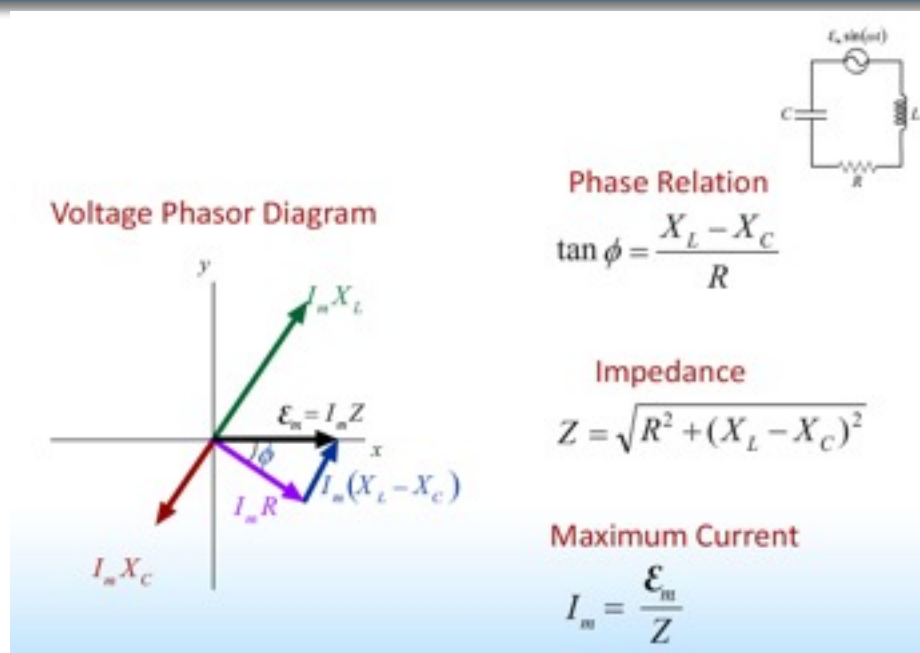
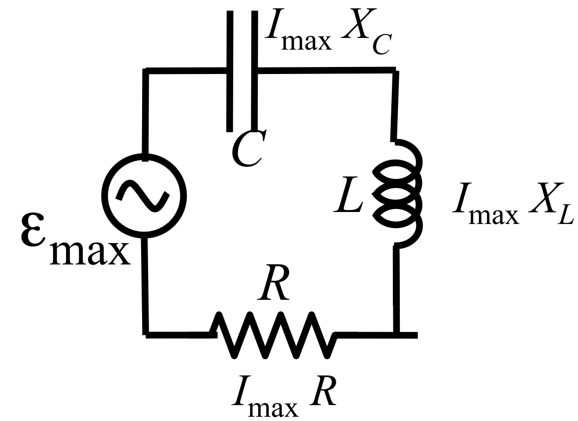
Electricity & Magnetism
Lecture 21

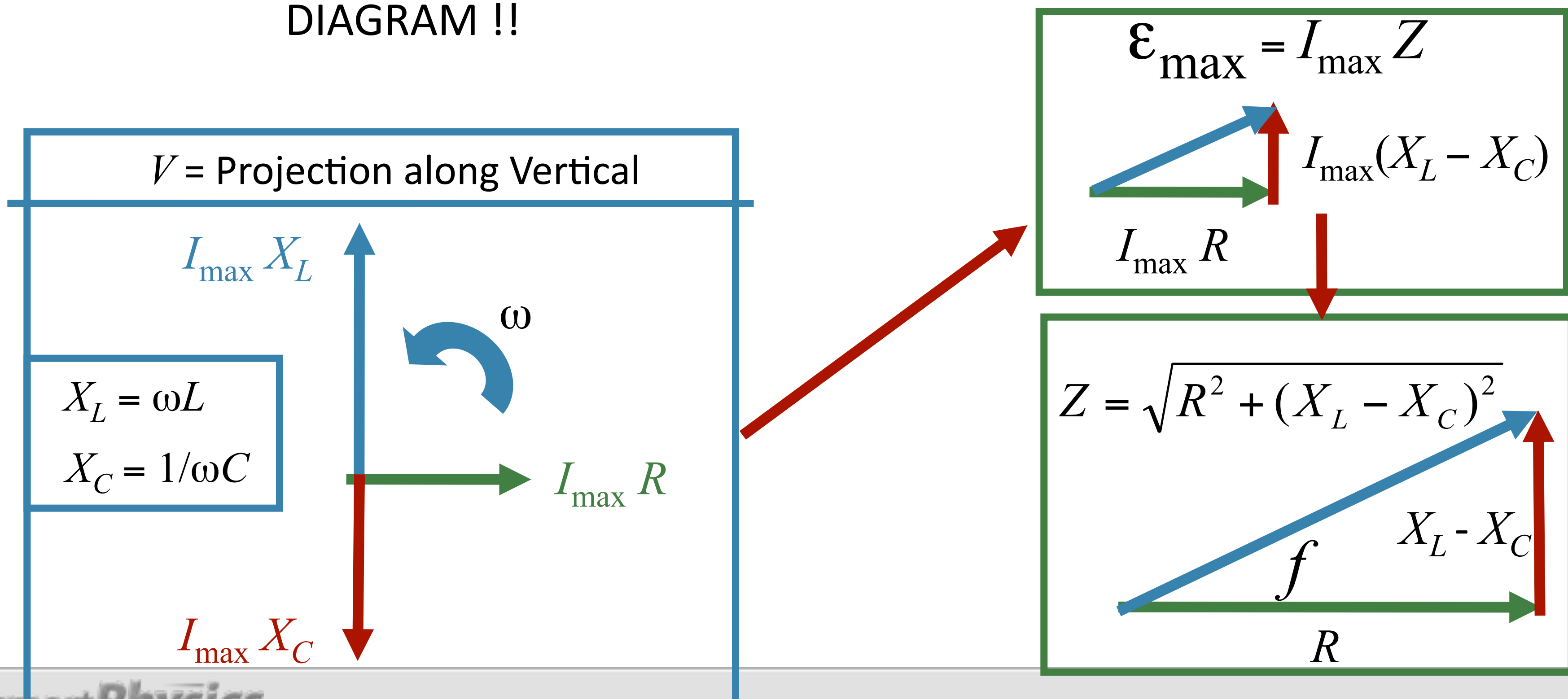


PHASORS ARE THE KEY !
FORMULAS ARE NOT !

