Electricity & Magnetism
Lecture 9: Electric Current

Today's Concept:

Electric Current

Your stuff

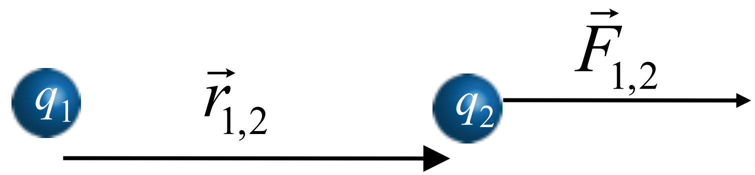
- “So simple i thought i did something wrong.”
- “current density”
- “Since $R = \rho L/A$, the greater the cross sectional area, the smaller the resistance, but the greater the length the higher the resistance. Is that why long cables have to be very thick?”

A Big Idea Review

Coulomb's Law

Force law between point charges

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2}$$



Electric Field

Force per unit charge

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric Field

Property of Space

Created by Charges

Superposition

Gauss' Law

Flux through closed surface is always proportional to charge enclosed

$$\oint_{\text{surface}} \vec{E}\cdot\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Gauss' Law

Can be used to determine E field → Spheres, Cylinders, Infinite Planes

Electric Potential

Potential energy per unit charge

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Capacitance

Relates charge and potential for two conductor system

$$C \equiv \frac{Q}{V}$$

Electric Potential

Scalar Function that can be used to determine E

$$\vec{E} = -\vec{\nabla}V$$

Applications of Big Ideas

Conductors
Charges free to move → What Determines How They Move? → They move until $E = 0$!

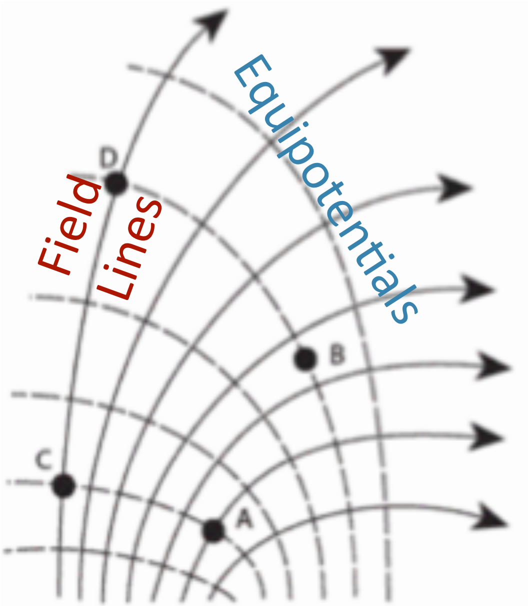
↓

$E = 0$ in conductor determines charge densities on surfaces

Gauss' Law

Spheres
Cylinders
Infinite Planes

Field Lines & Equipotentials



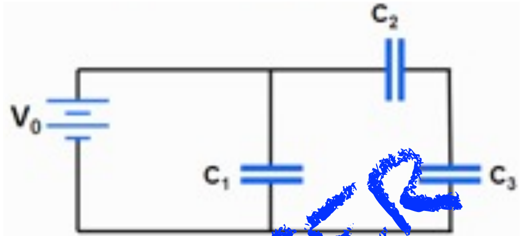
Work Done By E Field

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q\vec{E}\cdot d\vec{l}$$

Change in Potential Energy

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Capacitor Networks



LATER

Series:
$(1/C_{23}) = (1/C_2) + (1/C_3)$

Parallel
$C_{123} = C_1 + C_{23}$

Key Concepts:

1) How resistance depends on A, L, σ, ρ

σ is **conductivity** here (not surface charge density)
ρ is **resistivity** here (not volume charge density).

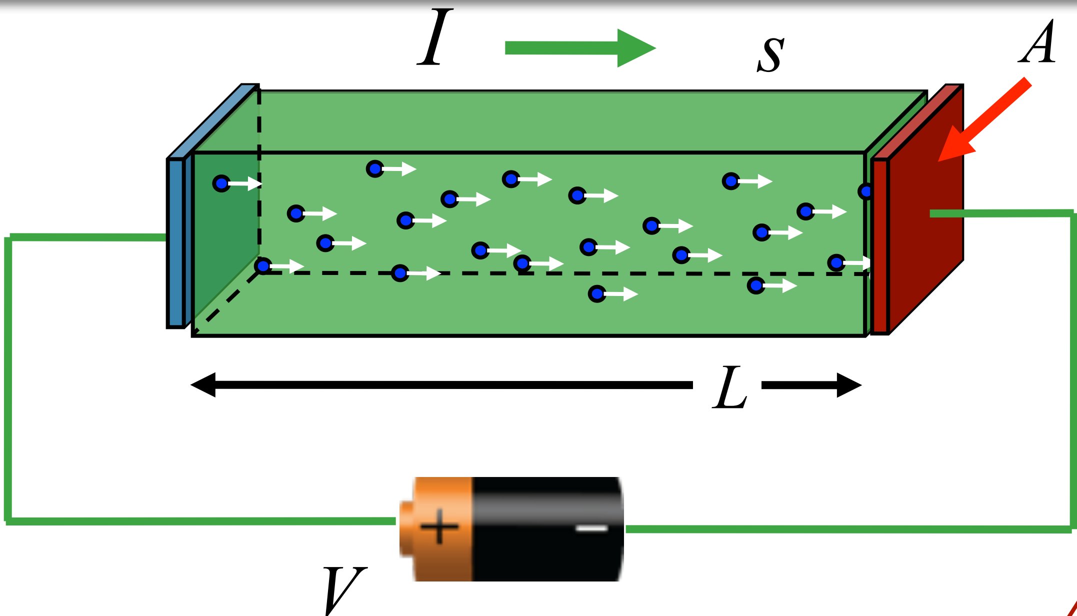
2) How to combine resistors in series and parallel

3) Understanding resistors in circuits

Today's Plan:

1) Review of resistance & preflights

2) Work out a circuit problem in detail



Conductivity – high for good conductors.

Ohm's Law: $J = \sigma E$

Observables:

$$V = EL$$
$$I = JA$$

$\rightarrow$ $I/A = \sigma V/L$ $\rightarrow$ $I = V/(L/\sigma A)$

$$R = \frac{L}{\sigma A}$$

$\rightarrow$ $I = V/R$

R = Resistance

$$\rho = 1/\sigma$$

This is just like Plumbing!

