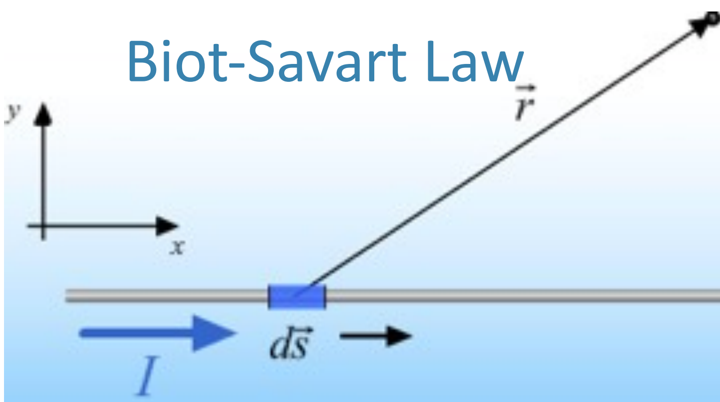
Electricity & Magnetism
Lecture 14

Today's Concept:

Biot-Savart Law

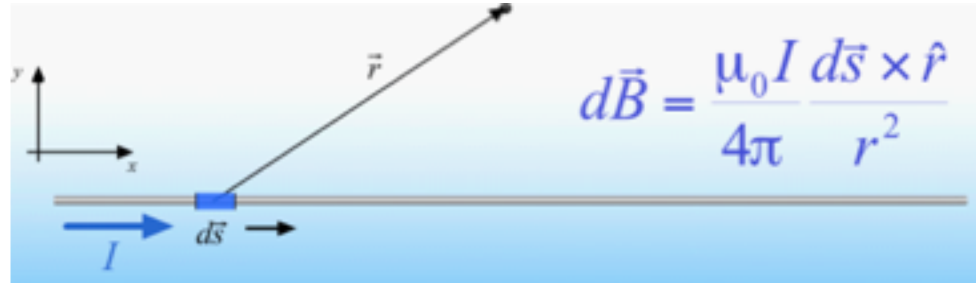


$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$

Biot-Savart Law:

What is it?

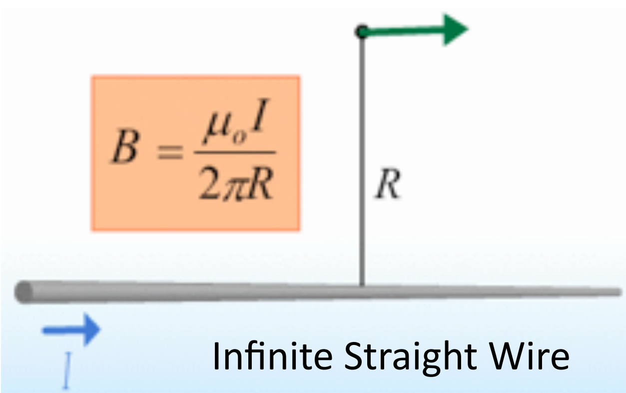
Fundamental law for determining the direction and magnitude of the magnetic field due to an element of current

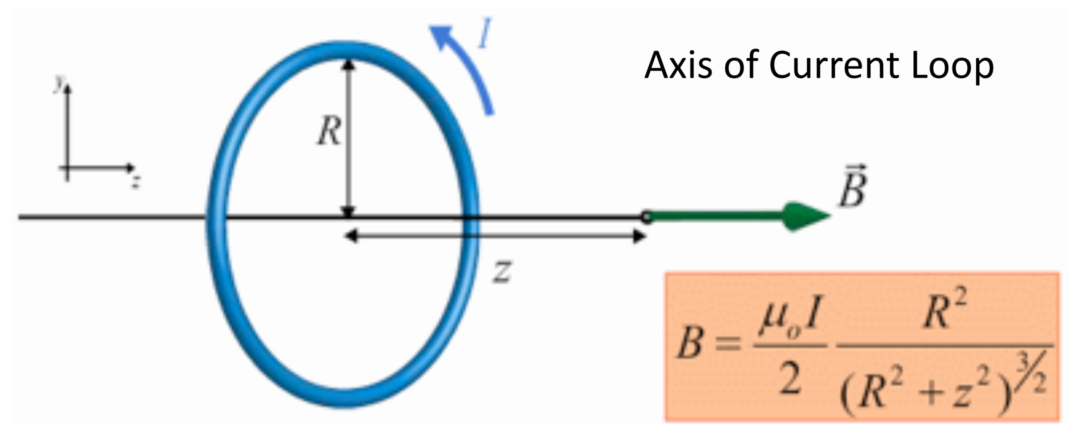


We can use this law to calculate the magnetic field produced by ANY current distribution

BUT

Easy analytic calculations are possible only for a few distributions:





Plan for Today: Mainly use the results of these calculations!

GOOD NEWS: Remember Gauss' Law? Allowed us to calculate E for symmetrical charge distributions

NEXT TIME: Introduce Ampere's Law Allows us to calculate B for symmetrical current distributions

B from Infinite Line of Current

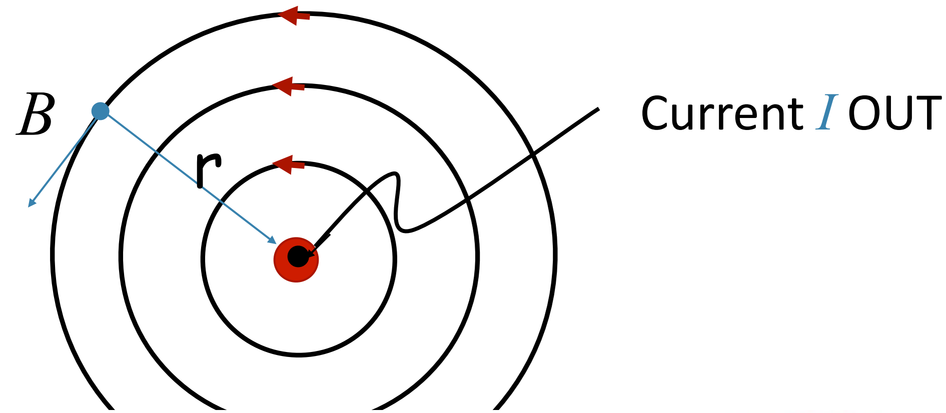
Integrating $d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$ gives result

Magnitude:

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\mu_0 = 4\pi \times 10^{-7}\, Tm/A$$