START WITH PHASOR DIAGRAM

DEVELOP FORMULAS FROM THE DIAGRAM !!





Peak AC Problems

"Ohms" Law for each element

NOTE: Good for PEAK values only)

$$V_{gen} = I_{max} Z$$

$$V_{Resistor} = I_{max} R$$

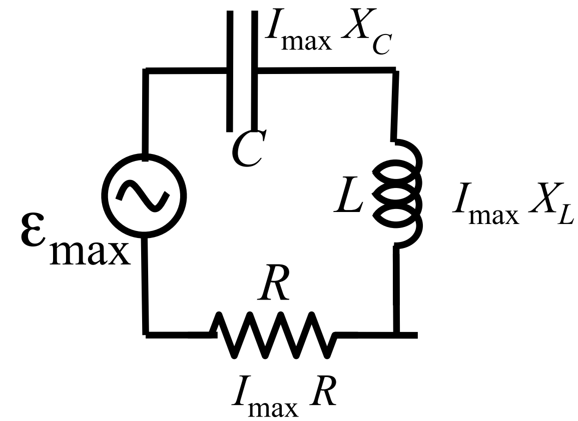
$$V_{inductor} = I_{max} X_L$$

$$V_{Capacitor} = I_{max} X_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$



Typical Problem

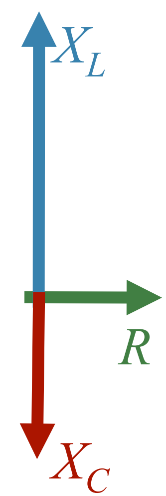
A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

$$X_L = \omega L = 200\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 122\,\Omega$$

$$X_C = \frac{1}{\omega C} = 100\,\Omega$$

$$I_{max} = \frac{V_{gen}}{Z} = 0.13\,A$$



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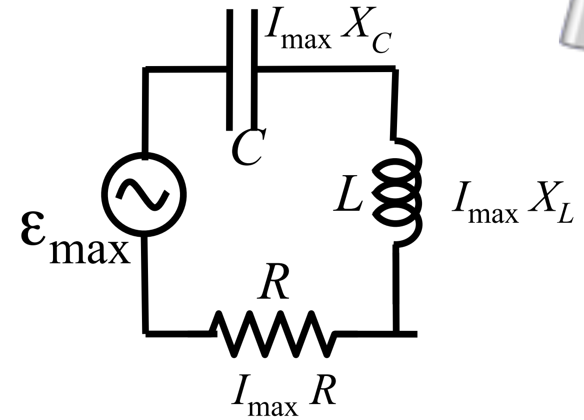


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

Which element has the largest peak voltage across it?

A) Generator

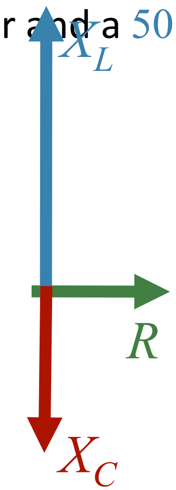
B) Inductor

C) Resistor

D) Capacitor

E) All the same.

$$V_{max} = I_{max} X$$



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Peak AC Problems

"Ohms" Law for each element

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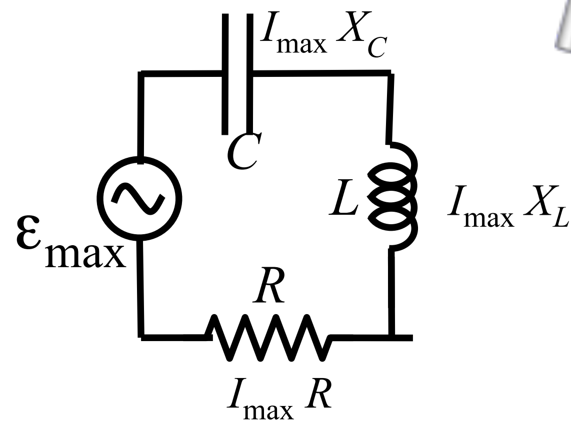


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

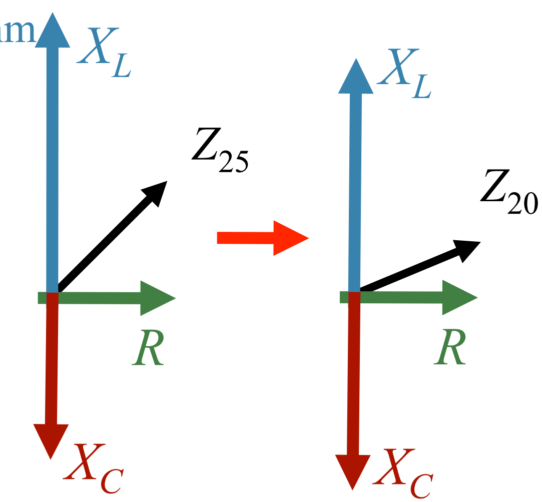
What happens to the impedance if we decrease the angular frequency to 20 rad/sec?

A) Z increases

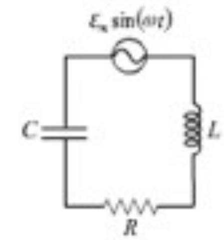
B) Z remains the same

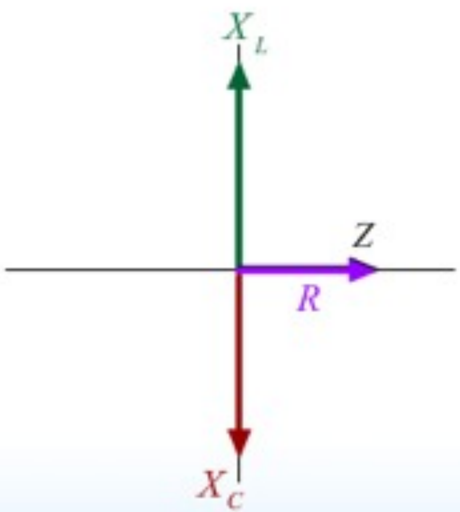
C) Z decreases

$(X_L - X_C)$: (200 – 100) → (160 – 125)



