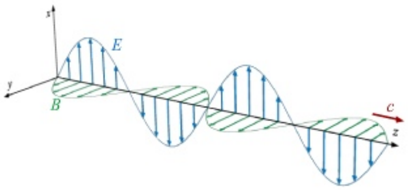
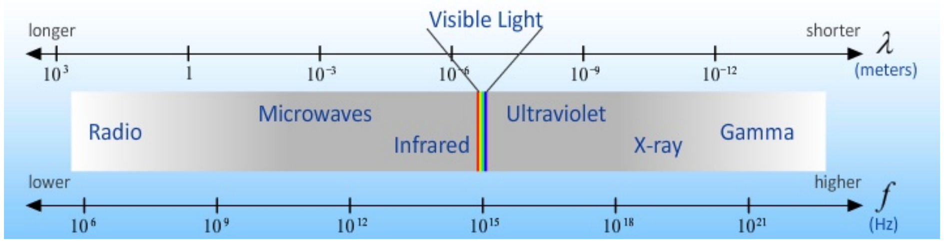
Electricity & Magnetism Lecture 23

PROPERTIES of ELECTROMAGNETIC WAVES



Electromagnetic Spectrum



Your Comments

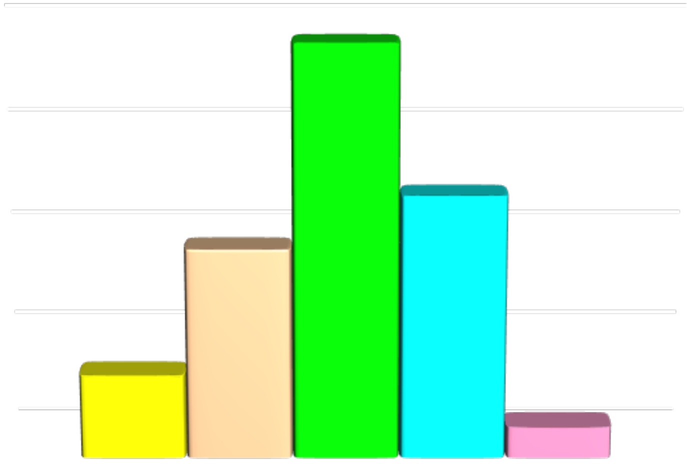
"the whole E-M wave graph, I still don't understand what it is trying to tell us and where the sin(kz-ωt) even comes from or how I'm even supposed to use it"

It certainly can be confusing..
We will try to make it clear !!

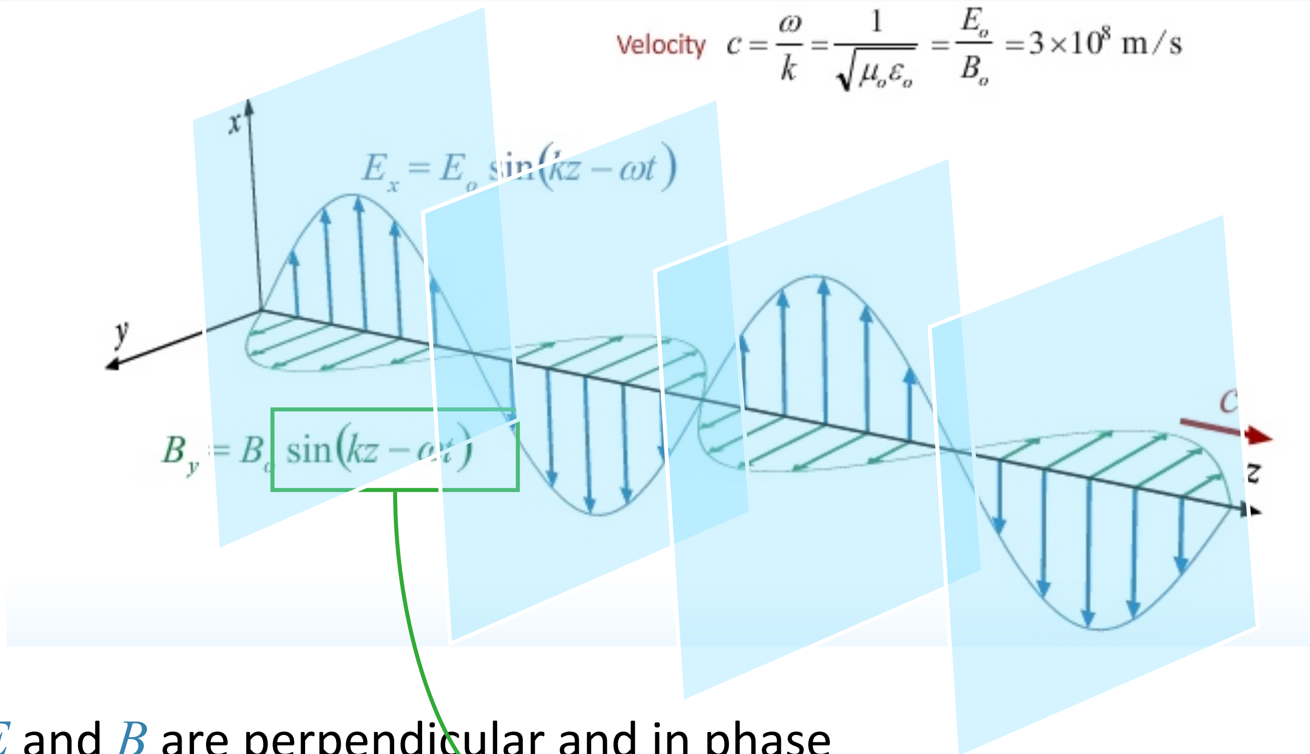
"The doppler thingy ma bobber."

We'll work an example
& discuss approximations

Since radio waves have long wavelengths does that mean the longer the wavelength, the longer it can travel, but the longer the wavelength, the less energy? How come waves with less energy travels longer?



Not a whole lot of difficult concepts. Would like to experience the challenge in physics for once.