I is like flow rate of water

V is like pressure

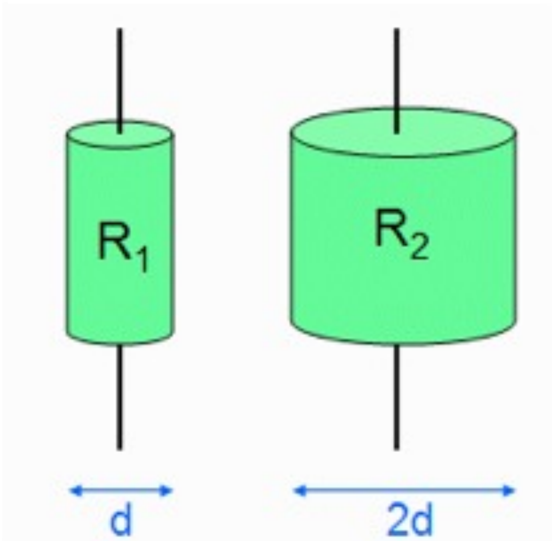
R is how hard it is for water to flow in a pipe

$$R = \frac{L}{\sigma A}$$

To make R big, make L long or A small

To make R small, make L short or A big

1 CheckPoint: Two Resistors 2



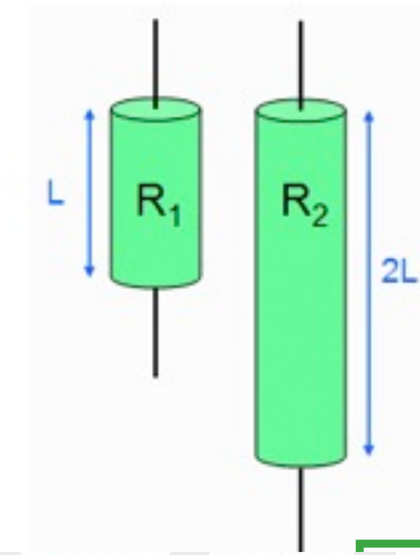
A $V_1 > V_2$ B $V_1 = V_2$ C $V_1 < V_2$

Same current through both resistors

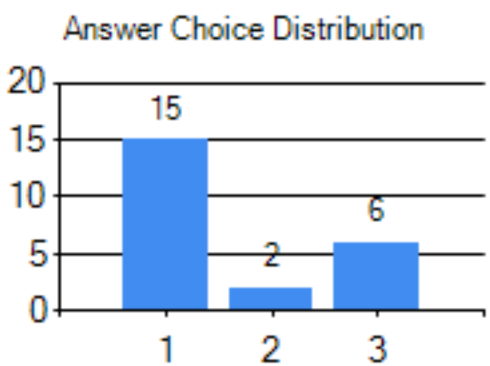
Compare voltages across resistors

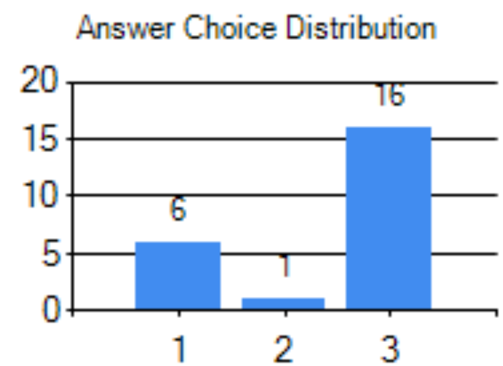
$$R \propto \frac{L}{A}$$

$$V = IR \propto \frac{L}{A}$$



A $V_1 > V_2$ B $V_1 = V_2$ C $V_1 < V_2$



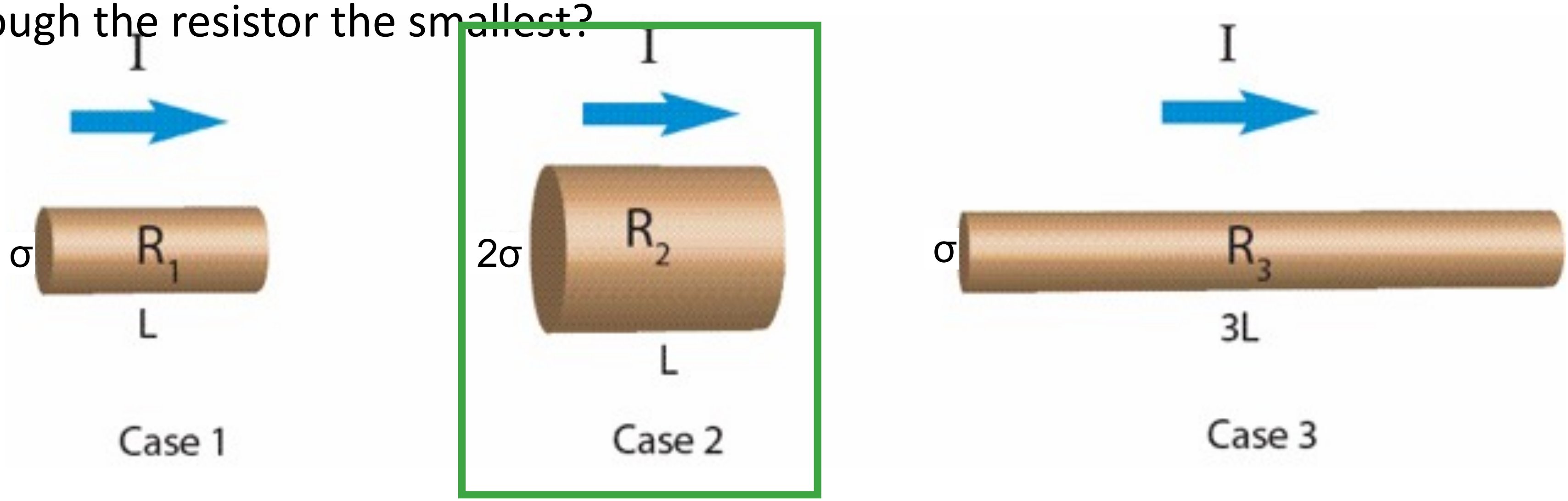


$$A_2 = 4A_1 \Rightarrow V_2 = \tfrac{1}{4}V_1$$

$$L_2 = 2L_1 \Rightarrow V_2 = 2V_1$$

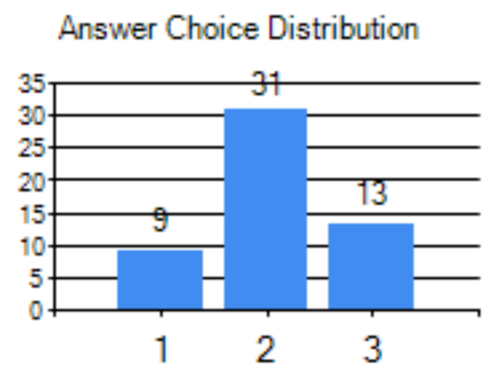
CheckPoint: Current Density

The SAME amount of current I passes through three different resistors. R_2 has twice the cross-sectional area and the same length as R_1, and R_3 is three times as long as R_1 but has the same cross-sectional area as R_1. In which case is the CURRENT DENSITY through the resistor the smallest?