Resonance





Resonance

I_m is a maximum → $I_m = \dfrac{\mathcal{E}_m}{R}$

$\omega = \omega_o$

Z minimized → $X_L = X_C$

$\phi = 0^\circ$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

Light-bulb Demo

Resonance in AC circuits

ω_0: Frequency at which voltage across inductor and capacitor cancel

R is independent of ω

X_L increases with ω

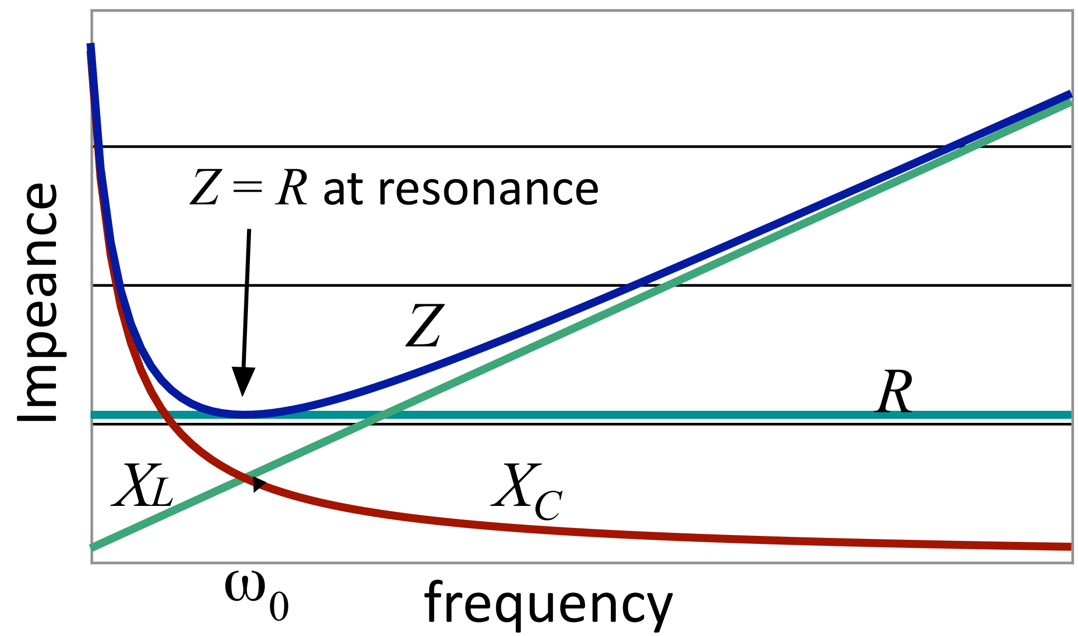
$$X_L = \omega L$$

X_C increases with $1/\omega$

$$X_C = \frac{1}{\omega C}$$

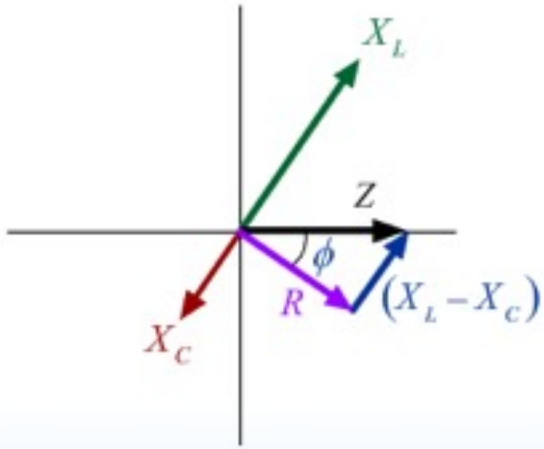
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

is minimum at resonance



Resonance: $X_L = X_C$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

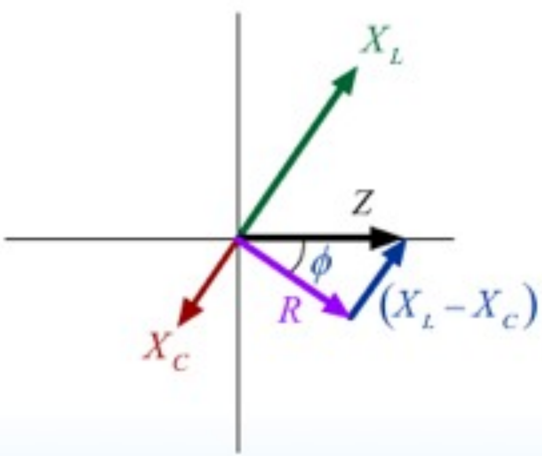
Off Resonance



$$I_m = \frac{\mathcal{E}_m}{Z}$$

$$I_m = \frac{\mathcal{E}_m}{R}\frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Z



$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C}$$

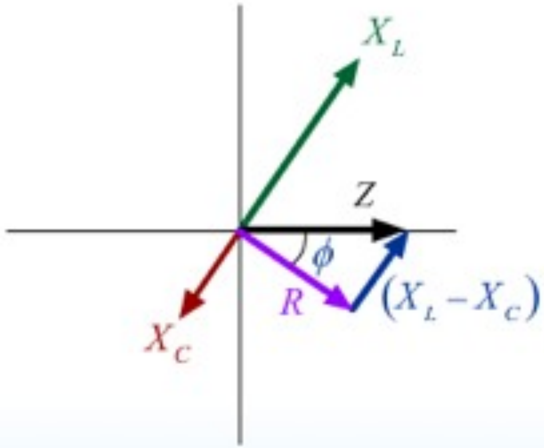
$$Q \equiv 2\pi \frac{U_{\max}}{\Delta U}$$

$$I_m = \frac{\mathcal{E}_m}{R}\frac{1}{\sqrt{1 + Q^2 \frac{(x^2 - 1)^2}{x^2}}}$$

