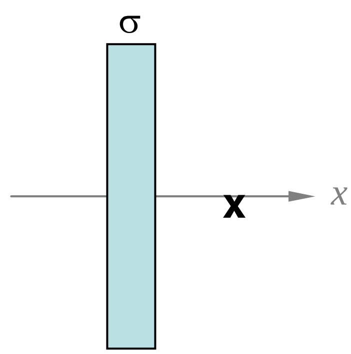
Discussion Question 3C

P212, Week 3

Electric Field due to Charged Sheets and Slabs

In the tutorial, you calculated the electric field of a charged plane of infinite area. The field turned out to be independent of distance. This property makes it easy to analyze systems of many charged sheets and slabs using **superposition**.

Here's a warmup exercise: consider the thick slab of infinite area shown at right. The slab is made of a non-conducting material. It carries a net charge per unit area of σ, which is distributed uniformly throughout the interior of the slab.



(a) Write down the electric field ***E*** at the point **x**.

$E = \sigma/2\varepsilon_o$

(b) Would the field change if the slab were moved **further to the left**?

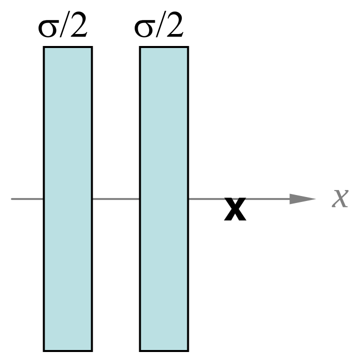
no

(c) Would the field change if the **thickness** of the slab were increased?

no

(d) Would the field change if the slab were replaced by **two slabs**, each carrying charge density σ/2?

no



See how easy superposition is with infinite planes? ☺

Let's also remind ourselves how to deal with **conductors**. Suppose that the slab described above were made of metal instead of non-conducting material … what would change?

(f) Would the field **outside** the slab (e.g. at the point **x**) change?

no, σ is still the same outside

(g) Would the field **inside** the slab change?

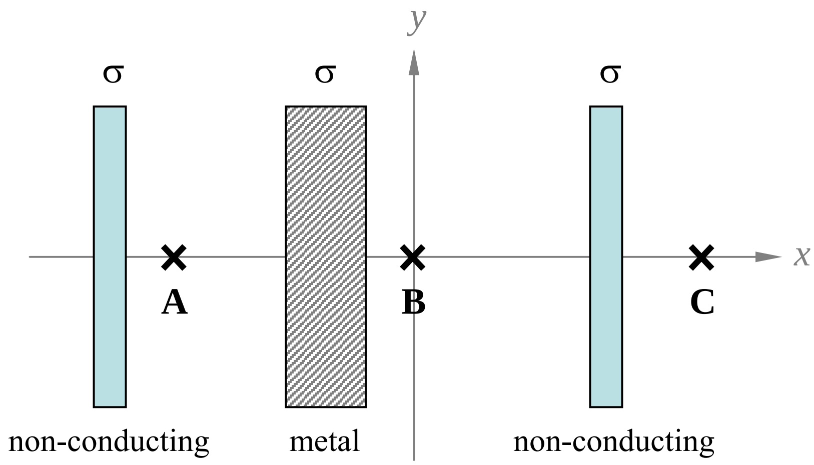
yes, field inside is zero

(h) Would the **distribution of charge** within the slab change? How so?

yes, charge is all on surface of the plates

Now that we have all the ingredients, let's analyze a more complicated system.

Two non-conducting slabs of infinite area are given a charge-per-unit area of $\sigma_a = -4\ C/m^2$ and $\sigma_b = +1.5\ C/m^2$ respectively. A third slab, made of metal, is placed between the first two plates. The charge density σ_m on the metal slab is 0 (i.e. the slab is uncharged).



$\sigma_a = -4\ C/m^2$
$\sigma_b = +1.5\ C/m^2$
$\sigma_m = 0$

(a) Find the magnitude and direction of the electric field at the points A, B, and C.

$E_A = E_B = (-5.5\ C/m^2)/2\varepsilon_o$
$E_C = (-2.5\ C/m^2)/2\varepsilon_o$
All in negative x-direction.

(b) Now let's think physically about the metal slab. The defining property of a conductor is that the charges within are *free to move*, and they will do so in response to an electric field. As a result, surface charges σ_L and σ_R will accumulate on the left- and right-hand surfaces of the slab. What are the *signs* of these surface charges?

σ_L – positive σ_R - negative

(c) Go ahead and use Gauss' Law plus your answers to question (a) to calculate the surface charges σ_L and σ_R induced on the metal slab.

$\sigma_L = 2.75\ C/m^2$ $\sigma_R = -2.75\ C/m^2$

(d) Finally, let's add a *net* charge per unit area of $\sigma_m = +3\ C/m^2$ to the metal slab. Now what are the induced surface charges σ_L and σ_R?

$\sigma_L = 4.25\ C/m^2$ $\sigma_R = -1.25\ C/m^2$