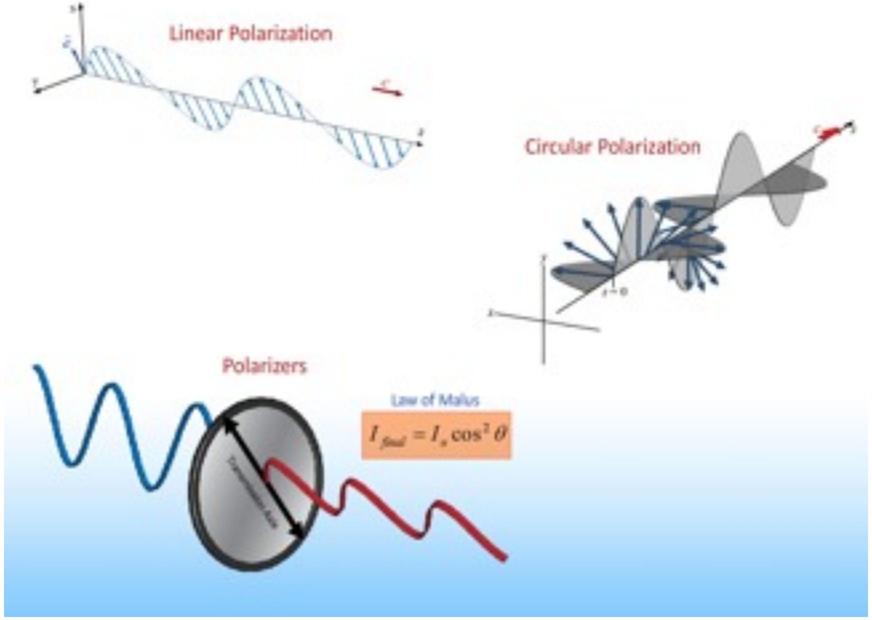
Electricity&Magnetism
Lecture 24



smartPhysics

Optics Kit

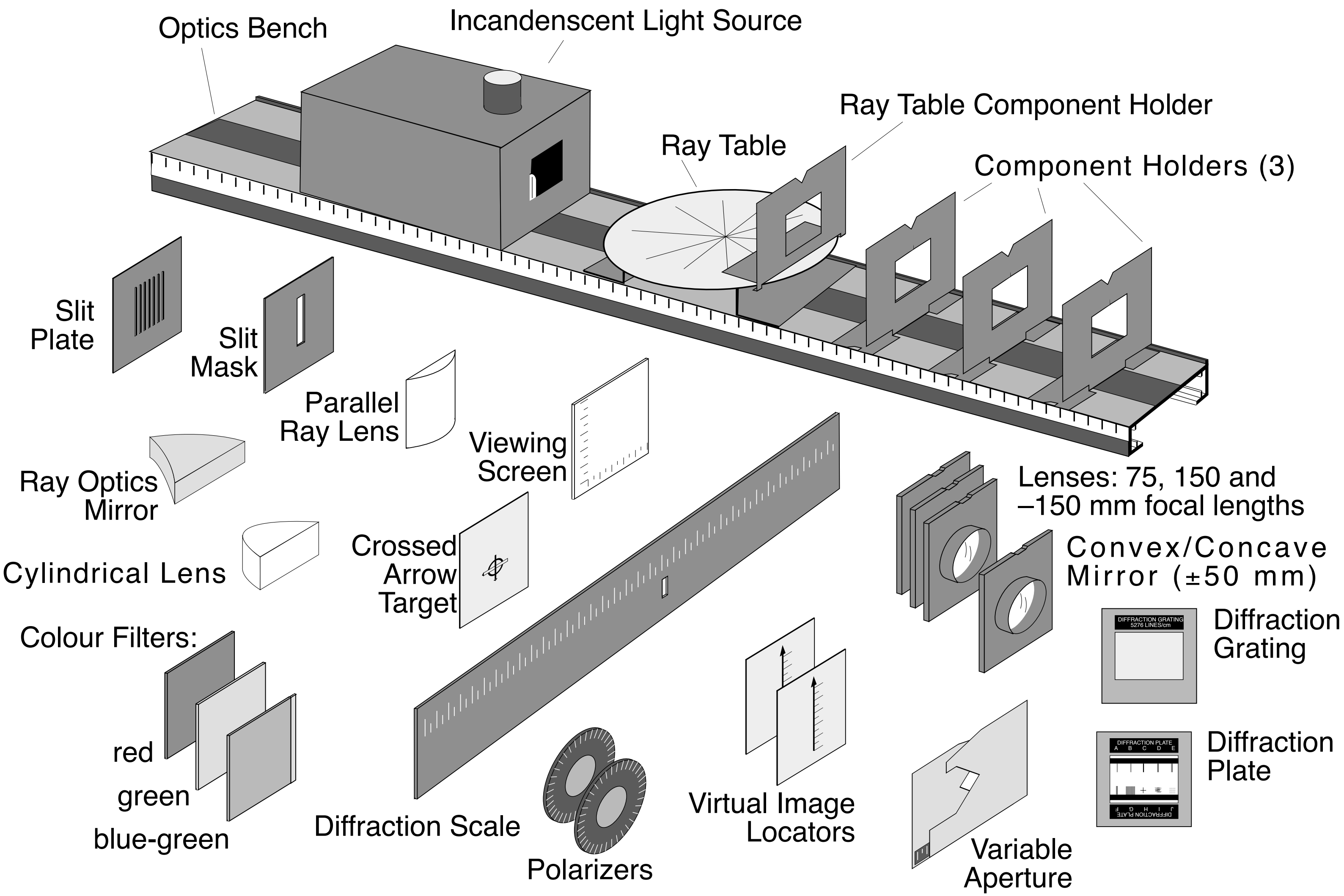


Figure 1: Equipment included in the OS-8500 Introductory Optics System

