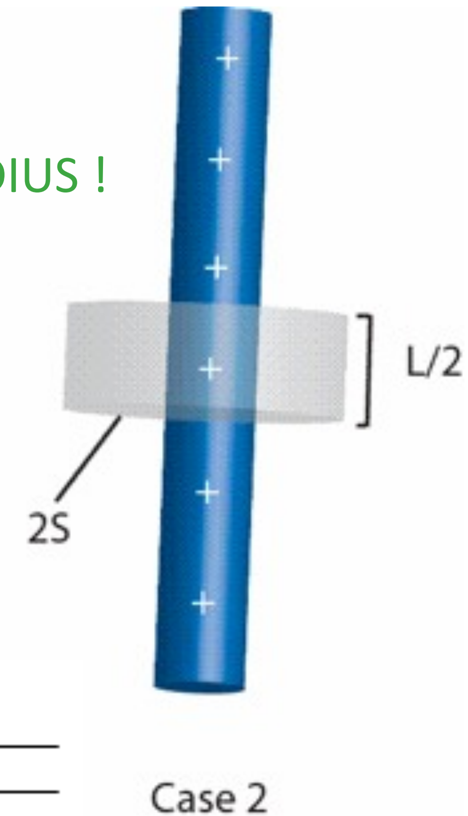
B. $\Phi_1 = \Phi_2$

C. $\Phi_1 = 1/2\,\Phi_2$

D. None of these



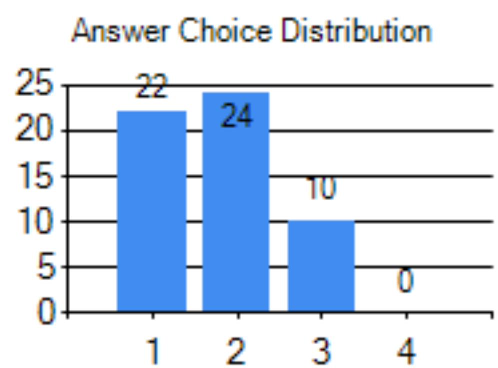
TAKE *s* TO BE RADIUS !



The first cylinder encloses twice the amount of charge as the second

flux doesn't depend on length or radius

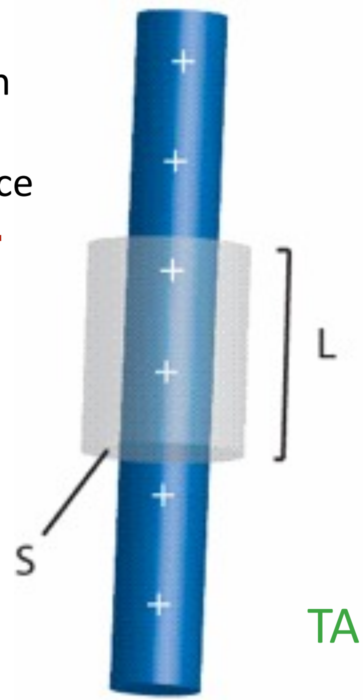
because case 1 have more surface area that is parallel to the electric field line than case 2 so case 2 is half case 1



CheckPoint Results: Flux Unif. Charged Rod

"The flux is proportional to the Area that the field is passing through. Although the radius is twice as long in the second case, it's length is half as long. Both cases have the same surface area that the field passes through, ~~so the fluxes are equal.~~"

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A}$$



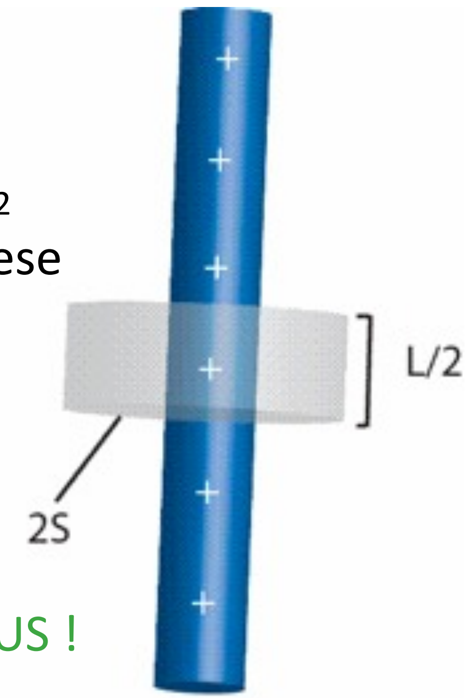
Case 1

$\boxed{A. \Phi_1 = 2\,\Phi_2}$

B. $\Phi_1 = \Phi_2$

C. $\Phi_1 = 1/2\ \Phi_2$

D. None of these



Case 2

TAKE s TO BE RADIUS !