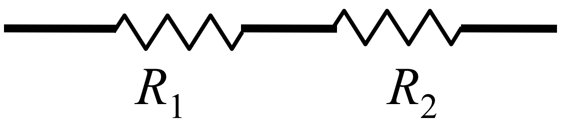
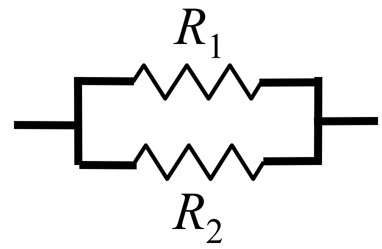


$$J \equiv \frac{I}{A} \quad \longrightarrow \quad J_1 = J_3 = 2J_2$$

Same Current $\longrightarrow$ $J \propto \frac{1}{A}$

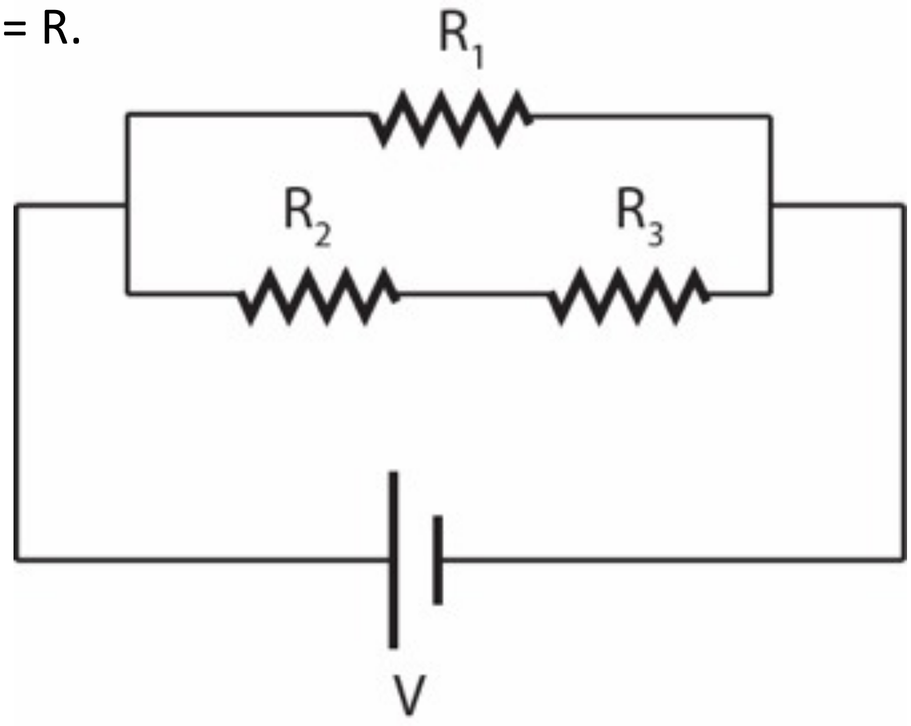


Resistor Summary

	Series	Parallel
		
Wiring	Each resistor on the <u>same</u> wire.	Each resistor on a <u>different</u> wire.
Voltage	<u>Different</u> for each resistor. $V_{total} = V_1 + V_2$ Voltage Divider	<u>Same</u> for each resistor. $V_{total} = V_1 = V_2$
Current	<u>Same</u> for each resistor $I_{total} = I_1 = I_2$	<u>Different</u> for each resistor $I_{total} = I_1 + I_2$ Current Divider
Resistance	<u>Increases</u> $R_{eq} = R_1 + R_2$	<u>Decreases</u> $1/R_{eq} = 1/R_1 + 1/R_2$

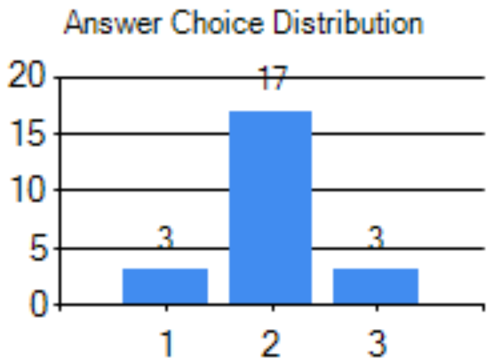
CheckPoint: Resistor Network 1

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



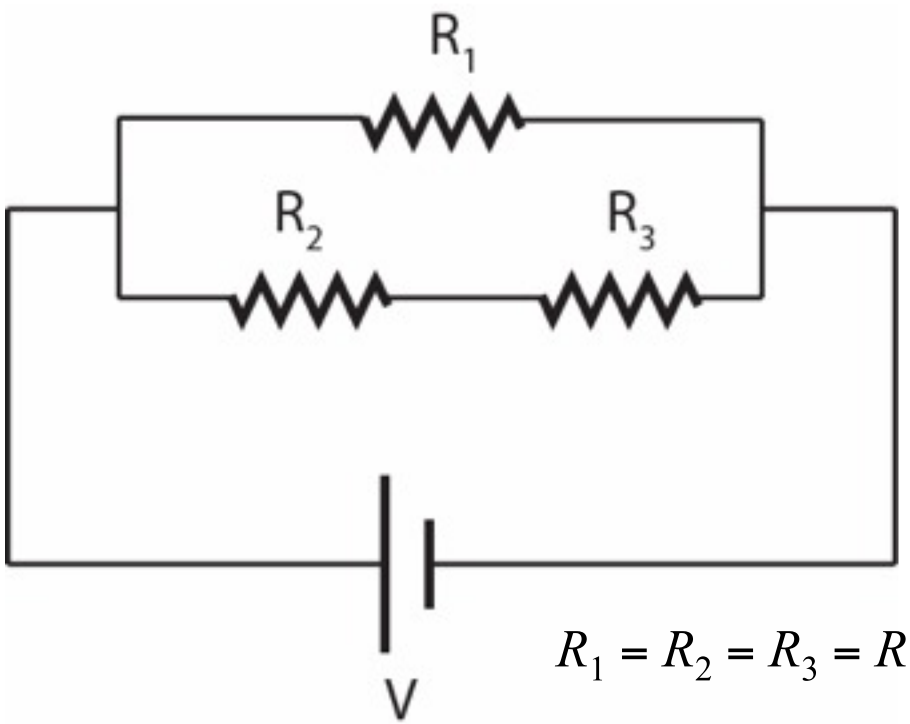
Compare the current through R_2 with the current through R_3:

A. $I_2 > I_3$
B. $I_2 = I_3$
C. $I_2 < I_3$



R_2 in series with R_3 → Current through R_2 and R_3 is the same

$$I_{23} = \frac{V}{R_2 + R_3}$$



$$R_1 = R_2 = R_3 = R$$

CheckPoint 2

Compare the current through R_1 with the current through R_2

$$I_1 \longleftrightarrow I_2$$

CheckPoint 3

Compare the voltage across R_2 with the voltage across R_3

$$V_2 \longleftrightarrow V_3$$

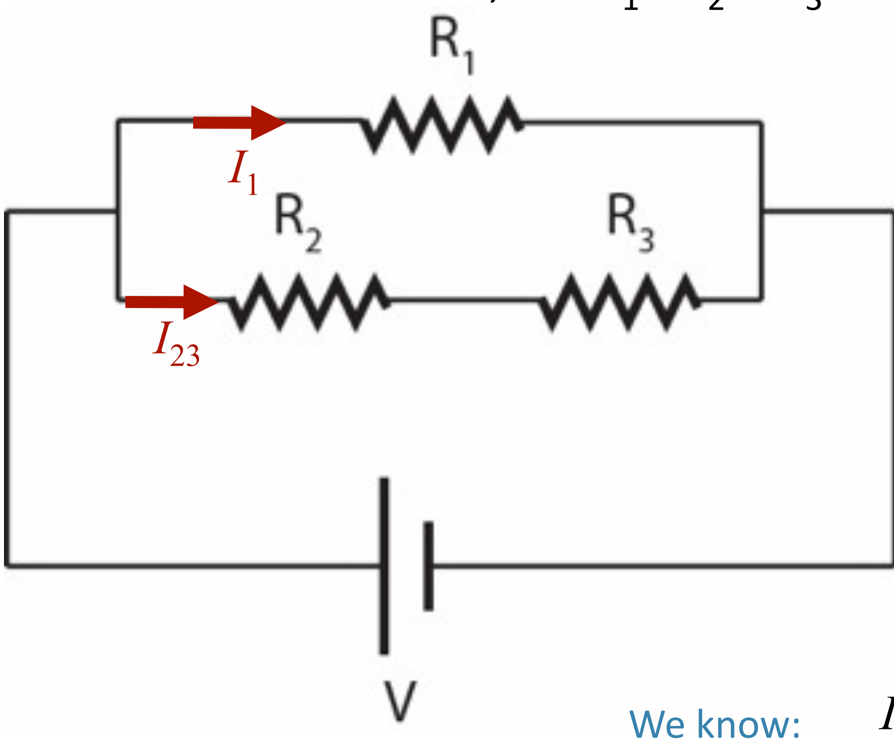
CheckPoint 4

Compare the voltage across R_1 with the voltage across R_2

$$V_1 \longleftrightarrow V_2$$

CheckPoint: Resistor Network 2

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



Compare the current through R_1 with the current through R_2:

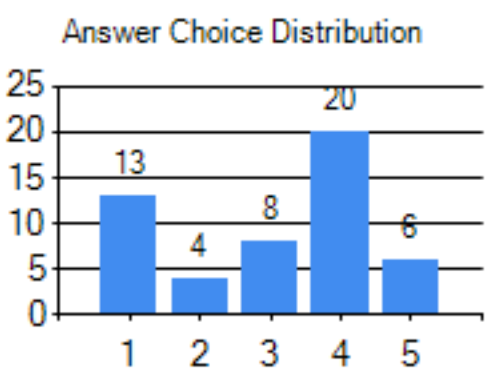
A. $I_1/I_{23}=1/2$
B. $I_1/I_{23}=1/3$
C. $I_1 = I_{23}$
D. $I_1/I_{23}=2$
E. $I_1/I_{23}=3$

We know: $$I_{23} = \frac{V}{R_2 + R_3}$$

Similarly: $$I_1 = \frac{V}{R_1}$$

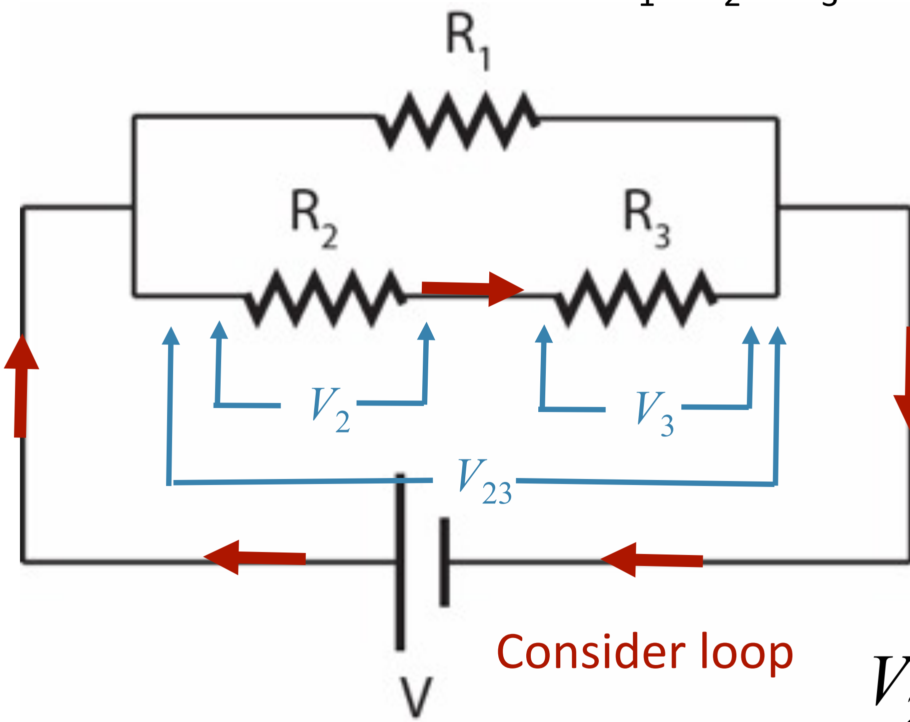
$$I_1 = I_{23}\frac{R_2 + R_3}{R_1}$$

$$\frac{I_1}{I_{23}} = \frac{R_2 + R_3}{R_1} = 2$$



CheckPoint: Resistor Network 3

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



Compare the voltage across R_2 with the voltage across R_3:

A. $V_2 > V_3$

B. $V_2 = V_3 = V$

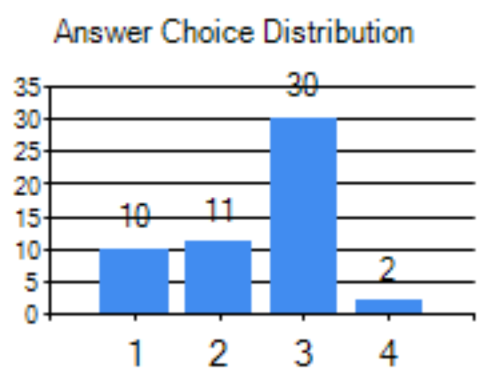
C. $V_2 = V_3 < V$

D. $V_2 < V_3$

$$V_{23} = V$$

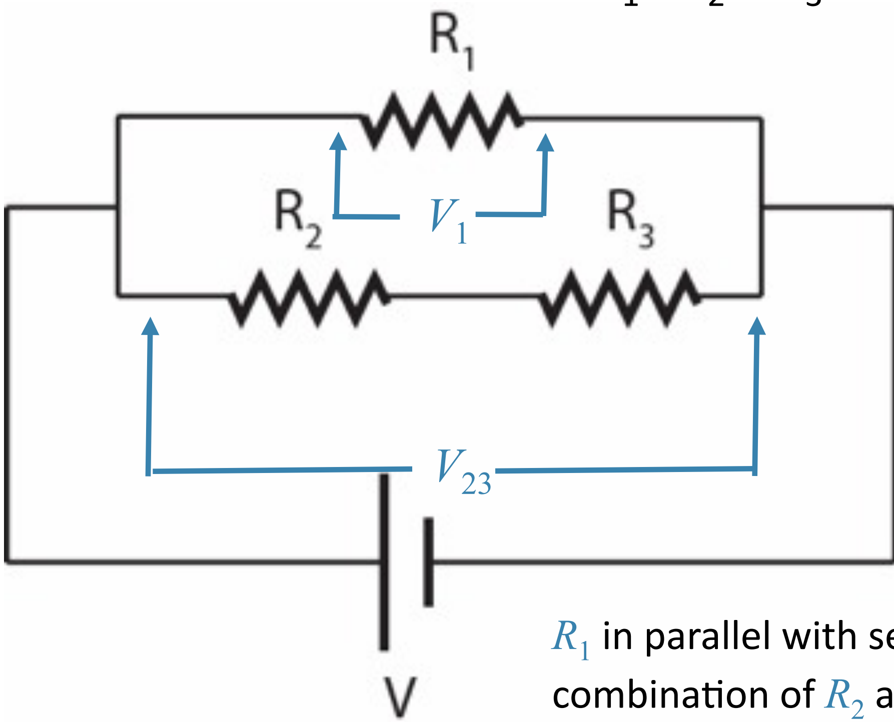
$$V_{23} = V_2 + V_3 \quad \longrightarrow \quad V_2 = V_3 = \frac{V}{2}$$

$$R_2 = R_3 \Rightarrow V_2 = V_3$$



CheckPoint: Resistor Network 4

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



Compare the voltage across R_1 with the voltage across R_2.

A. $V_1 = V_2 = V$

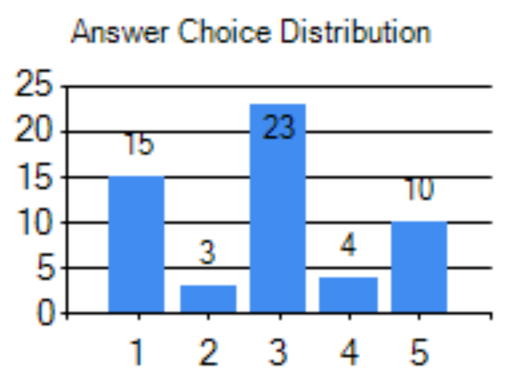
B. $V_1 = 1/2\ V_2 = V$

C. $V_1 = 2V_2 = V$

D. $V_1 = 1/2\ V_2 = 1/5\ V$

E. $V_1 = 1/2\ V_2 = 1/2\ V$

R_1 in parallel with series combination of R_2 and R_3

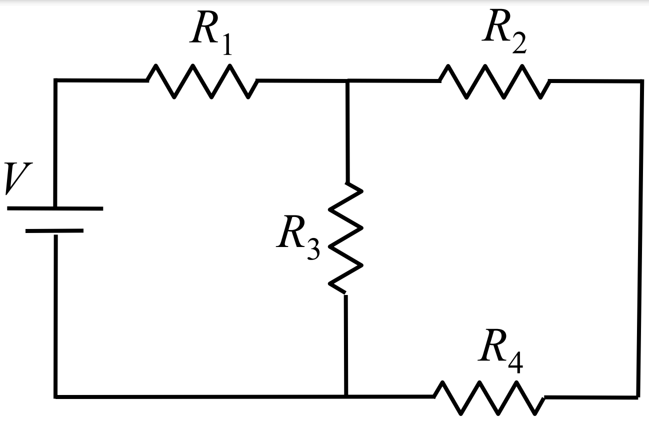


$$V_1 = V_{23}$$

$$R_2 = R_3 \Rightarrow V_2 = V_3 \longrightarrow V_1 = 2V_2 = V$$

$$V_{23} = V_2 + V_3 = 2V_2$$

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Conceptual Analysis:

Ohm's Law: when current *I* flows through resistance R, the potential drop V is given by:

$V = IR$.