U_{max} = max energy stored

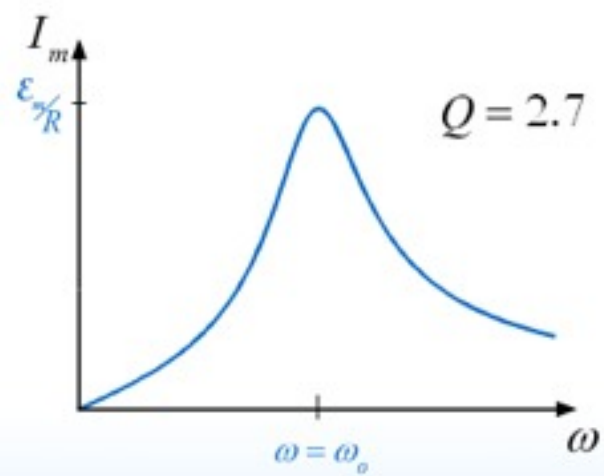
ΔU = energy dissipated

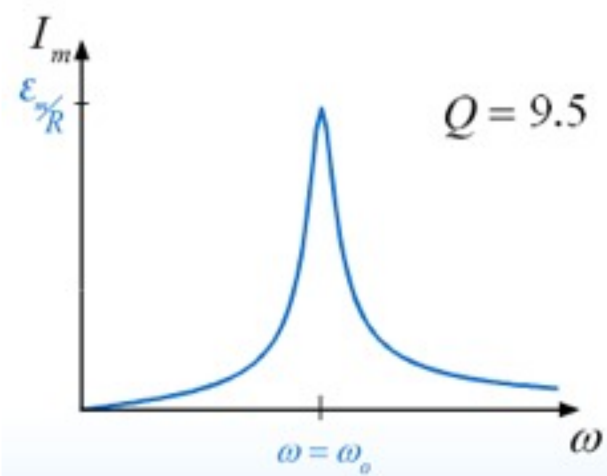
in one cycle at resonance



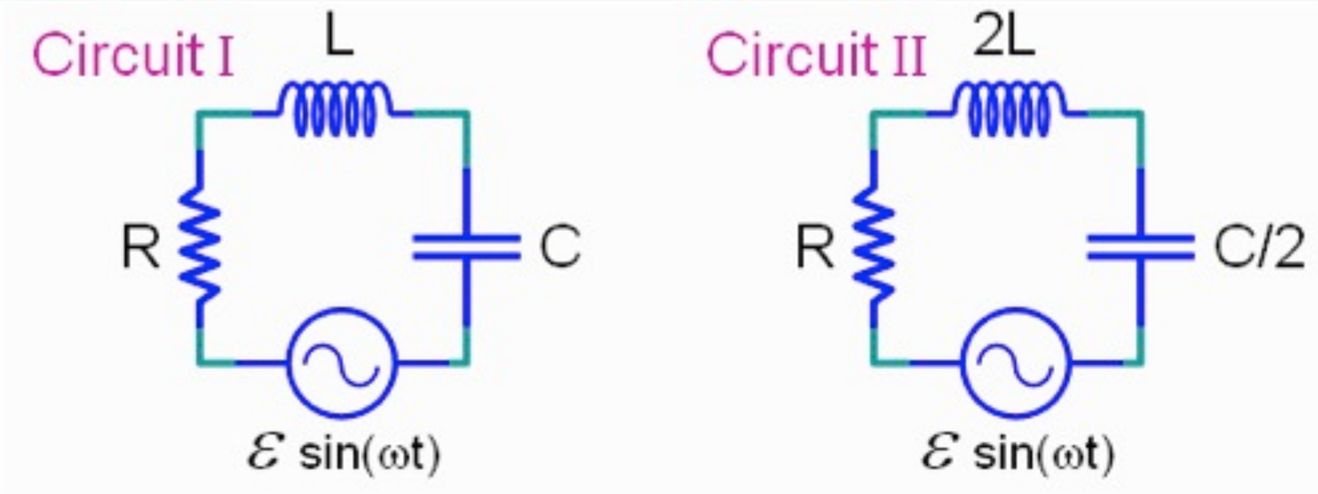
$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C}$$

$$I_m = \frac{\varepsilon_m}{R} \frac{1}{\sqrt{1 + Q^2 \frac{\left(x^2 - 1\right)^2}{x^2}}}$$





CheckPoint 2



Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

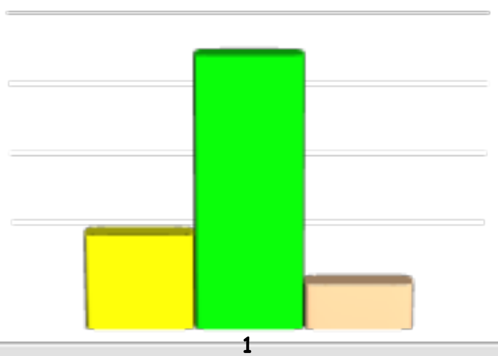
Compare the peak voltage across the resistor in the two circuits

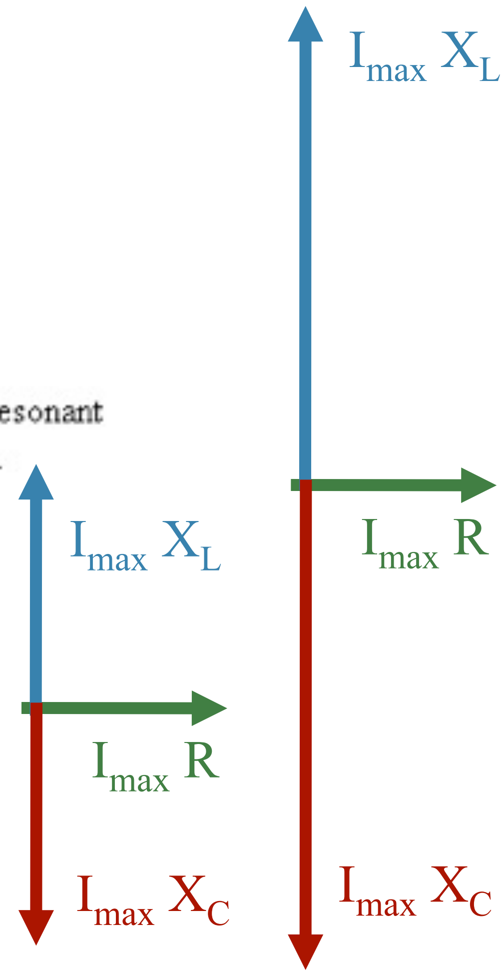
- $V_I > V_{II}$
- $V_I = V_{II}$
- $V_I < V_{II}$

