Electricity & Magnetism
Lecture 15

Today's Concept:

Ampere's Law

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enclosed}$$

Your Comments

"For infinite sheets when you break up the integral to its segments, I don't know how the integral was worked out to have gotten BL for each segment. When are you going to put up the review material for the 2nd exam on the website?

We will use a simulation to illustrate the contribution to integral B dot dl from different segments.

Very soon..

"Can we discuss Ampere's Law and actually solving problems using it? Thank you!

"B fields inside wires"

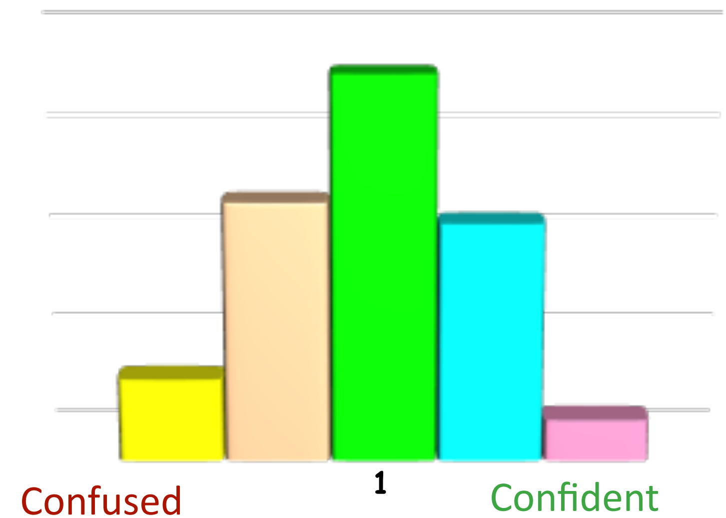
This will be our calculation today

"Let's talk about why our lecture needs to stop having side conversations during lecture. It's getting extremely irritating and distracting."

"People should really keep it down in lecture. It's getting really annoying."

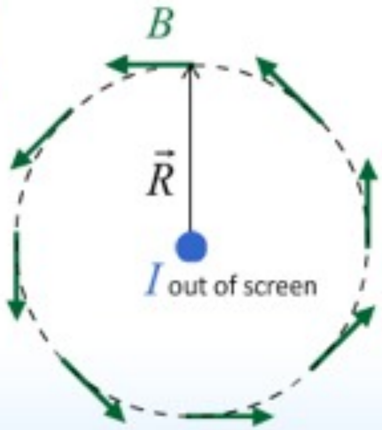
AGREED!

Talking is encouraged during clicker questions, but NOT OTHERWISE



Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I$$



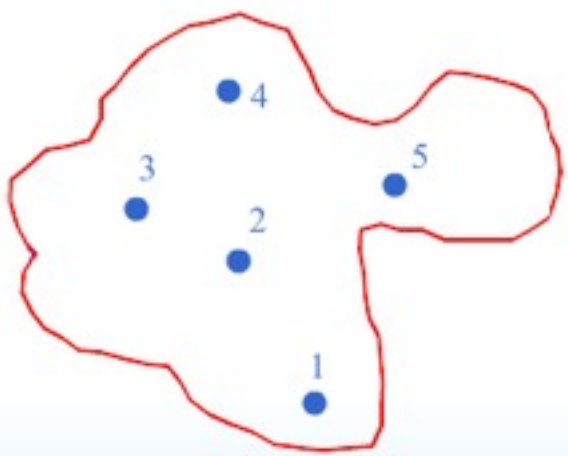
Infinite current-carrying wire

LHS: $\oint \vec{B} \bullet d\vec{\ell} = \oint B d\ell = B \oint d\ell = B \times 2\pi R$

RHS: $I_{enclosed} = I$

$$\longrightarrow \quad B = \frac{\mu_o I}{2\pi R}$$

General Case

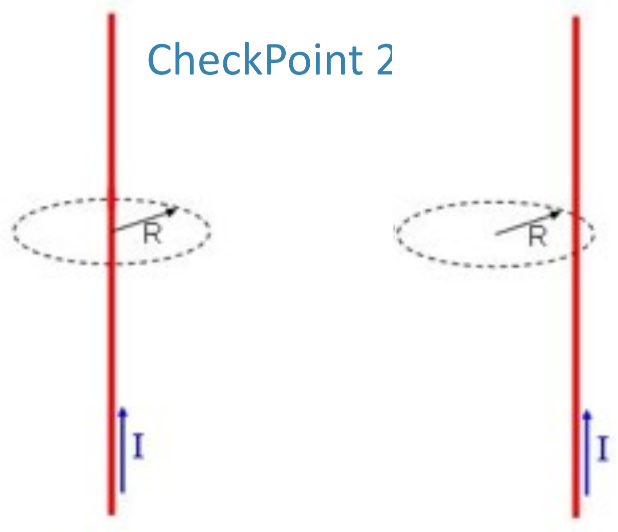


Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$

Practice on Enclosed Currents

CheckPoint 2

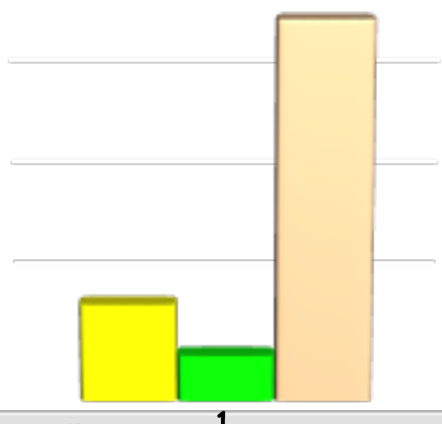


Case 1: $I_{enclosed} = I$

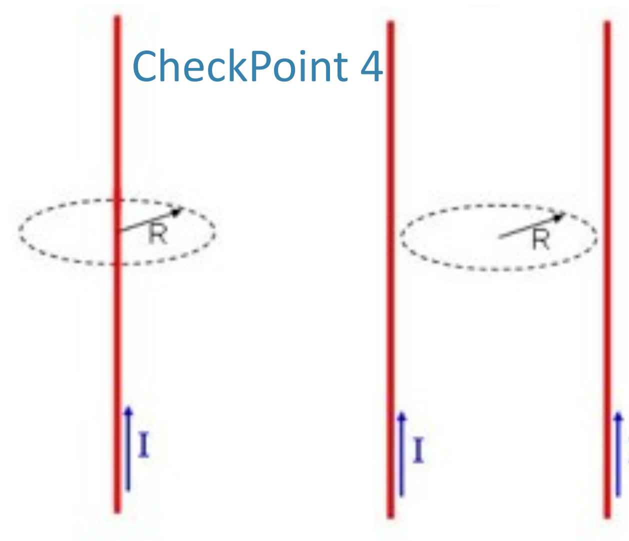
Case 2: $I_{enclosed} = I$

For which loop is §B·dl the greatest?

○Case 1 ○Case 2 ○the same



CheckPoint 4

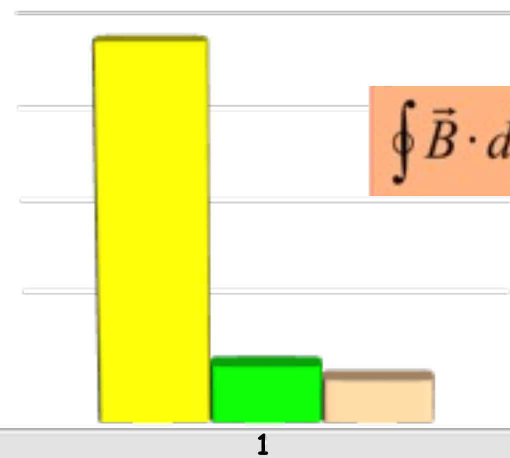


Case 1: $I_{enclosed} = I$

Case 2: $I_{enclosed} = 0$

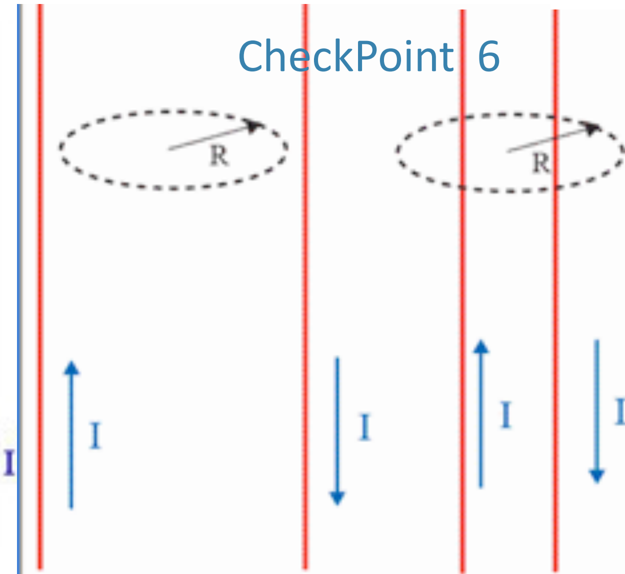
For which loop is §B·dl the greatest?

○Case 1 ○Case 2 ○the same



$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$

CheckPoint 6



Case 1: $I_{enclosed} = 0$

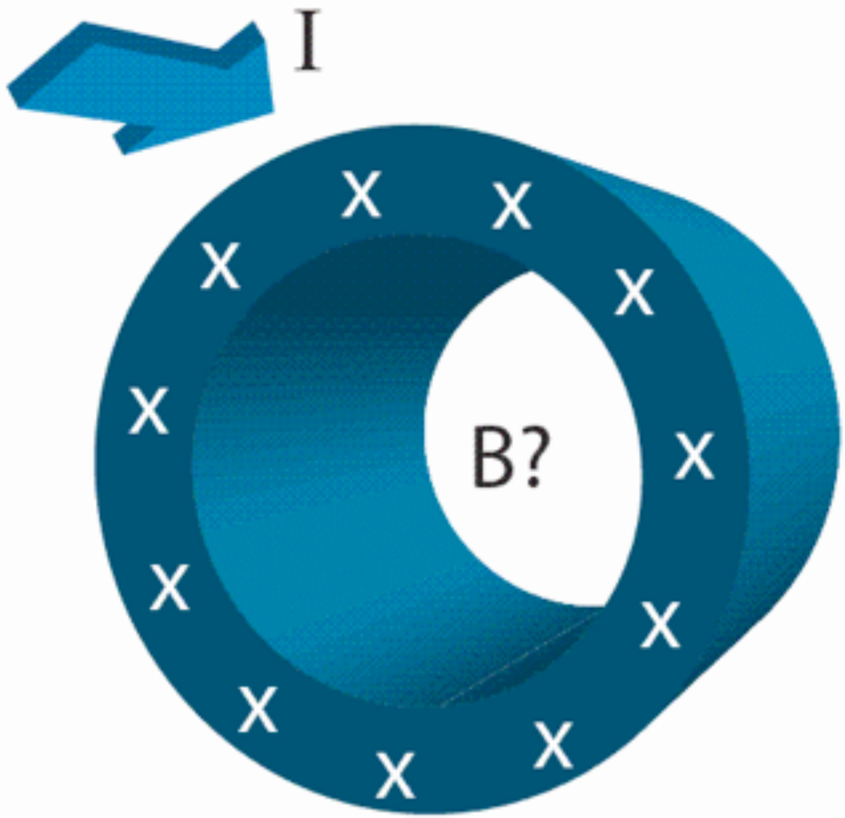
Case 2: $I_{enclosed} = 0$

For which loop is /;§B·dl the biggest?

