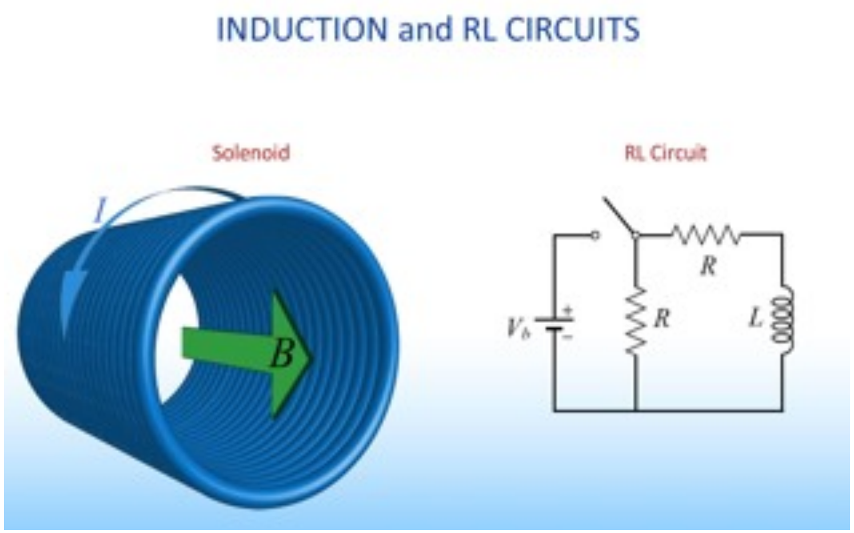
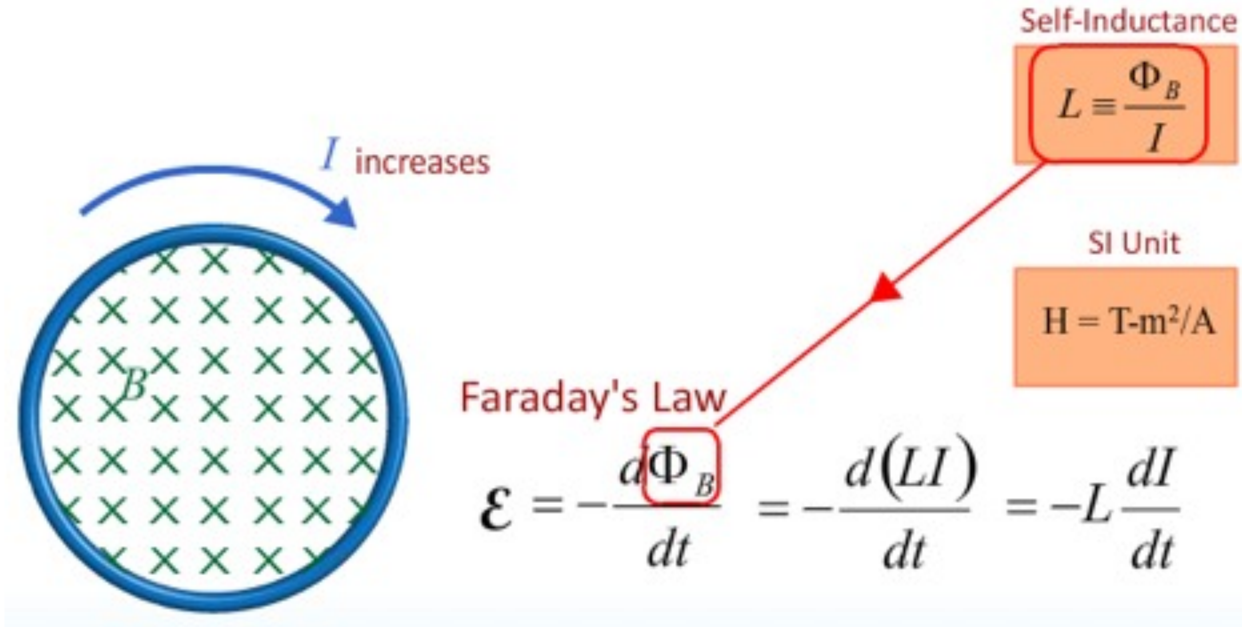
Electricity & Magnetism Lecture 18

Today's Concepts:

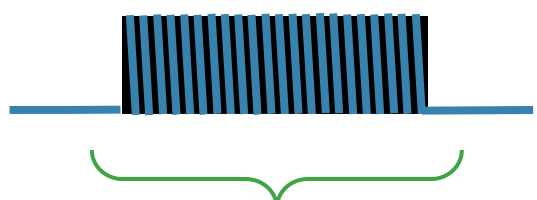
A) Induction

B) RL Circuits





Wrap a wire into a coil to make an "inductor"…

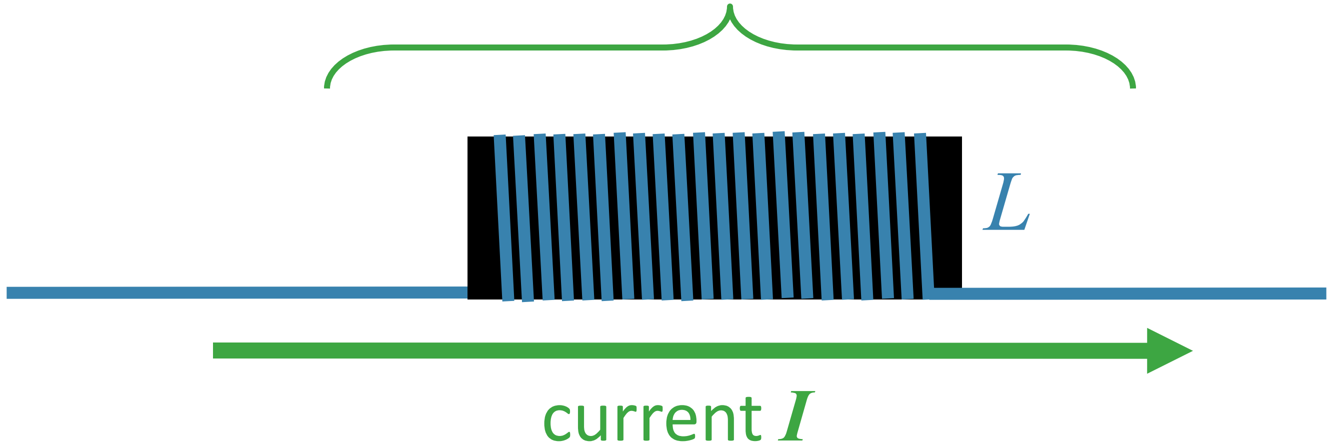


$$\mathcal{E} = -L\frac{dI}{dt}$$

What this really means:

emf induced across L tries to keep I constant.

$$\varepsilon_L = -L\frac{dI}{dt}$$



Inductors prevent discontinuous current changes!

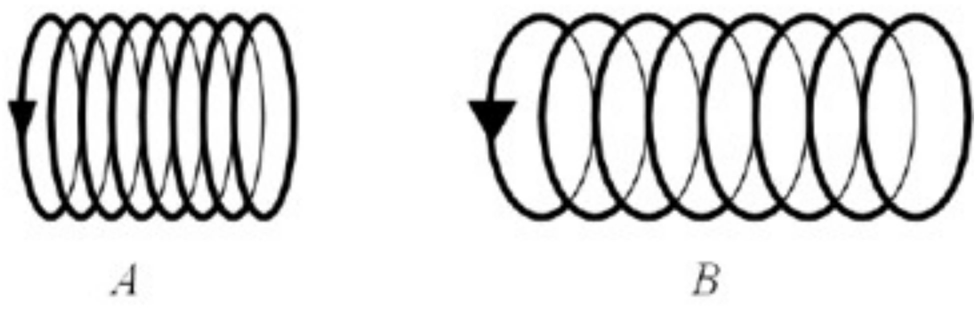
It's like inertia!

CheckPoint 2

Two solenoids are made with the same cross sectional area and total number of turns. Inductor B is twice as long as inductor A

$$L_B = \mu_0 n^2 \pi r^2 z$$

$(1/2)^2$ 2

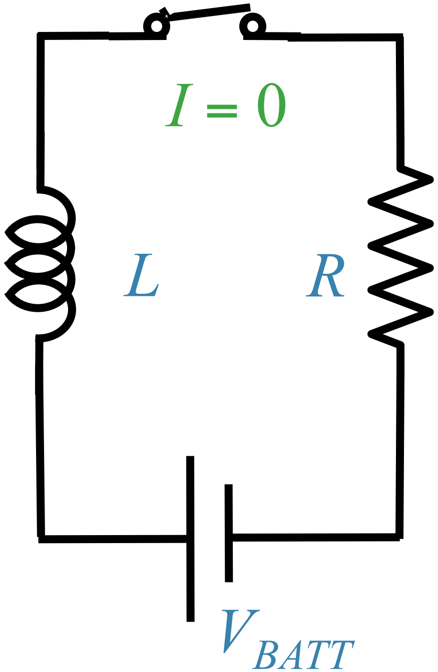


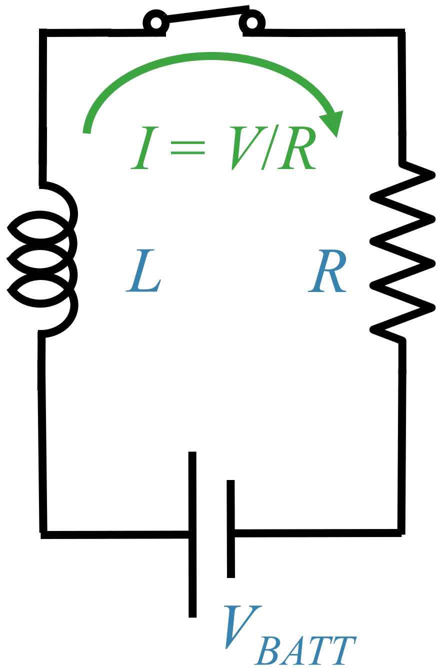
$$\longrightarrow \quad L_B = \tfrac{1}{2} L_A$$

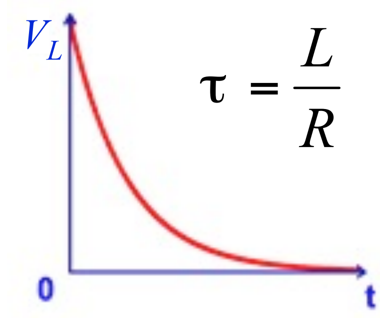
Compare the inductance of the two solenoids

A) $L_A = 4\ L_B$
B) $L_A = 2\ L_B$
C) $L_A = L_B$
D) $L_A = (1/2)\ L_B$
E) $L_A = (1/4)\ L_B$

When no current is flowing initially:







At $t = 0$:

$I = 0$

$V_L = V_{BATT}$

$V_R = 0$

(L is like a giant resistor)

At $t >> L/R$:

$V_L = 0$

$V_R = V_{BATT}$

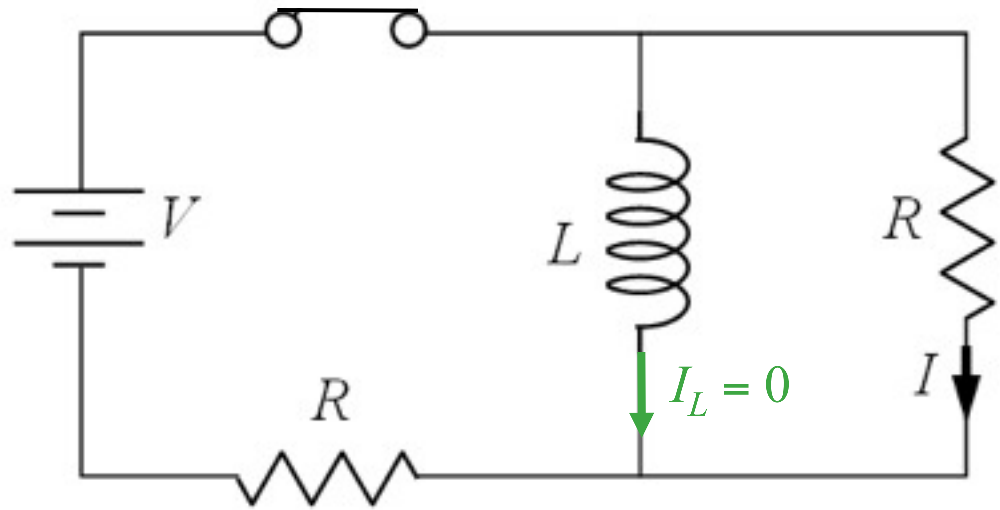
$I = V_{BATT}/R$

(L is like a short circuit)

CheckPoint 4

In the circuit, the switch has been open for a long time, and the current is zero everywhere.

At time $t = 0$ the switch is closed.



What is the current I through the vertical resistor immediately after the switch is closed?

(+ is in the direction of the arrow)

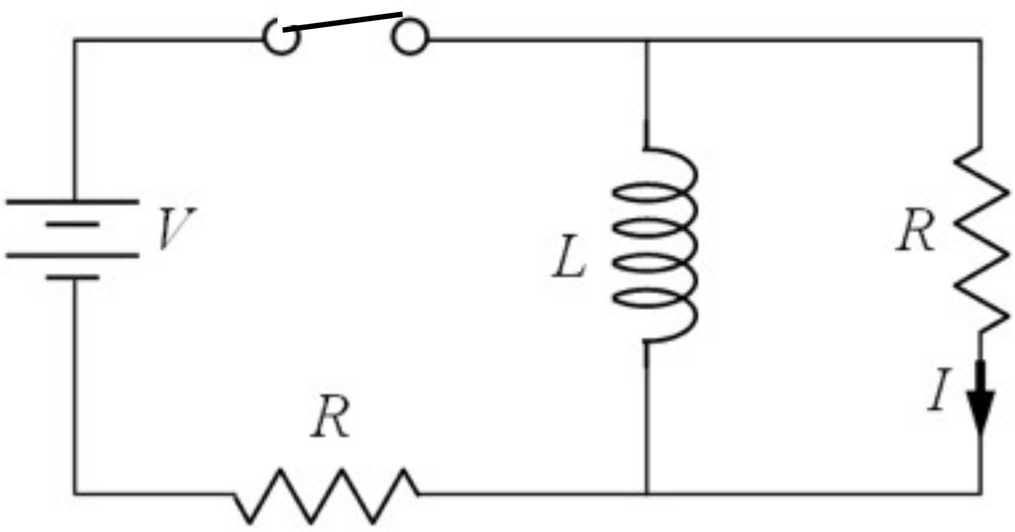
A) $I = V/R$
B) $I = V/2R$
C) $I = 0$
D) $I = -V/2R$
E) $I = -V/R$

RL Circuit (Long Time)

What is the current I through the vertical resistor after the switch has been closed for a long time?

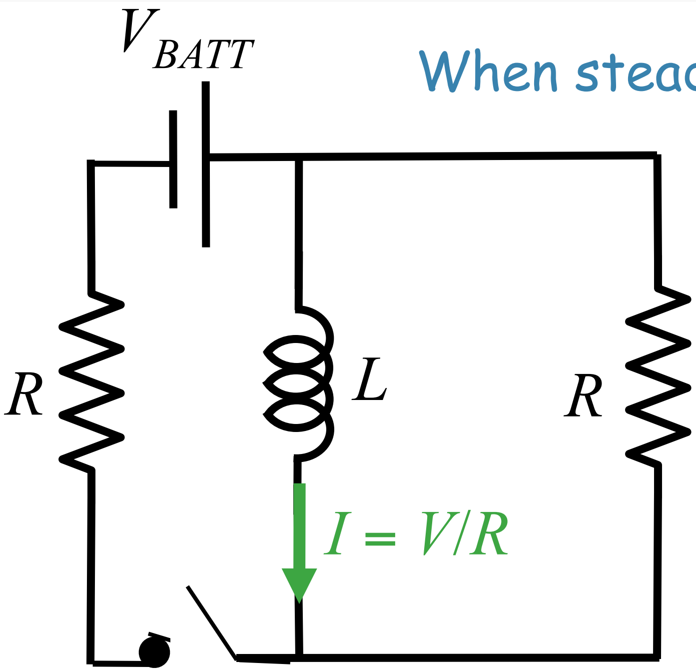
(+ is in the direction of the arrow)

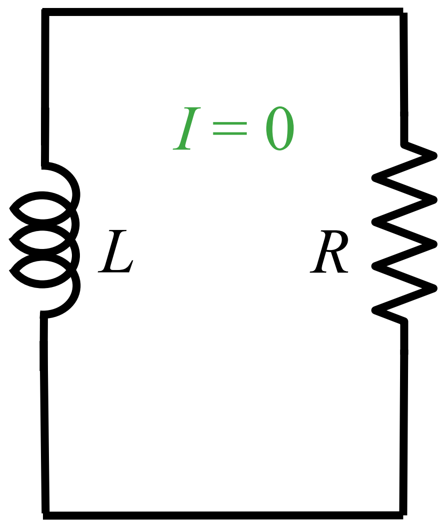
A) $I = V/R$
B) $I = V/2R$
C) $I = 0$
D) $I = -V/2R$
E) $I = -V/R$

