

Electricity & Magnetism

Lecture 7: Conductors and Capacitance

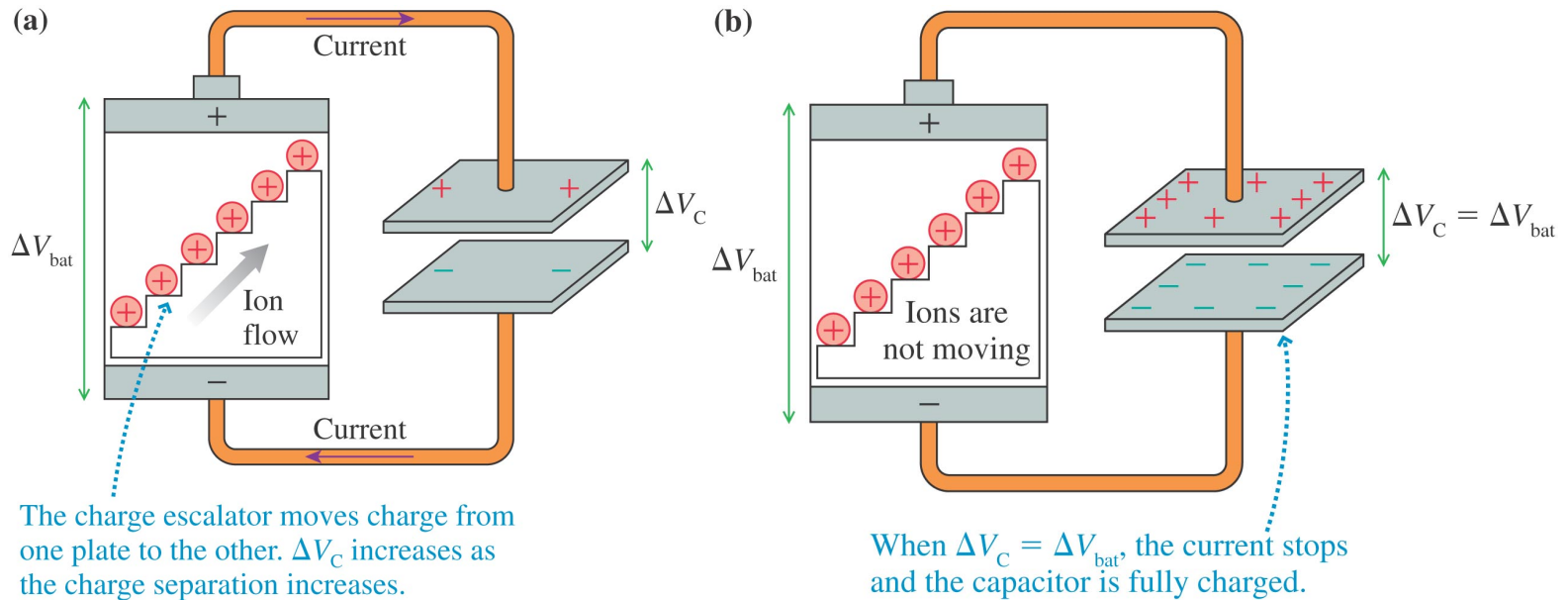
Today's Concept:

- A) Conductors
- B) Capacitance

Stuff you asked about:

- “They never said what capacitance is or what its used for”
- “How do I "charge" a capacitance? if I don't put a wire in between, the circuit is not complete. If I put a wire in between, I won't be able to build up a charge difference. Also, if I charge them as is (with no wire in between), won't they discharge immediately? Don't understand how this works.”
- “What is the difference between capacitors and batteries, although they both stores energy, can we say that capacitors are a kind a batteries or vice versa?”

Capacitors vs. Batteries



- ▶ A battery uses an electrochemical reaction to separate charges. ΔV between terminals is called “EMF”, symbol: \mathcal{E} or \mathcal{E} .
- ▶ Capacitors store charge on its plates.

Our Comments

WE BELIEVE THERE ARE ONLY THREE THINGS
YOU NEED TO KNOW TO DO ALL OF HOMEWORK!

1. $E = 0$ within the material of a conductor: (at static equilibrium)

Charges move inside a conductor in order to cancel out the fields that would be there in the absence of the conductor. This principle determines the induced charge densities on the surfaces of conductors.

2. Gauss' Law:

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

If charge distributions have sufficient symmetry (spherical, cylindrical, planar), then Gauss' law can be used to determine the electric field everywhere.

3. Definition of Potential:

$$\Delta V_{a \rightarrow b} \equiv \frac{\Delta U_{a \rightarrow b}}{q} = - \int_a^b \vec{E} \cdot d\vec{l}$$

CONCEPTS DETERMINE THE CALCULATION !

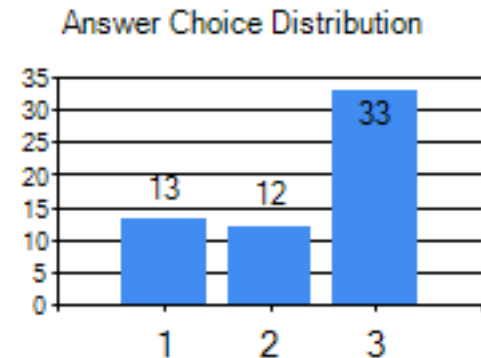
Conductors

The Main Points

- Charges free to move
- $E = 0$ in a conductor
- Surface = Equipotential
- E at surface perpendicular to surface

CheckPoint: Two Spherical Conductors 1

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q . Conductor A has a larger radius than conductor B.



Compare the potential at the surface of conductor A with the potential at the surface of conductor B.

A. $V_A > V_B$

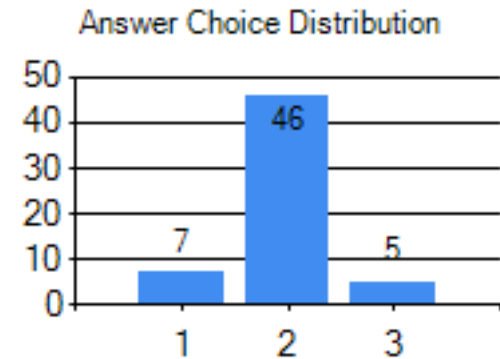
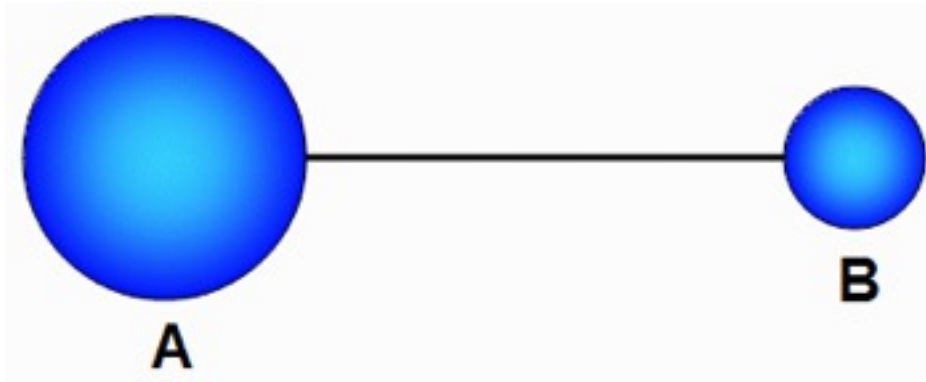
B. $V_A = V_B$