Resonance: $X_L = X_C$

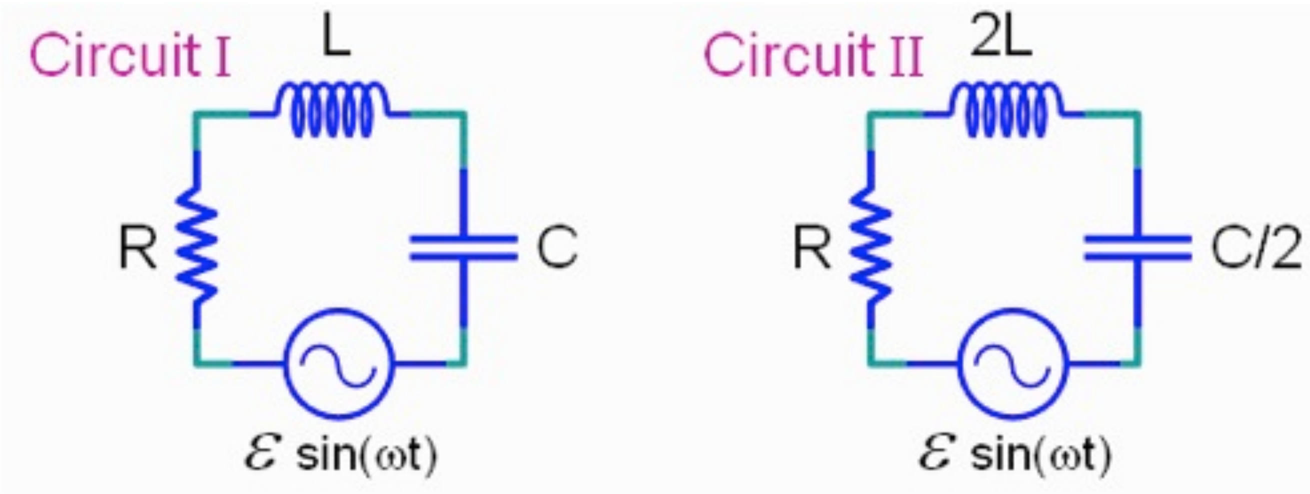
$Z = R$

Same since R doesn't change





CheckPoint 4

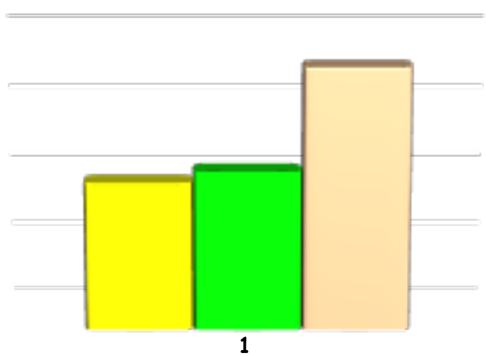


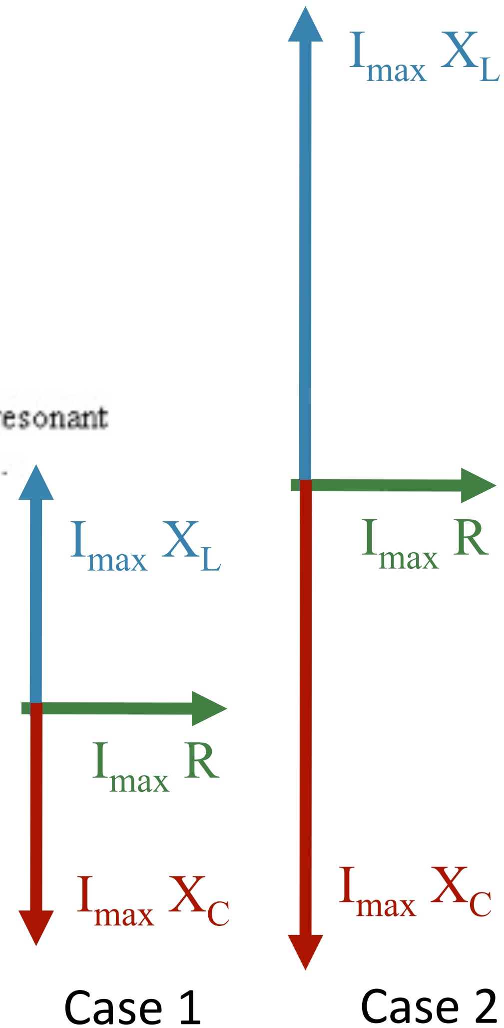
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

Compare the peak voltage across the inductor in the two circuits

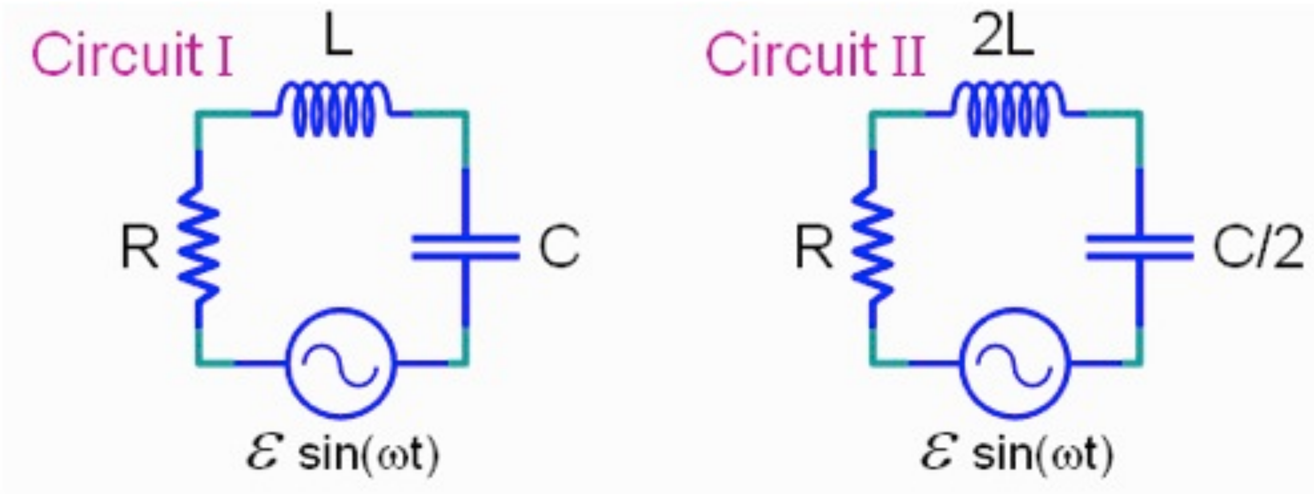
- ○ $V_I > V_{II}$
- ○ $V_I = V_{II}$
- ○ $V_I < V_{II}$

Voltage in second circuit will be twice that of the first because of the $2L$ compared to L.