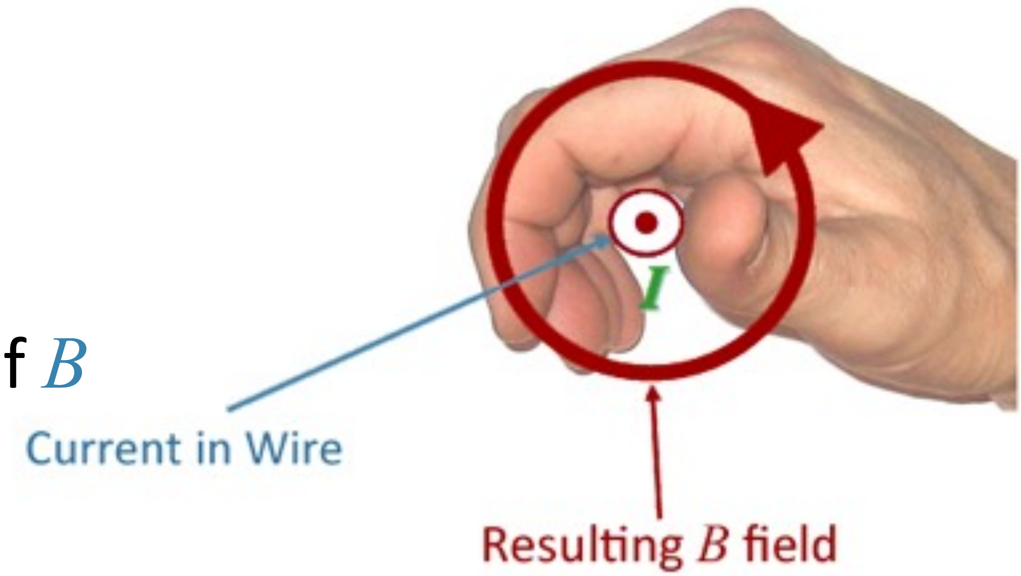
$r =$ distance from wire



Direction:

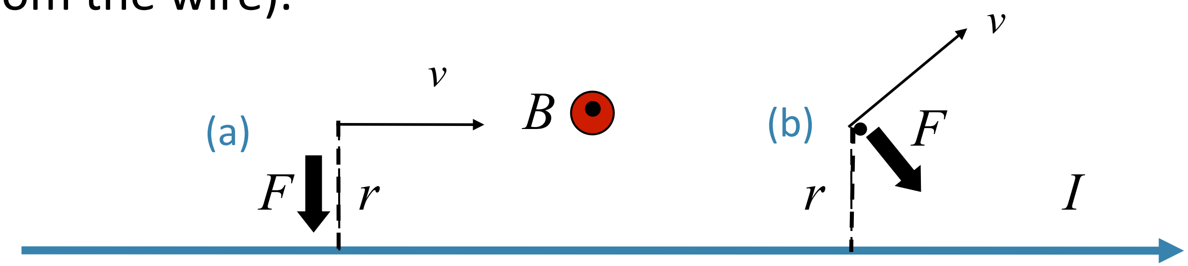
Thumb: on I

Fingers: curl in direction of B



Currents + Charges

A long straight wire is carrying current from left to right. Two identical charges are moving with equal speed. Compare the magnitude of the force on charge a moving directly to the right, to the magnitude of the force on charge b moving up and to the right at the instant shown (i.e. same distance from the wire).



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\left|\vec{F}\right| = qvB\sin\theta$$

A) $|F_a| > |F_b|$

B) $|F_a| = |F_b|$

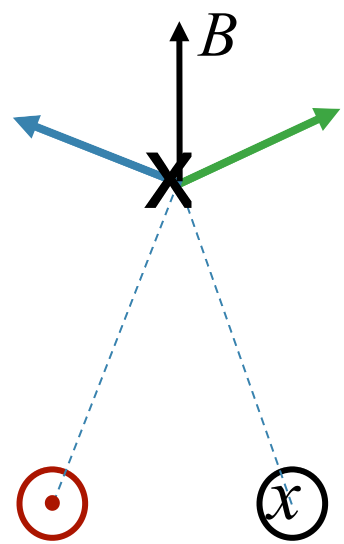
C) $|F_a| < |F_b|$

Same q, $|v|$, B and θ $(=90)$

Forces are in different directions

Adding Magnetic Fields

Two long wires carry opposite current

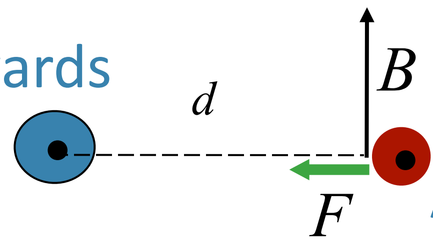


What is the direction of the magnetic field above, and midway between the two wires carrying current – at the point marked "X"?

A) Left B) Right C) Up D) Down E) Zero

Force Between Current-Carrying Wires

I towards us

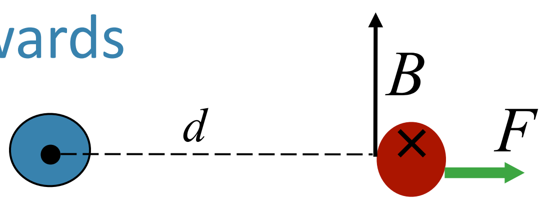


Another I towards us

$$\vec{F}_{12} = I_2\vec{L} \times \vec{B} \;\Rightarrow\; F_{12} = I_2 L \times \frac{\mu_o}{2\pi d} I_1$$

Conclusion: Currents in same direction attract!

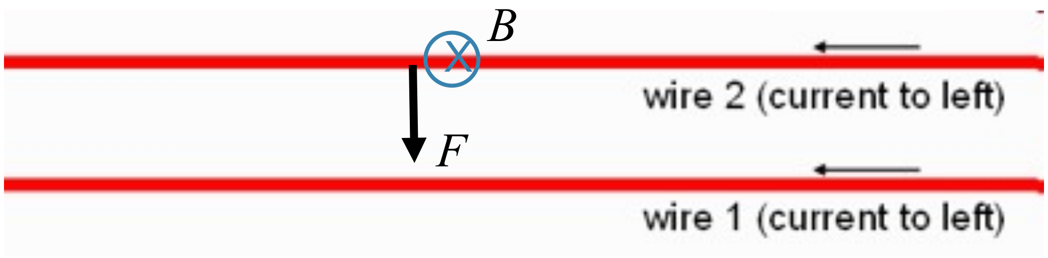
I towards us

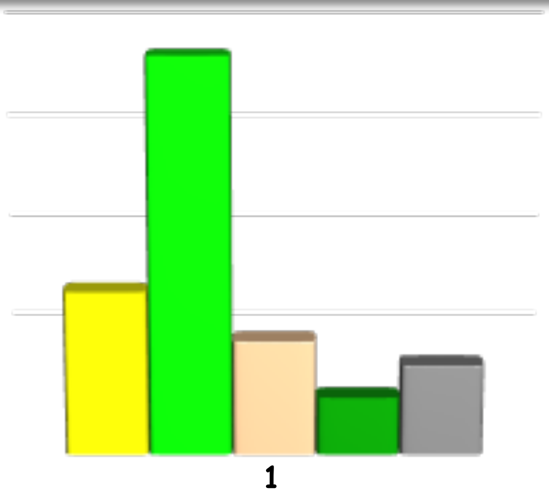


Another I away from us

Conclusion: Currents in opposite direction repel!

CheckPoint 2 & 4



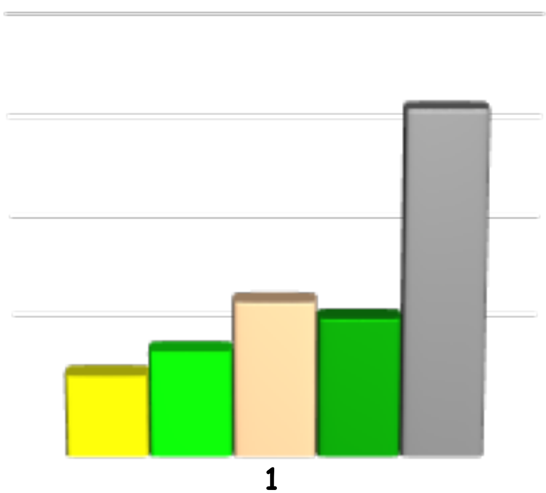


What is the direction of the force on wire 2 due to wire 1?