C. $V_A < V_B$

“The potential is the same as from a point charge at the center of A or B. So, following $V=kQ/r$, A would have a smaller potential “

CheckPoint: Two Spherical Conductors 2

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q . Conductor A has a larger radius than conductor B.



The two conductors are now connected by a wire. How do the potentials at the conductor surfaces compare now?

A. $V_A > V_B$

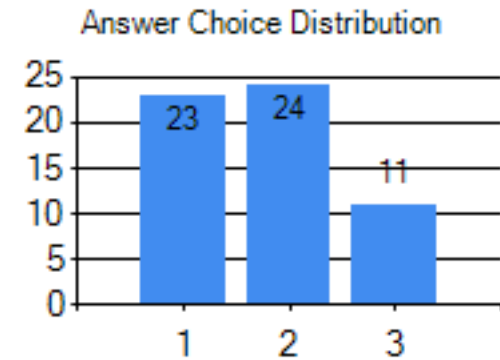
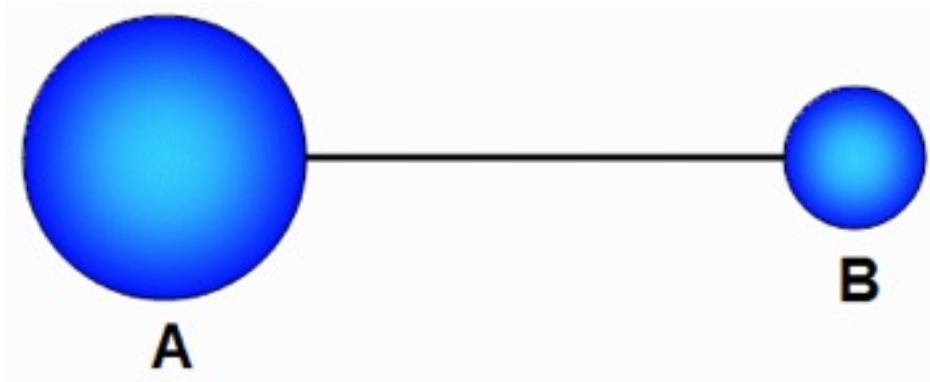
B. $V_A = V_B$

C. $V_A < V_B$

“The potentials will become equal since the charges will want to go to places of lower potential, until it balances out. “

CheckPoint: Two Spherical Conductors 3

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q . Conductor A has a larger radius than conductor B.



What happens to the charge on conductor A after it is connected to conductor B by the wire?

A. Q_A increases

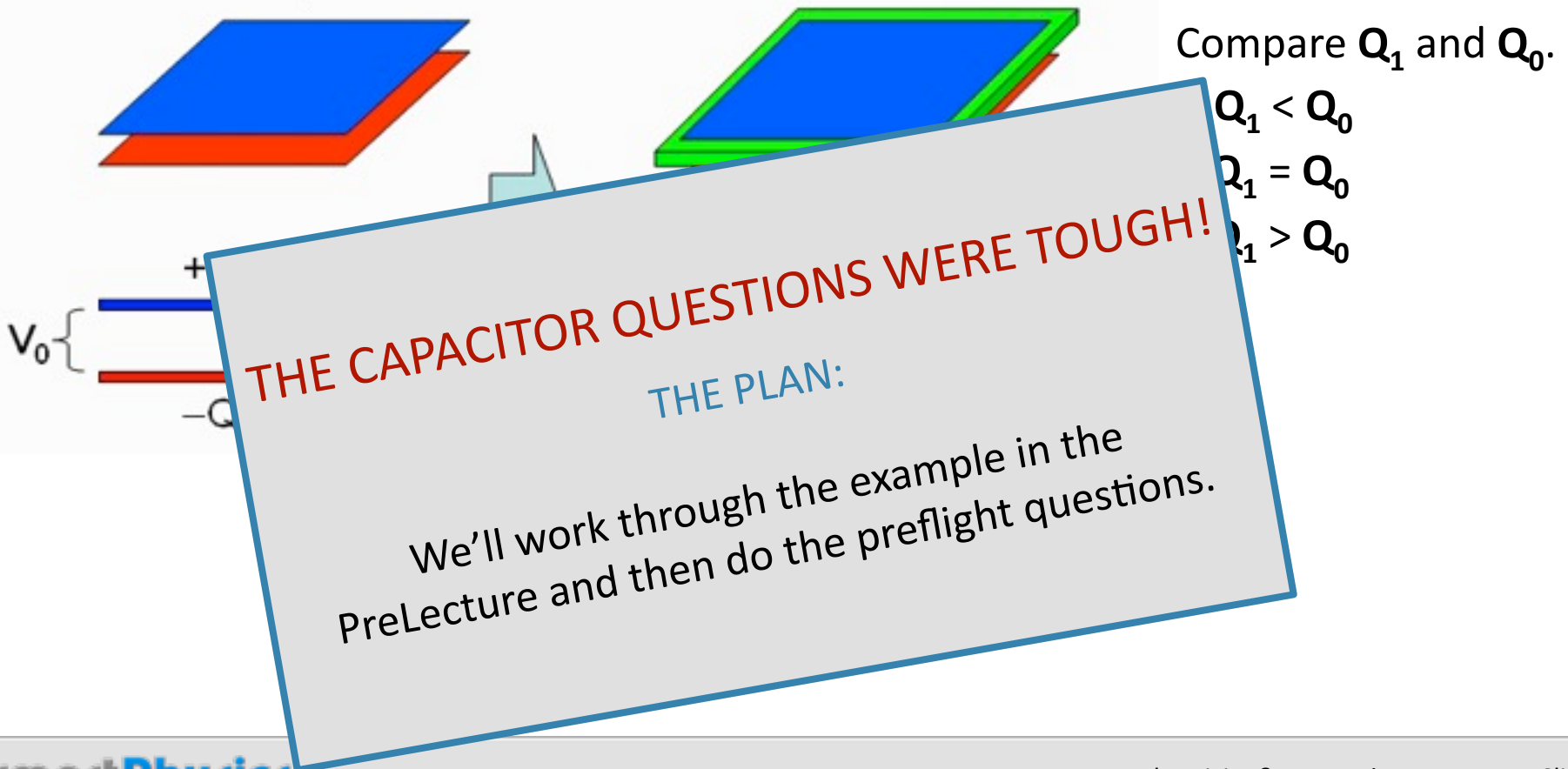
B. Q_A decreases

C. Q_A does not change

“Since B initially has a higher potential, charges move from B to A.”