CheckPoint 6

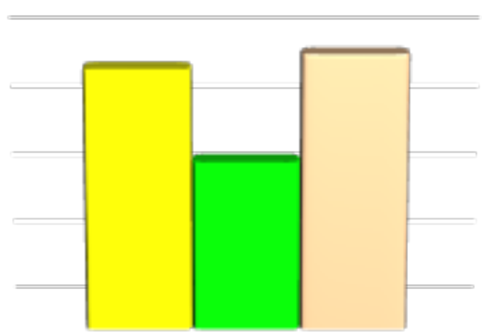


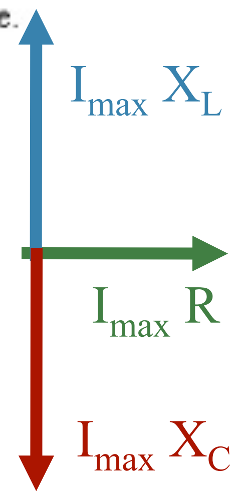
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

Compare the peak voltage across the capacitor in the two circuits

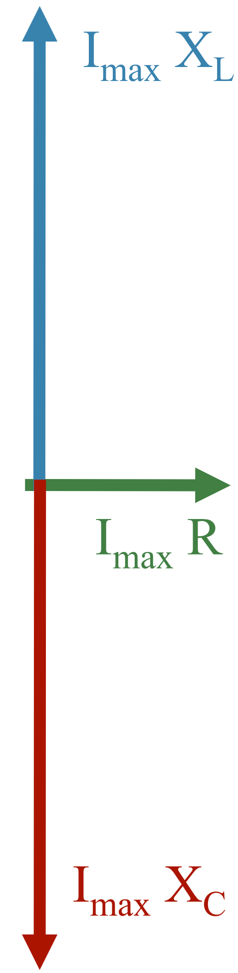
- ○ $V_I > V_{II}$
- ○ $V_I = V_{II}$
- ○ $V_I < V_{II}$

The peak voltage will be greater in circuit 2 because the value of X_C doubles.



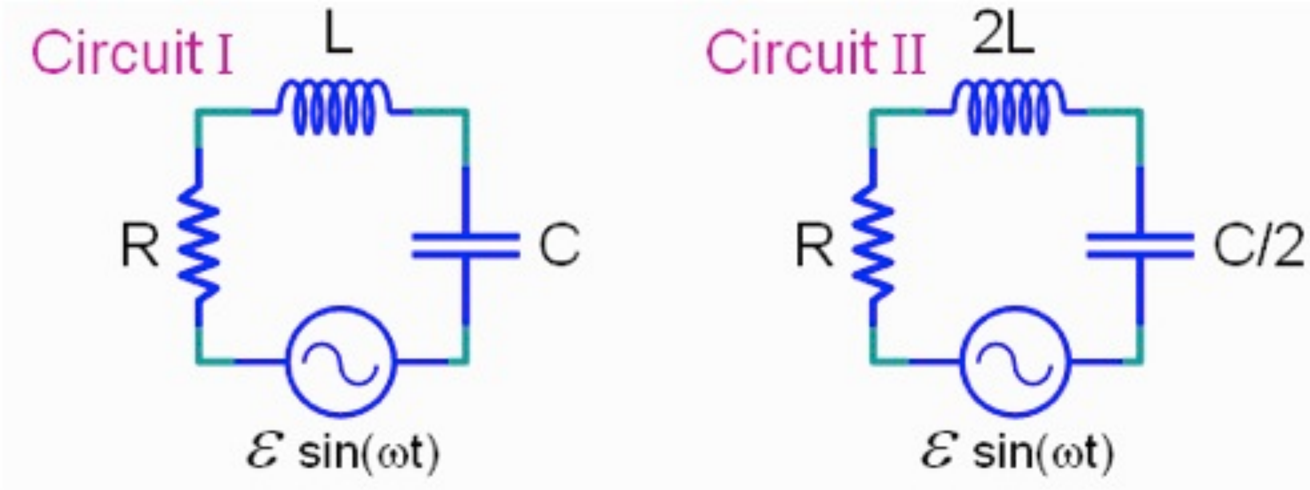


Case 1



Case 2

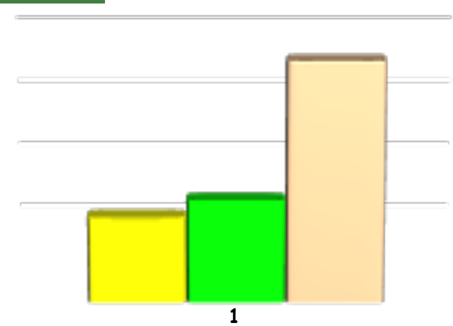
CheckPoint 8

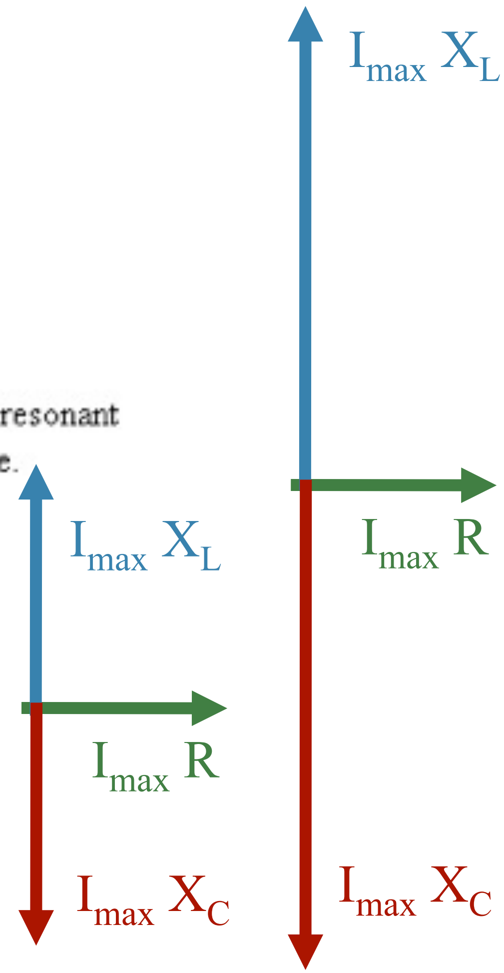


Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

At the resonant frequency, which of the following is true?