E constant on barrel of cylinder

E perpendicular to barrel surface

(E parallel to dA)

$$\Phi = E \int_{barrel} d\vec{A} = EA_{barrel}$$

Case 1

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 s} \rightarrow \boxed{\Phi_1 = \frac{\lambda L}{\varepsilon_0}}$$

Case 2

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 (2s)}$$

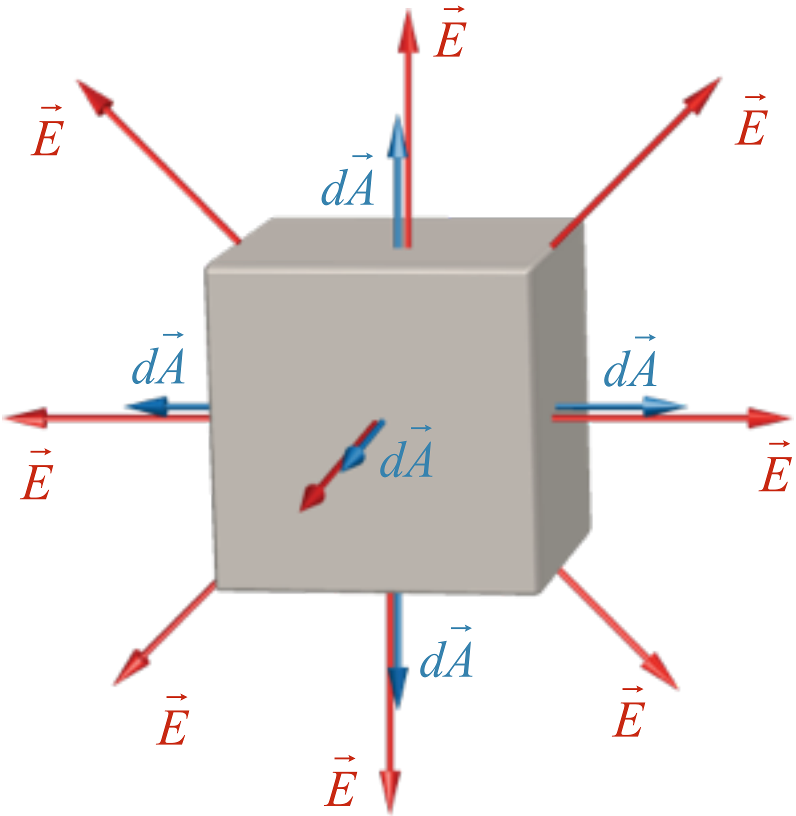
$$A_2 = (2\pi(2s))L/2 = 2\pi sL$$

$$\rightarrow \boxed{\Phi_2 = \frac{\lambda(L/2)}{\varepsilon_0}}$$

RESULT: GAUSS' LAW

Φ proportional to charge enclosed !

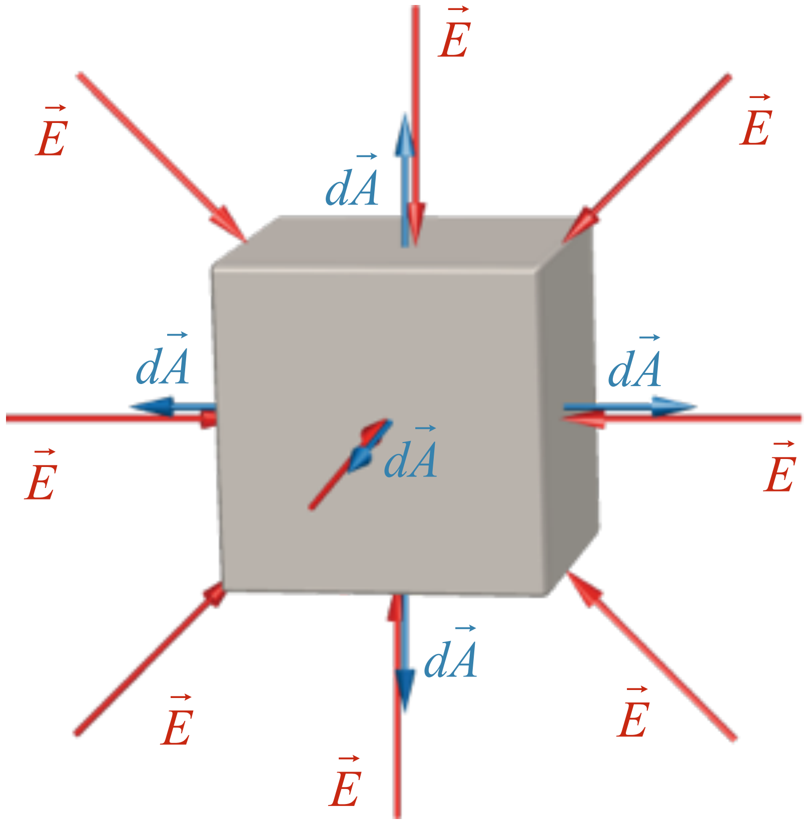
Direction Matters:



For a closed surface, $d\vec{A}$ points outward

$$\Phi_S = \oint \vec{E} \cdot \vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Direction Matters:



For a closed surface, $d\vec{A}$ points outward

$$\Phi_S = \oint_S \vec{E} \cdot d\vec{A} < 0$$

Clicker Question: Trapezoid in Constant Field

Label faces:

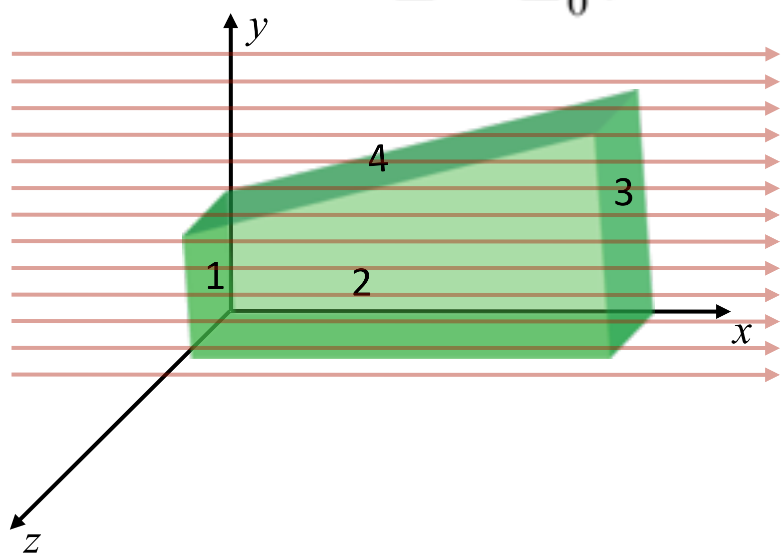
1: $x = 0$

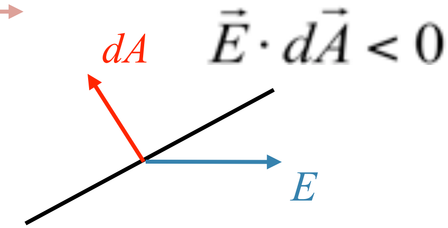
2: $z = +a$

3: $x = +a$

4: slanted

Define Φ_n = Flux through Face n





$\boxed{\text{A)}\quad \Phi_1 < 0}$

B) $\Phi_1 = 0$

C) $\Phi_1 > 0$

A) $\Phi_2 < 0$

$\boxed{\text{B)}\quad \Phi_2 = 0}$

C) $\Phi_2 > 0$

A) $\Phi_3 < 0$

B) $\Phi_3 = 0$

$\boxed{\text{C)}\quad \Phi_3 > 0}$

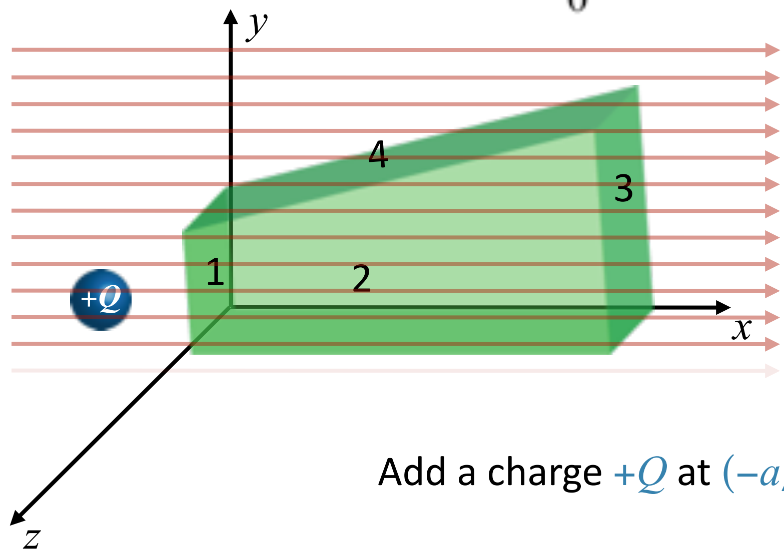
$\boxed{\text{A)}\quad \Phi_4 < 0}$

B) $\Phi_4 = 0$

C) $\Phi_4 > 0$

Clicker Question: Trap. in Constant Field + Q

$$\vec{E} = E_0\,\hat{\imath}$$



Label faces:

1: $x = 0$

2: $z = +a$

3: $x = +a$

4: slanted

Define Φ_n = Flux through Face n

Φ = Flux through Trapezoid

Add a charge $+Q$ at $(-a, a/2, a/2)$

How does Flux change? ("more negative" is "decreases", "less negative" is "increases")

A) Φ_1 increases

B) Φ_1 decreases

C) Φ_1 remains same

A) Φ_3 increases

B) Φ_3 decreases

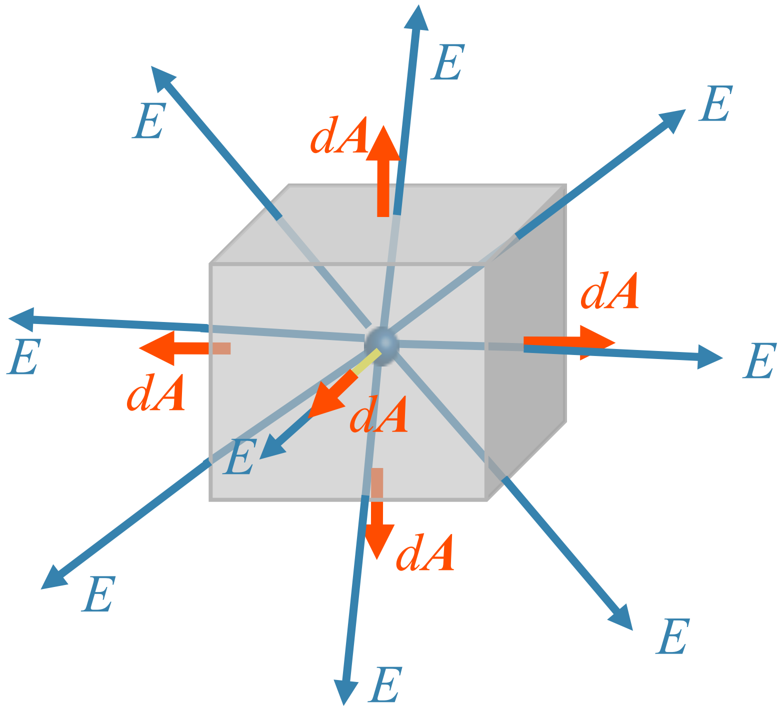
C) Φ_3 remains same

A) Φ increases

B) Φ decreases

C) Φ remains same

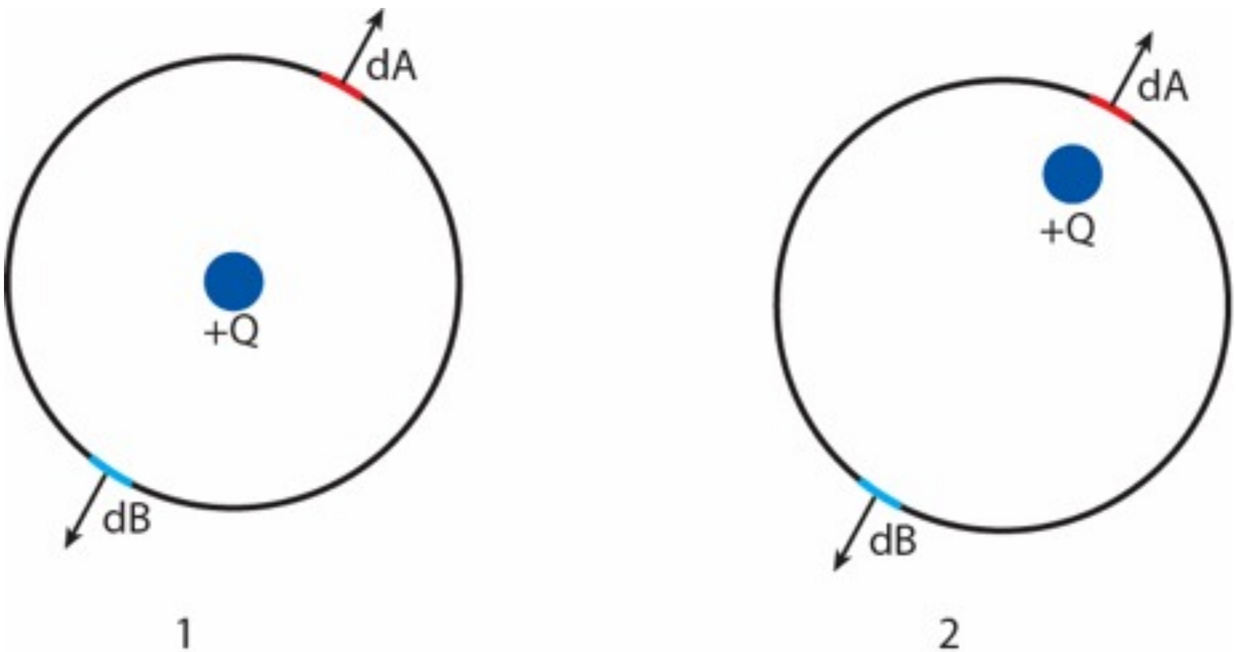
Gauss Law



$$\Phi_S = \oint_{\text{closed surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

CheckPoint: Flux from Point Charge (Sphere) 1

A positive charge (blue) is contained inside a spherical shell (black).