E and B are perpendicular and in phase

Oscillate in time and space

Direction of propagation given by $E \times B$

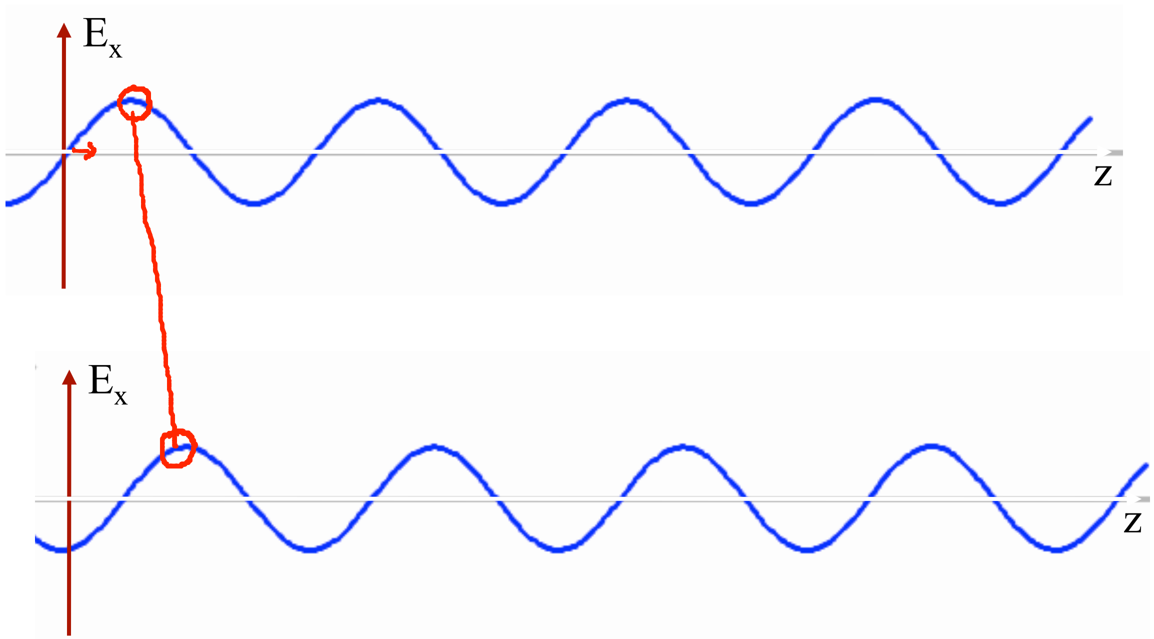
$E_0 = cB_0$

Argument of $\sin/\cos$ gives direction of propagation

Understanding the speed and direction of the wave

$$E_x = E_o \sin(kz - \omega t)$$

$t = 0$



$\sin(kz - \frac{\pi}{2}) = -\cos(kz)$

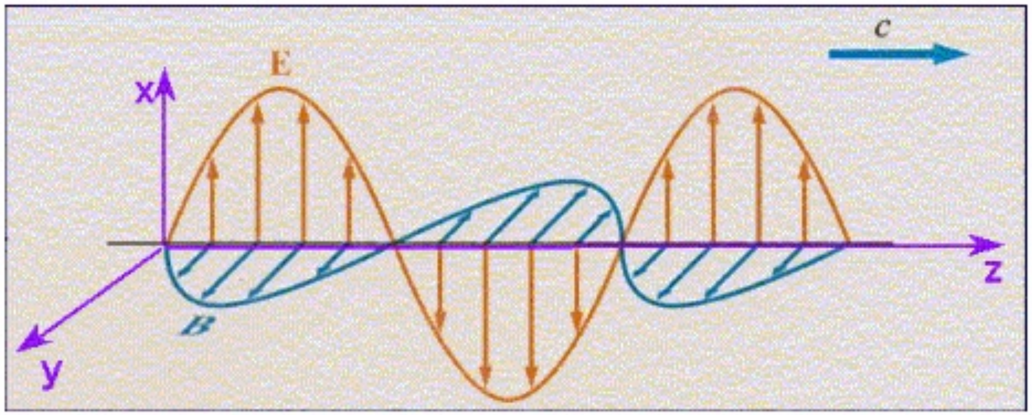
$t = \pi/2\omega$

What has happened to the wave form in this time interval?

It has MOVED TO THE RIGHT by $\lambda/4$

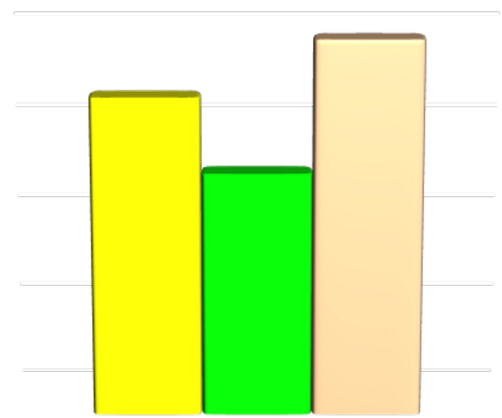
$$speed = c = \frac{\lambda/4}{\pi/2\omega} = \lambda\frac{\omega}{2\pi} = \lambda f$$

CheckPoint 2



Which equation correctly describes this electromagnetic wave?

- ○ $\mathbf{E}_x = \mathbf{E}_o \sin(kz + \omega t)$ No – moving in the minus z direction
- ○ $\mathbf{E}_y = \mathbf{E}_o \sin(kz - \omega t)$ No – has E_y rather than E_x
- ○ $\mathbf{B}_y = \mathbf{B}_o \sin(kz - \omega t)$



CheckPoint 6

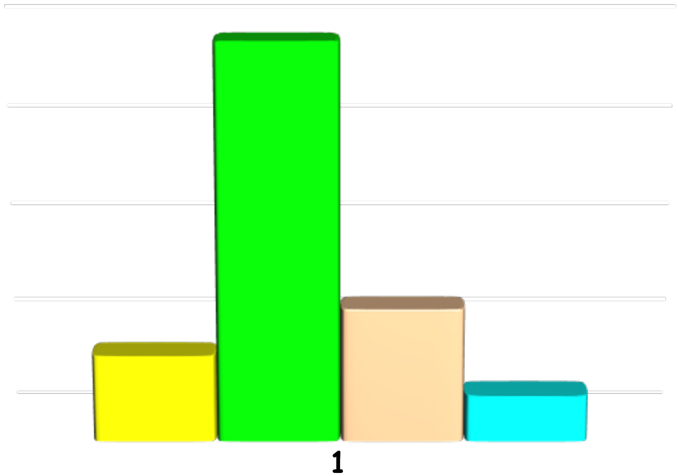
Your iclicker operates at a frequency of approximately 900 MHz (900×10^6 Hz). What is the approximately wavelength of the EM wave produced by your iclicker?