○Case 1 ○Case 2 ○the same

CheckPoint 8

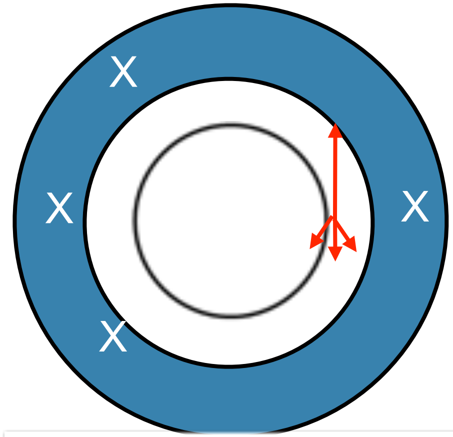
8) An infinitely long hollow conducting tube carries current I in the direction shown.



What is the direction of the magnetic field inside the tube?

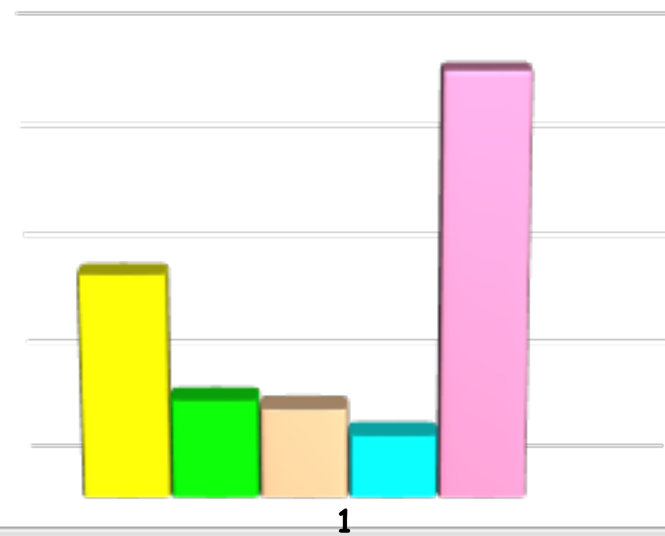
- clockwise
- counterclockwise
- radially inward to the center
- radially outward from the center
- the magnetic field is zero