How does the electric flux through the two surface elements, dΦA and dΦB change when the charge is moved from position 1 to position 2?

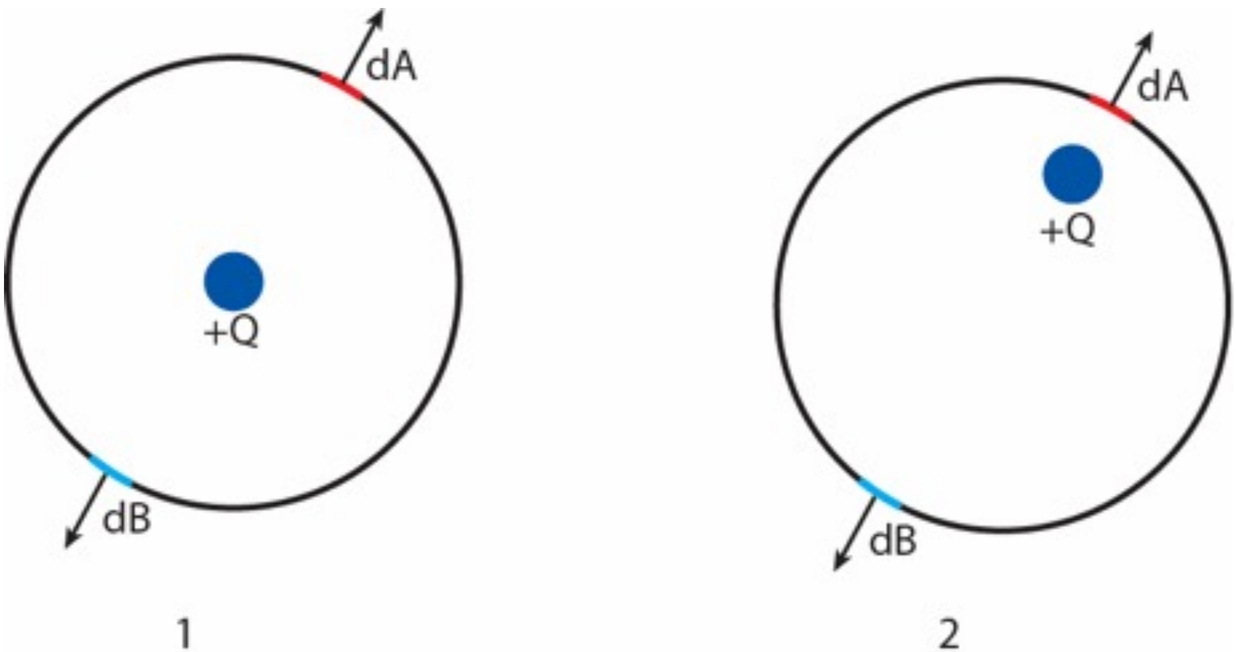
○ dΦA increases and dΦB decreases

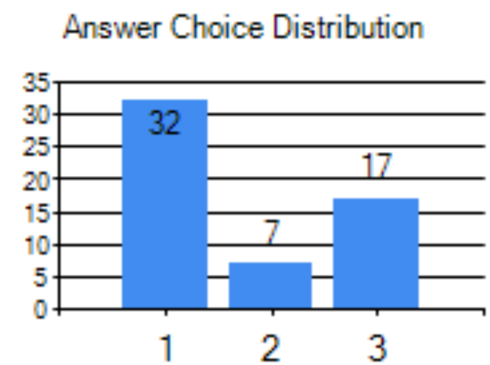
○ dΦA decreases and dΦB increases

○ Both dΦA and dΦB do not change

CheckPoint Results: Flux Point Charge (Sphere) 1

A positive charge (blue) is contained inside a spherical shell (black).





How does the electric flux through the two surface elements, dΦA and dΦB change when the charge is moved from position 1 to position 2?