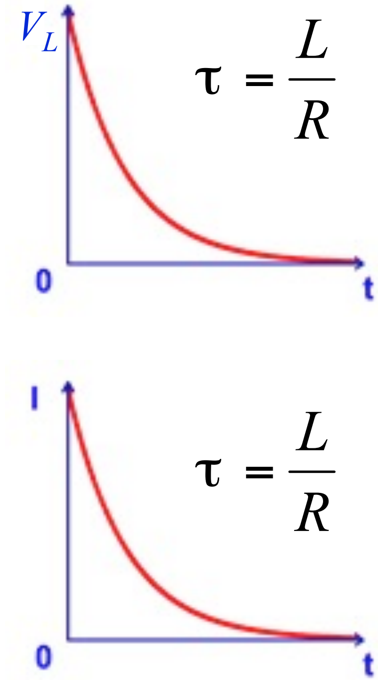


How to Think about RL Circuits Episode 2:

When steady current is flowing initially:







At $t = 0$:

$I = V_{BATT}/R$

$V_R = IR$

$V_L = V_R$

At $t >> L/R$:

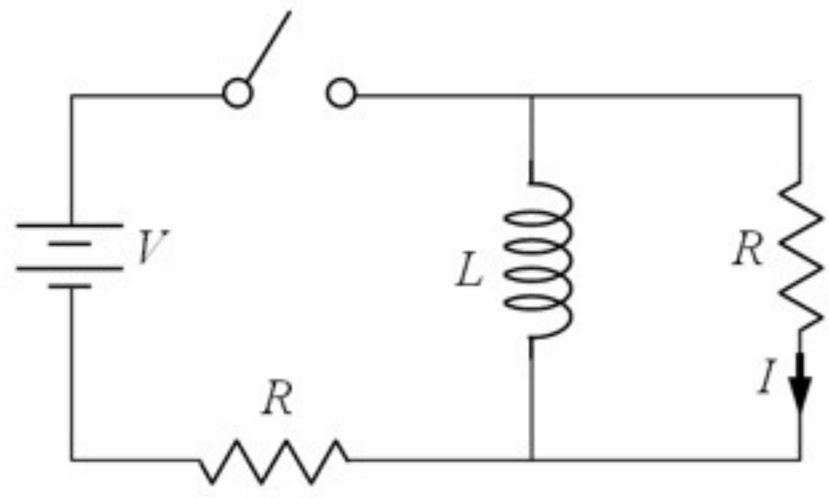
$I = 0$

$V_L = 0$

$V_R = 0$

CheckPoint 6

After a long time, the switch is opened, abruptly disconnecting the battery from the circuit.



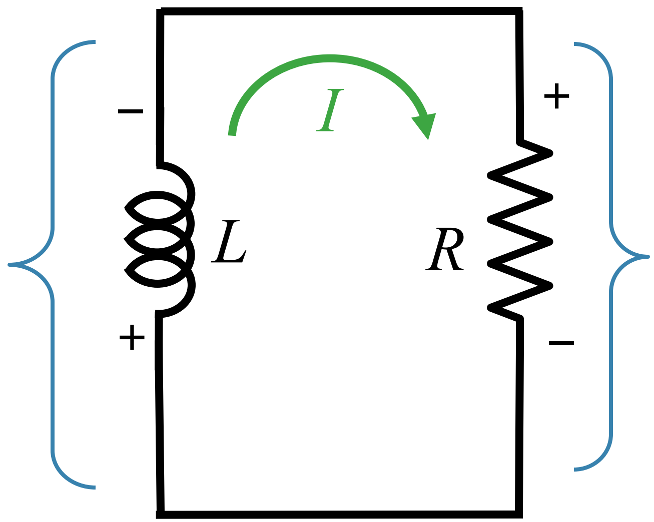
What is the current I through the vertical resistor immediately after the switch is opened?

(+ is in the direction of the arrow)

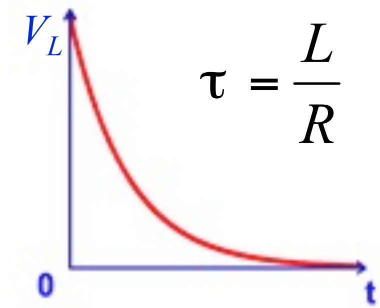
A) $I = V/R$
B) $I = V/2R$
C) $I = 0$
D) $I = -V/2R$
E) $I = -V/R$

Why is there Exponential Behavior?