CheckPoint: Charged Parallel Plates 1

Two parallel plates of equal area carry equal and opposite charge Q_0 . The potential difference between the two plates is measured to be V_0 . An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same.



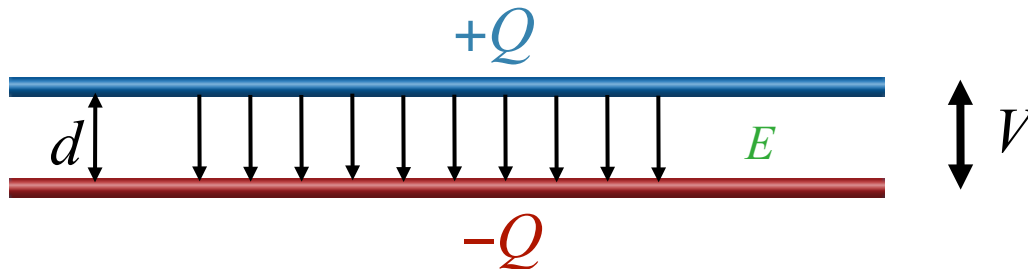
Capacitance

Capacitance is defined for any pair of spatially separated conductors.

$$C \equiv \frac{Q}{V}$$

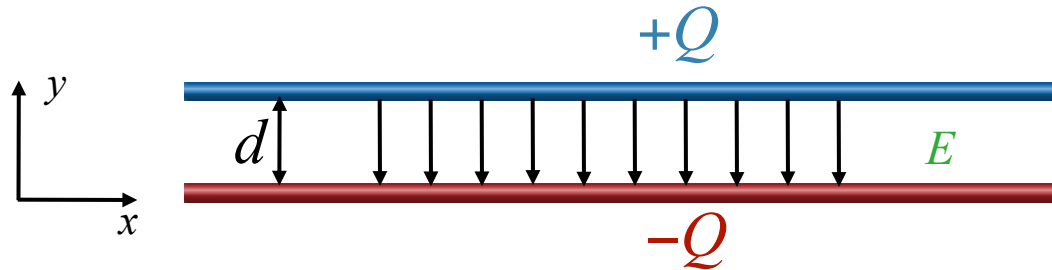
How do we understand this definition ?

- Consider two conductors, one with excess charge = $+Q$ and the other with excess charge = $-Q$



- These charges create an electric field in the space between them
- We can integrate the electric field between them to find the potential difference between the conductor
- This potential difference should be proportional to Q !
 - The ratio of Q to the potential difference is the capacitance and only depends on the geometry of the conductors

Example (done in Prelecture 7)



What is σ ?

$$E = \frac{\sigma}{\epsilon_o}$$

$$\sigma = \frac{Q}{A}$$

A = area of plate

Second, integrate E to find the potential difference V

$$V = - \int_0^d \vec{E} \cdot d\vec{y} \longrightarrow V = - \int_0^d (-E dy) = E \int_0^d dy = \frac{Q}{\epsilon_o A} d$$

As promised, V is proportional to Q !

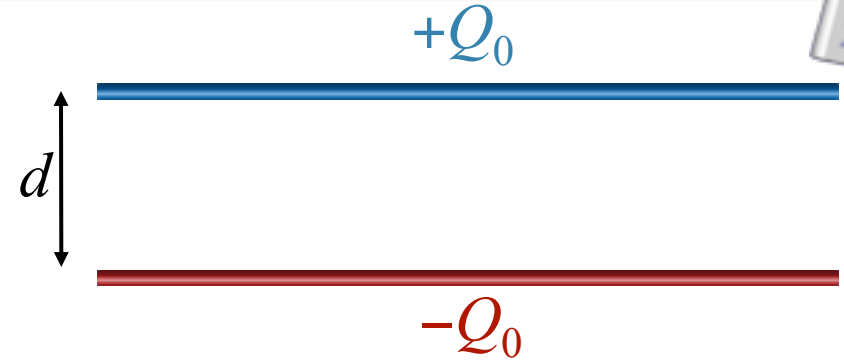
$$C \equiv \frac{Q}{V} = \frac{\cancel{Q}}{\cancel{Qd} / \epsilon_o A} \longrightarrow C = \frac{\epsilon_o A}{d}$$

C determined by
geometry !

Clicker Question Related to CheckPoint

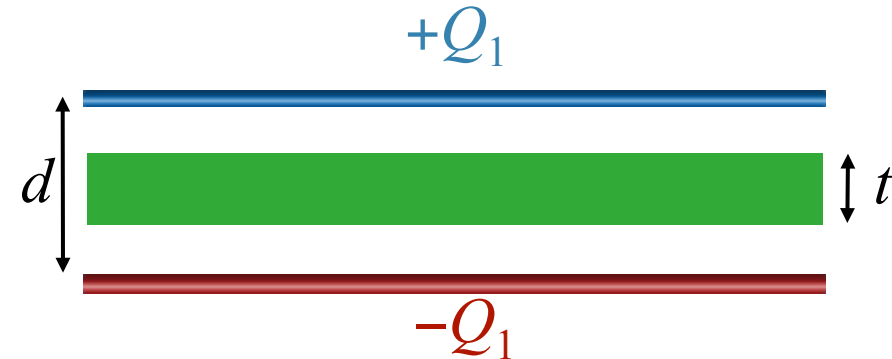


Initial charge on capacitor = Q_0



Insert uncharged conductor

Charge on capacitor now = Q_1



How is Q_1 related to Q_0 ?