- ○ current leads voltage across the generator
- ○ current lags voltage across the generator
- ○ current is in phase with the voltage across the generator



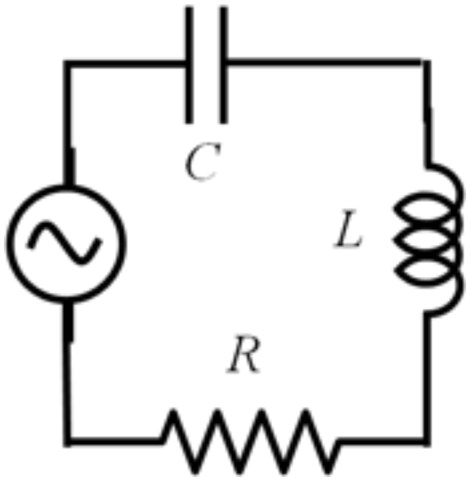


The voltage across the inductor and the capacitor are equal when at resonant frequency, so there is no lag or lead.

Power

$P = IV$ instantaneous always true

- Difficult for Generator, Inductor and Capacitor because of phase
- Resistor I, V are always in phase!



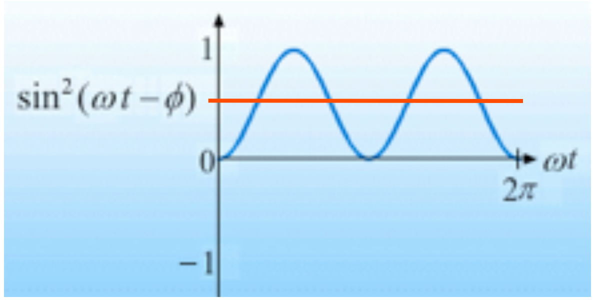
$$P = IV$$
$$= I^2 R$$

Average Power

Inductor and Capacitor = 0 ($< \sin\omega t \cos\omega t > = 0$)

Resistor

$<I^2R> = <I^2> R = ½ I^2_{peak} R$



RMS = Root Mean Square

$I_{peak} = I_{rms} \sqrt{2}$

→ $< I^2 R > = I_{rms}^{\ 2} R$

Power Line Calculation

If you want to deliver 1500 Watts at 100 Volts over transmission lines w/ resistance of 5 Ohms. How much power is lost in the lines?

- Current Delivered: $I = P/V = 15$ Amps
- Loss = IV (on line) = $I^2 R = 15 \times 15 \times 5 = 1125$ Watts!

If you deliver 1,500 Watts at 10,000 Volts over the same transmission lines. How much power is lost?

- Current Delivered: $I = P/V = .15$ Amps
- Loss = IV (on line) = $I^2 R = 0.125$ Watts

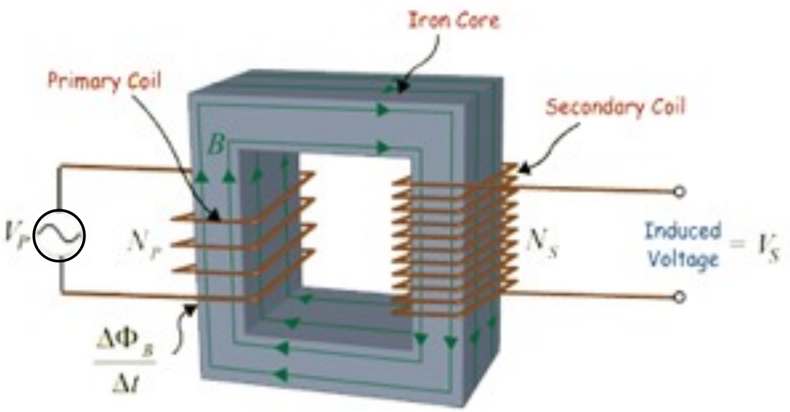
DEMO

Transformers

Application of Faraday's Law

- Changing EMF in Primary creates changing flux
- Changing flux, creates EMF in secondary

$$\frac{V_p}{N_p} = \frac{V_s}{N_s}$$



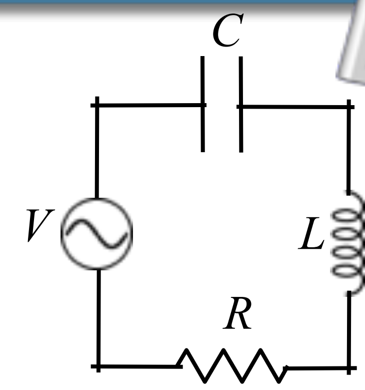
Efficient method to change voltage for AC.

Power Transmission   Loss = I^2R

Power electronics

Demo

Follow-Up from Last Lecture



Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2))

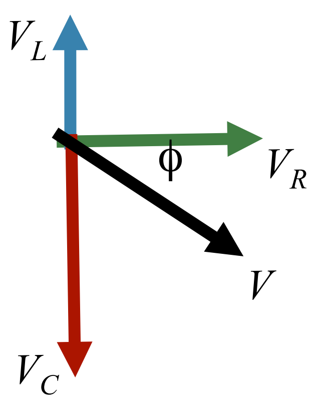
The current leads generator voltage by 45^o (cos = sin = $1/\sqrt{2}$)

L and R are unknown.

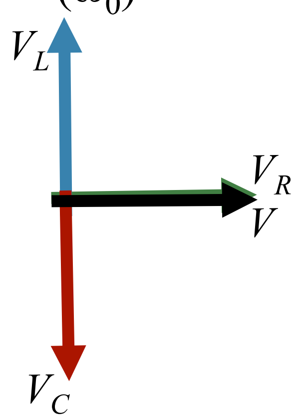
How should we change ω to bring circuit to resonance?

A) decrease ω B) increase ω C) Not enough info

Original ω



At resonance (ω_0)



At resonance $X_L = X_C$

↓

X_L increases

X_C decreases

→ ω increases

More Follow-Up

Consider the harmonically driven series LCR circuit shown.