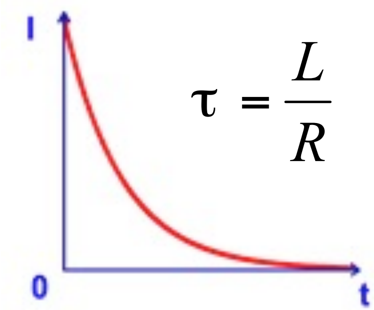
$$V = L\frac{dI}{dt}$$



$$V = IR$$



$$\tau = \frac{L}{R}$$

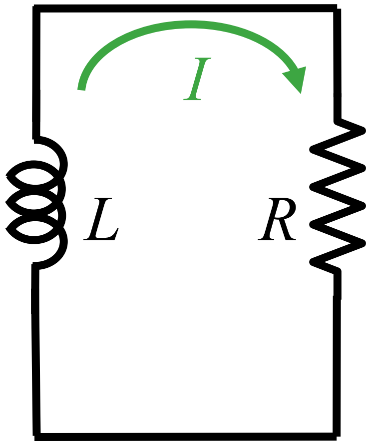


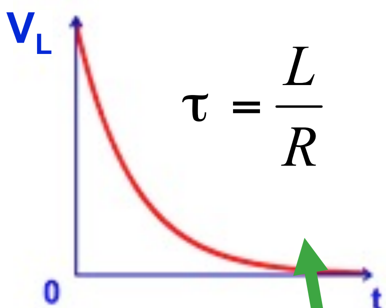
$$\tau = \frac{L}{R}$$

$$L\frac{dI}{dt} + IR = 0$$

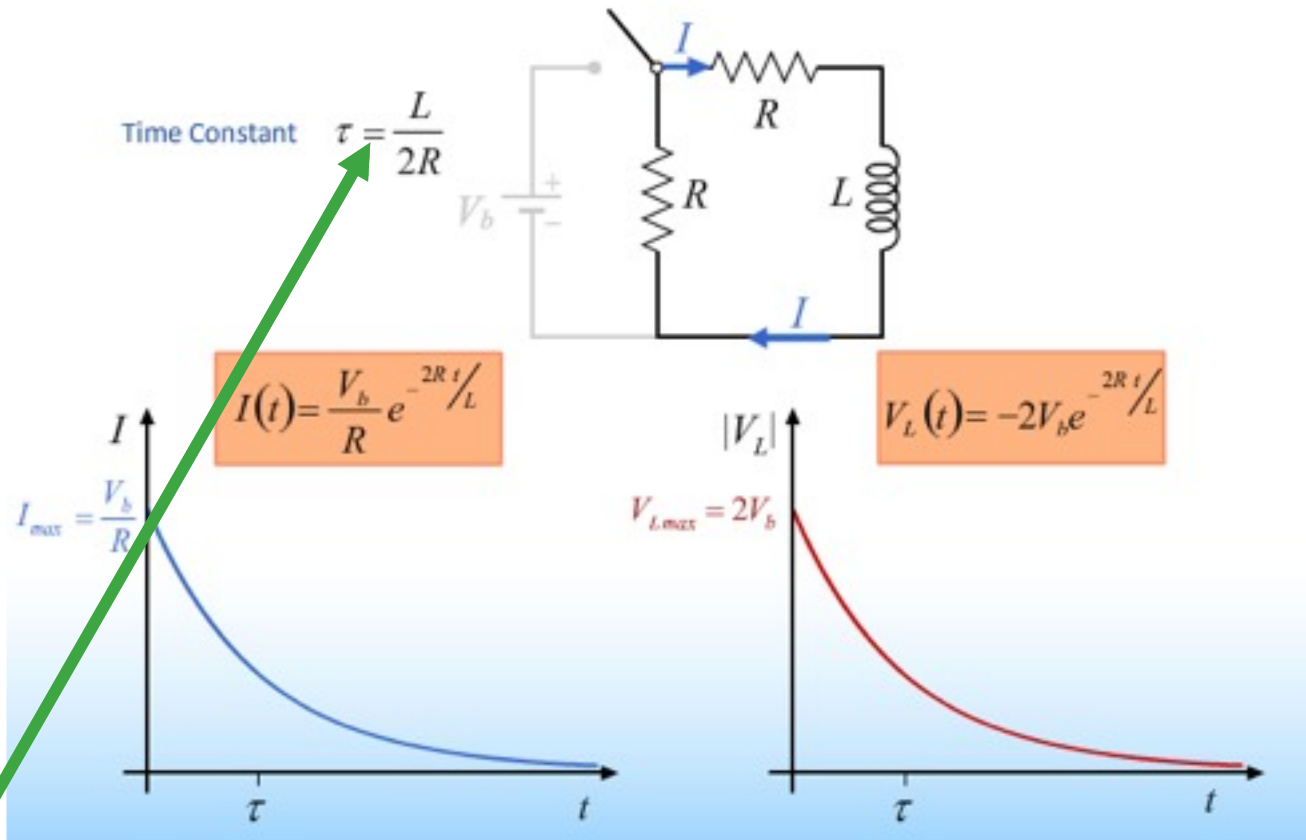
$$I(t) = I_0 e^{-tR/L} = I_0 e^{-t/\tau}$$

where $\tau = \dfrac{L}{R}$





Lecture:

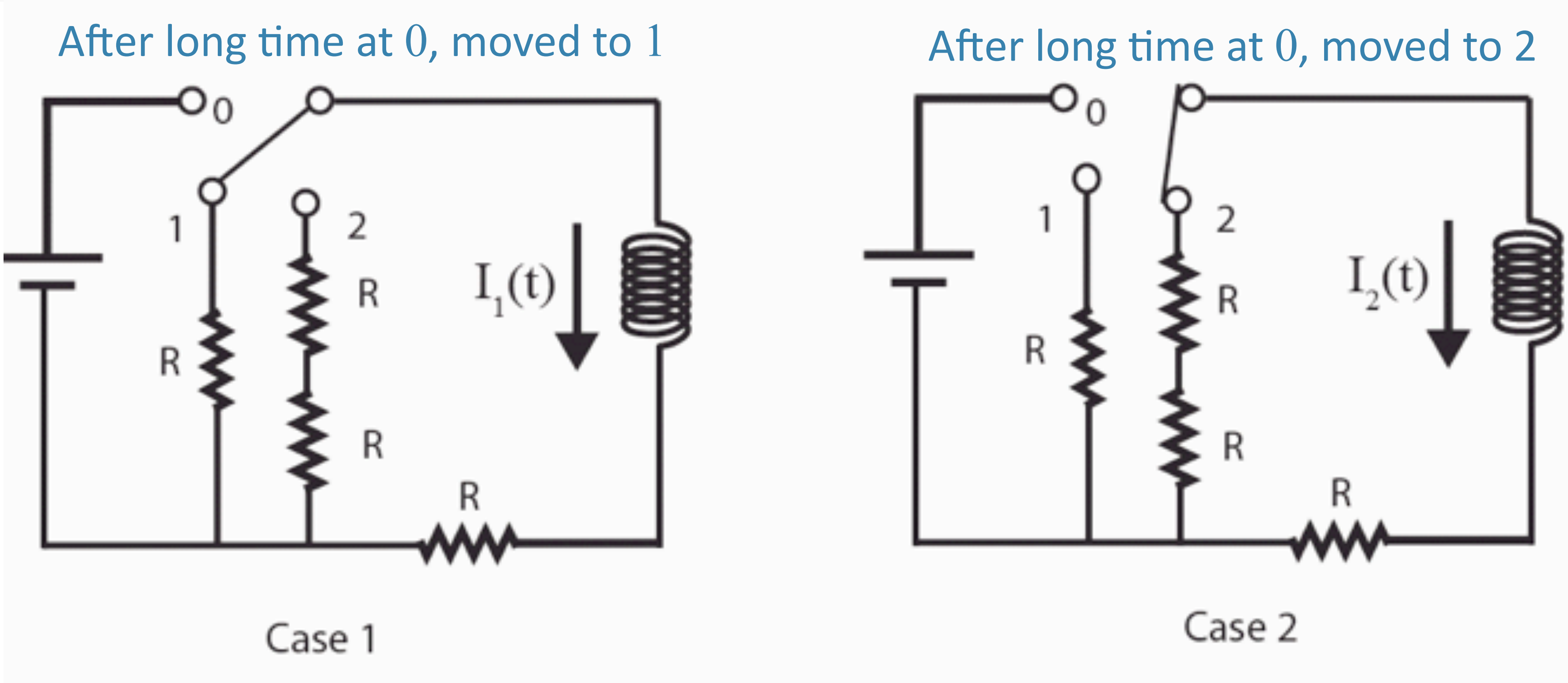


Prelecture:

Did we mess up?

No: The resistance is simply twice as big in one case.

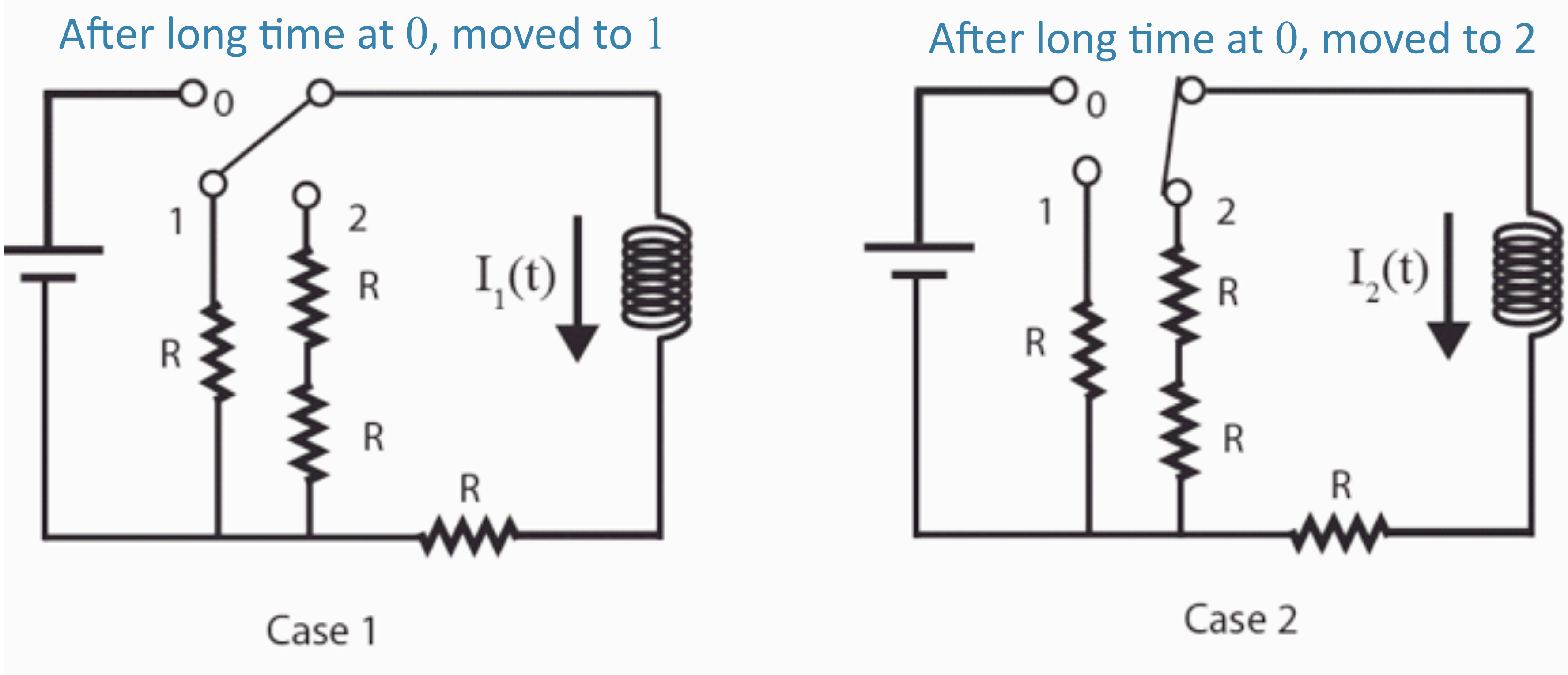
CheckPoint 8



After switch moved, which case has larger time constant?