Resistances are combined in series and parallel combinations

$R_{series} = R_a + R_b$

$(1/R_{parallel}) = (1/R_a) + (1/R_b)$

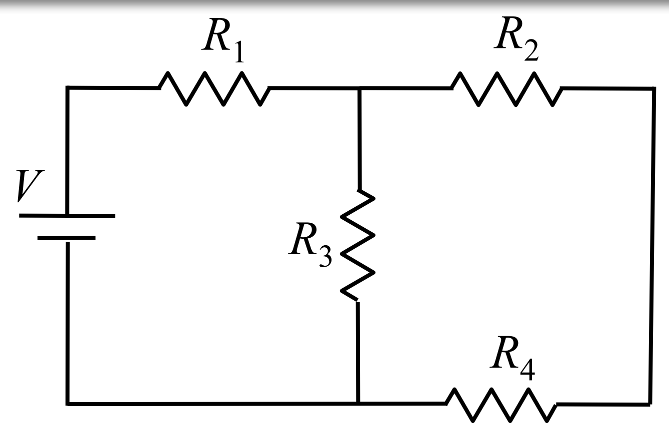
Strategic Analysis:

Combine resistances to form equivalent resistances

Evaluate voltages or currents from Ohm's Law

Expand circuit back using knowledge of voltages and currents

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Combine Resistances:

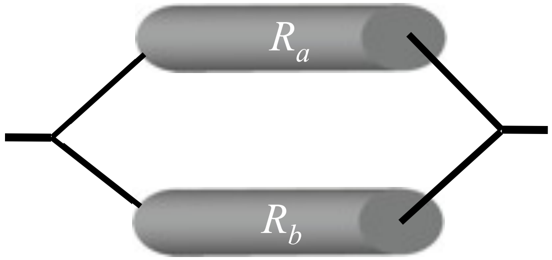
R_1 and R_2 are connected:

A) in series

B) in parallel

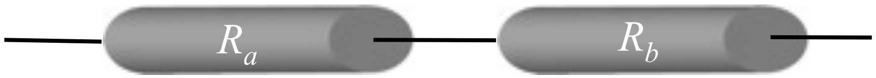
C) neither in series nor in parallel

Parallel Combination



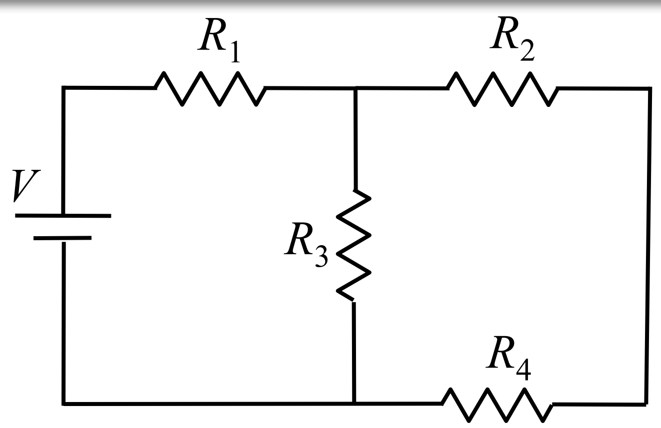
Parallel: Can make a loop that contains only those two resistors

Series Combination



Series : Every loop with resistor 1 also has resistor 2.

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

We first will combine resistances $R_2 + R_3 + R_4$:

Which of the following is true?

A) R_2, R_3 and R_4 are connected in series

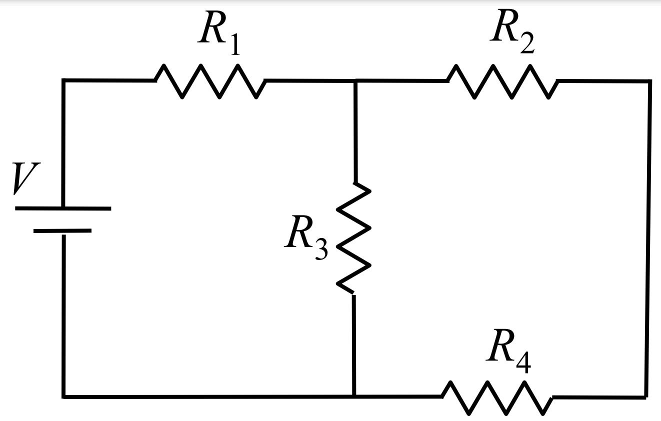
B) R_2, R_3, and R_4 are connected in parallel

C) R_3 and R_4 are connected in series (R_{34}) which is connected in parallel with R_2

D) R_2 and R_4 are connected in series (R_{24}) which is connected in parallel with R_3

E) R_2 and R_4 are connected in parallel (R_{24}) which is connected in parallel with R_3

Calculation



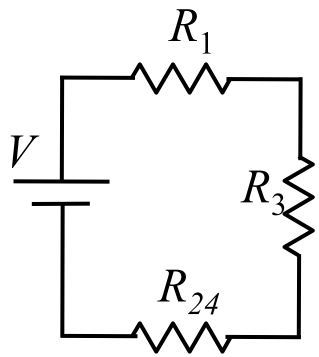
In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

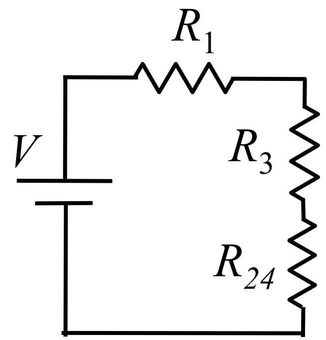
What is V_2, the voltage across R_2?

R_2 and R_4 are connected in series (R_{24}) which is connected in parallel with R_3

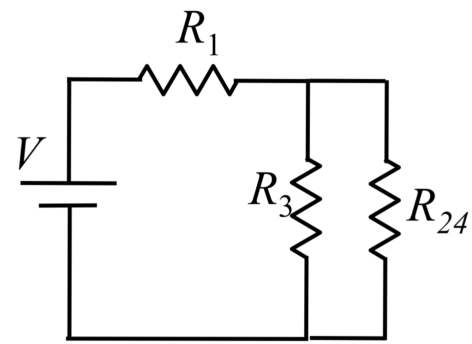
Redraw the circuit using the equivalent resistor R_{24} = series combination of R_2 and R_4.