- ○ 0.03 meters
- ○ 0.3 meters
- ○ 3.0 meters
- ○ 30. meters

$C = 3.0 \times 10^8$ m/s

Wavelength is equal to the speed of light divided by the frequency.

$$\lambda = \frac{c}{f} = \frac{300{,}000{,}000}{900{,}000{,}000} = \frac{1}{3}$$



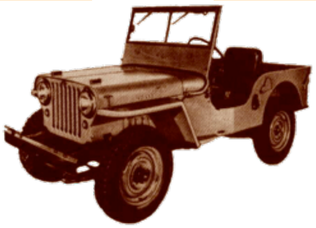
Check:

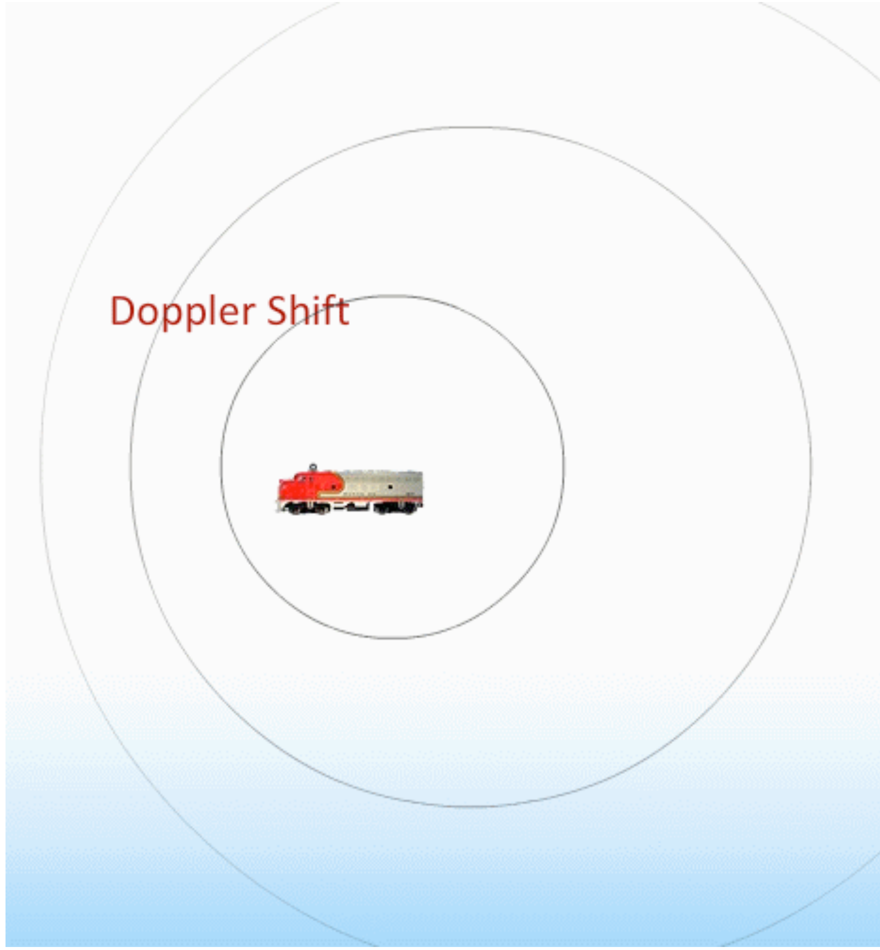
Look at size of antenna on base unit

Doppler Shift

Paris Ambulance

Dr Chai Acela





BigBang

The Big Idea

As source approaches:

Wavelength decreases

Frequency Increases

Doppler Shift for E-M Waves

What's Different from Sound or Water Waves ?

Sound /Water Waves :
You can calculate (no relativity needed)
BUT
Result is somewhat complicated: is source or observer moving wrt medium?

Electromagnetic Waves :
You need relativity (time dilation) to calculate
BUT
Result is simple: only depends on relative motion of source & observer

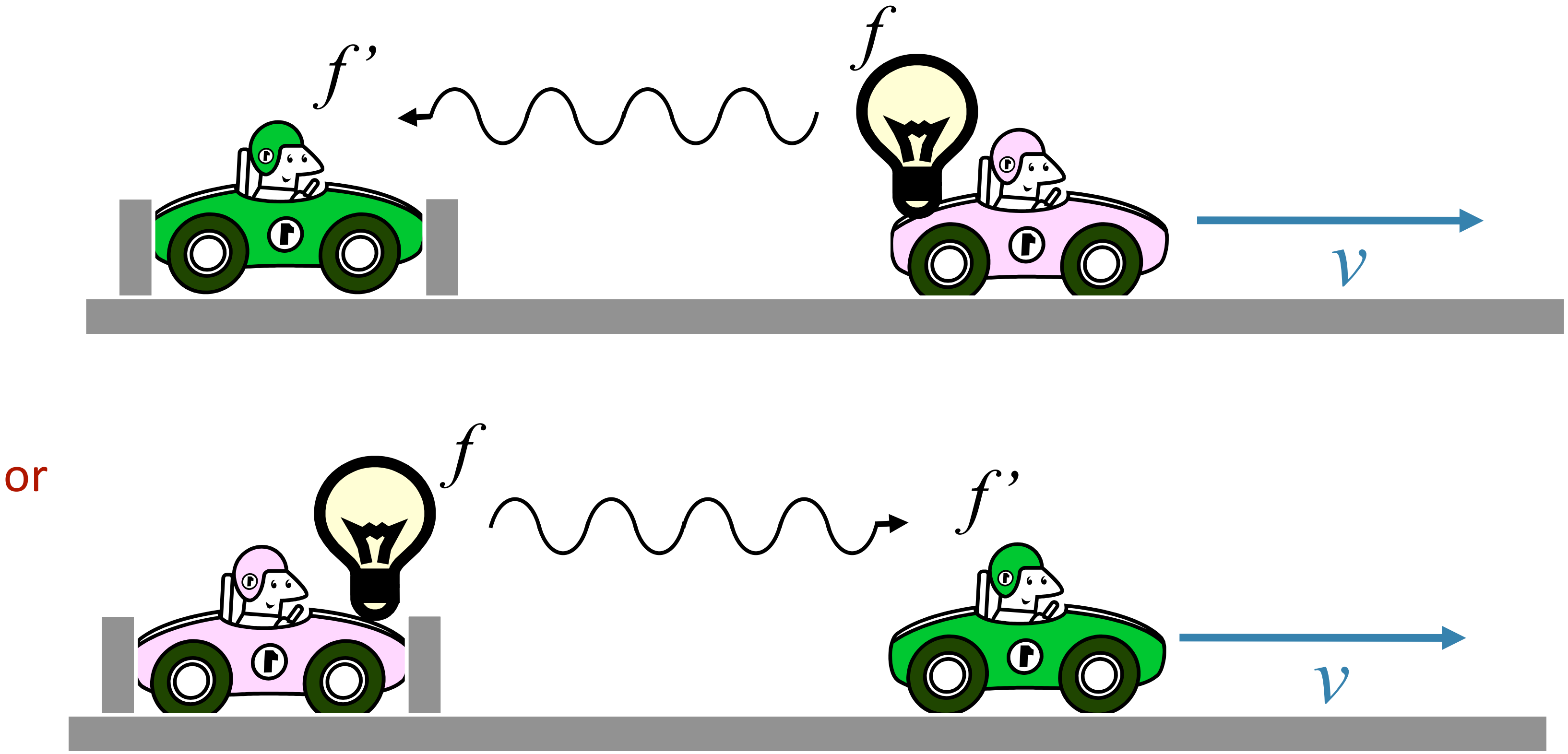
$$f' = f\sqrt{\frac{1+\beta}{1-\beta}}$$