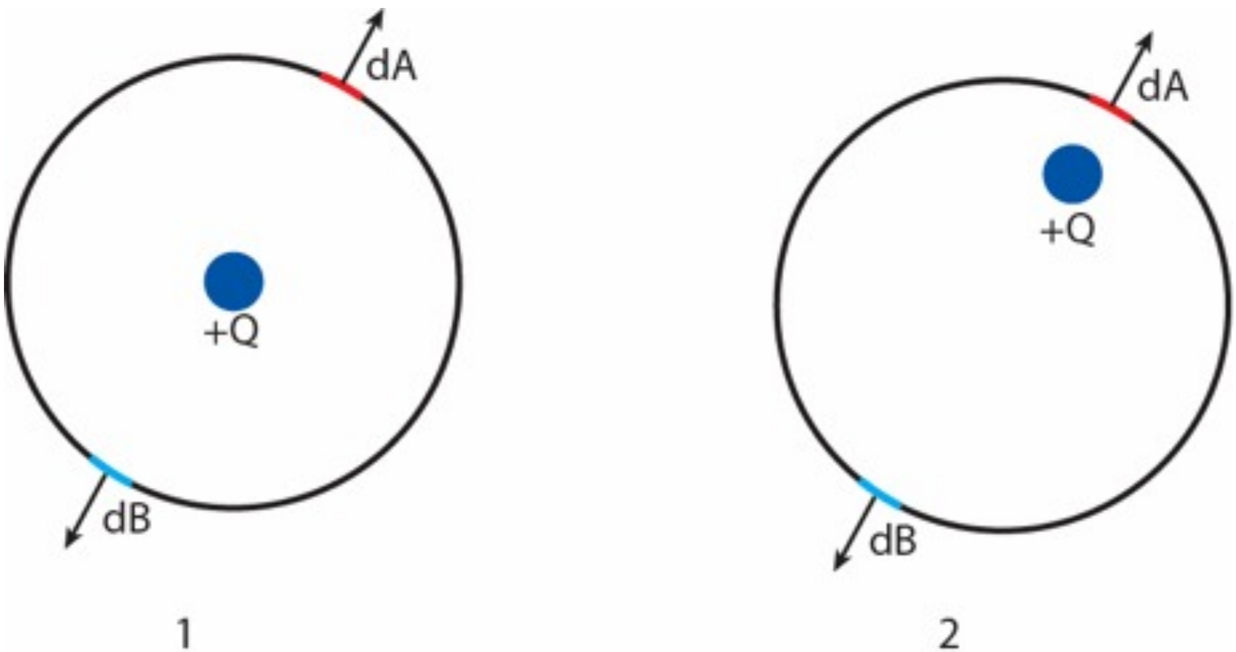
○ dΦA increases and dΦB decreases

○ dΦA decreases and dΦB increases

○ Both dΦA and dΦB do not change

CheckPoint: Flux from Point Charge (Sphere) 2

A positive charge (blue) is contained inside a spherical shell (black).