A) Case 1
B) Case 2
C) The same

CheckPoint 10

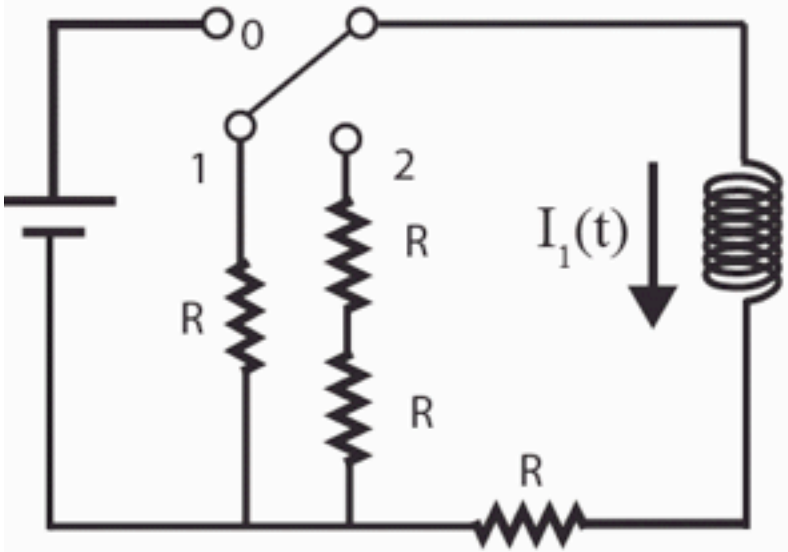


Immediately after switch moved, in which case is the voltage across the inductor larger?

A) Case 1
B) Case 2
C) The same

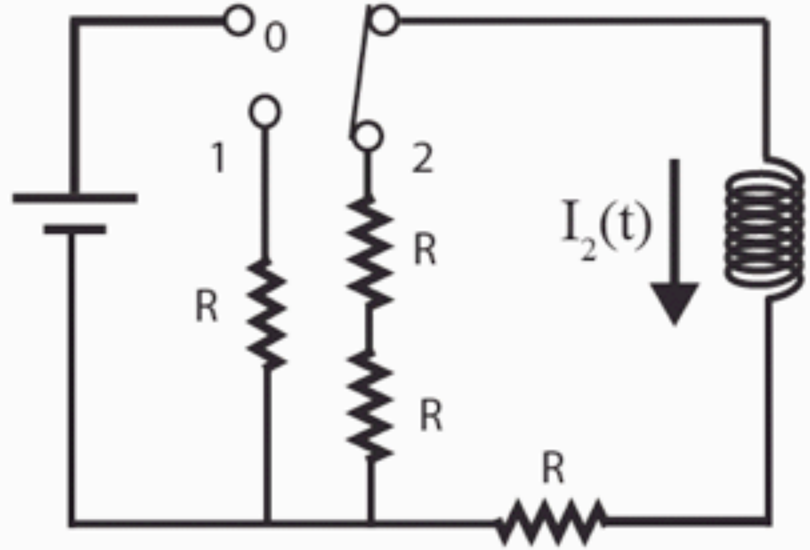
CheckPoint 12

After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



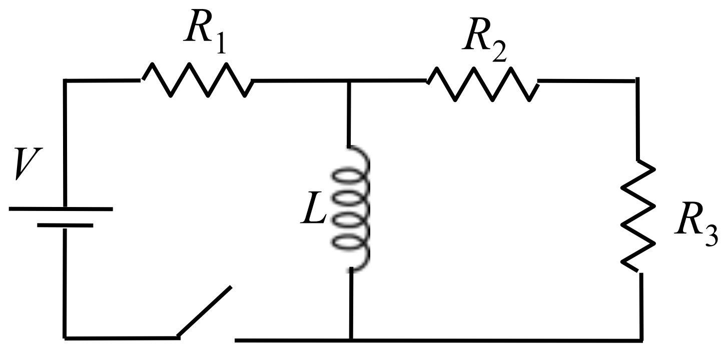
Case 2

After switch moved for finite time, in which case is the current through the inductor larger?

A) Case 1
B) Case 2
C) The same

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



What is dI_L/dt, the time rate of change of the current through the inductor immediately after switch is closed

Conceptual Analysis

Once switch is closed, currents will flow through this 2-loop circuit.

KVR and KCR can be used to determine currents as a function of time.

Strategic Analysis

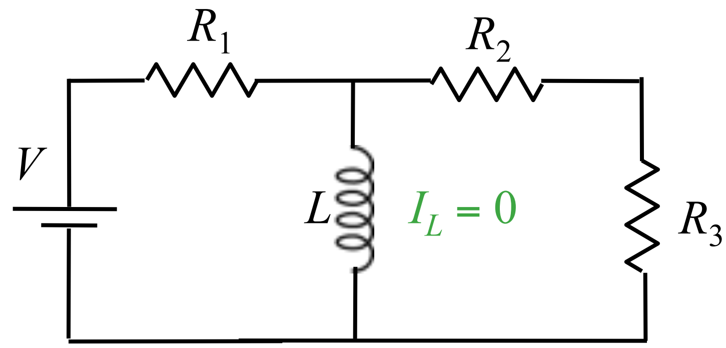
Determine currents immediately after switch is closed.

Determine voltage across inductor immediately after switch is closed.

Determine dI_L/dt immediately after switch is closed.

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



What is I_L, the current in the inductor, immediately after the switch is closed?

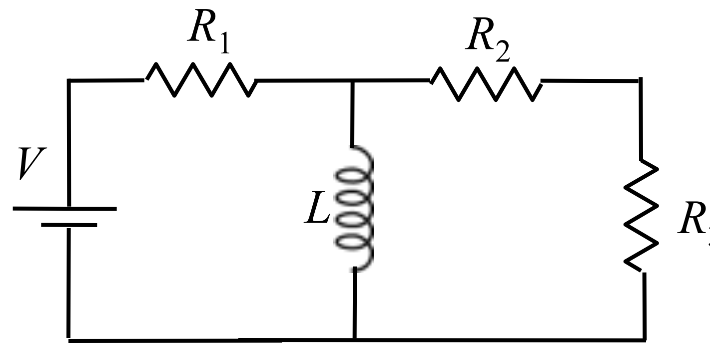
A) $I_L = V/R_1$ up

B) $I_L = V/R_1$ down

C) $I_L = 0$

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.