$\beta = v/c$

$\beta > 0$ if source & observer are approaching

$\beta < 0$ if source & observer are separating

Doppler Shift for E-M Waves



The Doppler Shift is the SAME for both cases!

f'/f only depends on the relative velocity

$$f' = f\sqrt{\frac{1+\beta}{1-\beta}}$$

Doppler Shift for E-M Waves

A Note on Approximations

$$f' = f\sqrt{\frac{1+\beta}{1-\beta}}$$

$\beta << 1$ →

$$f' \approx f(1+\beta)$$

why?

Taylor Series: Expand $F(\beta) = \left(\frac{1+\beta}{1-\beta}\right)^{1/2}$ around $\beta = 0$

$$F(\beta) = F(0) + \frac{F'(0)}{1!}\beta + \frac{F''(0)}{2!}\beta^2 + \ldots$$

Evaluate:

$F(0) = 1$

$F'(0) = 1$

→

$$F(\beta) \approx 1 + \beta$$

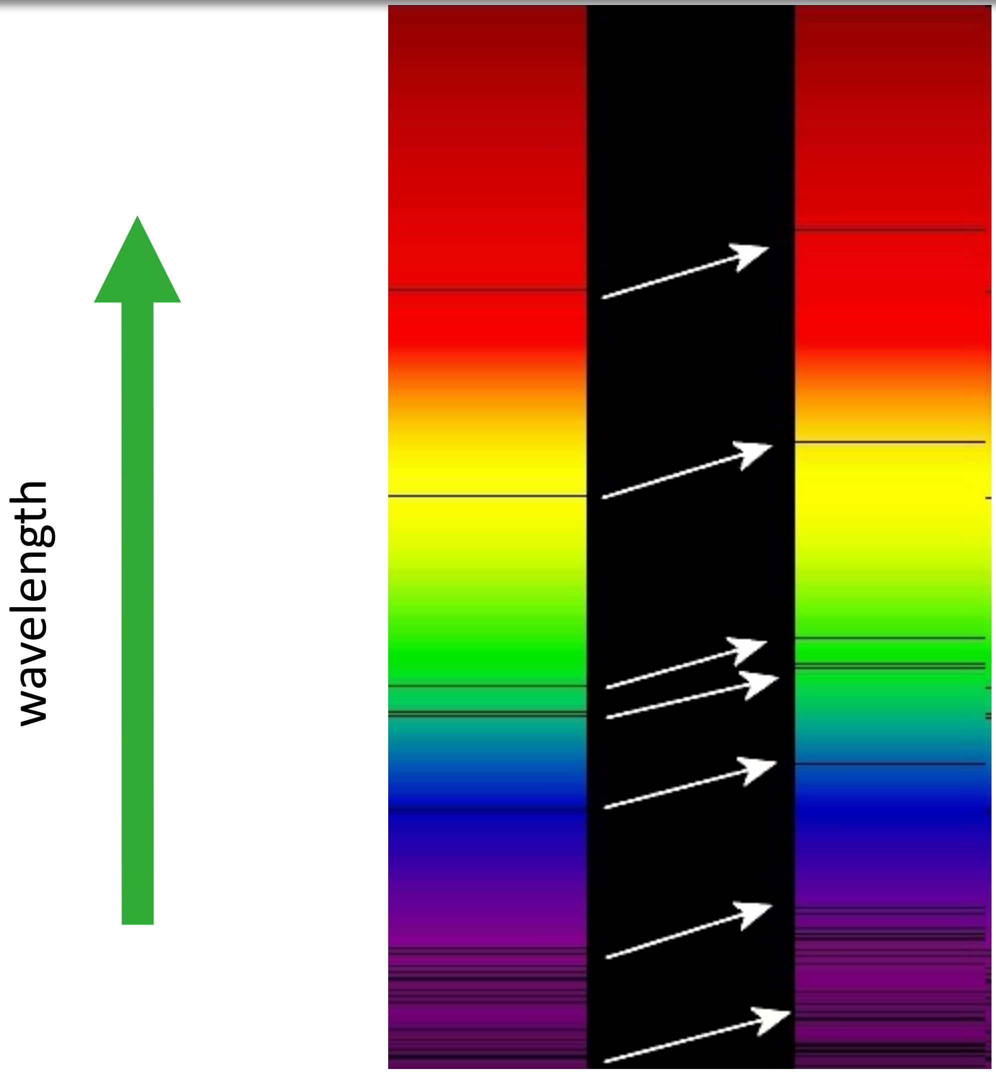
NOTE:

$$F(\beta) = (1+\beta)^{1/2}$$

↓

$$F(\beta) \approx 1 + \frac{1}{2}\beta$$

Red Shift of Stellar Spectra



Wavelengths shifted higher

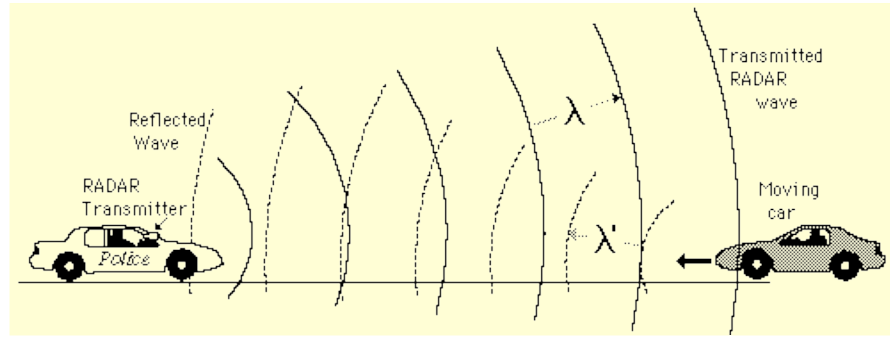
Frequencies shifted lower

Star separating from us
(Expanding Universe)

Our Sun

Star in a distant galaxy

Example



Police radars get twice the effect since the EM waves make a round trip:

$$f' \approx f(1 + 2\beta)$$

If f = 24,000,000,000 Hz (k-band radar gun)

c = 300,000,000 m/s

v	β	f'	$f'-f$
30 m/s (108 km/h)	1.000×10^{-7}	24,000,004,800	4800 Hz
31 m/s (112 km/h)	1.033×10^{-7}	24,000,004,959	4959 Hz

Waves Carry Energy

Total Energy Density

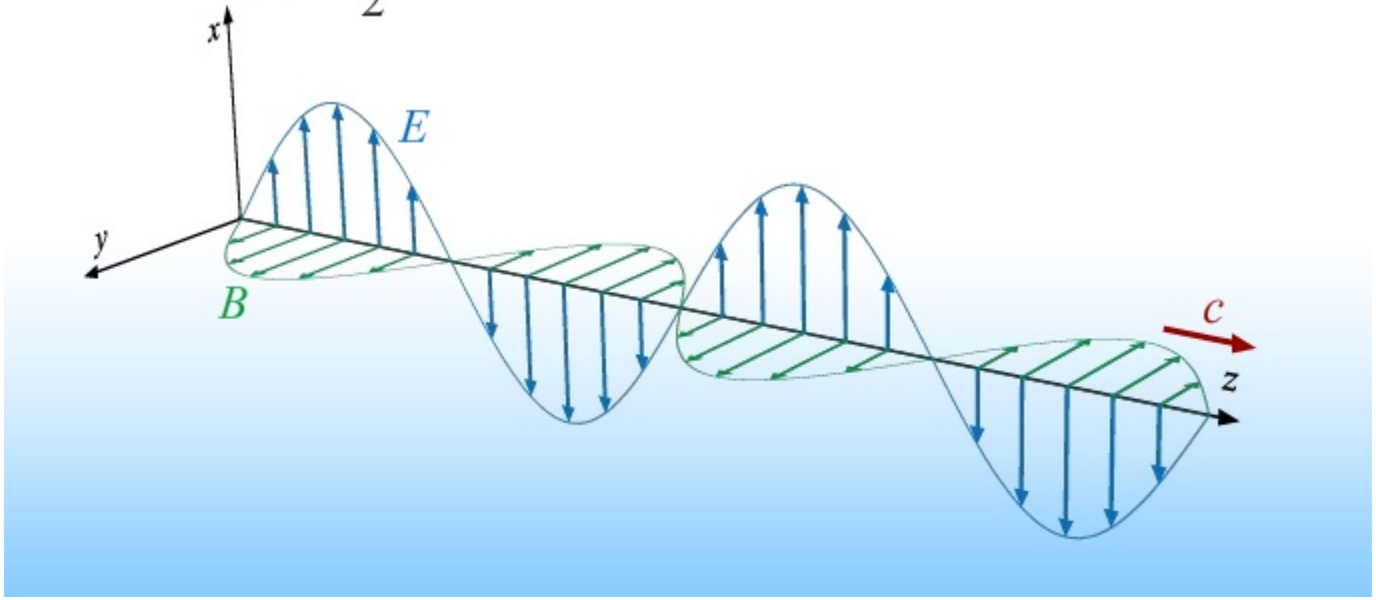
$$u = \varepsilon_o E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o{}^2$$

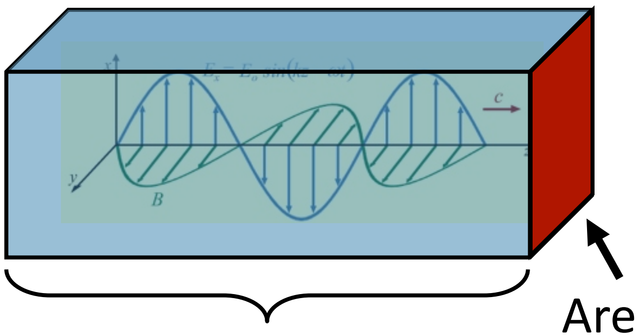
Intensity

$$I = \frac{1}{2} c\,\varepsilon_o E_o{}^2 = c\,\langle u \rangle$$



Intensity

Intensity = Average energy delivered per unit time, per unit area



Area = A

Length = $c\ dt$

$$I \equiv \frac{1}{A}\left\langle \frac{dU}{dt} \right\rangle$$

$$\langle dU \rangle = \langle u \rangle \times volume = \langle u \rangle A c dt$$