(A)

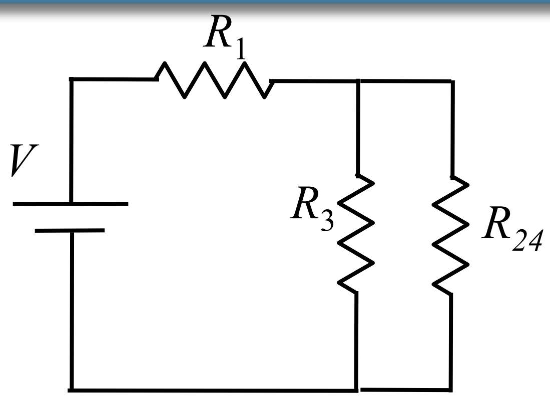


(B)



(C)

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Combine Resistances:

R_2 and R_4 are connected in series = R_{24}

R_3 and R_{24} are connected in parallel = R_{234}

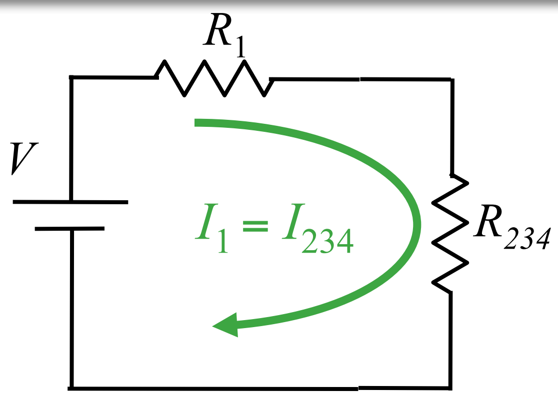
What is the value of R_{234}?

A) $R_{234} = 1\ \Omega$ **B) $R_{234} = 2\ \Omega$** C) $R_{234} = 4\ \Omega$ D) $R_{234} = 6\ \Omega$

R_2 and R_4 in series → $R_{24} = R_2 + R_4 = 2\Omega + 4\Omega = 6\Omega$

$(1/R_{parallel}) = (1/R_a) + (1/R_b)$ → $1/R_{234} = (1/3) + (1/6) = (3/6)\ \Omega^{-1}$ → $R_{234} = 2\ \Omega$

Calculation



In the circuit shown: $V = 18V$,

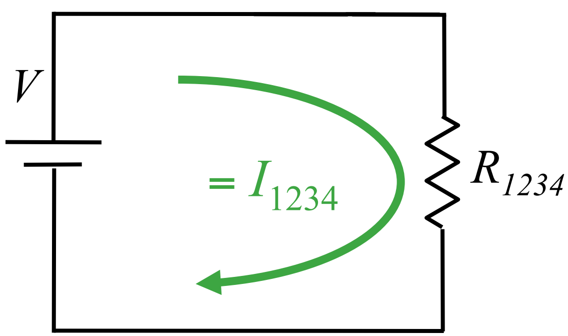
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

$R_{24} = 6\Omega$ $R_{234} = 2\Omega$

What is V_2, the voltage across R_2?

R_1 and R_{234} are in series. $R_{1234} = 1 + 2 = 3\ \Omega$

Our next task is to calculate the total current in the circuit

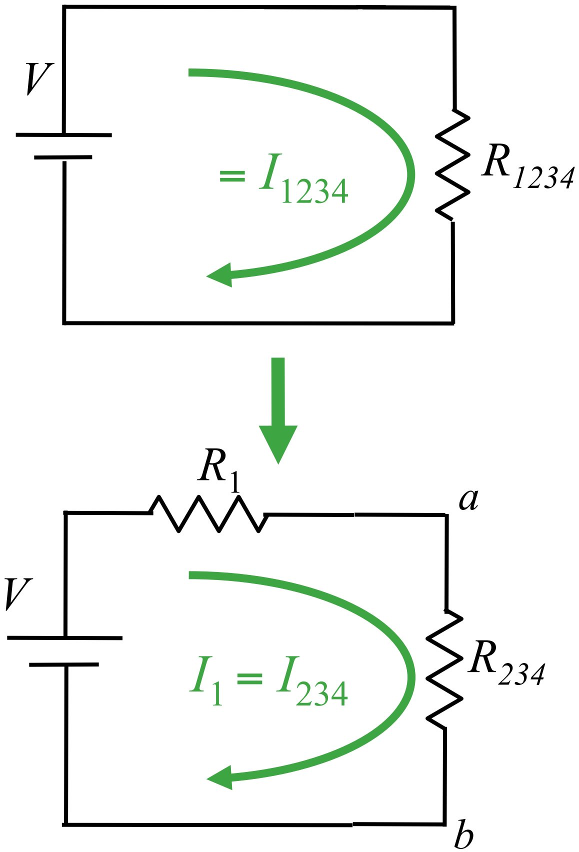


Ohm's Law tells us: $I_{1234} = V/R_{1234}$

$= 18 / 3$

$= 6$ Amps

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

$R_{24} = 6\Omega$ $R_{234} = 2\Omega$ $I_{1234} = 6\ A$

What is V_2, the voltage across R_2?

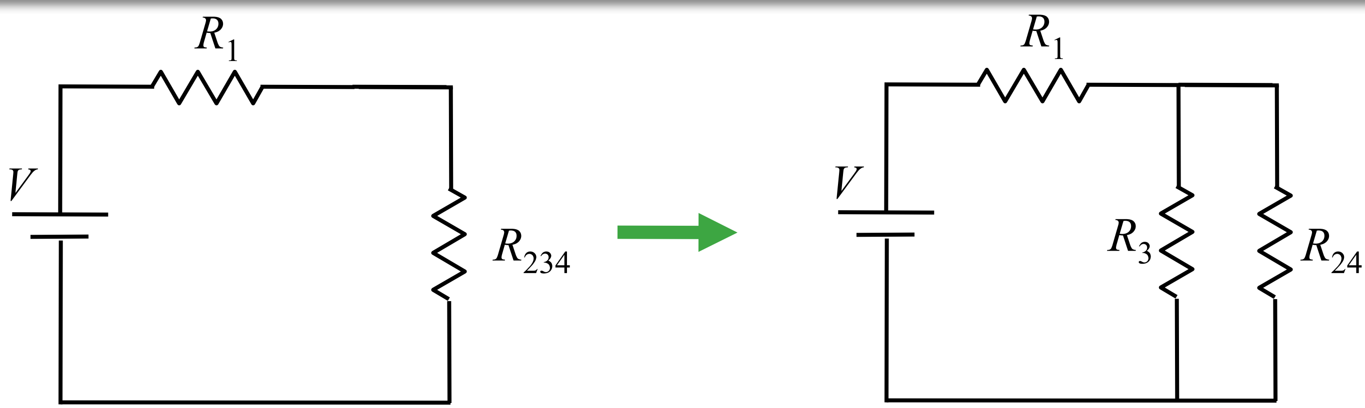
$I_{234} = I_{1234}$ Since R_1 in series with R_{234}

$$V_{234} = I_{234}\, R_{234}$$
$$= 6 \times 2$$
$$= 12 \text{ Volts}$$

What is V_{ab}, the voltage across R_{234} ?