A) $Q_1 < Q_0$

B) $Q_1 = Q_0$

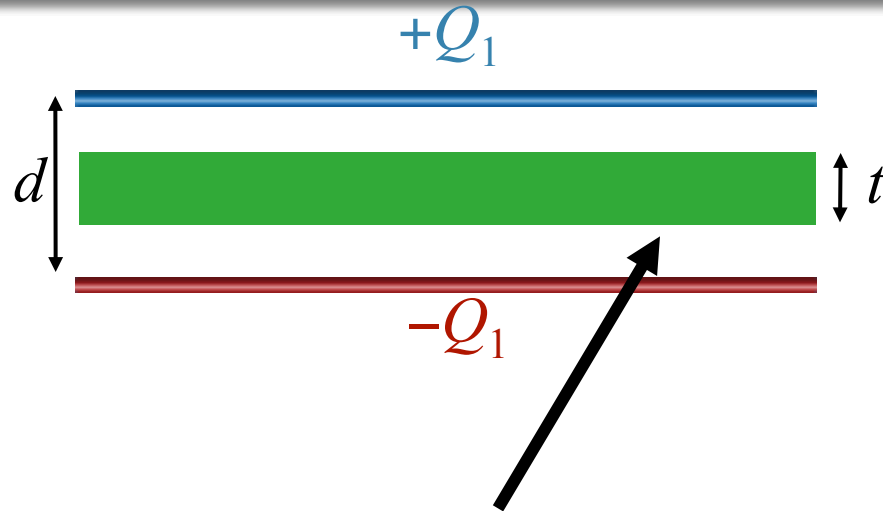
C) $Q_1 > Q_0$

Plates not connected to anything



CHARGE CANNOT CHANGE !

Where to Start ?



What is the total charge induced on the bottom surface of the conductor?

A) $+Q_0$

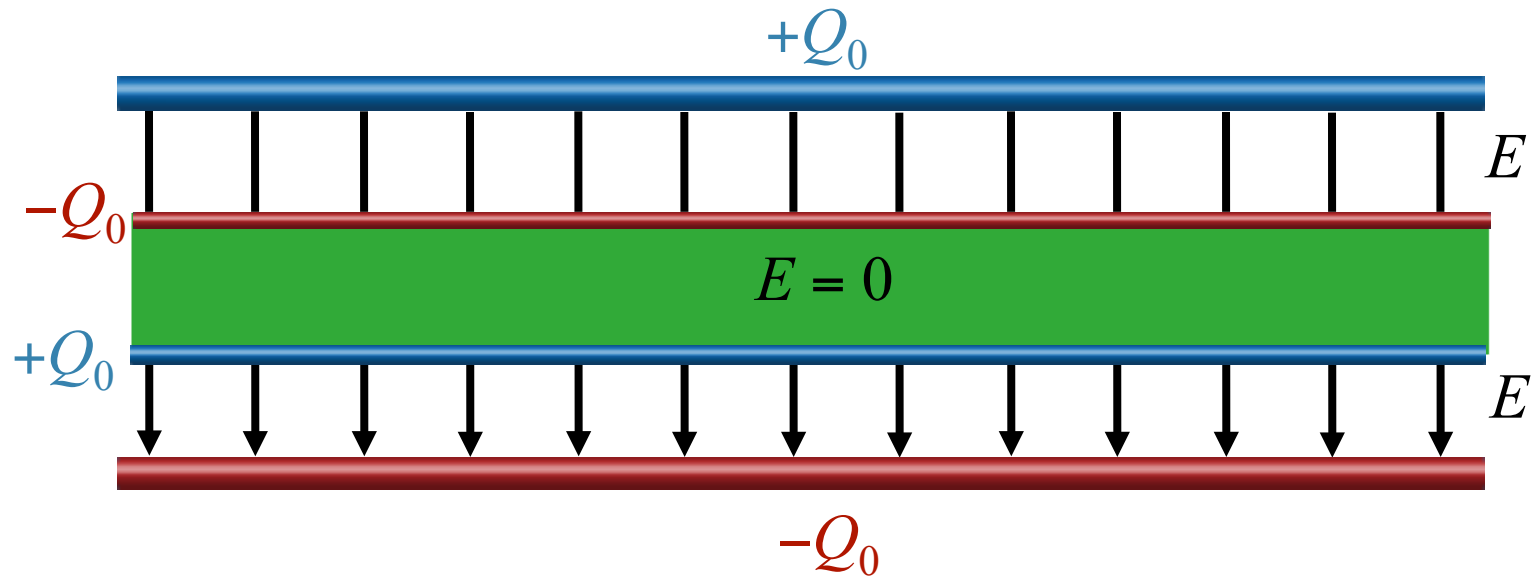
B) $-Q_0$

C) 0

D) Positive but the magnitude unknown

E) Negative but the magnitude unknown

Why ?



WHAT DO WE KNOW ?

E must be $= 0$ in conductor !

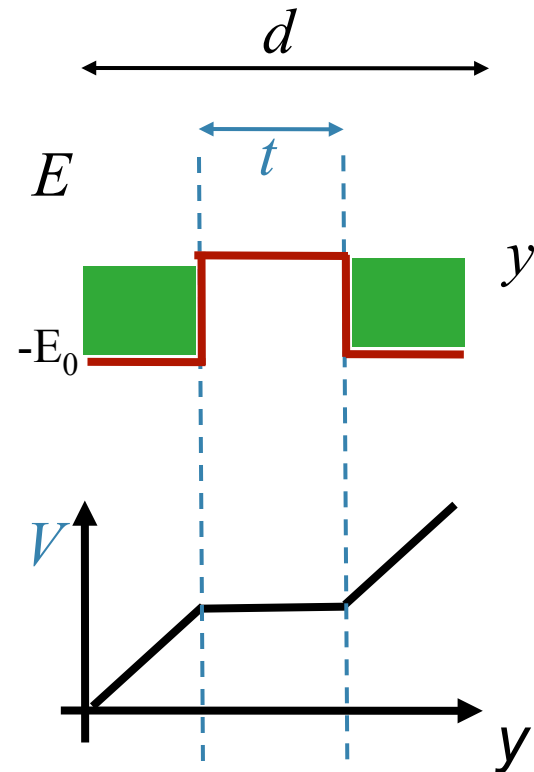
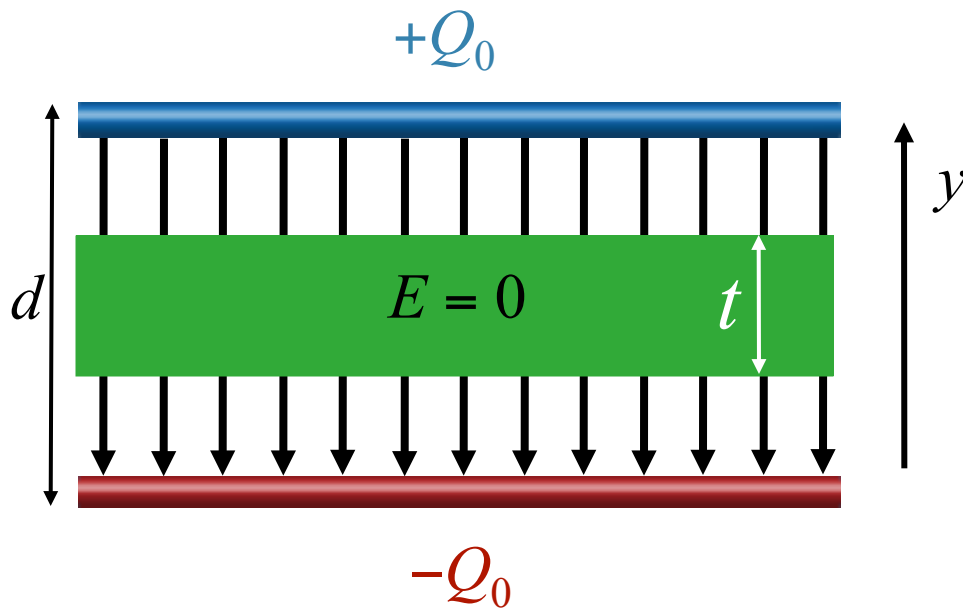


Charges inside conductor move to cancel E field from top & bottom plates.