A) Up B) Down C) Into Screen D) Out of screen E) Zero

What is the direction of the torque on wire 2 due to wire 1?

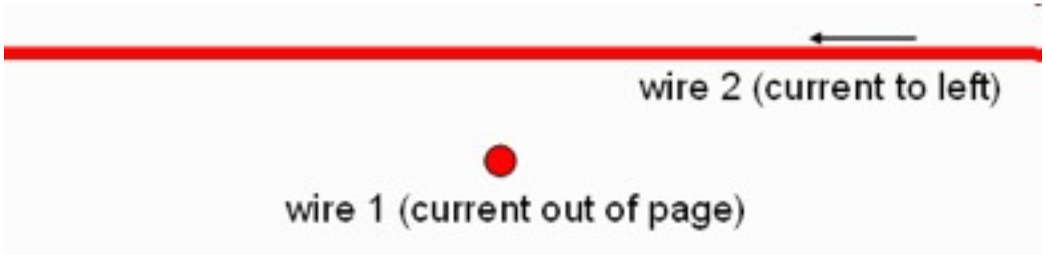
A) Up B) Down C) Into Screen D) Out of screen E) Zero

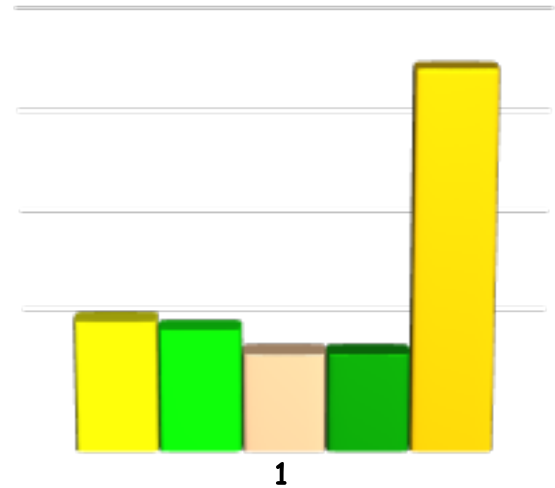


Uniform force at every segment of wire

No torque about any axis

CheckPoint 7



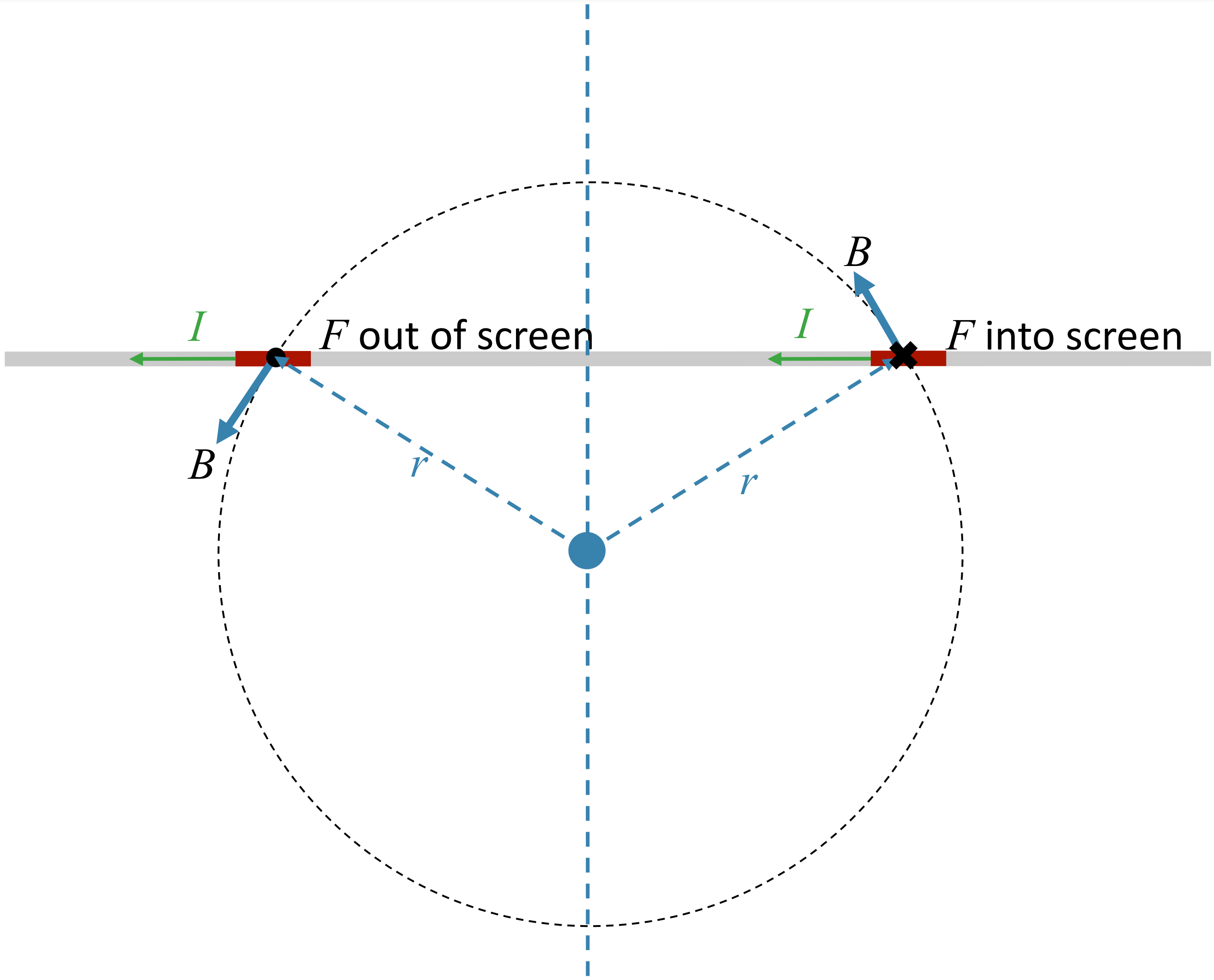


What is the direction of the force on wire 2 due to wire 1?

A) Up B) Down C) Into Screen D) Out of screen E) Zero

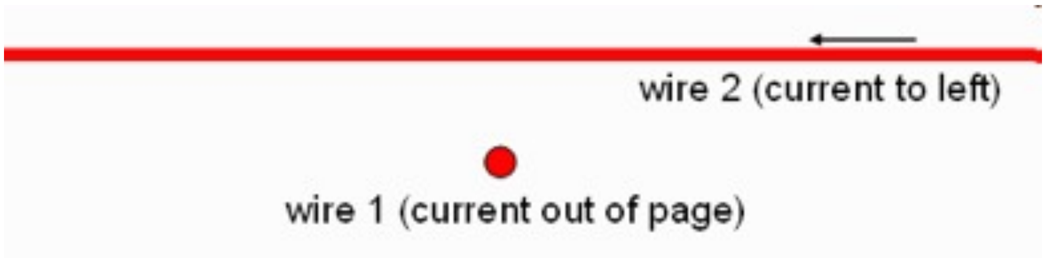
WHY?
DRAW PICTURE!