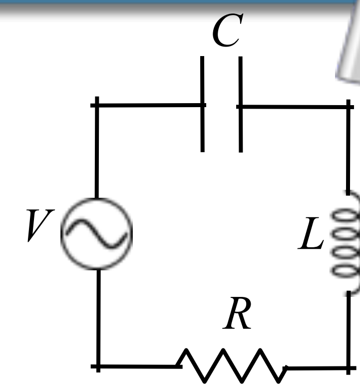
$V_{max} = 100\ V$

$I_{max} = 2\ mA$ → $X_C = 40\sqrt{2}\ k\Omega$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2))

The current leads generator voltage by 45^o (cos = sin = 1/sqrt(2))

L and R are unknown.



$R = 25\sqrt{2}\ k\Omega$

$X_L = 15\sqrt{2}\ k\Omega$

By what factor should we increase ω to bring circuit to resonance? i.e. if $\omega_0 = f\omega$, what is f?

A) $f = \sqrt{2}$ B) $f = 2\sqrt{2}$ C) $f = \sqrt{\frac{8}{3}}$ D) $f = \sqrt{\frac{8}{5}}$

If ω is increased by a factor of f:

X_L increases by factor of f

X_C decreases by factor of f

→ X_L ® $f \times 15\sqrt{2}$

X_C ® $(1/f) \times 40\sqrt{2}$

At resonance $X_L = X_C$ → $15f = \frac{40}{f}$ → $f^2 = \frac{40}{15}$ → $f = \sqrt{\frac{8}{3}}$

Current Follow-Up

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

$I_{max} = 2\ mA$

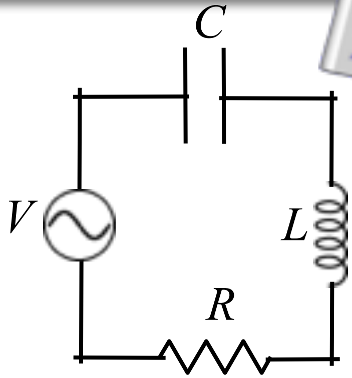
$V_{Cmax} = 113\ V$ (= 80 sqrt(2))

→ $X_C = 40\sqrt{2}\ k\Omega$

The current leads generator voltage by 45^o (cos = sin = 1/sqrt(2))

L and R are unknown.

What is the maximum current at resonance



$R = 25\sqrt{2}\ k\Omega$

$X_L = 15\sqrt{2}\ k\Omega$

$\omega_0 = \sqrt{\frac{8}{3}}\omega$

A) $I_{\max}(\omega_0) = \sqrt{2}\ mA$

B) $I_{\max}(\omega_0) = 2\sqrt{2}\ mA$

C) $I_{\max}(\omega_0) = \sqrt{\frac{8}{3}}\ mA$

At resonance
$X_L = X_C$ → $Z = R$ → $I_{\max}(\omega_0) = \frac{V_{\max}}{R} = \frac{100}{25\sqrt{2}} = 2\sqrt{2}\ mA$

Phasor Follow-Up

Consider the harmonically driven series LCR circuit shown.

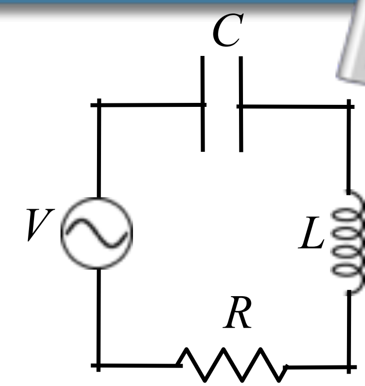
$V_{max} = 100\ V$

$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V\ (= 80\sqrt{2})$

The current leads generator voltage by 45^o $(\cos = \sin = 1/\sqrt{2})$

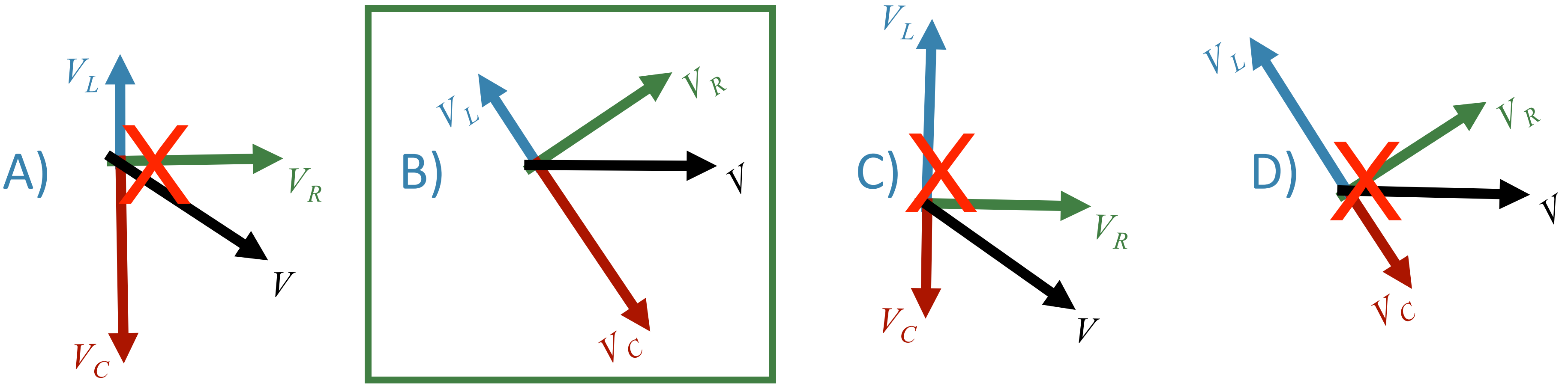
L and R are unknown.



$R = 25\sqrt{2}\ k\Omega$

$X_L = 15\sqrt{2}\ k\Omega$

What does the phasor diagram look like at t = 0? (assume $V = V_{max}\sin(\omega t)$)



$V = V_{max}\sin(\omega t) \Rightarrow V$ is horizontal at $t = 0$ $(V = 0)$

$\vec{V} = \vec{V}_L + \vec{V}_C + \vec{V}_R$ → $V_L < V_C$ if current leads generator voltage