Calculate V

Now calculate V as a function of distance from the bottom conductor.

$$V(y) = - \int_0^y \vec{E} \cdot d\vec{y}$$



What is $\Delta V = V(d)$?

A) $\Delta V = E_0 d$

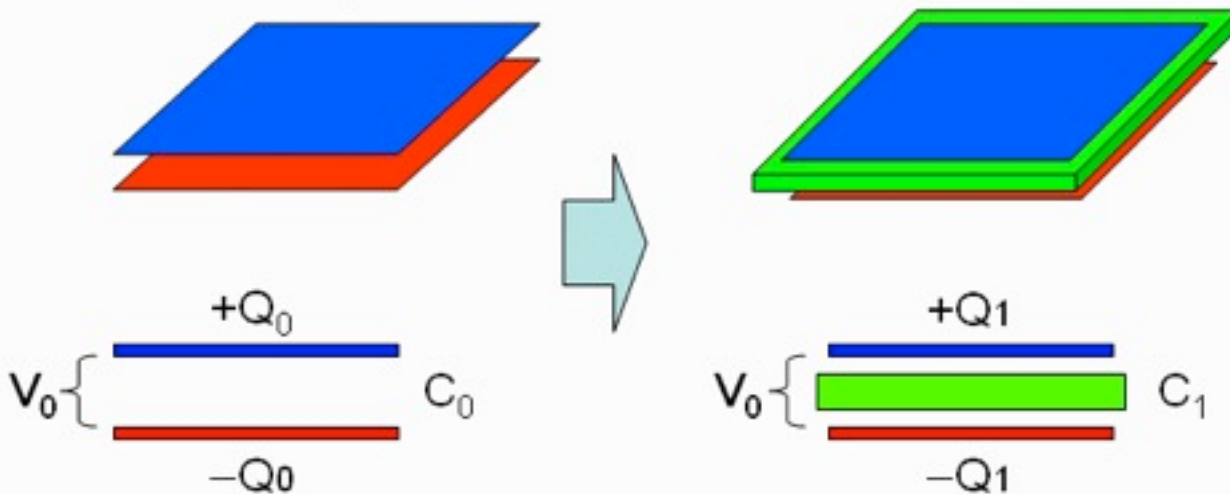
B) $\Delta V = E_0 (d - t)$

C) $\Delta V = E_0 (d + t)$

The integral = area under the curve

CheckPoint Results: Charged Parallel Plates 1

Two parallel plates of equal area carry equal and opposite charge Q_0 . The potential difference between the two plates is measured to be V_0 . An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same.



Compare Q_1 and Q_0 .

A. $Q_1 < Q_0$

B. $Q_1 = Q_0$

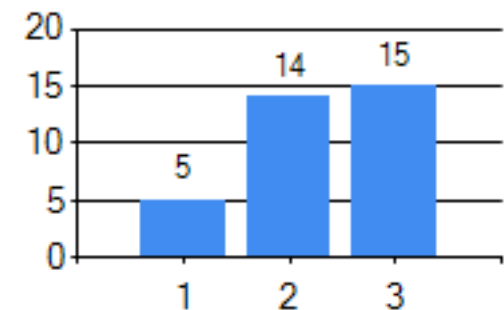
C. $Q_1 > Q_0$

“The distance for Q_1 is smaller therefore the charge must decrease to compensate for the change in distance “

“Since the potential remains the same, there is no change in the charge of Q . “

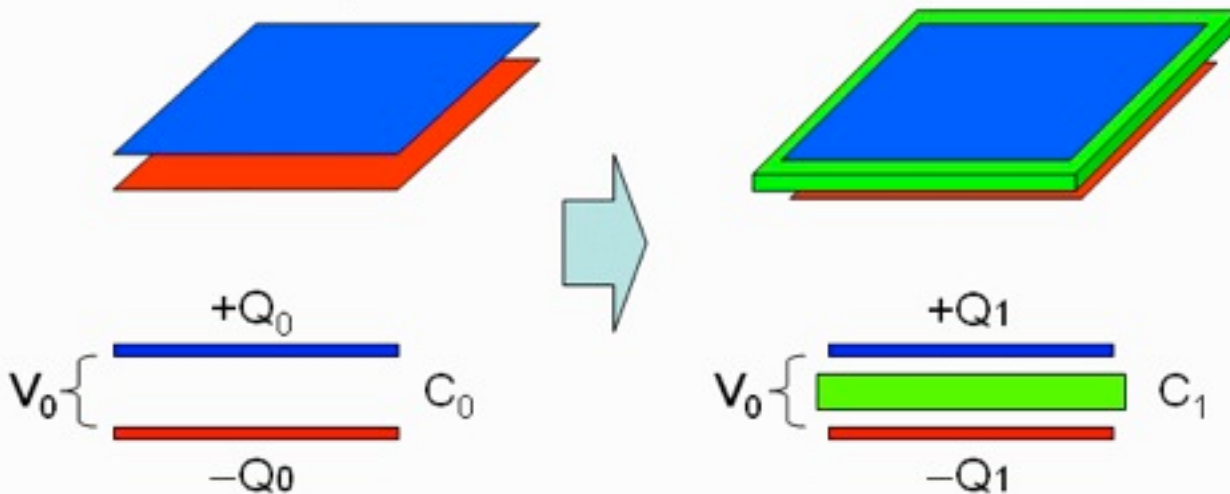
“the field through the conductor is zero, so it has constant potential. Because of this it must have greater charge so the total V is that same. “

Answer Choice Distribution



CheckPoint Results: Charged Parallel Plates 2

Two parallel plates of equal area carry equal and opposite charge Q_0 . The potential difference between the two plates is measured to be V_0 . An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same.



Compare the capacitance of the two configurations in the above problem.