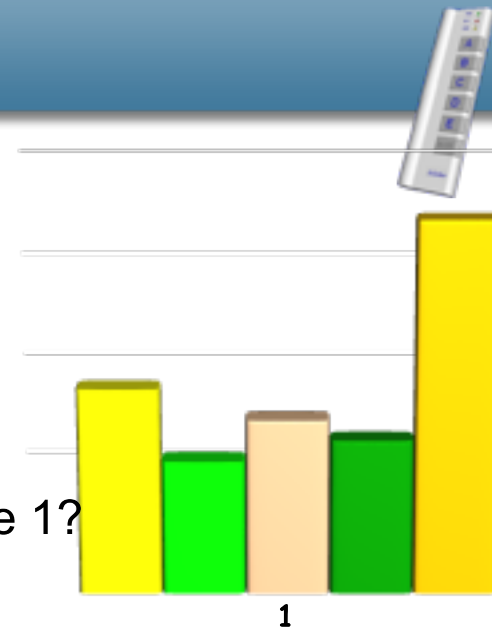
Consider Force on Symmetric Segments



Net Force is Zero!

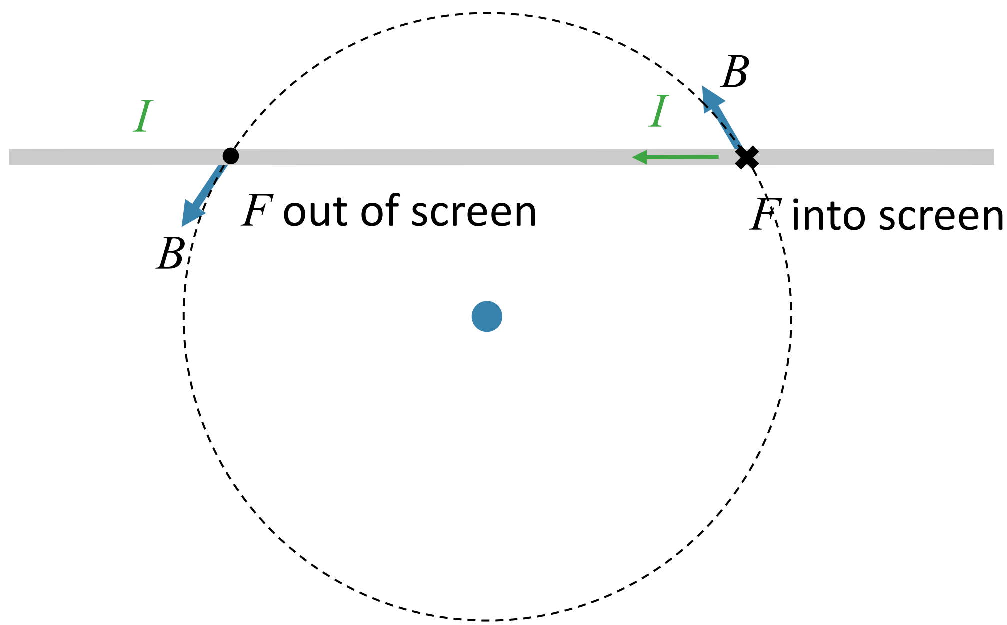
CheckPoint 9





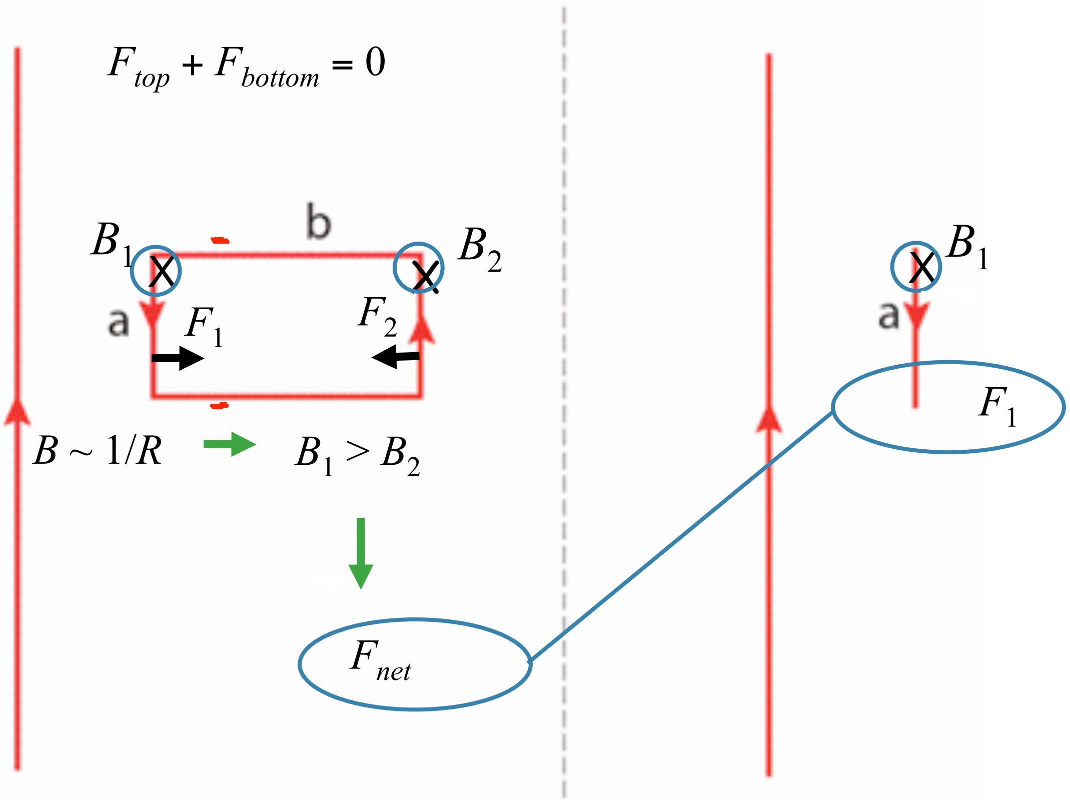
In the above situation, what is the net torque on wire 2 due to wire 1?