Cylindrical Symmetry

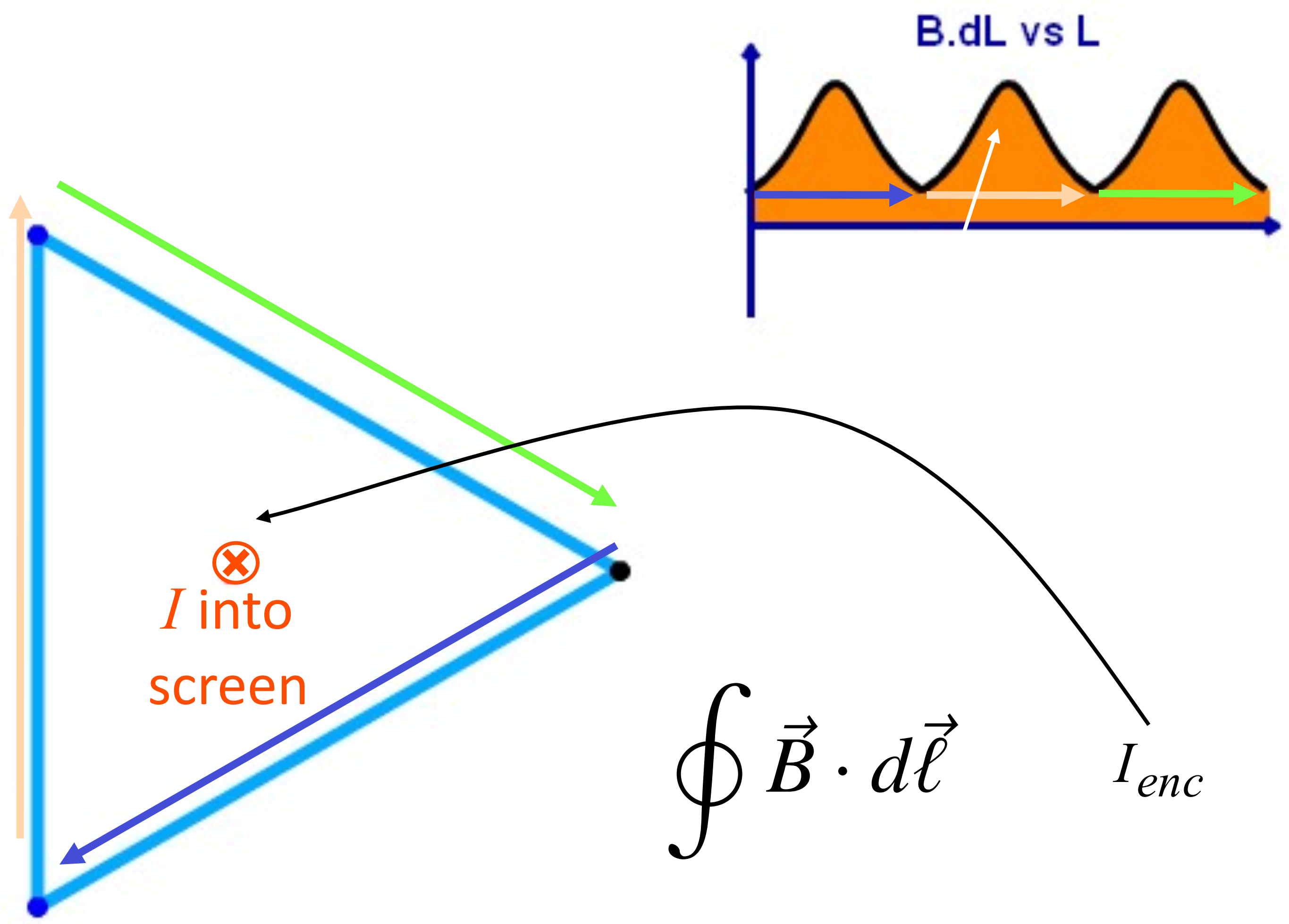


Enclosed Current = 0

Check cancellations



Ampere's Law

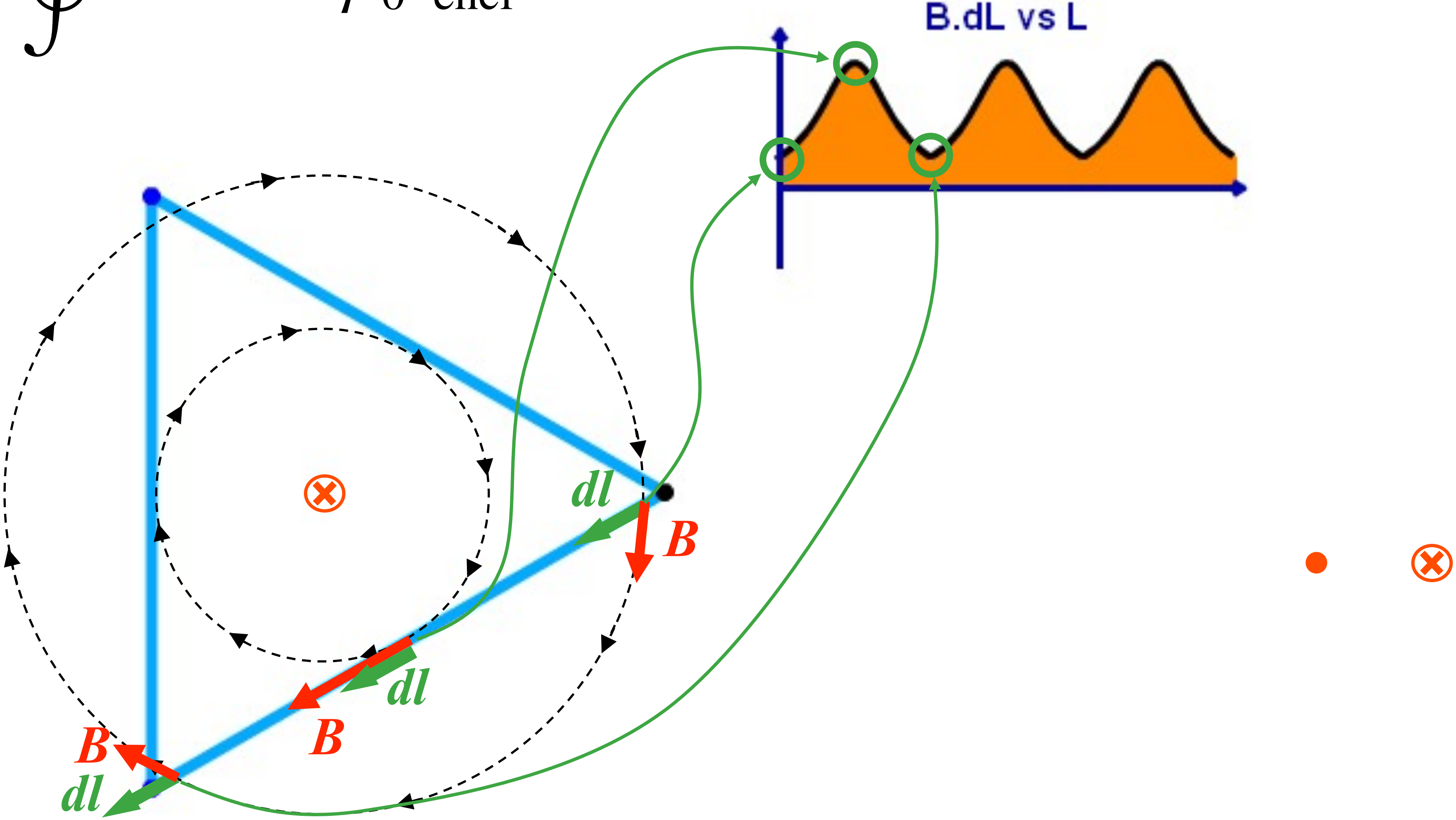


$$\oint \vec{B} \cdot d\vec{\ell}$$

I_{enc}

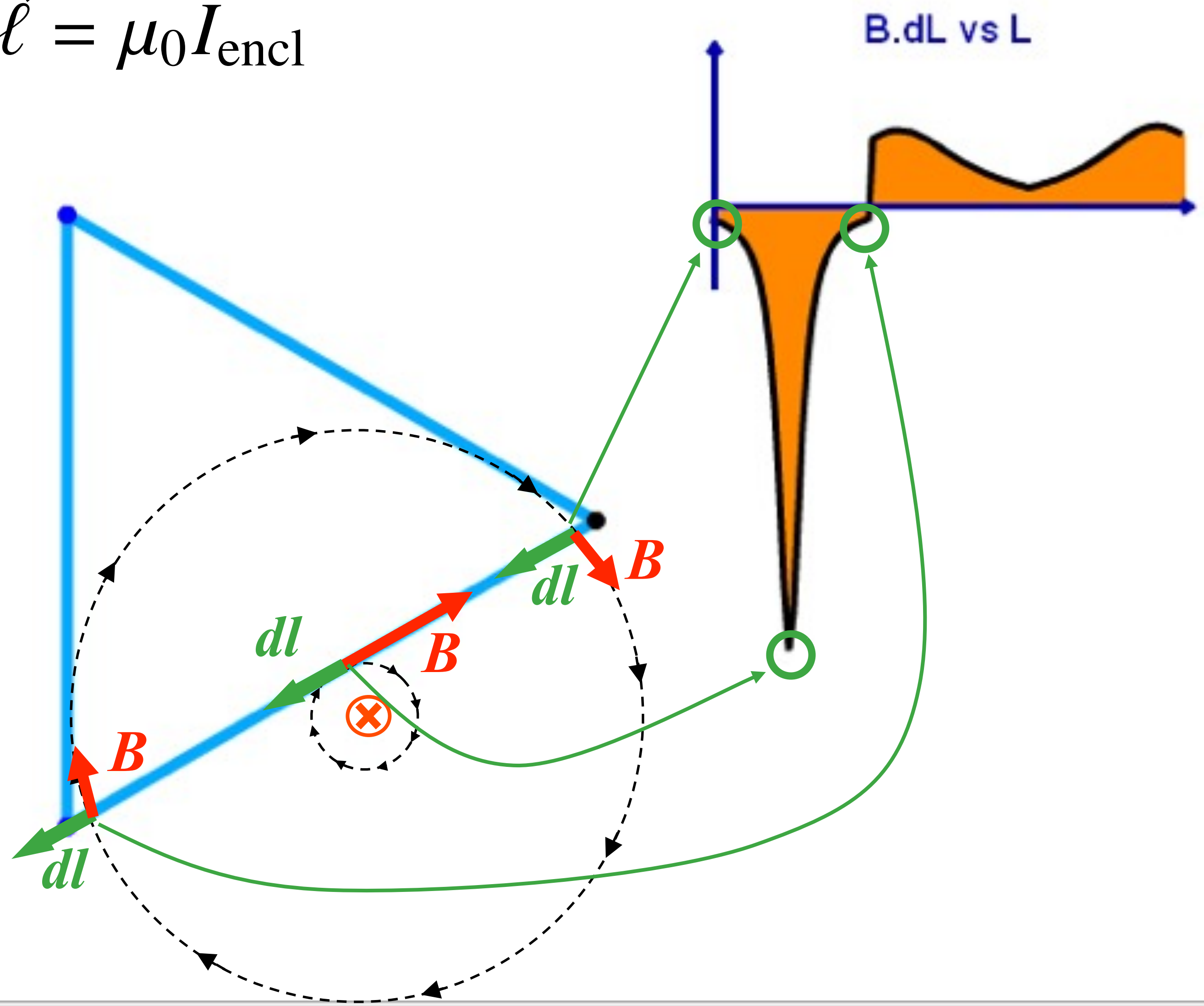
Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$$



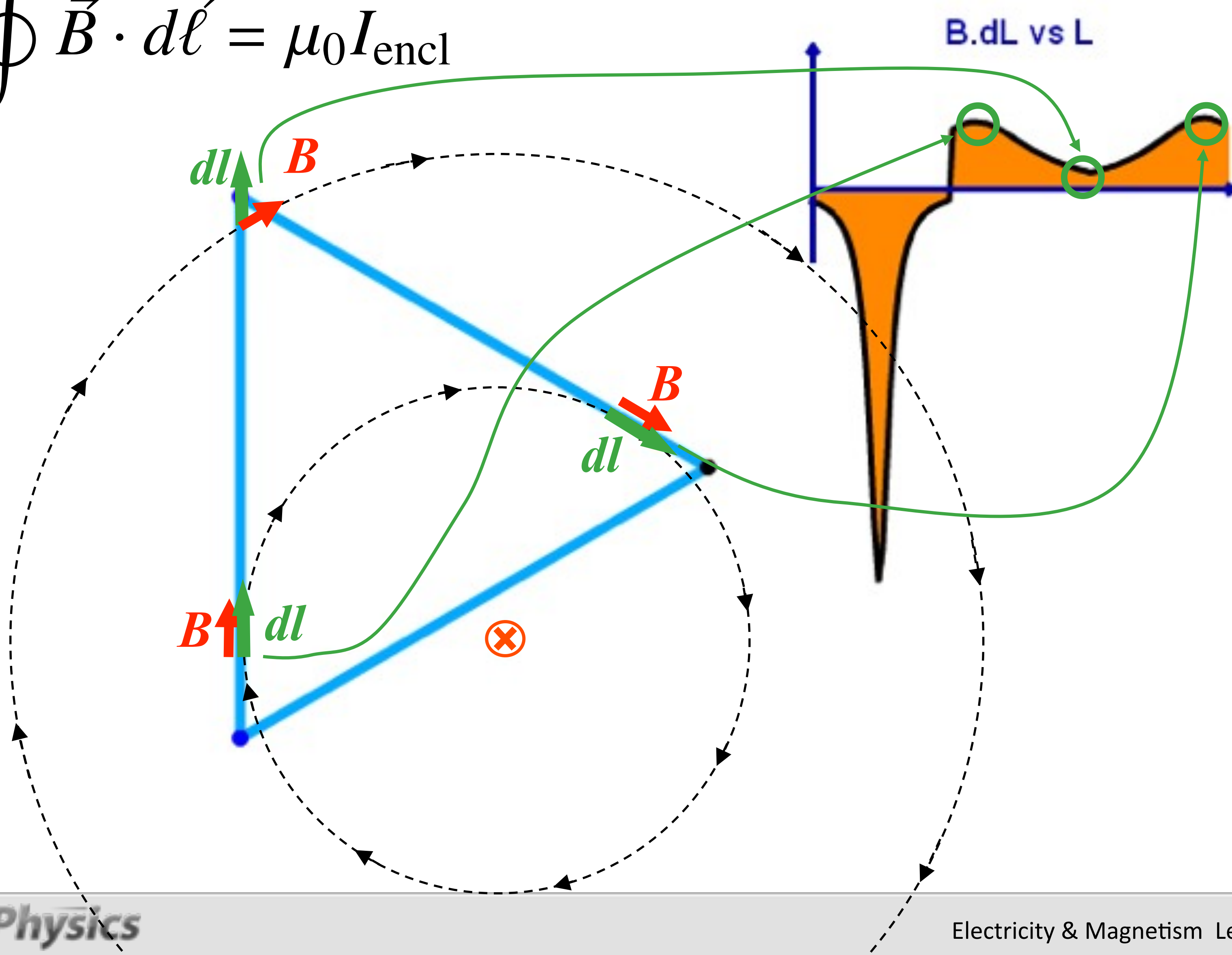
Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$$

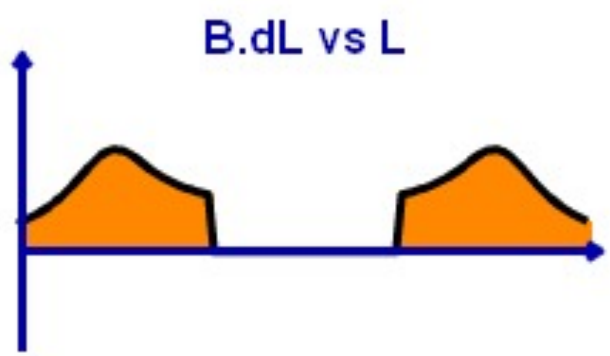


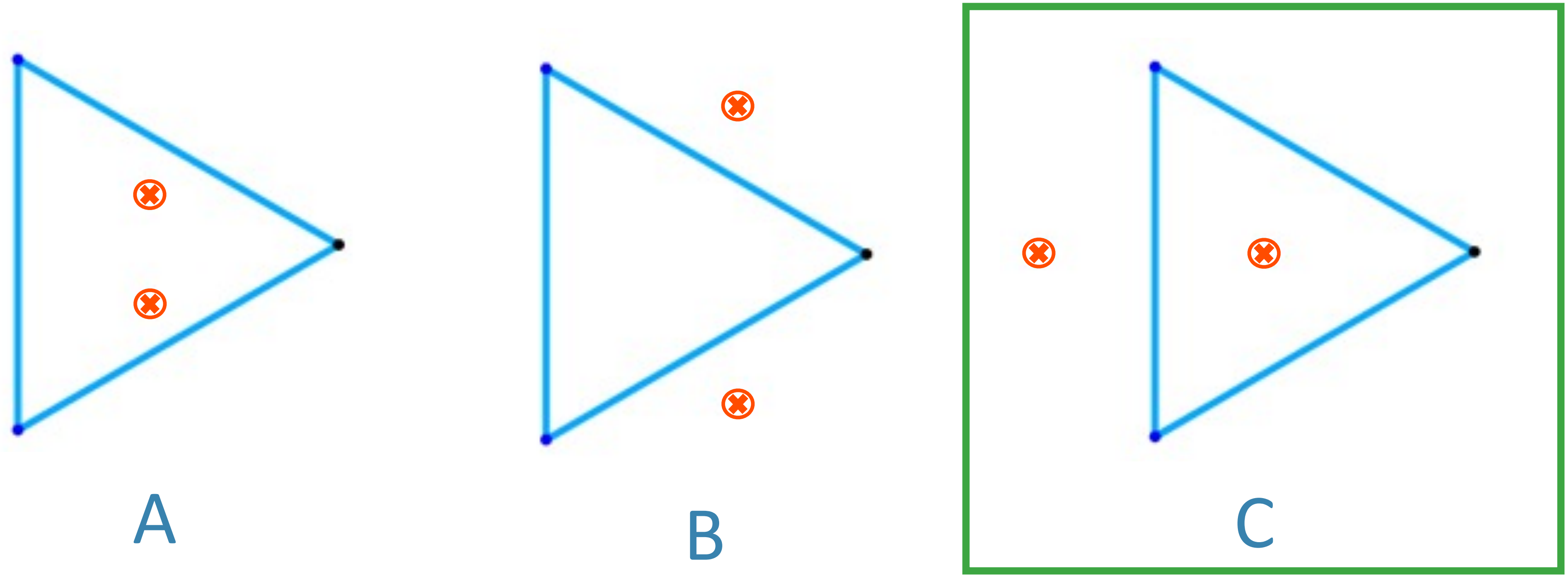
Ampere's Law

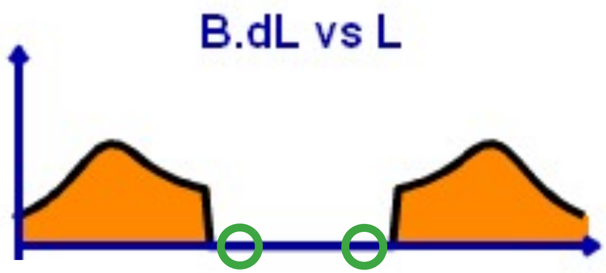
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$$