A. $C_1 > C_0$

B. $C_1 = C_0$

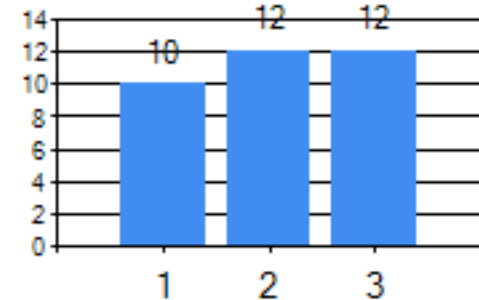
C. $C_1 < C_0$

"The distance for Q_1 is smaller therefore the charge must decrease to compensate for the change in distance "

"Since the potential remains the same, there is no change in the charge of Q . "

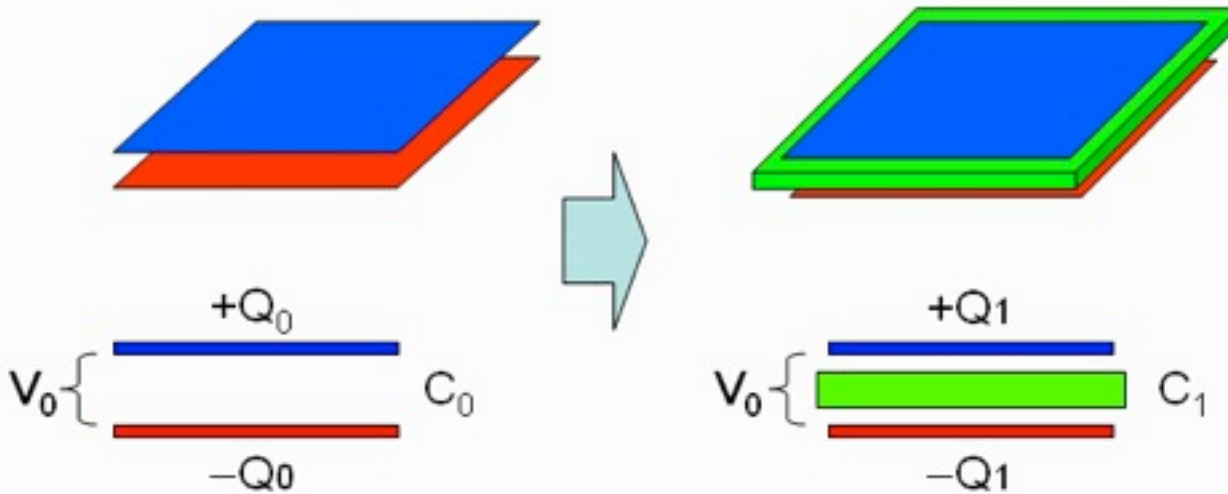
"the field through the conductor is zero, so it has constant potential. Because of this it must have greater charge so the total V is that same. "

Answer Choice Distribution



CheckPoint Results: Charged Parallel Plates 2

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Compare the capacitance of the two configurations in the above problem.

A. $C_1 > C_0$

B. $C_1 = C_0$

C. $C_1 < C_0$

We can determine C from either case

same V (preflight)

same Q (lecture)

C depends only on geometry !

$$E_0 = Q_0 / \epsilon_0 A$$

Same Q :

$$V_0 = E_0 d$$
$$V_1 = E_0 (d - t)$$



$$C_0 = Q_0 / E_0 d$$
$$C_1 = Q_0 / (E_0 (d - t))$$



$$C_0 = \epsilon_0 A / d$$
$$C_1 = \epsilon_0 A / (d - t)$$

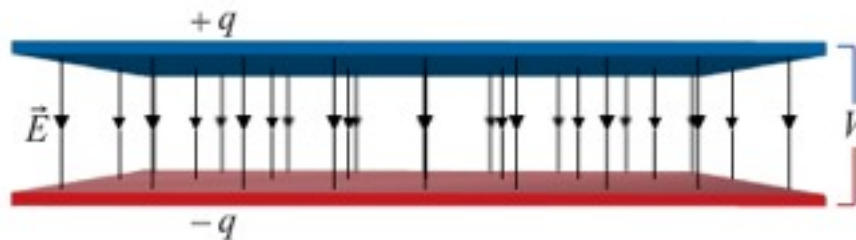
Energy in Capacitors

Energy Stored in Capacitors

$$U = \frac{1}{2} QV \quad \text{or} \quad U = \frac{1}{2} \frac{Q^2}{C} \quad \text{or} \quad U = \frac{1}{2} CV^2$$

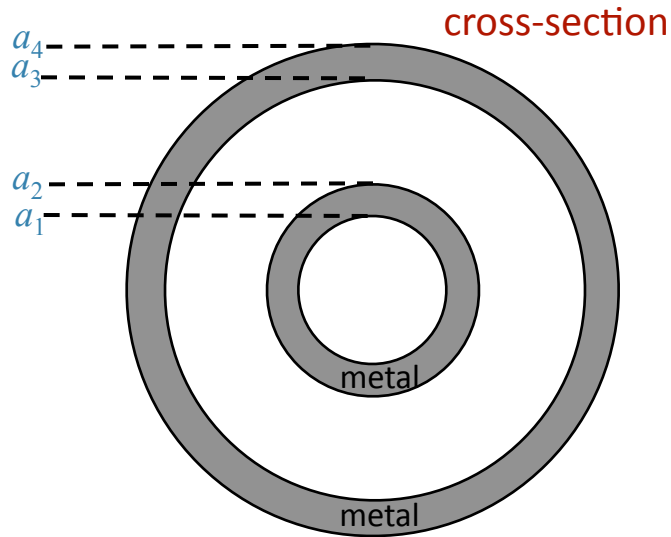
Energy Density

$$u = \frac{1}{2} \epsilon_0 E^2$$



BANG

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1 , a_2 , a_3 , and a_4 and length L ($L \gg a_i$).

What is the capacitance C of this capacitor ?