A) Up   B) Down   C) Into Screen   D) Out of screen   E) Zero

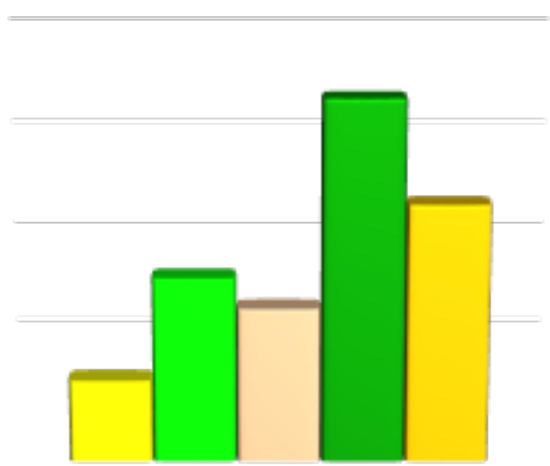


CheckPoint 6

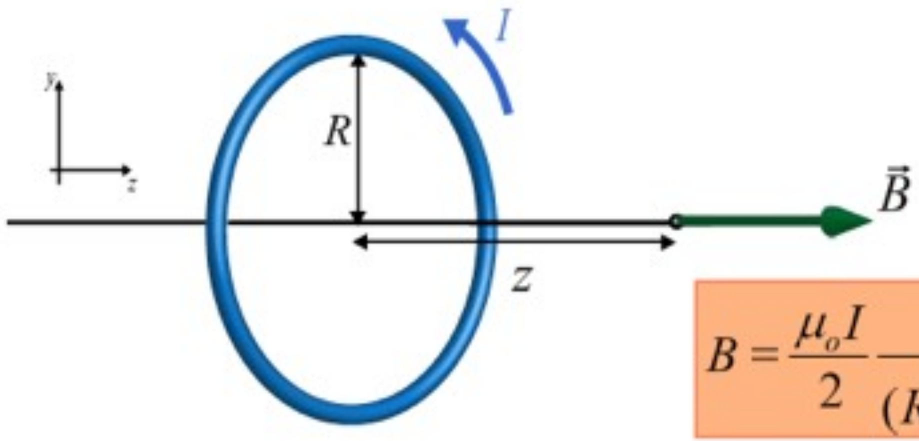
6) A current carrying loop of width a and length b is placed near a current carrying wire. How does the net force on the loop compare to the net force on a single wire segment of length a carrying the same amount of current placed at the same distance from the wire?



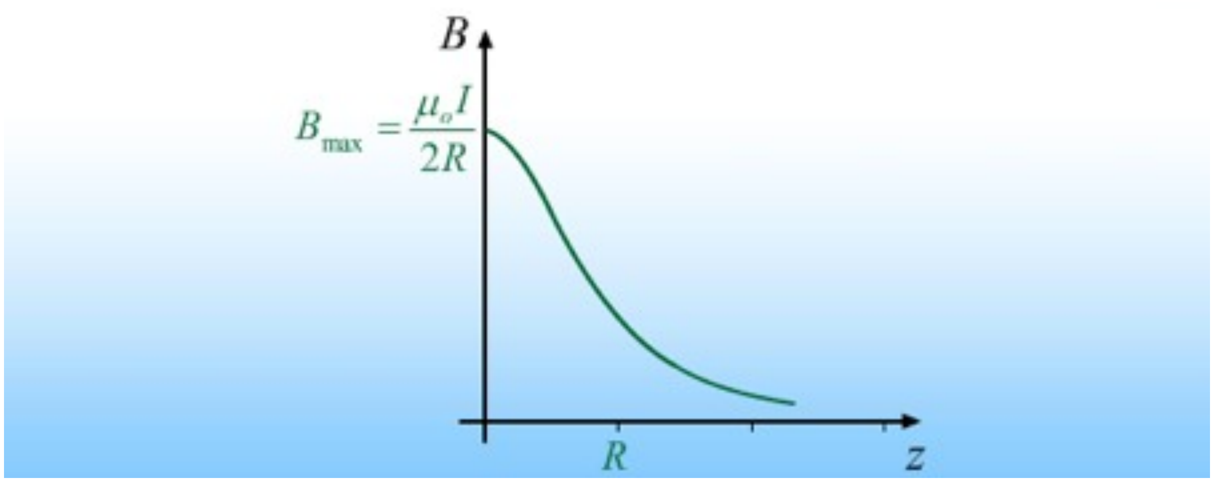
A ○ the forces are in opposite directions
B ○ the net forces are the same.
C ○ the net force on the loop is greater than the net force on the wire segment
D ○ the net force on the loop is smaller than the net force on the wire segment
E ○ there is no net force on the loop