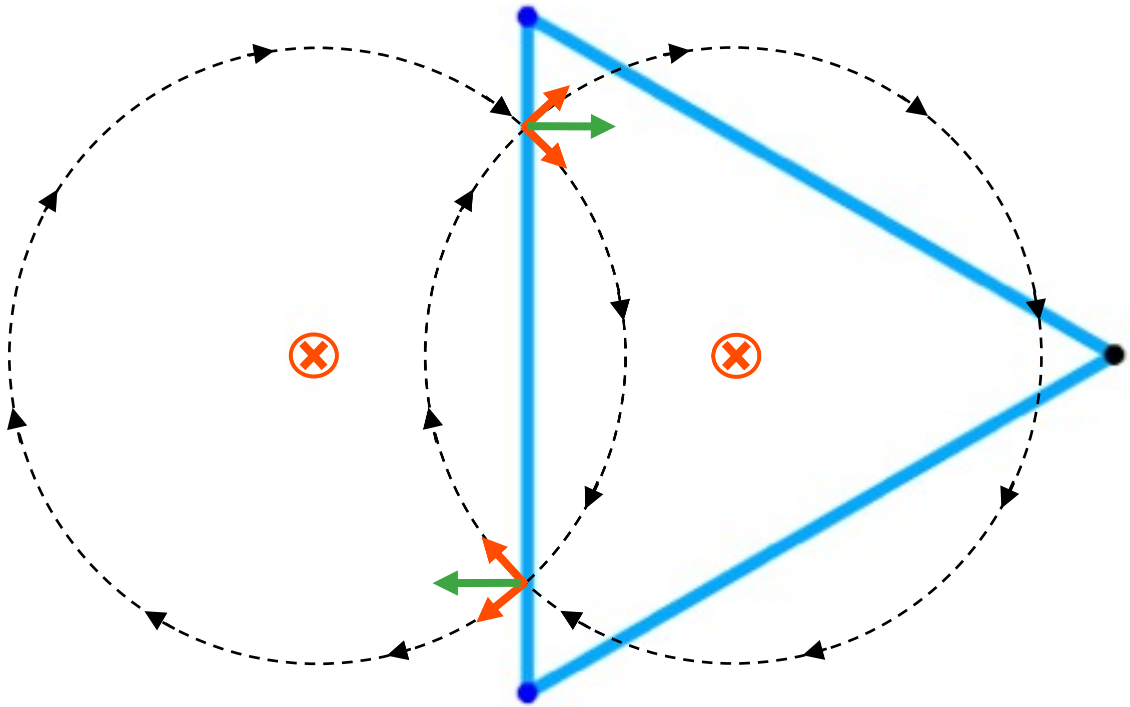


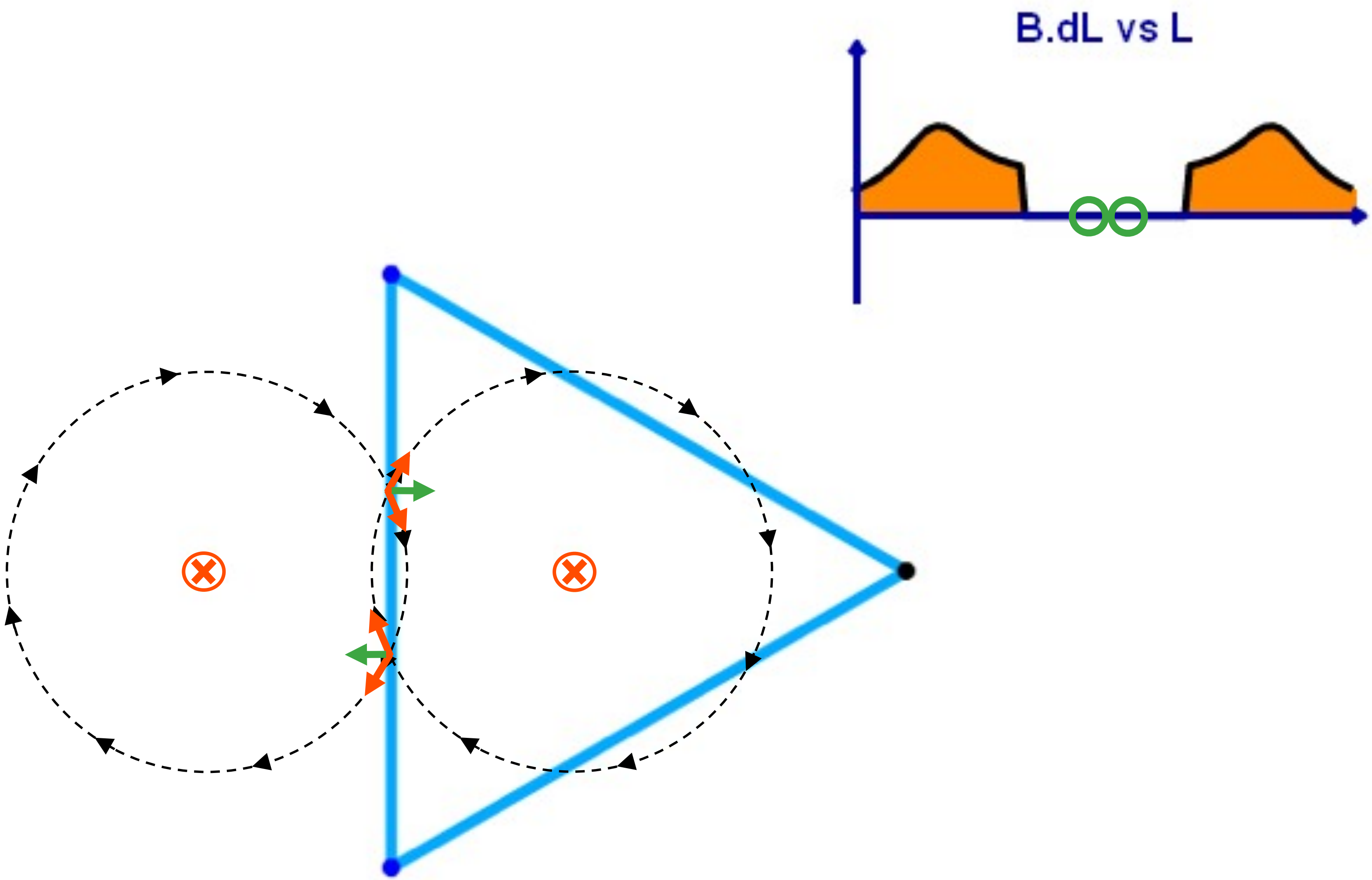
Which of the following current distributions would give rise to the $B \cdot dL$ distribution at the right?