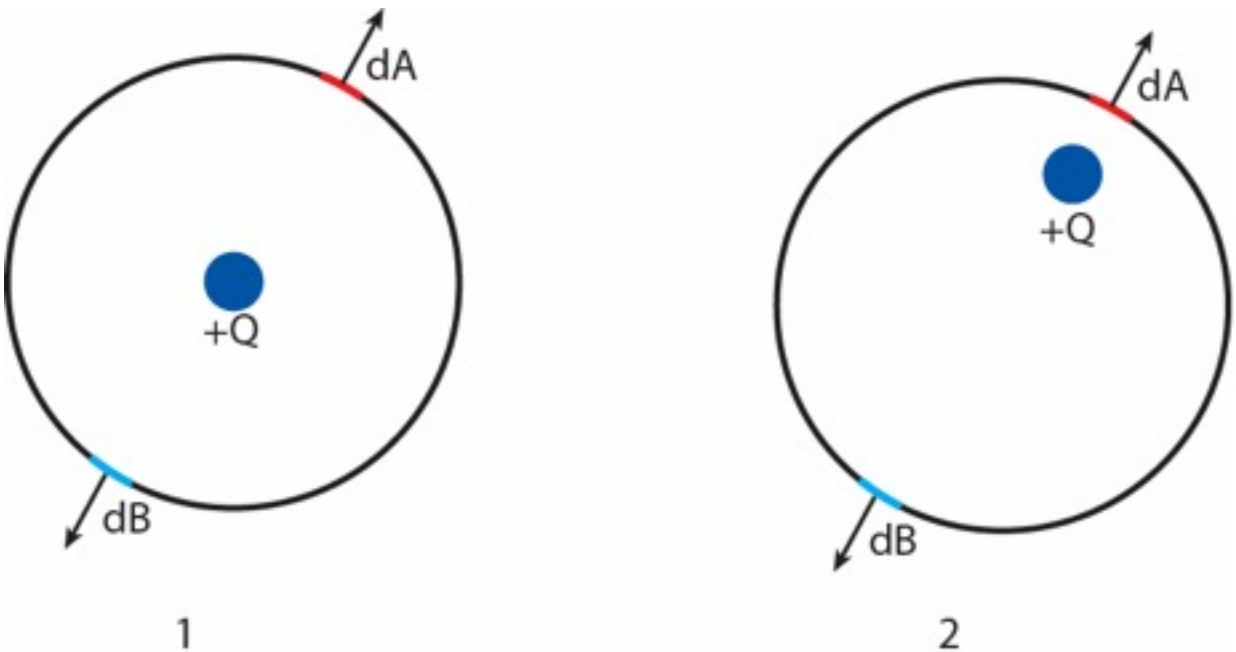


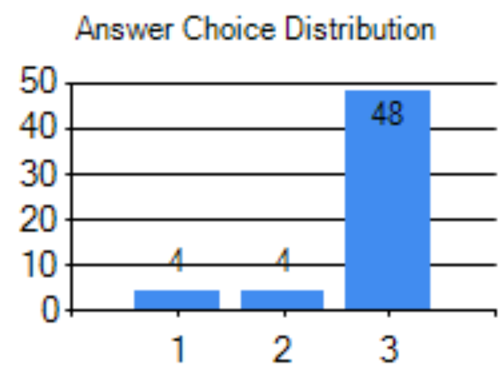
How does the flux Φ_E through the entire surface change when the charge is moved from position 1 to position 2?

- ○ Φ_E increases
- ○ Φ_E decreases
- ○ Φ_E does not change

CheckPoint Results: Flux Point Charge (Sphere) 2

A positive charge (blue) is contained inside a spherical shell (black).



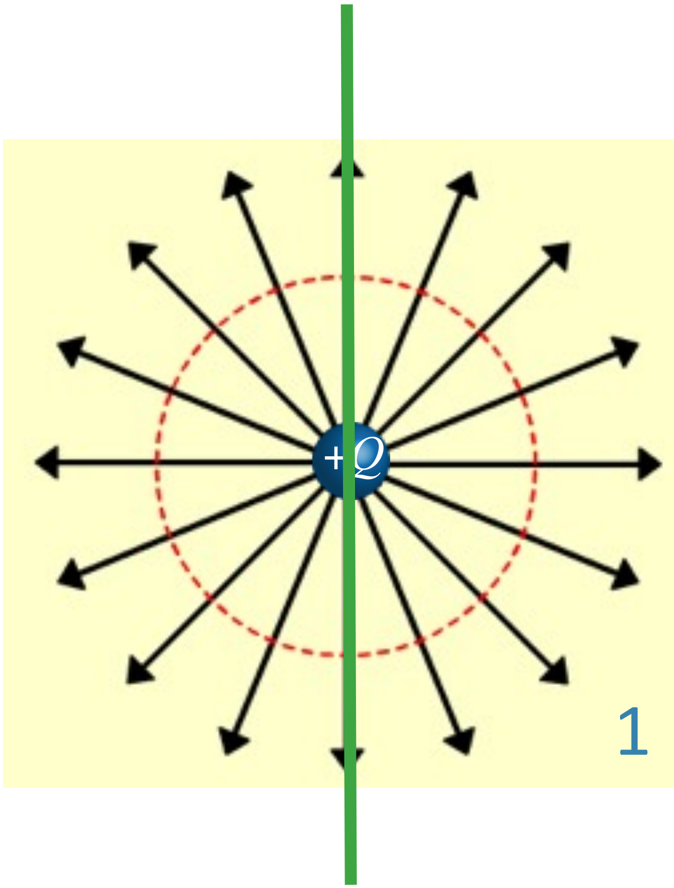


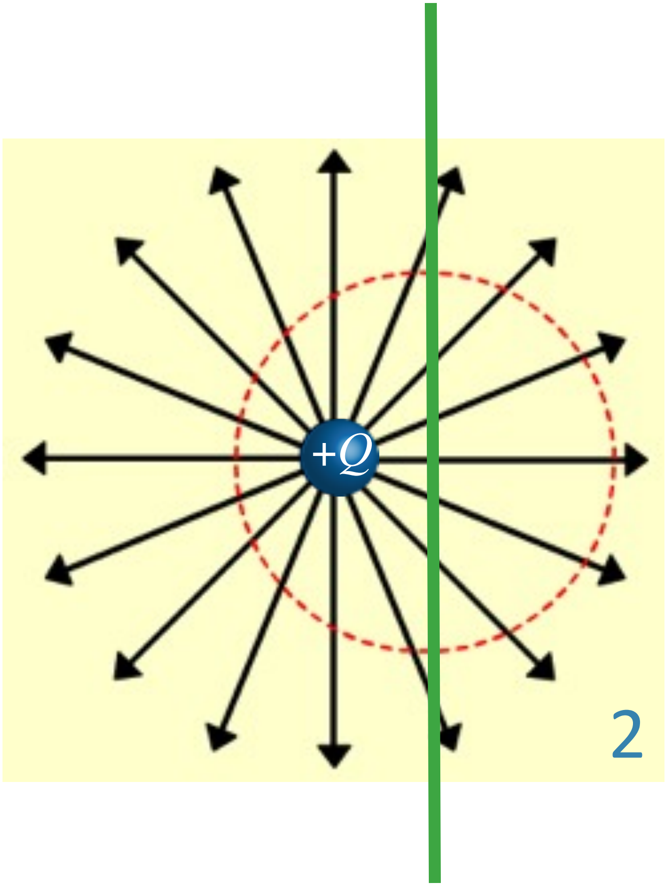
How does the flux Φ_E through the entire surface change when the charge is moved from position 1 to position 2?