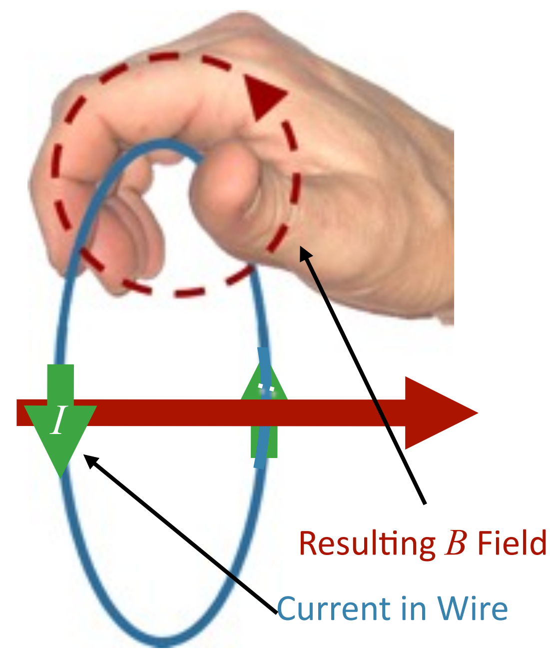


B on axis from Current Loop

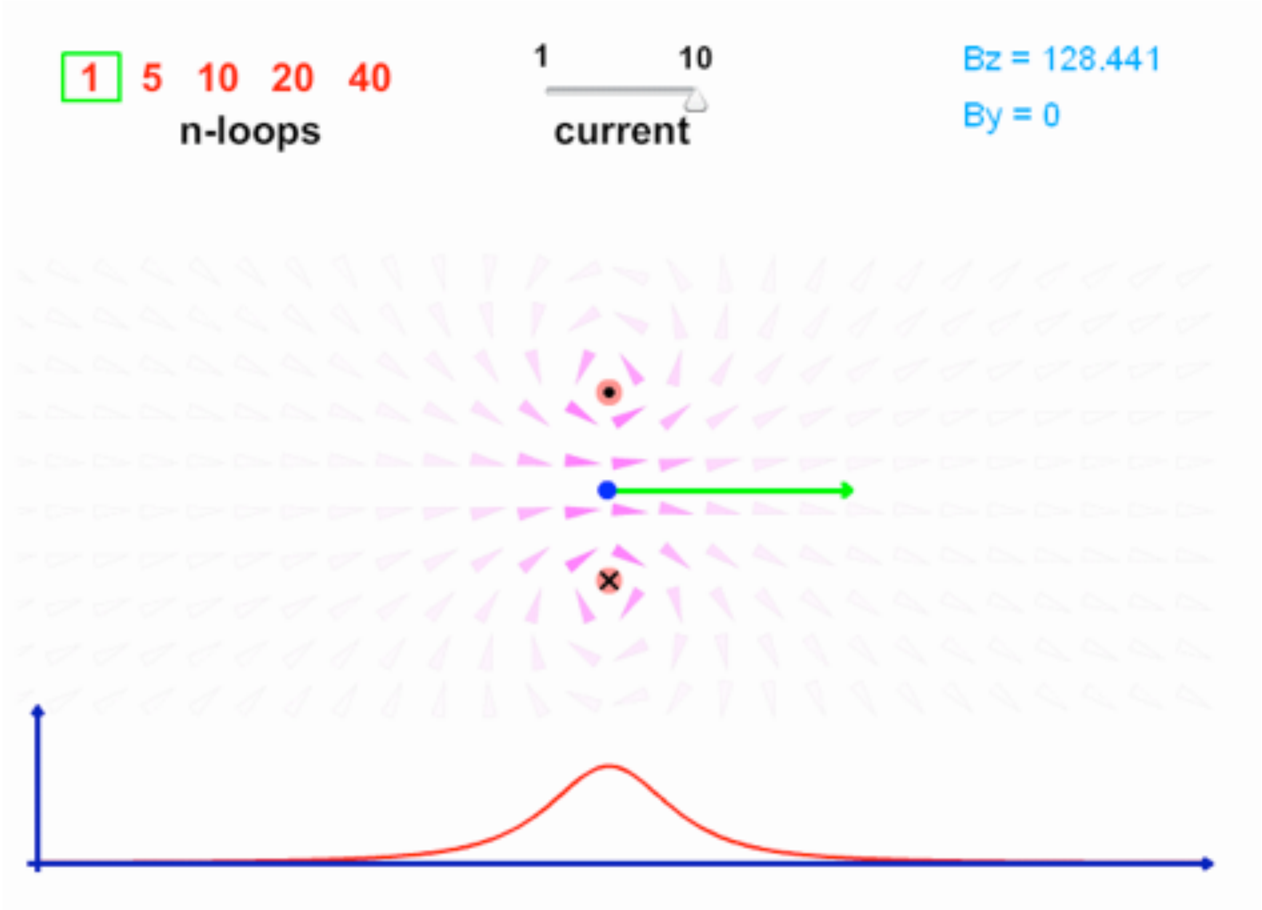


$$B = \frac{\mu_o I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$$





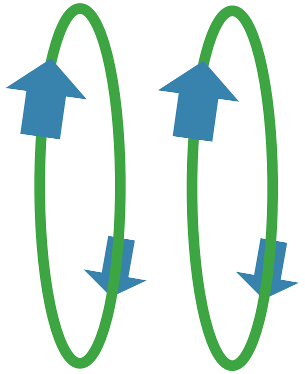
What about Off-Axis ?



See Simulation!

Two Current Loops

Two identical loops are hung next to each other. Current flows in the same direction in both.



The loops will:

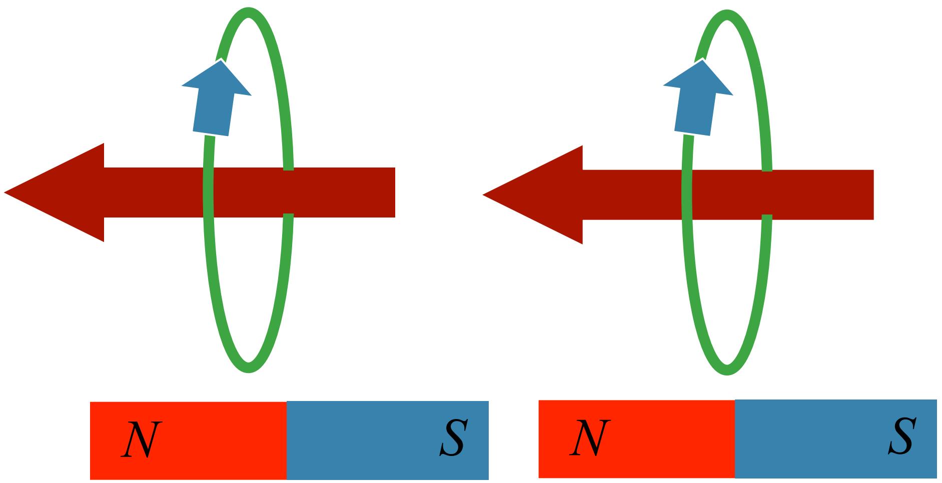
A) Attract each other

B) Repel each other

Two ways to see this:

1) Like currents attract

2) Look like bar magnets



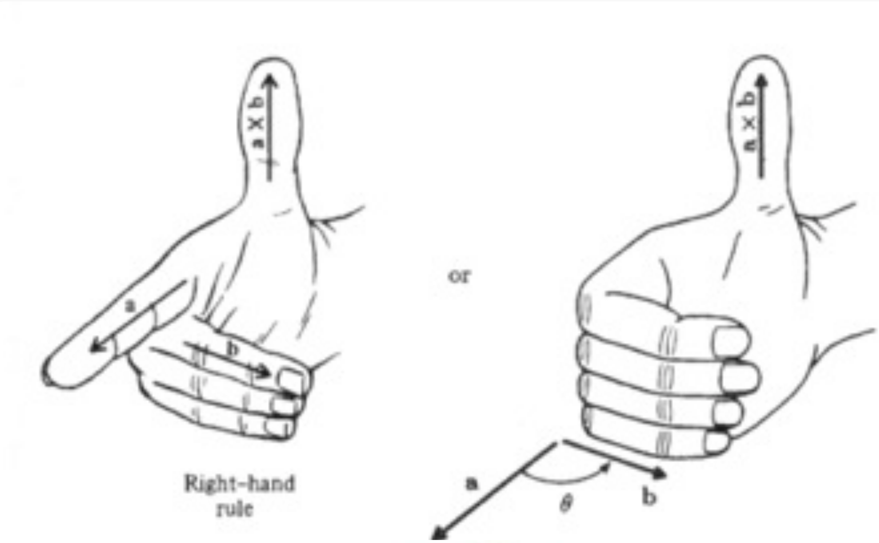
Right Hand Rule Review

1. ANY CROSS PRODUCT

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = I\vec{L} \times \vec{B}$$

$$\tau = \vec{r} \times \vec{F} \qquad \tau = \vec{\mu} \times \vec{B}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$



2. Direction of Magnetic Moment

Fingers: Current in Loop

Thumb: Magnetic Moment

3. Direction of Magnetic Field from Wire

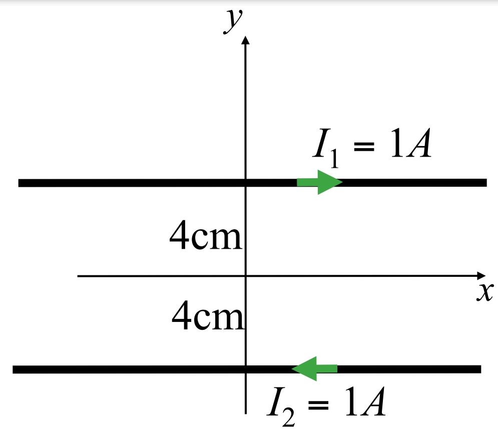
Fingers: Magnetic Field

Thumb: Current

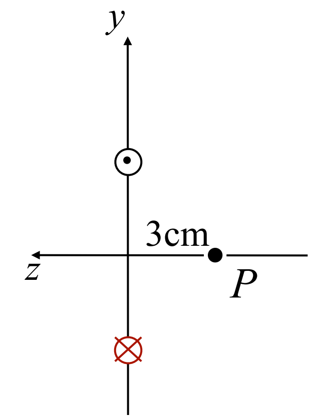
Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?