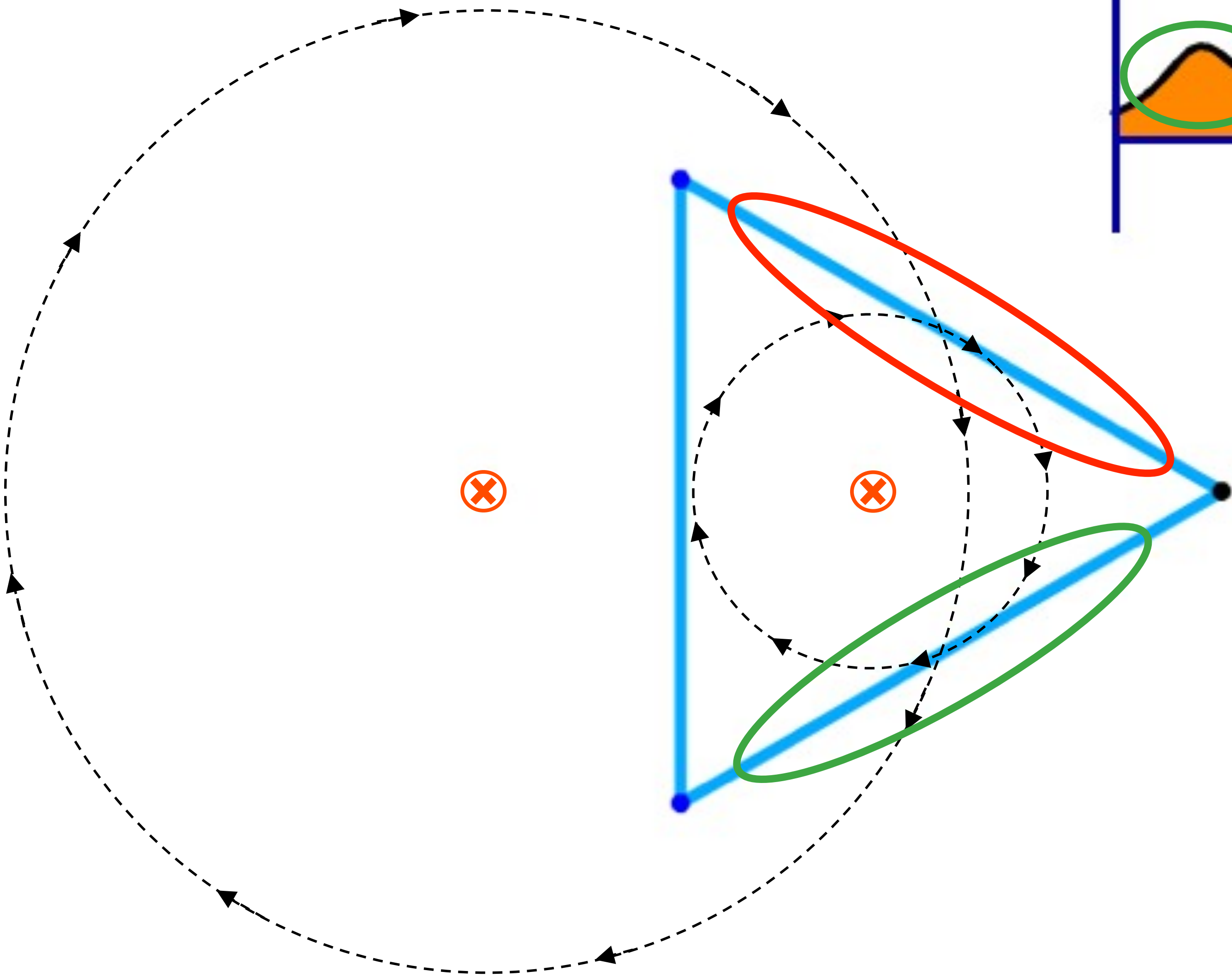


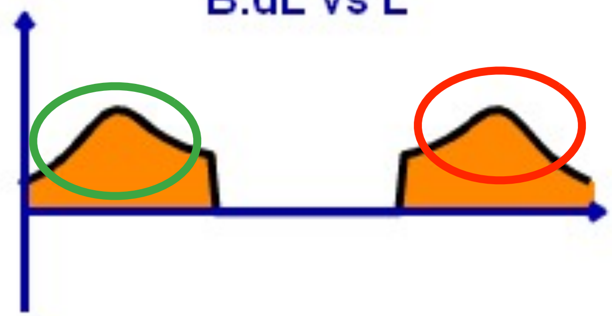



B.dL vs L

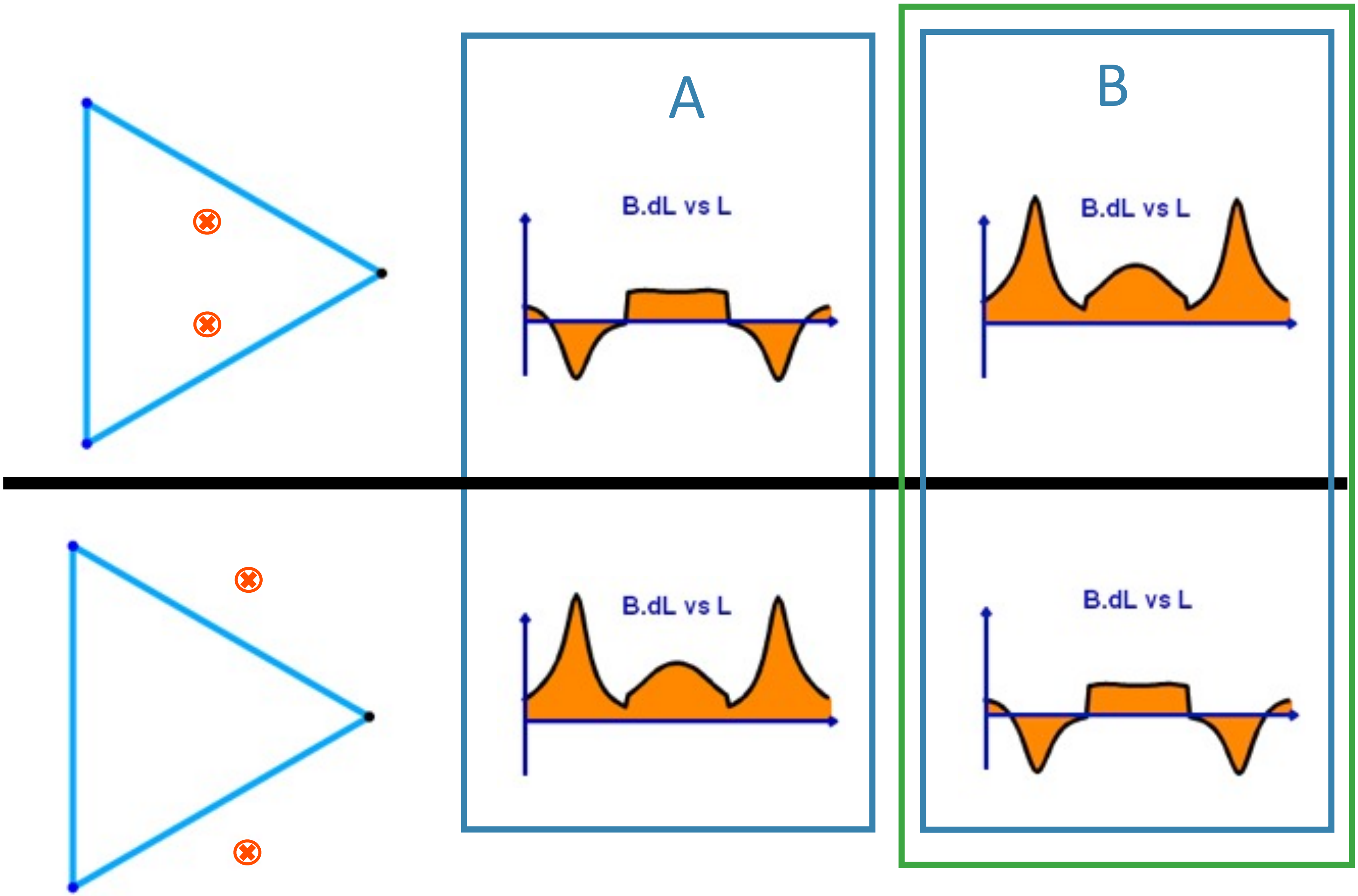



B.dL vs L



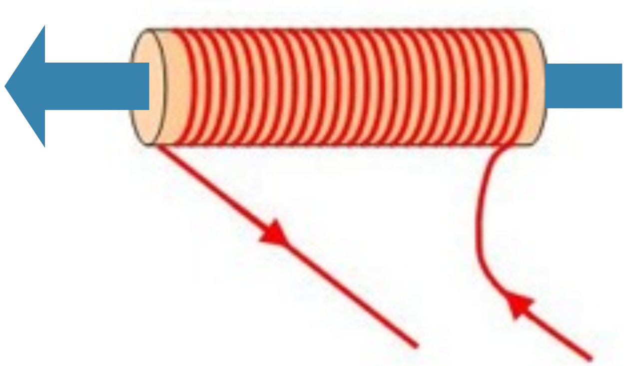

B.dL vs L

Match the other two:



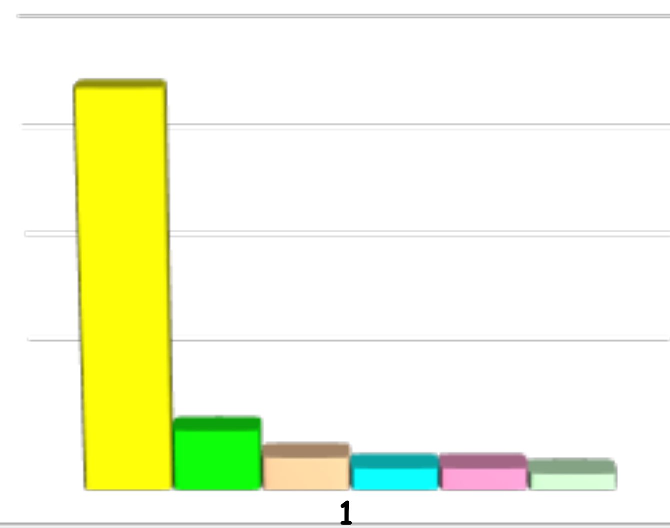
CheckPoint 10

A current carrying wire is wrapped around cardboard tube as shown below.



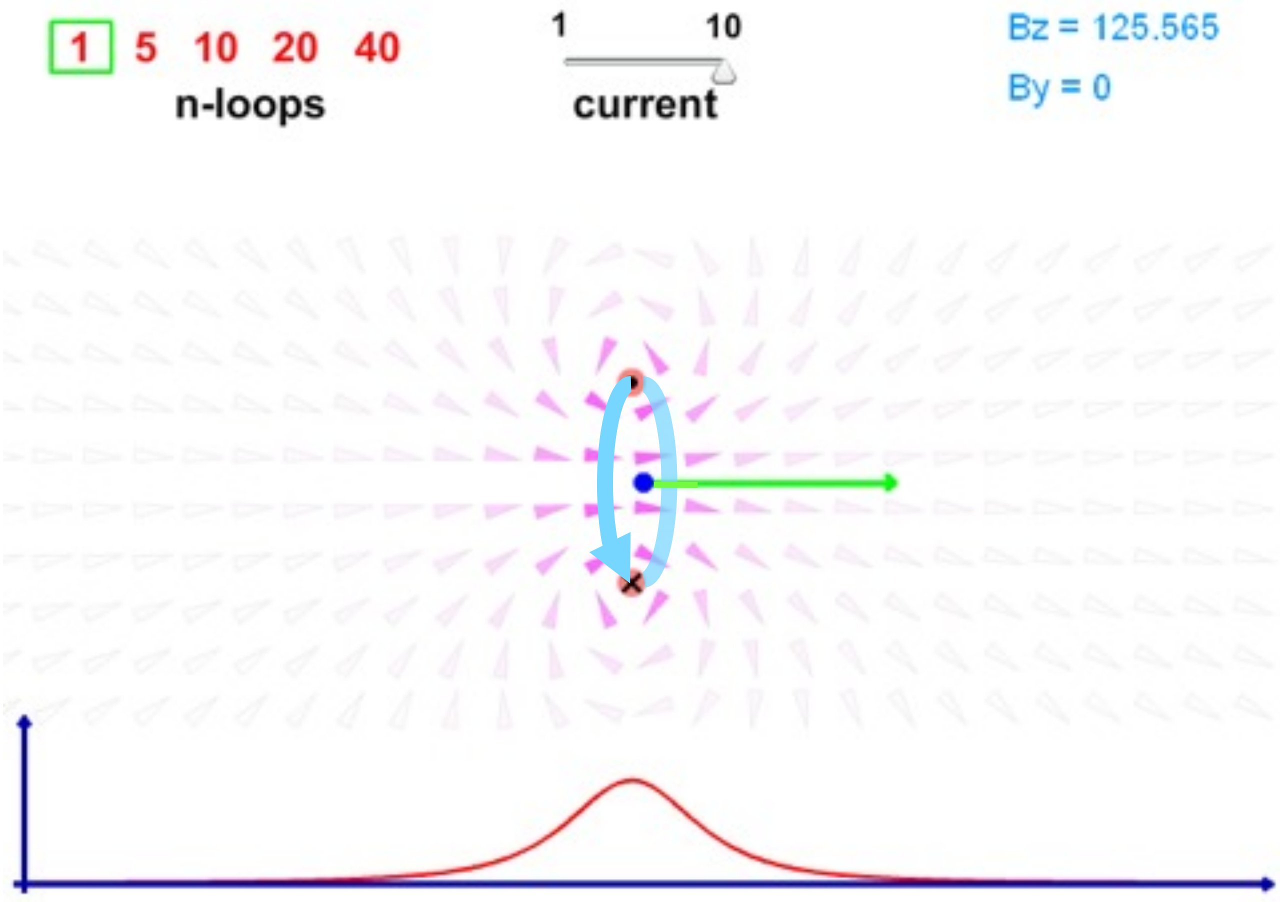
In which direction does the magnetic field point inside the tube?