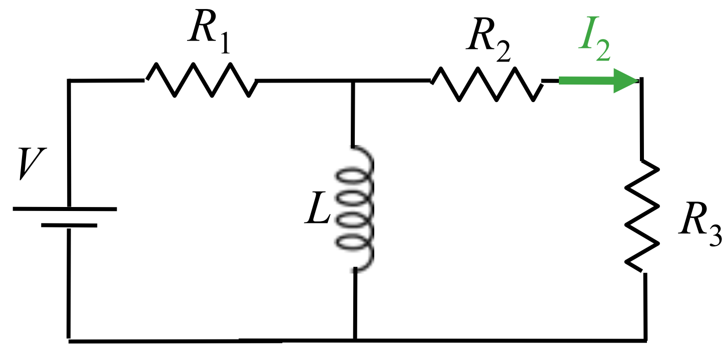


$I_L(t = 0+) = 0$

What is the magnitude of I_2, the current in R_2, immediately after the switch is closed?

A) $I_2 = \frac{V}{R_1}$ B) $I_2 = \frac{V}{R_2 + R_3}$ C) $I_2 = \frac{V}{R_1 + R_2 + R_3}$ D) $I_2 = \frac{VR_2R_3}{R_2 + R_3}$

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



$I_L(t = 0 +) = 0 \quad I_2(t = 0 +) = V/(R_1 + R_2 + R_3)$

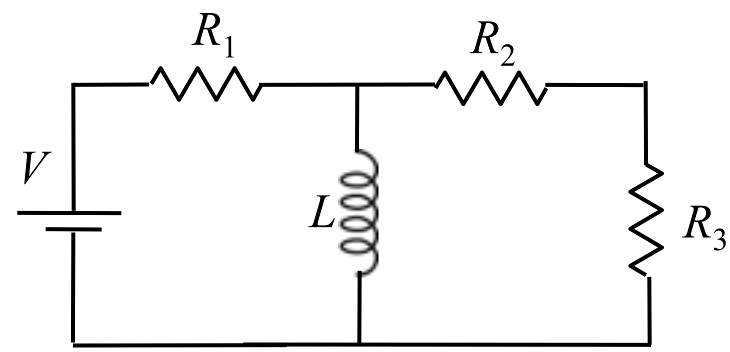
What is the magnitude of V_L, the voltage across the inductor, immediately after the switch is closed?

A) $V_L = V \dfrac{R_2 R_3}{R_1}$ B) $V_L = V$ C) $V_L = 0$ D) $V_L = V \dfrac{R_2 R_3}{R_1(R_2 + R_3)}$ E) $V_L = V \dfrac{R_2 + R_3}{R_1 + R_2 + R_3}$

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.

What is dI_L/dt, the time rate of change of the current through the inductor immediately after switch is closed



$V_L(t = 0+) = V(R_2 + R_3)/(R_1 + R_2 + R_3)$

A) $\frac{dI_L}{dt} = \frac{V}{L}\frac{R_2 + R_3}{R_1}$

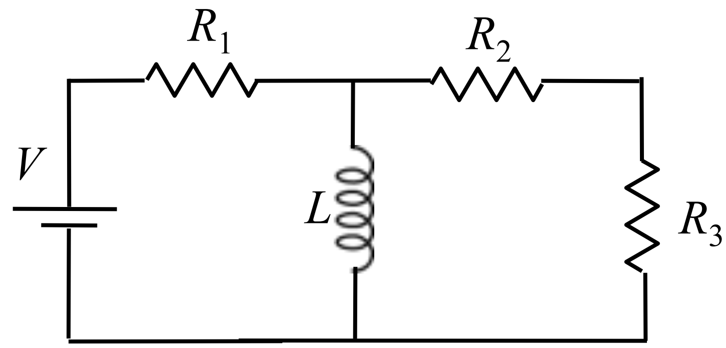
B) $\frac{dI_L}{dt} = 0$

C) $\frac{dI_L}{dt} = \frac{V}{L}\frac{R_2 + R_3}{R_1 + R_2 + R_3}$

D) $\frac{dI_L}{dt} = \frac{V}{L}$

Follow Up

The switch in the circuit shown has been closed for a long time.



What is I_2, the current through R_2 ?

(Positive values indicate current flows to the right)

A) $I_2 = +\frac{V}{R_2 + R_3}$

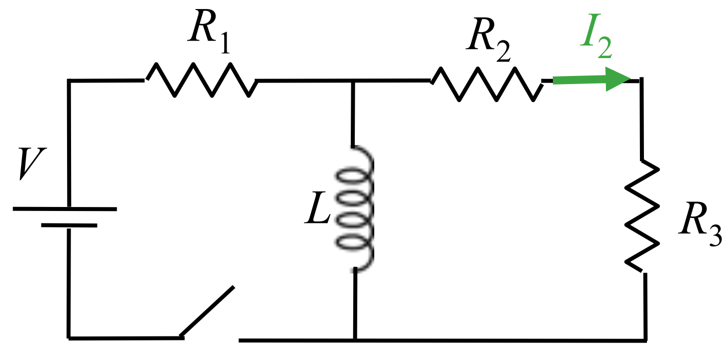
B) $I_2 = +\frac{V(R_2 R_3)}{R_1 + R_2 + R_3}$

C) $I_2 = 0$

D) $I_2 = -\frac{V}{R_2 + R_3}$

Follow Up 2

The switch in the circuit shown has been closed for a long time at which point, the switch is opened.



What is I_2, the current through R_2 immediately after switch is opened ?
(Positive values indicate current flows to the right)

A) $I_2 = +\frac{V}{R_1 + R_2 + R_3}$

B) $I_2 = +\frac{V}{R_1}$

C) $I_2 = 0$

D) $I_2 = -\frac{V}{R_1}$

E) $I_2 = -\frac{V}{R_1 + R_2 + R_3}$