Front view



Side view

Conceptual Analysis

Each wire creates a magnetic field at P

B from infinite wire: $B = m_0 I / 2pr$

Total magnetic field at P obtained from superposition

Strategic Analysis

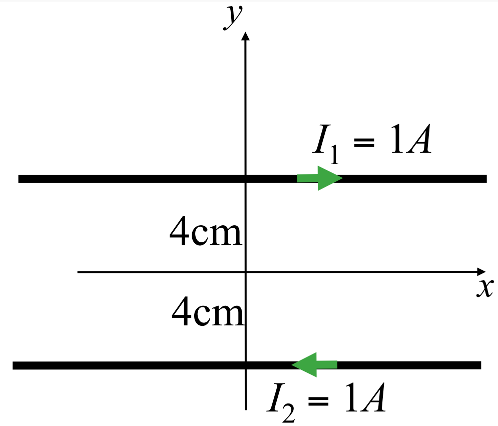
Calculate B at P from each wire separately

Total B = vector sum of individual B fields

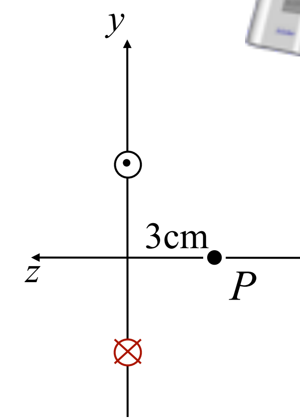
Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?

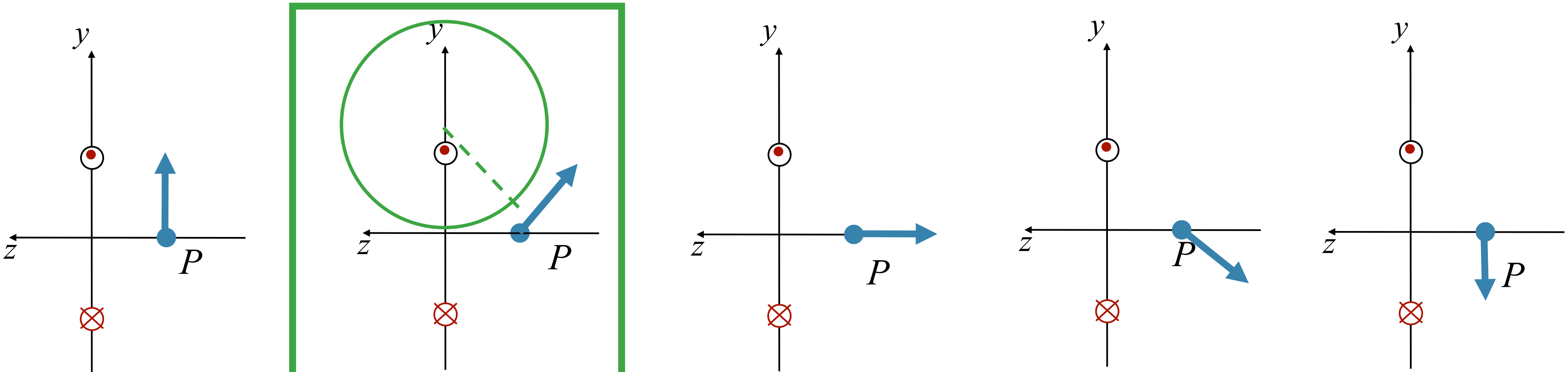


Front view



Side view

What is the direction of B at P produced by the top current I_1?

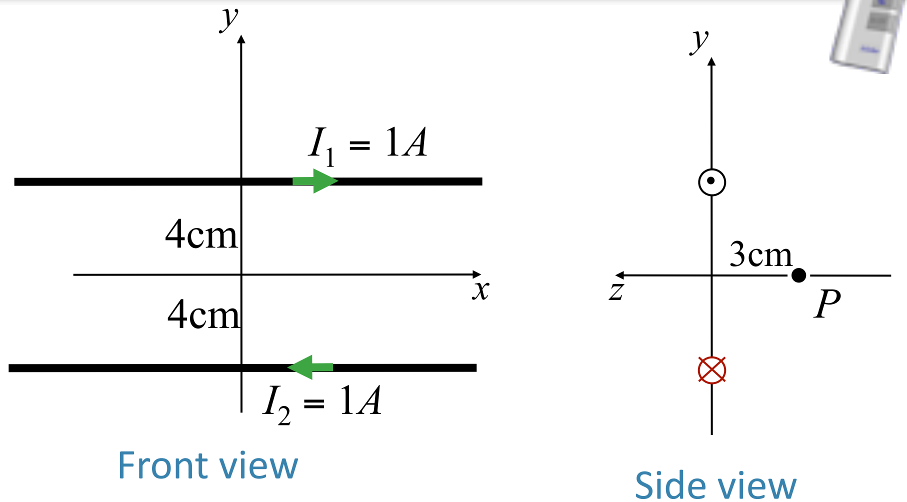


A B C D E

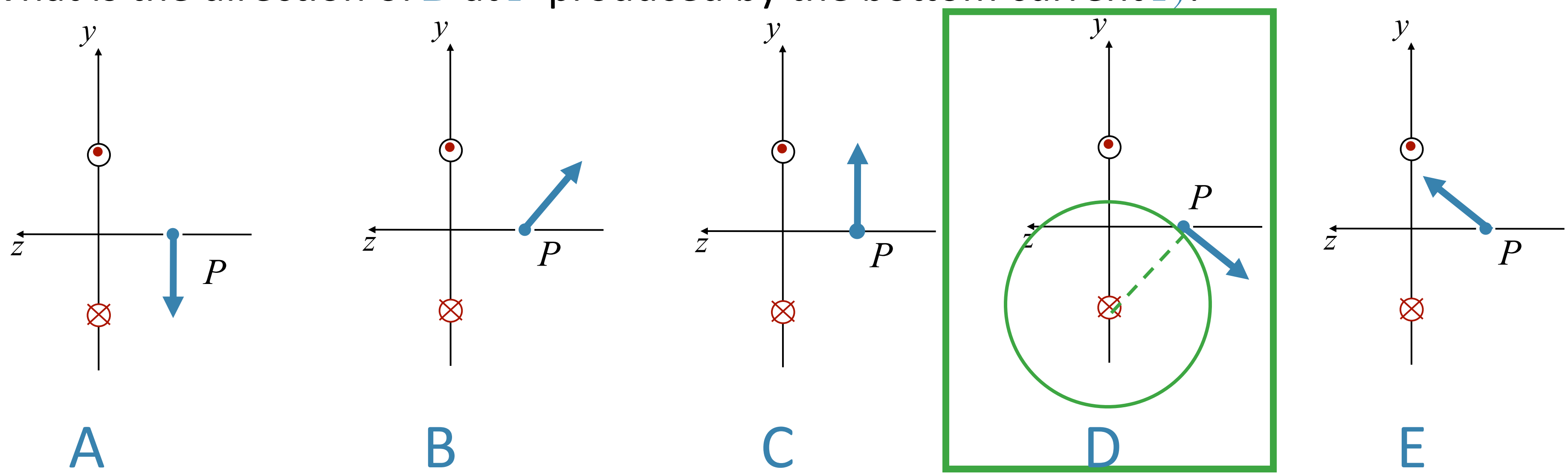
Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?



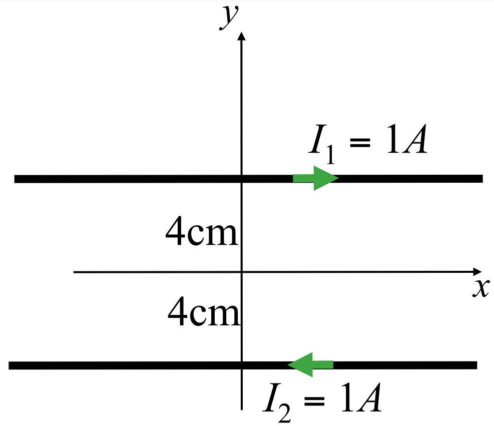
What is the direction of B at P produced by the bottom current I_2?