- left
- right
- up
- down
- out of the screen
- into the screen



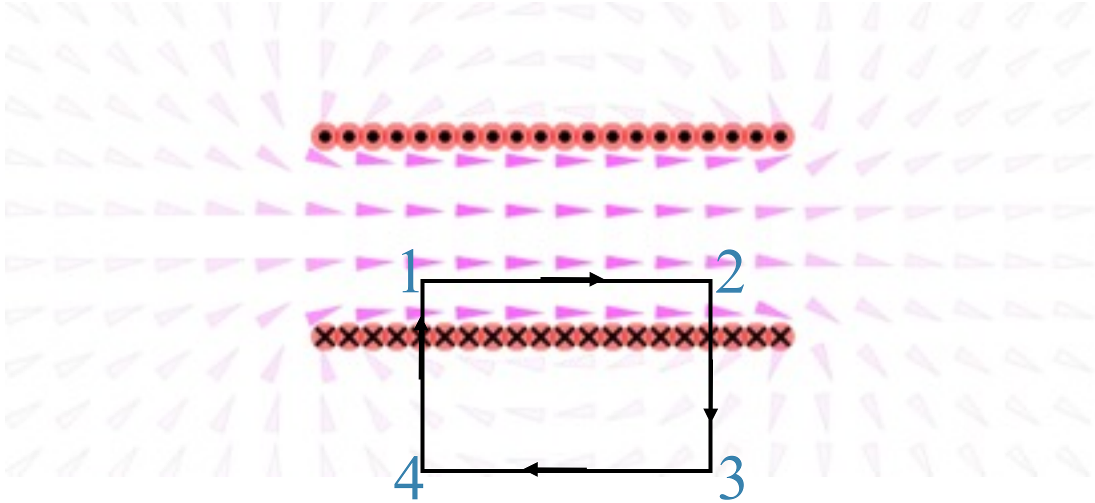
Use the right hand rule and curl your fingers along the direction of the current.

Simulation



Solenoid

Several loops packed tightly together form a uniform magnetic field inside, and nearly zero magnetic field outside.



From this simulation, we can assume a constant field inside the solenoid and zero field outside the solenoid, and apply Ampere's law to find the magnitude of the constant field inside the solenoid!

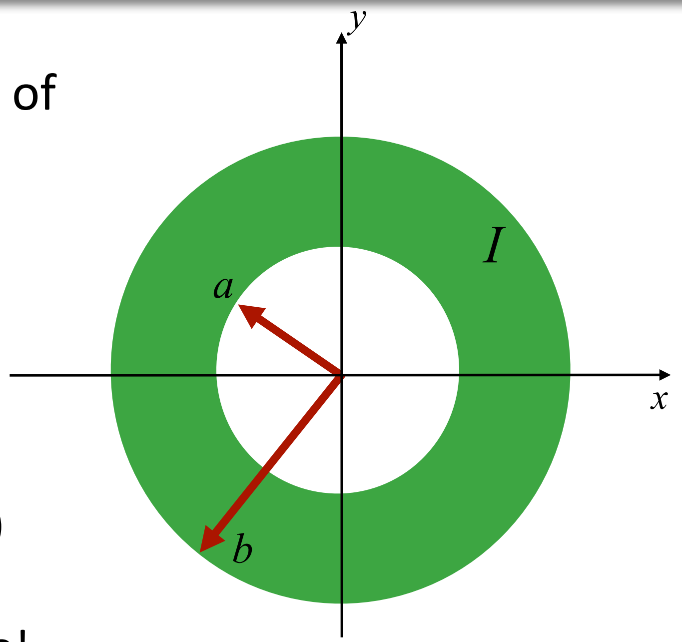
$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc} \longrightarrow \int_1^2 \vec{B} \bullet d\vec{\ell} + \int_2^3 \vec{B} \bullet d\vec{\ell} + \int_3^4 \vec{B} \bullet d\vec{\ell} + \int_4^1 \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

$$BL + 0 + 0 + 0 = \mu_o I_{enc} \longrightarrow BL = \mu_o nLI \longrightarrow B = \mu_o nI$$

n = # turns/length

Example Problem

An infinitely long cylindrical shell with inner radius a and outer radius b carries a uniformly distributed current I out of the screen.



Sketch $|B|$ as a function of r.

Conceptual Analysis

Complete cylindrical symmetry (can only depend on r)

$\Rightarrow$ can use Ampere's law to calculate B

B field can only be clockwise, counterclockwise or zero!

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

$$B\oint d\ell = \mu_o I_{enc}$$ For circular path concentric with shell.

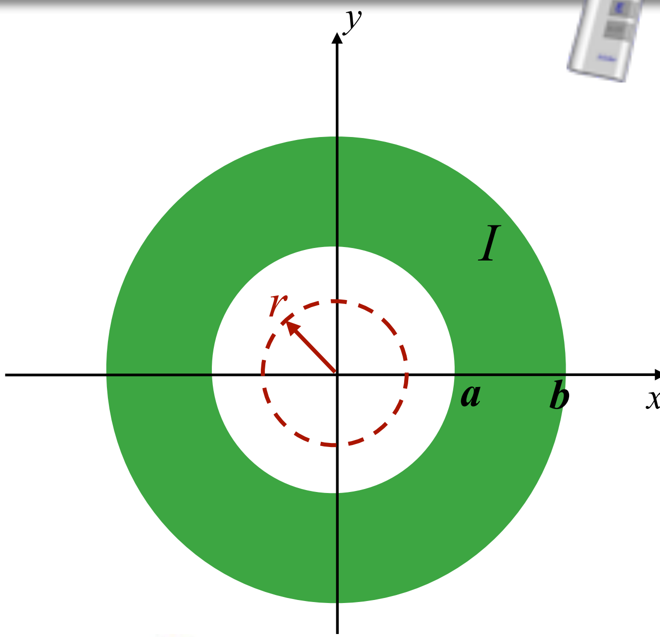
Strategic Analysis

Calculate B for the three regions separately:

1) $r < a$
2) $a < r < b$
3) $r > b$

Example Problem

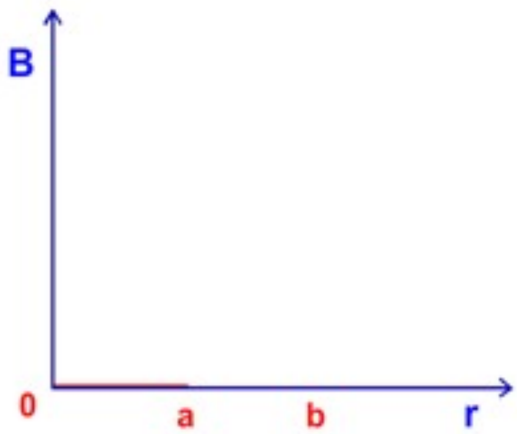
What does $|B|$ look like for $r < a$?



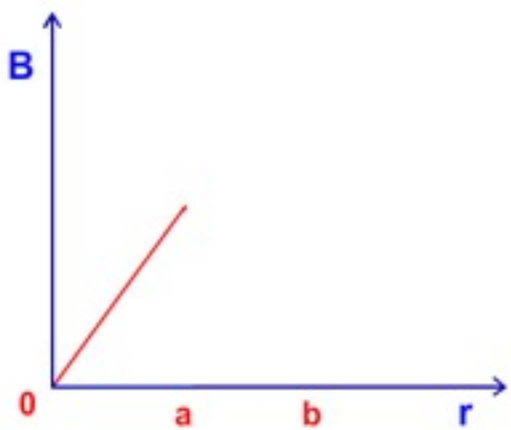
$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{inc}$$

0

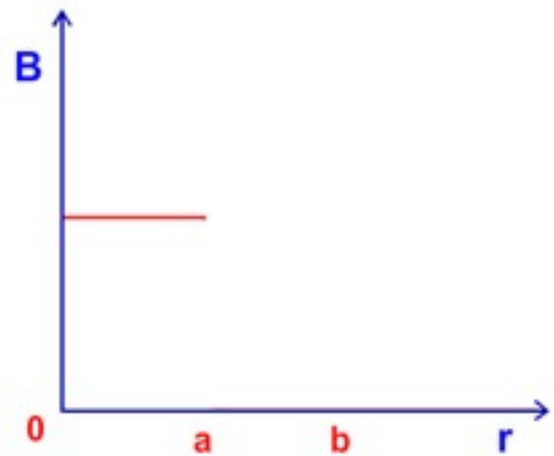
so $\vec{B} = 0$



A



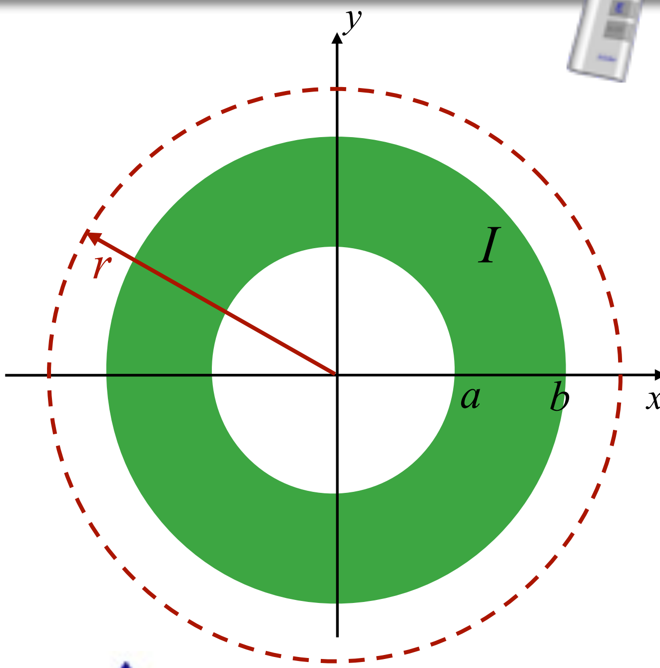
B



C

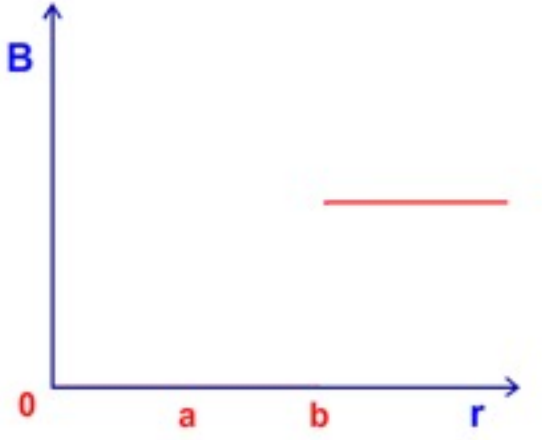
Example Problem

What does $|B|$ look like for $r < b$?