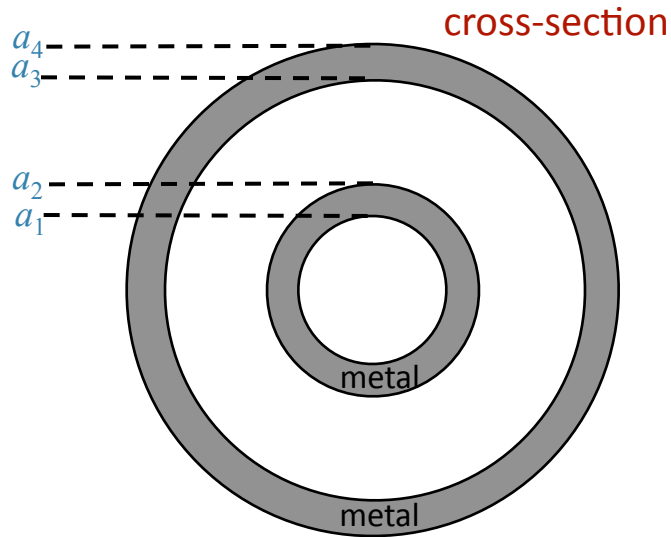
➤ Conceptual Analysis:

$$C \equiv \frac{Q}{V} \quad \text{But what is } Q \text{ and what is } V? \text{ They are not given?}$$

➤ Important Point: C is a property of the object! (concentric cylinders here)

- Assume some Q (i.e., $+Q$ on one conductor and $-Q$ on the other)
- These charges create E field in region between conductors
- This E field determines a potential difference V between the conductors
- V should be proportional to Q ; the ratio Q/V is the capacitance.

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1 , a_2 , a_3 , and a_4 and length L ($L \gg a_i$).

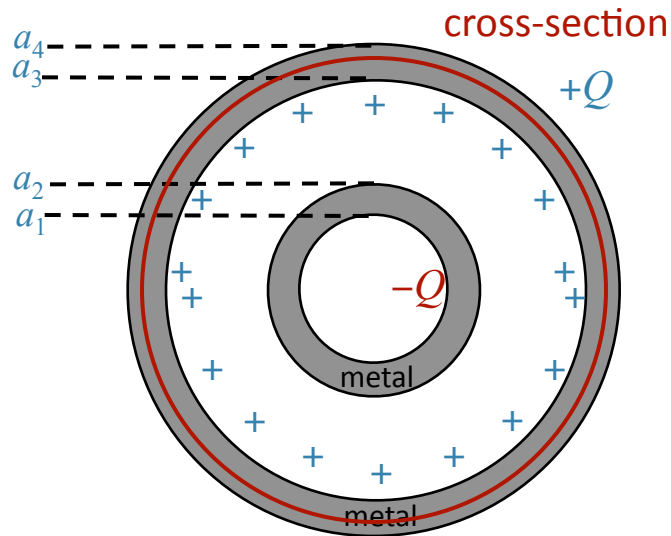
What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

➤ Strategic Analysis:

- Put $+Q$ on outer shell and $-Q$ on inner shell
- Cylindrical symmetry: Use Gauss' Law to calculate E everywhere
- Integrate E to get V
- Take ratio Q/V : should get expression only using geometric parameters (a_i , L)

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1 , a_2 , a_3 , and a_4 and length L ($L \gg a_i$).

What is the capacitance C of this capacitor?

$$C \equiv \frac{Q}{V}$$

Where is $+Q$ on outer conductor located?

- A) at $r = a_4$ **B) at $r = a_3$** C) both surfaces D) throughout shell

Why?

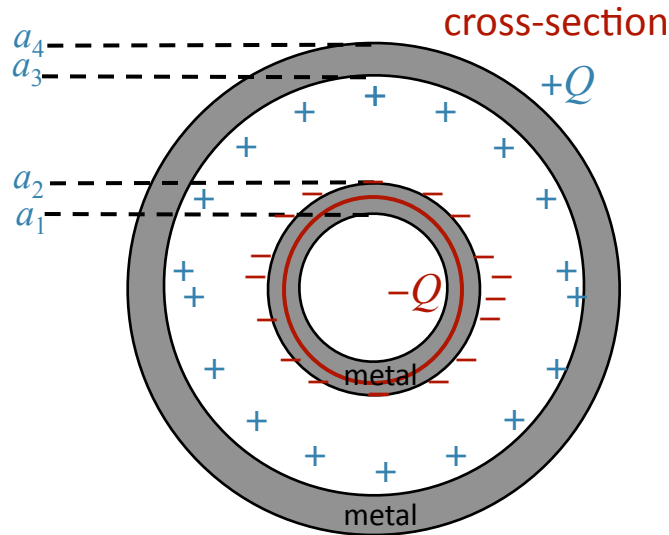
Gauss' law: $\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$

We know that $E = 0$ in conductor (between a_3 and a_4)

$$\longrightarrow Q_{\text{enclosed}} = 0$$

$$Q_{\text{enclosed}} = 0 \longrightarrow +Q \text{ must be on inside surface } (a_3), \text{ so that } Q_{\text{enclosed}} = +Q - Q = 0$$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1 , a_2 , a_3 , and a_4 and length L ($L \gg a_i$).

What is the capacitance C of this capacitor?

$$C \equiv \frac{Q}{V}$$

Where is $-Q$ on outer conductor located?

- A) at $r = a_2$ B) at $r = a_1$ C) both surfaces D) throughout shell

Why?

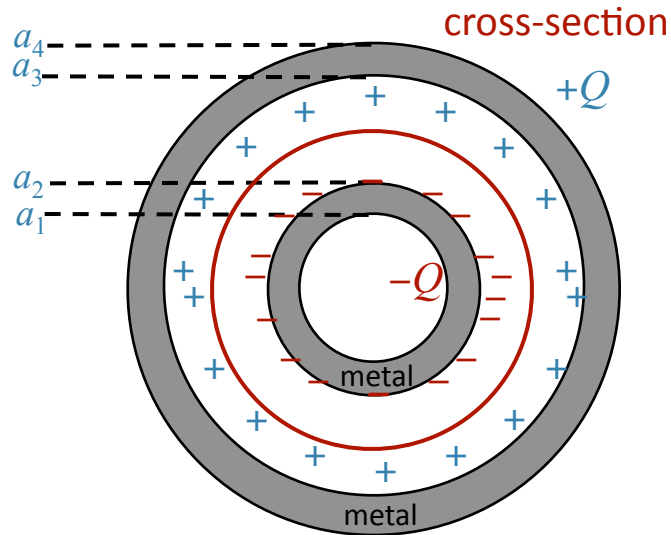
Gauss' law: $\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$

We know that $E = 0$ in conductor (between a_1 and a_2)

$$\longrightarrow Q_{\text{enclosed}} = 0$$

$$Q_{\text{enclosed}} = 0 \longrightarrow +Q \text{ must be on outer surface } (a_2), \text{ so that } Q_{\text{enclosed}} = 0$$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1 , a_2 , a_3 , and a_4 and length L ($L \gg a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

$a_2 < r < a_3$: What is $E(r)$?