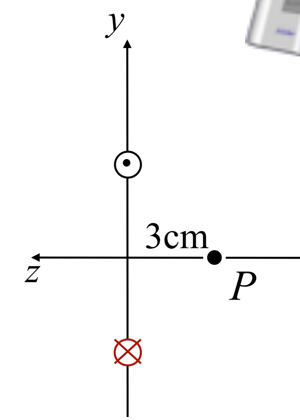
Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?

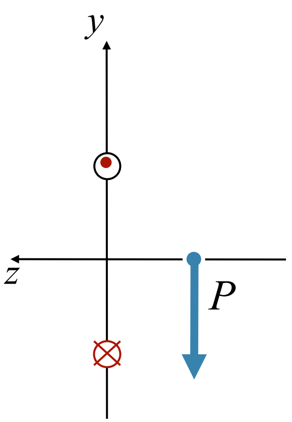


Front view

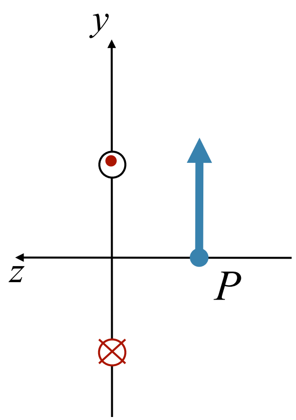


Side view

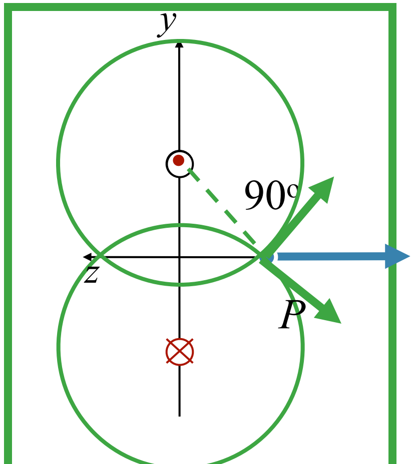
What is the direction of B at P?



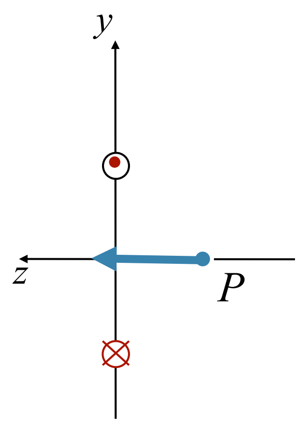
A



B



C



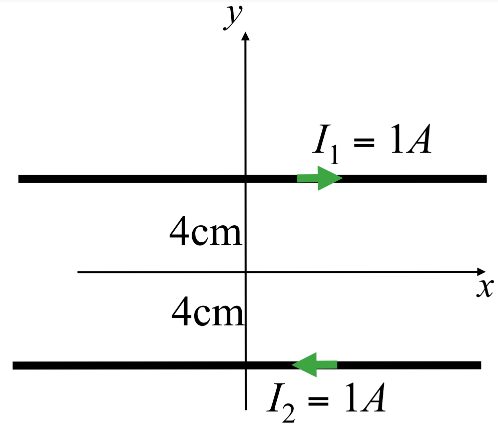
D

Calculation

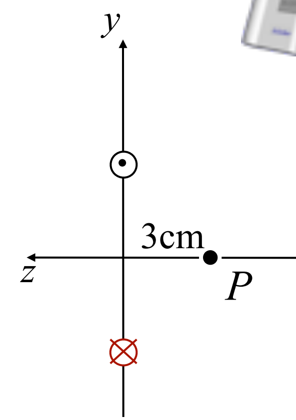
Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?

$$B = \frac{\mu_0 I}{2\pi r}$$



Front view



Side view

What is the magnitude of B at P produced by the top current I_1?

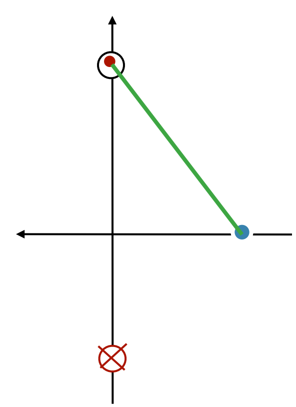
$(\mu_0 = 4\pi \times 10^{-7}\ T\text{–}m/A)$

A) $4.0 \times 10^{-6}\ T$ B) $5.0 \times 10^{-6}\ T$ C) $6.7 \times 10^{-6}\ T$

What is r?

r = distance from wire axis to P

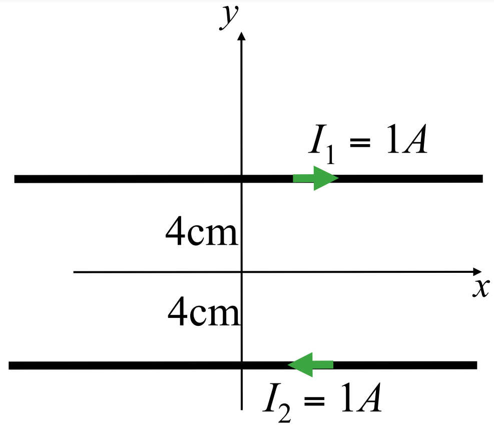
$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}) \times 1}{2\pi r} = 40 \times 10^{-7}$$



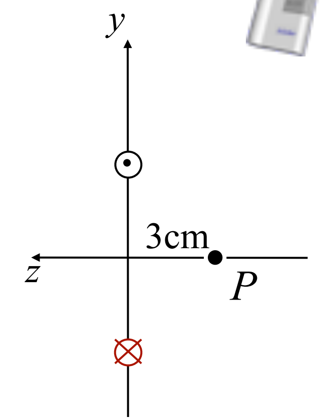
$$r = \sqrt{3^2 + 4^2} = 5cm$$

Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.



Front view

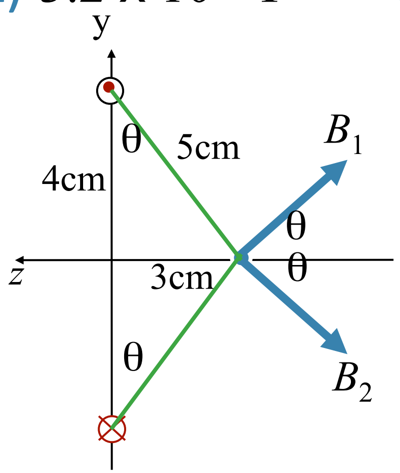


What is the B field at point P?

$B_{top} = 4 \times 10^{-6}\ T$

What is the magnitude of B at P? $(\mu_0 = 4\pi \times 10^{-7}\ T - m/A)$

A) $3.2 \times 10^{-6}\ T$ B) $4.8 \times 10^{-6}\ T$ C) $6.4 \times 10^{-6}\ T$ D) $8.0 \times 10^{-6}\ T$



$$B_{1x} = B_1 \cos\theta$$

$$B_{2x} = B_2 \cos\theta$$

$$B_x = 2B_1 \cos\theta = 2 \times 4 \times 10^{-6} \times \left(\frac{4}{5}\right) = 6.4 \times 10^{-6}$$