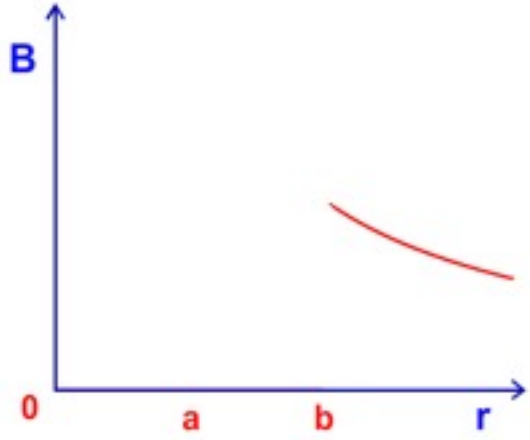


$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{inc}$$

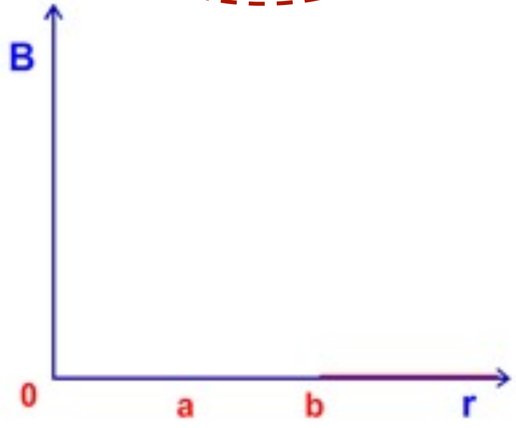
I



A



B



C

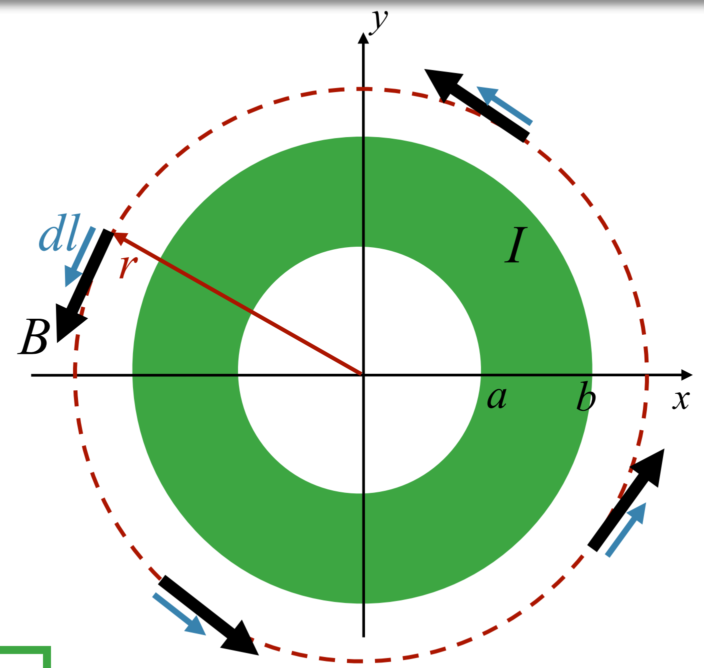
Example Problem

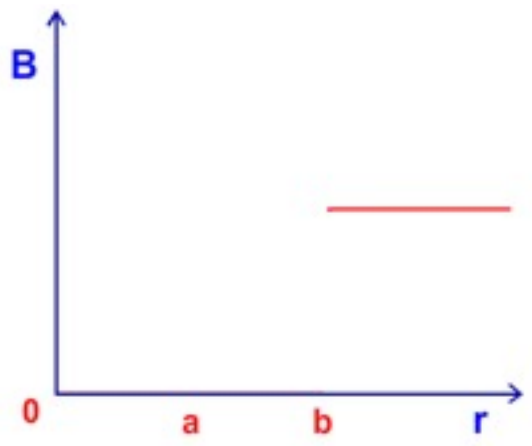
What does $|B|$ look like for $r < b$?

LHS: $\oint \vec{B} \cdot d\vec{\ell} = \oint B d\ell = B \oint d\ell = B \cdot 2\pi r$

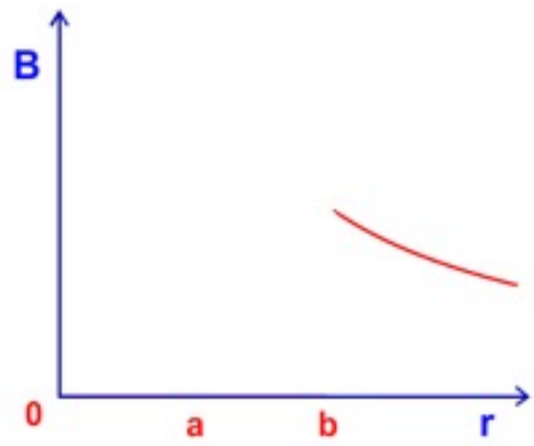
RHS: $I_{enclosed} = I$

$$B = \frac{\mu_o I}{2\pi r}$$

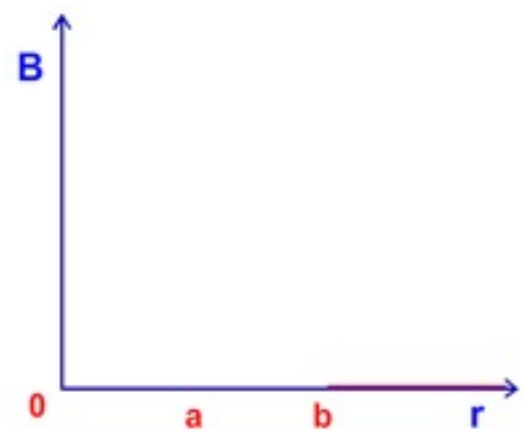




A



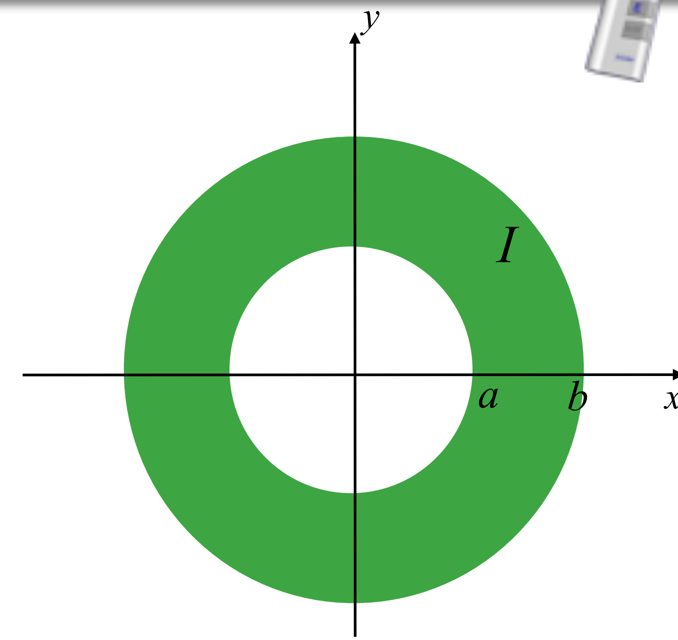
B



C

Example Problem

What is the current density j (Amp/m^2) in the conductor?



A) $j = \dfrac{I}{\pi b^2}$

B) $j = \dfrac{I}{\pi b^2 + \pi a^2}$

C) $j = \dfrac{I}{\pi b^2 - \pi a^2}$

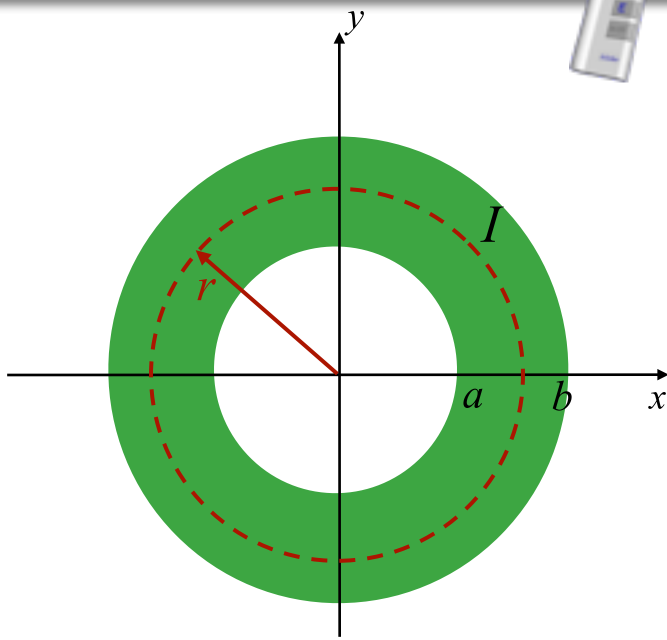
$j = I / area$

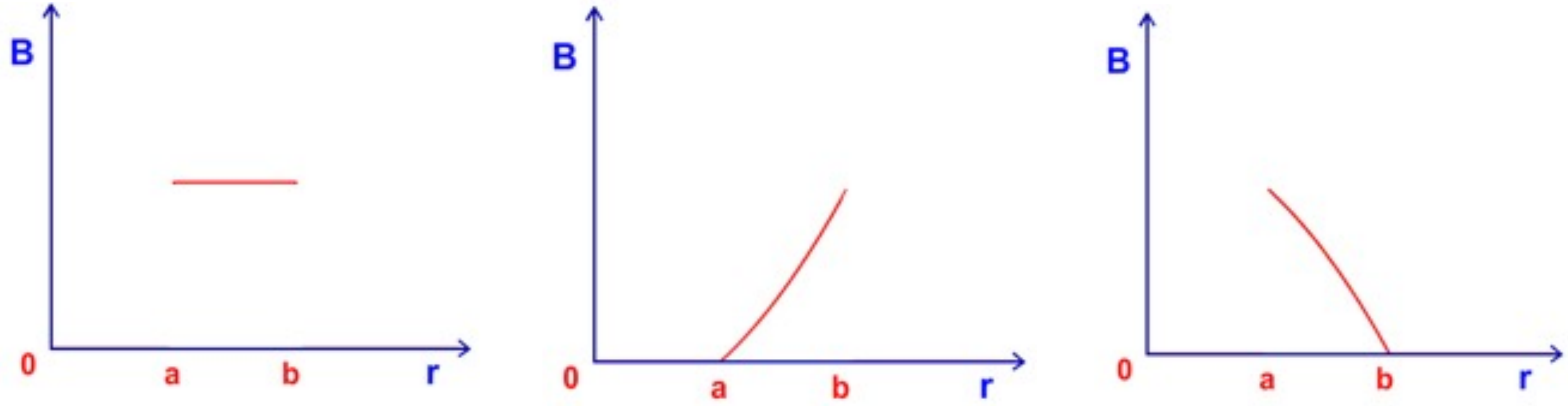
$area = \pi b^2 - \pi a^2$

$$j = \frac{I}{\pi b^2 - \pi a^2}$$

Example Problem

What does $|B|$ look like for $a < r < b$?