- ○ Φ_E increases
- ○ Φ_E decreases
- ○ Φ_E does not change

gauss law refers to the total charge enclosed, regardless of where it is.

Think of it this way:

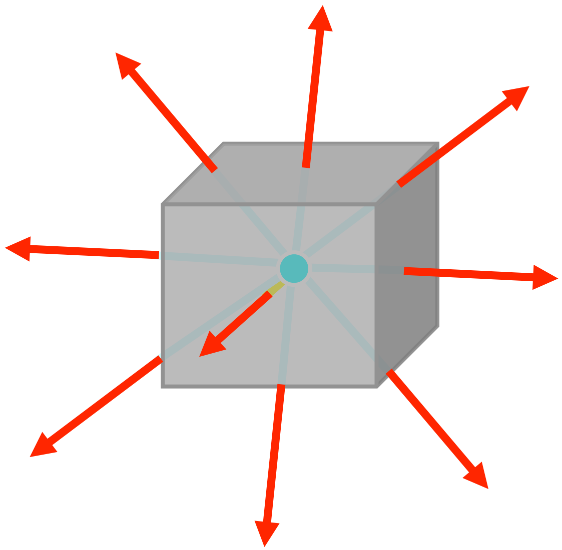




The total flux is the same in both cases (just the total number of lines)

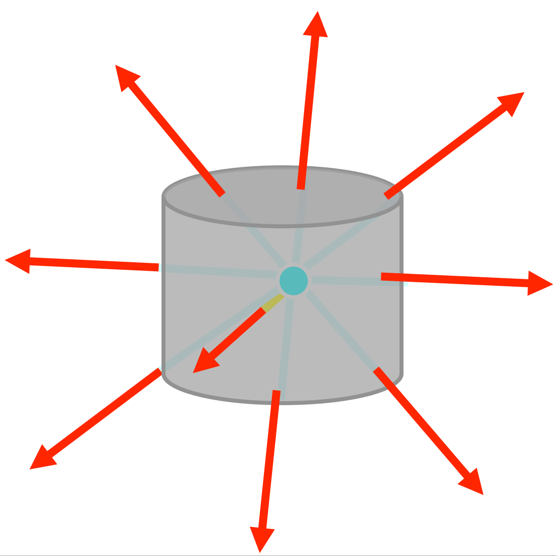
The flux through the right (left) hemisphere is smaller (bigger) for case 2.

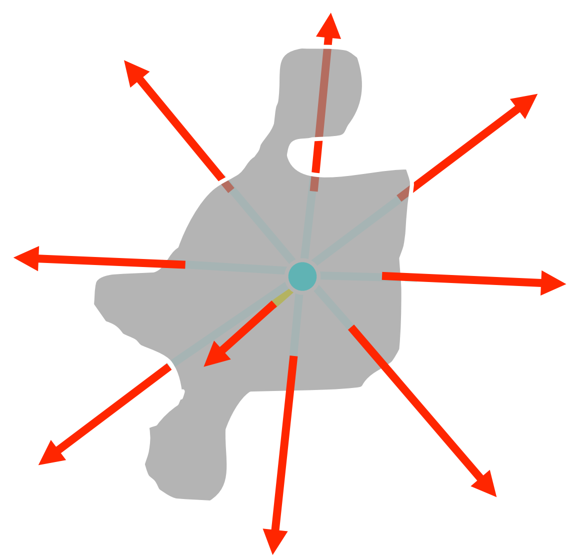
Things to notice about Gauss Law



$$\Phi_S = \int_{\substack{\text{closed}\\\text{surface}}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

If $Q_{enclosed}$ is the same, the flux has to be the same, which means that the integral must yield the same result for any surface.





Things to notice about Gauss Law

$$\Phi_S = \int_{\substack{\text{closed}\\\text{surface}}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

In cases of high symmetry it may be possible to bring E outside the integral. In these cases we can solve Gauss Law for E

$$E\int dA = EA = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

So - if we can figure out $Q_{enclosed}$ and the area of the surface A, then we know E!

This is the topic of the next lecture.