Total Energy Density

$$u = \varepsilon_o E^2$$

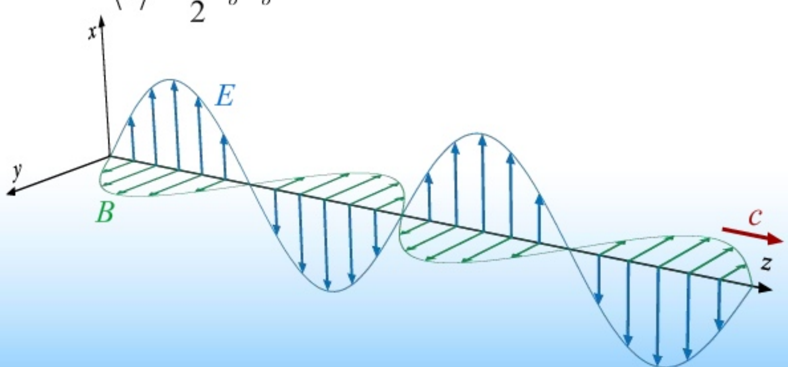
Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\ 2}$$

Intensity

$$I = \frac{1}{2} c \varepsilon_o E_o^{\ 2} = c \langle u \rangle$$

$$I = c\langle u \rangle$$



Sunlight on Earth:

$I \sim 1000\ \mathrm{J/s/m^2}$

$\sim 1\ \mathrm{kW/m^2}$

Waves Carry Energy

Total Energy Density

$$u = \varepsilon_o E^2$$

Average Energy Density

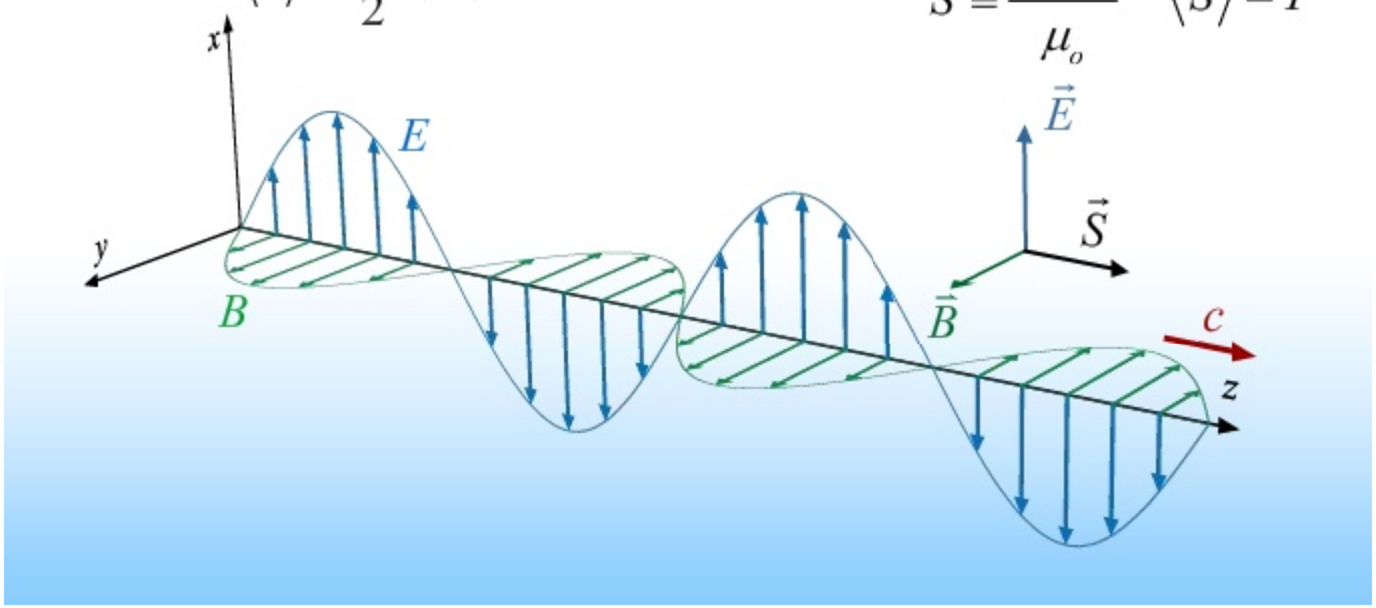
$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\ 2}$$

Intensity

$$I = \frac{1}{2} c \varepsilon_o E_o^{\ 2} = c\,\langle u \rangle$$

Poynting Vector

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_o} \qquad \langle S \rangle = I$$