A) 0

B) $\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

C) $\frac{1}{2\pi\epsilon_0} \frac{Q}{Lr}$

D) $\frac{1}{2\pi\epsilon_0} \frac{2Q}{Lr}$

E) $\frac{1}{4\pi\epsilon_0} \frac{2Q}{r^2}$

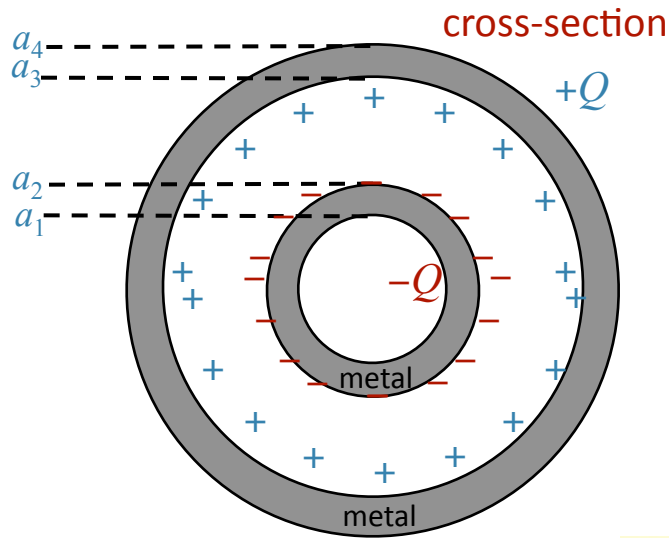
Why?

Gauss' law:

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} \rightarrow E(2\pi rL) = \frac{Q}{\epsilon_0} \rightarrow E = \frac{1}{2\pi\epsilon_0} \frac{Q}{Lr}$$

Direction: Radially In

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1 , a_2 , a_3 , and a_4 and length L ($L \gg a_i$).

What is the capacitance C of this capacitor?

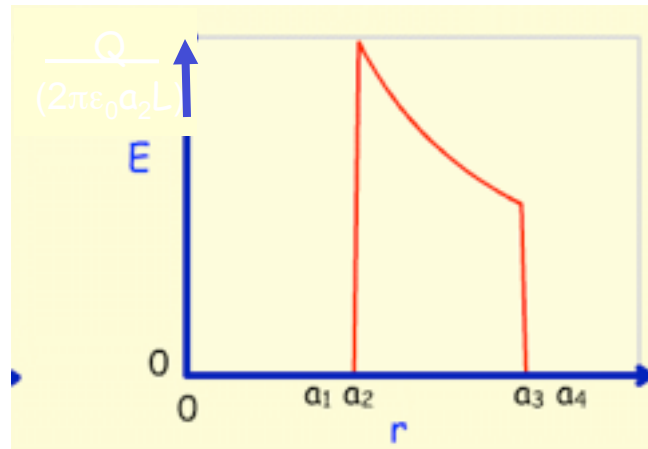
$$C \equiv \frac{Q}{V} \quad a_2 < r < a_3: \quad E = \frac{1}{2\pi\epsilon_0} \frac{Q}{Lr}$$

$$r < a_2: E(r) = 0$$

since $Q_{\text{enclosed}} = 0$

What is V ?

The potential difference between the conductors.



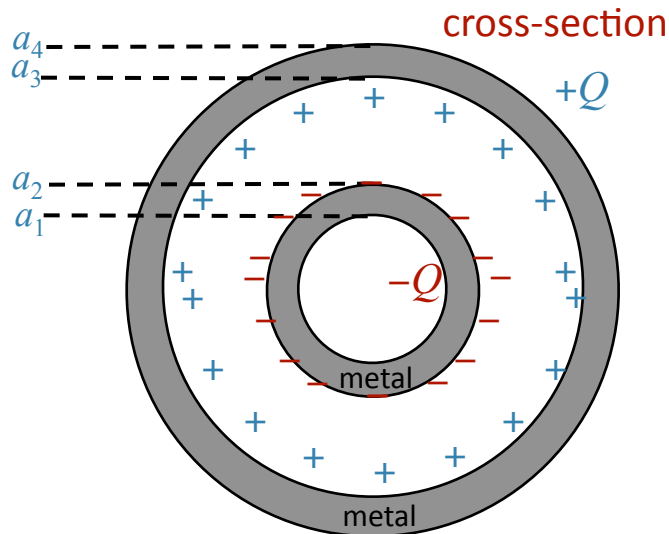
What is the sign of $V = V_{\text{outer}} - V_{\text{inner}}$?

A) $V_{\text{outer}} - V_{\text{inner}} < 0$

B) $V_{\text{outer}} - V_{\text{inner}} = 0$

C) $V_{\text{outer}} - V_{\text{inner}} > 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1 , a_2 , a_3 , and a_4 and length L ($L \gg a_i$).

What is the capacitance C of this capacitor?

$$C \equiv \frac{Q}{V} \quad a_2 < r < a_3: \quad E = \frac{1}{2\pi\epsilon_0} \frac{Q}{Lr}$$

What is $V \equiv V_{outer} - V_{inner}$?

$$\frac{Q}{2\pi\epsilon_0 L} \ln \frac{a_1}{a_4}$$

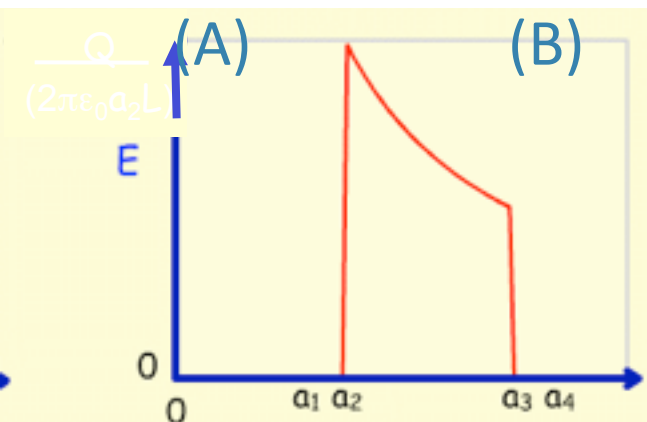
$$\frac{Q}{2\pi\epsilon_0 L} \ln \frac{a_4}{a_1}$$

$$\frac{Q}{2\pi\epsilon_0 L} \ln \frac{a_3}{a_2}$$

$$\frac{Q}{2\pi\epsilon_0 L} \ln \frac{a_2}{a_3}$$

(C)

(D)



$$V = -\int_{a_2}^{a_3} \frac{-Q}{2\pi\epsilon_0 L} \frac{dr}{r} \rightarrow V = \frac{Q}{2\pi\epsilon_0 L} \int_{a_2}^{a_3} \frac{dr}{r} \rightarrow V = \frac{Q}{2\pi\epsilon_0 L} \ln \frac{a_3}{a_2}$$

V proportional to Q , as promised

$$\rightarrow C \equiv \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln(a_3 / a_2)}$$