A) $V_{ab} = 1\ V$ B) $V_{ab} = 2\ V$ C) $V_{ab} = 9\ V$ D) $V_{ab} = 12\ V$ E) $V_{ab} = 16\ V$

Calculation



$V = 18V$

$R_1 = 1\Omega$

$R_2 = 2\Omega$

$R_3 = 3\Omega$

$R_4 = 4\Omega$

$R_{24} = 6\Omega$

$R_{234} = 2\Omega$

$I_{1234} = 6$ Amps

$I_{234} = 6$ Amps

$V_{234} = 12V$

What is V_2?

Which of the following are true?

A) $V_{234} = V_{24}$ B) $I_{234} = I_{24}$ C) Both A+B D) None

R_3 and R_{24} were combined in parallel to get R_{234} → Voltages are same!

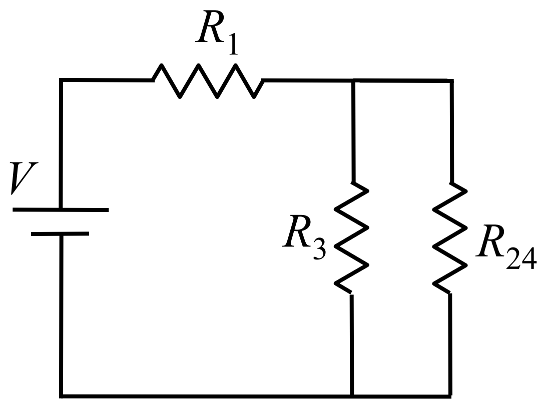
Ohm's Law

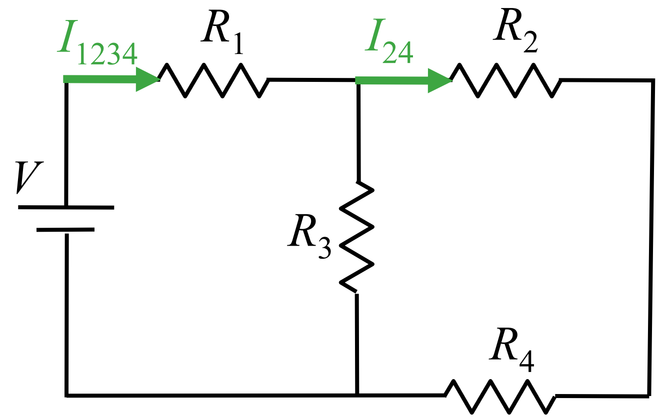
$$I_{24} = V_{24} / R_{24}$$

$$= 12 / 6$$

$$= 2 \text{ Amps}$$

Calculation





$V = 18V$

$R_1 = 1\Omega$

$R_2 = 2\Omega$

$R_3 = 3\Omega$

$R_4 = 4\Omega$.

$R_{24} = 6\Omega$

$R_{234} = 2\Omega$

$I_{1234} = 6$ Amps

$I_{234} = 6$ Amps

$V_{234} = 12V$

$V_{24} = 12V$

$I_{24} = 2$ Amps

What is V_2?

Which of the following are true?

A) $V_{24} = V_2$ B) $I_{24} = I_2$ C) Both A+B D) None

R_2 and R_4 where combined in series to get R_{24} → Currents are same!

The Problem Can Now Be Solved!

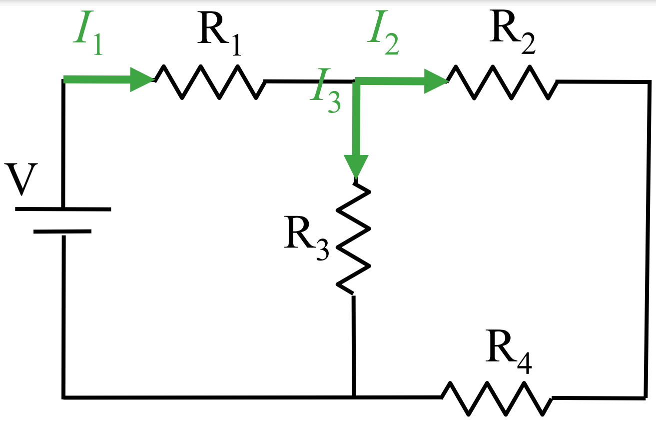
Ohm's Law

$V_2 = I_2 \; R_2$

$= \; 2 \times 2$

$= 4$ Volts!

Quick Follow-Ups



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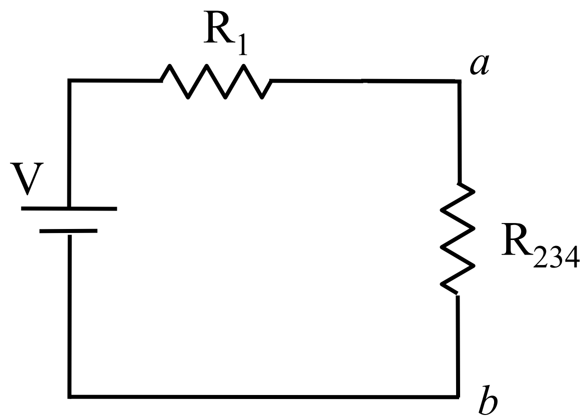

$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$V_{234} = 12V$
$V_2 = 4V$
$I_{1234} = 6$ Amps

What is I_3 ?

A) $I_3 = 2\ A$ B) $I_3 = 3\ A$ C) $I_3 = 4\ A$

$V_3 = V_{234} = 12V$ → $I_3 = V_3/R_3 = 12V/3\Omega = 4A$

What is I_1 ?

We know $I_1 = I_{1234} = 6\ A$

NOTE: $I_2 = V_2/R_2 = 4/2 = 2\ A$ → $I_1 = I_2 + I_3$ Make Sense?