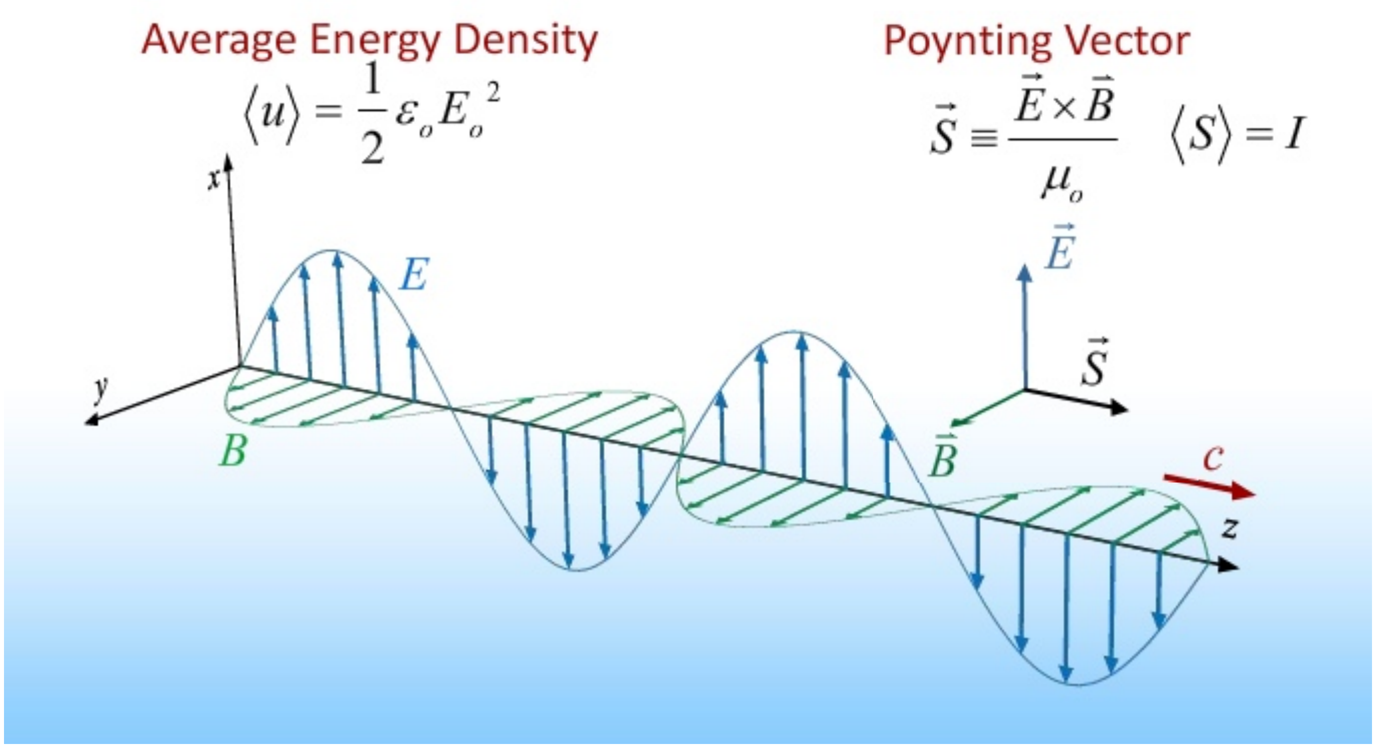


Comment on Poynting Vector

Just another way to keep track of all this:

Its magnitude is equal to I

Its direction is the direction of propagation of the wave



Light has Momentum!

If it has energy and its moving, then it also has momentum:

Analogy from mechanics:

$$E = \frac{p^2}{2m}$$

$$\frac{dE}{dt} = \frac{2p}{2m}\frac{dp}{dt} = \frac{mv}{m}\frac{dp}{dt} = vF$$

For $E - M$ waves: $\frac{dE}{dt} \rightarrow \frac{dU}{dt} = IA$

$v \rightarrow c$

$$IA = cF$$

$$\frac{I}{c} = \frac{F}{A}$$ pressure

$$P = \frac{I}{c}$$

Radiation pressure

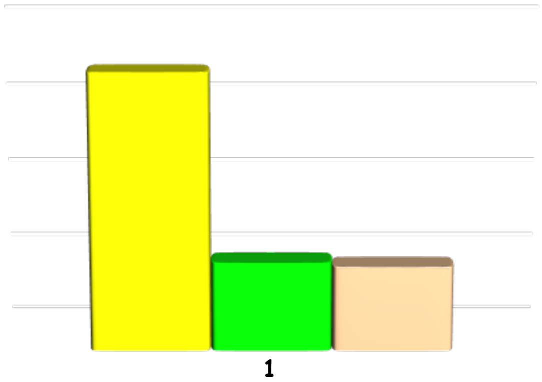
CheckPoint 4

An electromagnetic wave has electric field amplitude E, wavelength λ, and frequency ω. Which should we increase if we want the energy carried by the wave to increase (you can mark more than one answer).

- ☐ E
- ☐ λ
- ☐ ω

Intensity

$$I = \frac{1}{2} c \varepsilon_o E_o^{\,2}$$



But then again, what are we keeping constant here?

WHAT ABOUT PHOTONS?

Photons

We believe the energy in an e-m wave is carried by photons

Question: What are Photons?

Answer: Photons are Photons.

Photons possess both wave and particle properties

Particle:

Energy and Momentum localized

Wave:

They have definite frequency & wavelength $(f\lambda = c)$

Connections seen in equations:

$$E = hf$$

$$p = h/\lambda$$

Planck's constant

$$h = 6.63\times10^{-34}\ J\cdot s$$

Question: How can something be both a particle and a wave?

Answer: It can't (when we observe it)

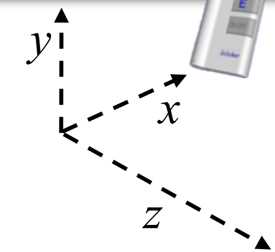
What we see depends on how we choose to measure it!

The mystery of quantum mechanics: More on this in PHYS 285

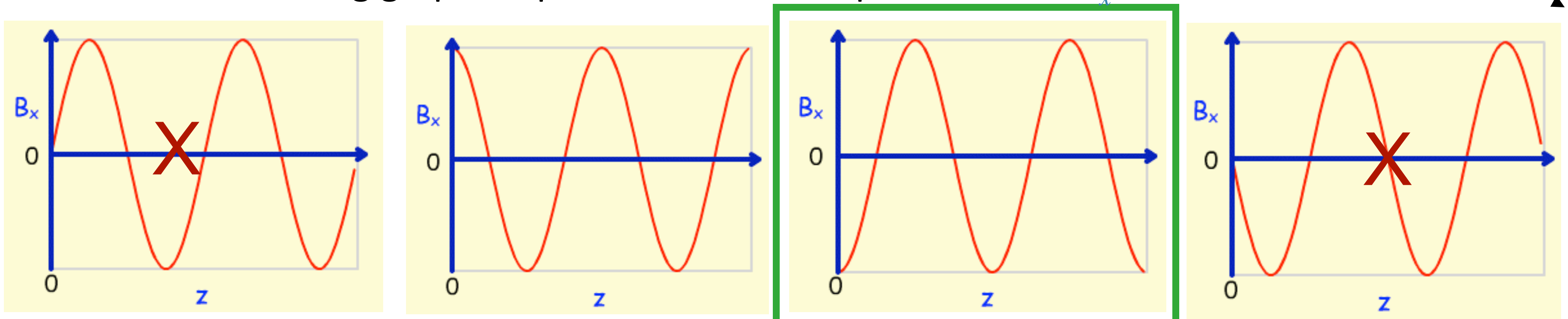
Exercise

An electromagnetic wave is described by: $\vec{E} = \hat{j}E_0\cos(kz - \omega t)$

where $\hat{j}$ is the unit vector in the $+y$ direction.



Which of the following graphs represents the z – dependence of B_x at $t = 0$?