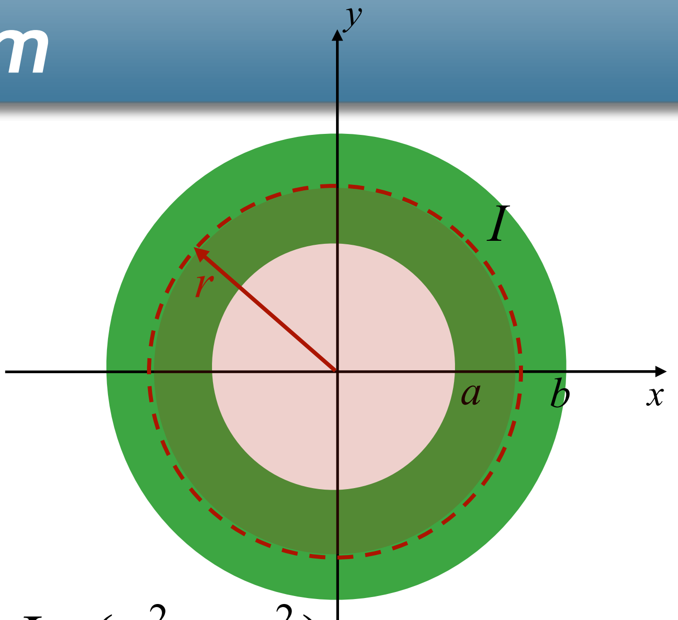


A B C

Example Problem

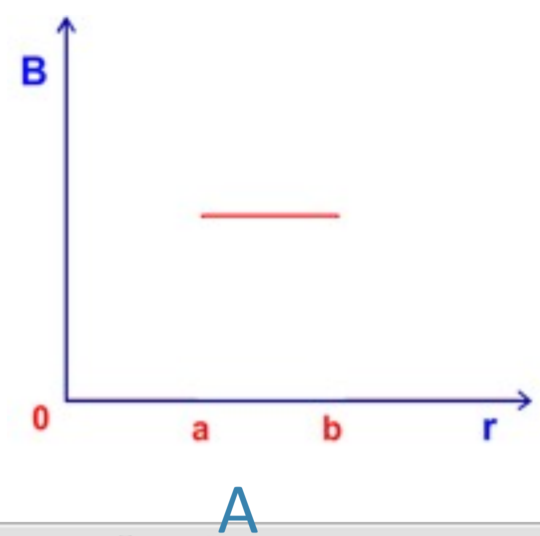
What does $|B|$ look like for $a < r < b$?

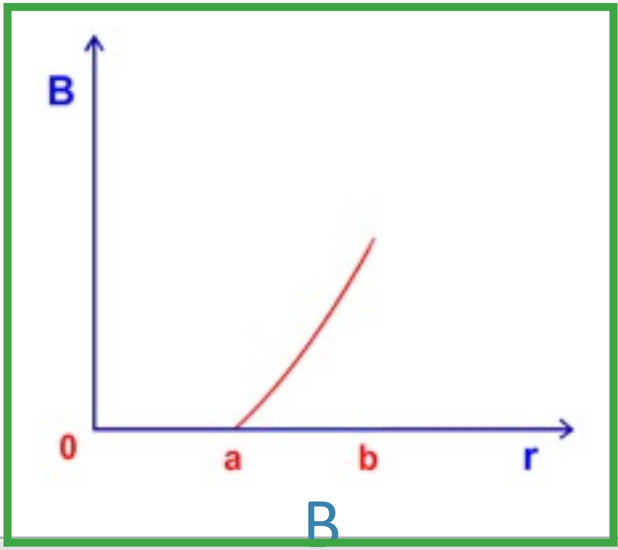
$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc} \rightarrow B \times 2\pi r = \mu_o \times j A_{enc}$$

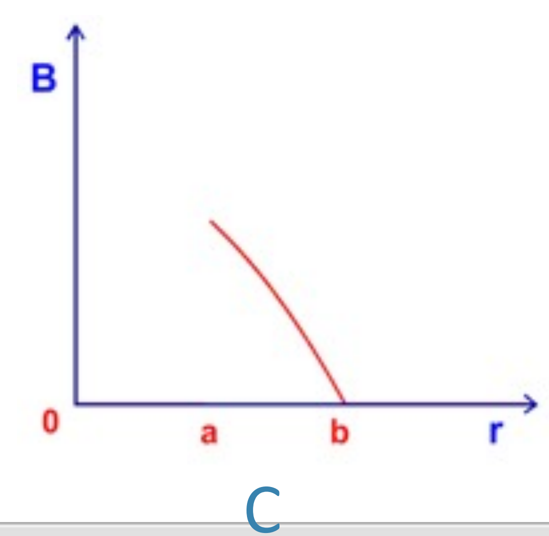


$$B \times 2\pi r = \mu_o \times \frac{I}{\pi(b^2 - a^2)} \times \pi(r^2 - a^2) \rightarrow B = \frac{\mu_o I}{2\pi r} \times \frac{(r^2 - a^2)}{(b^2 - a^2)}$$

Starts at 0 and increases almost linearly

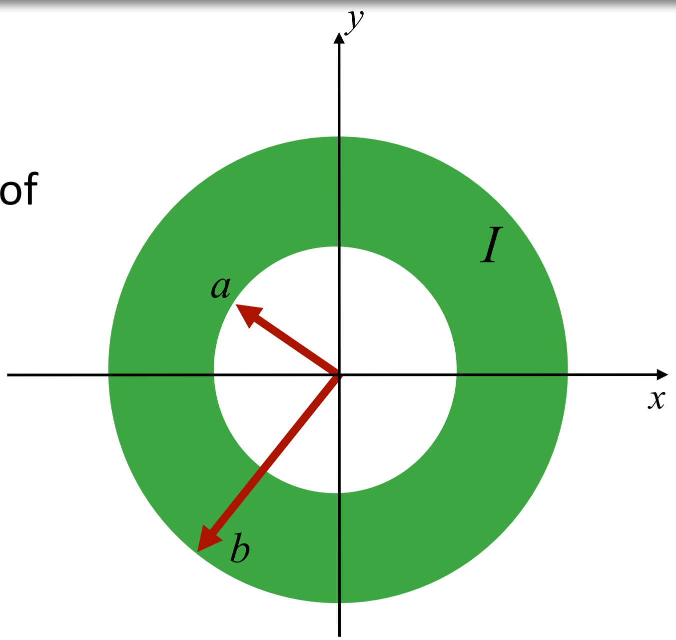




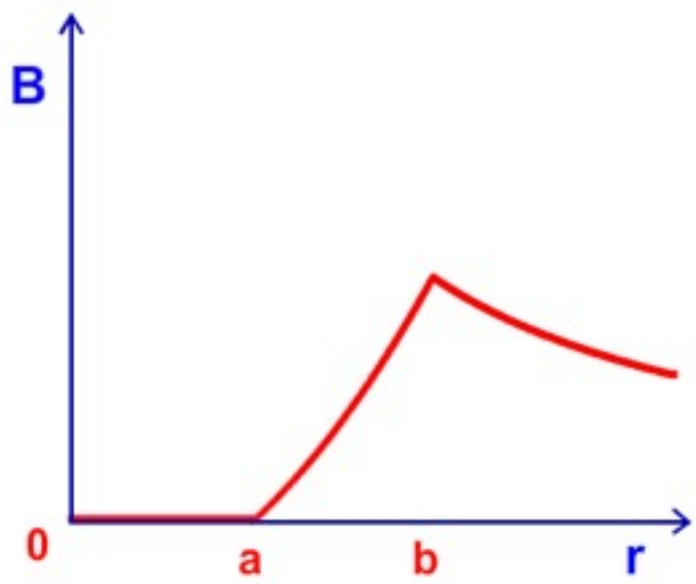


Example Problem

An infinitely long cylindrical shell with inner radius a and outer radius b carries a uniformly distributed current I out of the screen.



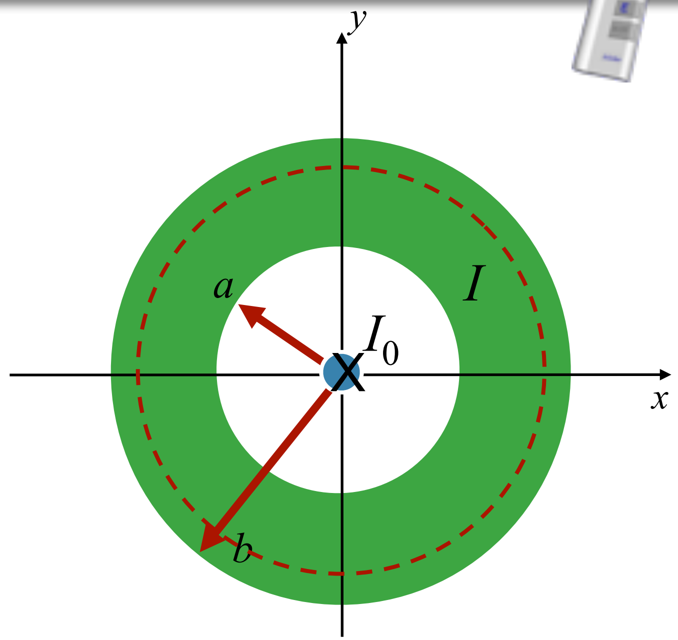
Sketch $|B|$ as a function of r.



Follow-Up

Add an infinite wire along the z axis carrying current I_0.

What must be true about I_0 such that there is some value of r, $a < r < b$, such that $B(r) = 0$?



A) $|I_0| > |I|$ AND I_0 into screen

B) $|I_0| > |I|$ AND I_0 out of screen

C) $|I_0| < |I|$ AND I_0 into screen

D) $|I_0| < |I|$ AND I_0 out of screen

E) There is no current I_0 that can produce $B = 0$ there

B will be zero if total current enclosed = 0