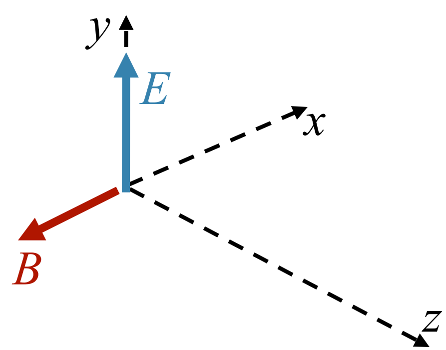


A B C D

E and B are "in phase" (or 180^o out of phase)

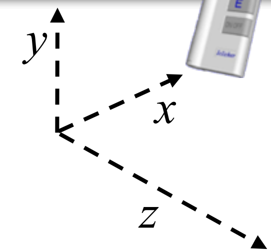
$\vec{E} = \hat{j}E_0\cos(kz - \omega t)$ → Wave moves in $+z$ direction

$\vec{E} \times \vec{B}$ Points in direction of propagation



$\vec{B} = -\hat{i}B_0\cos(kz - \omega t)$

Exercise



An electromagnetic wave is described by: $$\vec{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$$

What is the form of B for this wave?

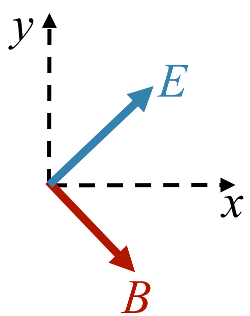
A) $\vec{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

B) $\vec{B} = \frac{\hat{i} - \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

C) $\vec{B} = \frac{-\hat{i} + \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

D) $\vec{B} = \frac{-\hat{i} - \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

$$\vec{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$$ → Wave moves in $-z$ direction



$+z$ points out of screen

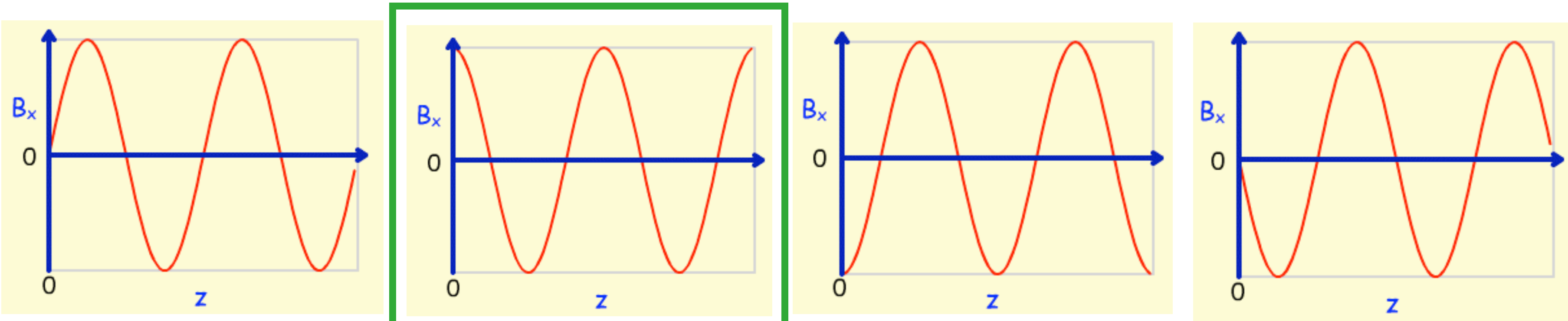
$-z$ points into screen

$\vec{E} \times \vec{B}$ Points in direction of propagation

Exercise

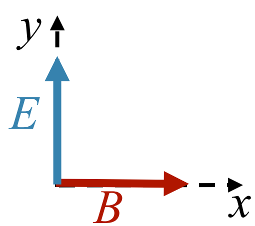
An electromagnetic wave is described by: $\vec{E} = \hat{j}E_0 \sin(kz + \omega t)$

Which of the following plots represents $B_x(z)$ at time $t = \pi/2\omega$?



A B C D

Wave moves in negative z – direction → $\vec{B} = \hat{i}(E_0/c)\sin(kz + \omega t)$



$+z$ points out of screen

$-z$ points into screen

at $\omega t = \pi/2$:

$$B_x = (E_0/c)\sin(kz + \pi/2)$$

$$B_x = (E_0/c)\{\sin kz \cos(\pi/2) + \cos kz \sin(\pi/2)\}$$

$$B_x = (E_0/c)\cos(kz)$$

$\vec{E} \times \vec{B}$ Points in direction of propagation

Exercise

A certain unnamed physics professor was arrested for running a stoplight. He said the light was green. A pedestrian said it was red. The professor then said: “We are both being truthful; you just need to account for the Doppler effect !”

Is it possible that the professor’s argument is correct?

$$(\lambda_{green} = 500\ nm,\ \ \lambda_{red} = 600\ nm)$$

A) YES B) NO

As professor approaches stoplight, the frequency of its emitted light will be shifted UP

The speed of light does not change

Therefore, the wavelength (c/f) would be shifted DOWN

If he goes fast enough, he could observe a green light !

Follow-Up

A certain unnamed physics professor was arrested for running a stoplight. He said the light was green. A pedestrian said it was red. The professor then said: "We are both being truthful; you just need to account for the Doppler effect !"

How fast would the professor have to go to see the light as green?

$$(\lambda_{green} = 500\ nm,\ \ \lambda_{red} = 600\ nm)$$

A) $540\ m/s$ B) $5.4 \times 10^4\ m/s$ **C) $5.4 \times 10^7\ m/s$** D) $5.4 \times 10^8\ m/s$

Relativistic Doppler effect: $f' = f\sqrt{\dfrac{1+\beta}{1-\beta}}$

$$\frac{f'}{f} = \frac{600}{500} = \sqrt{\frac{1+\beta}{1-\beta}} \rightarrow 36(1-\beta) = 25(1+\beta) \rightarrow \beta = \frac{11}{61} = 0.18$$

Note approximation for small β is not bad: $f' = f(1+\beta) \rightarrow \beta = \dfrac{1}{5} = 0.2$

$c = 3 \times 10^8\ m/s$ Þ $v = 5.4 \times 10^7\ m/s$ → Change the charge to speeding!