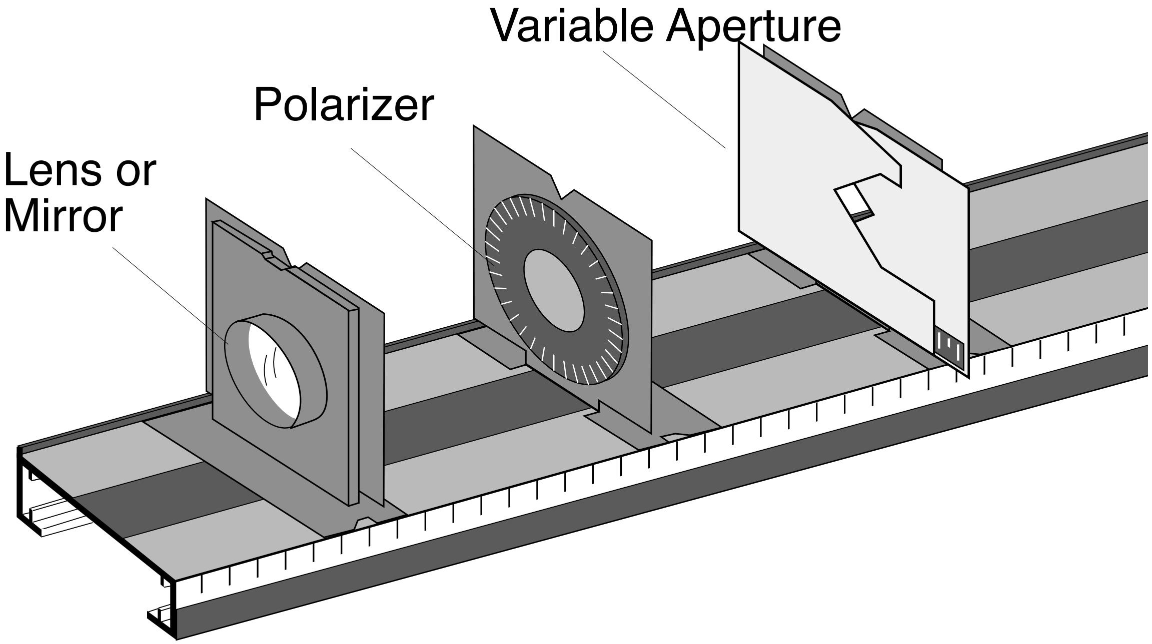
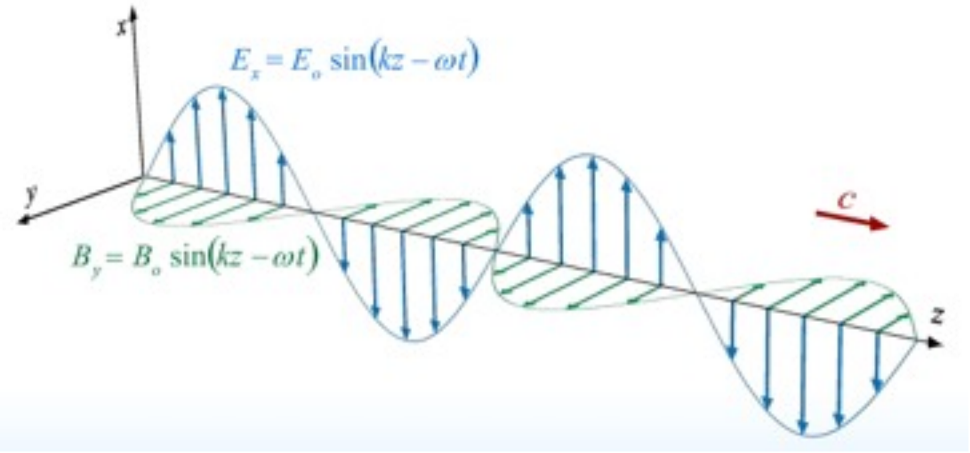


Figure 5: Using The Component Holder

So far we have considered plane waves that look like this:



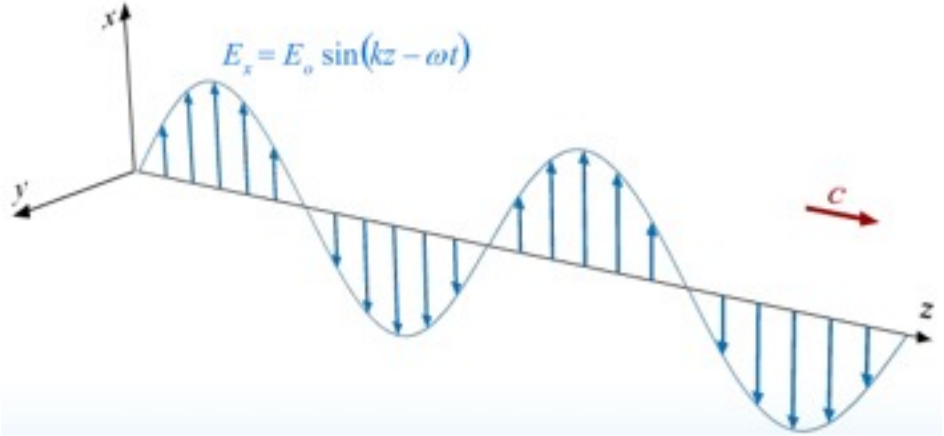
$$\omega = kc$$

$$E_o = cB_o$$

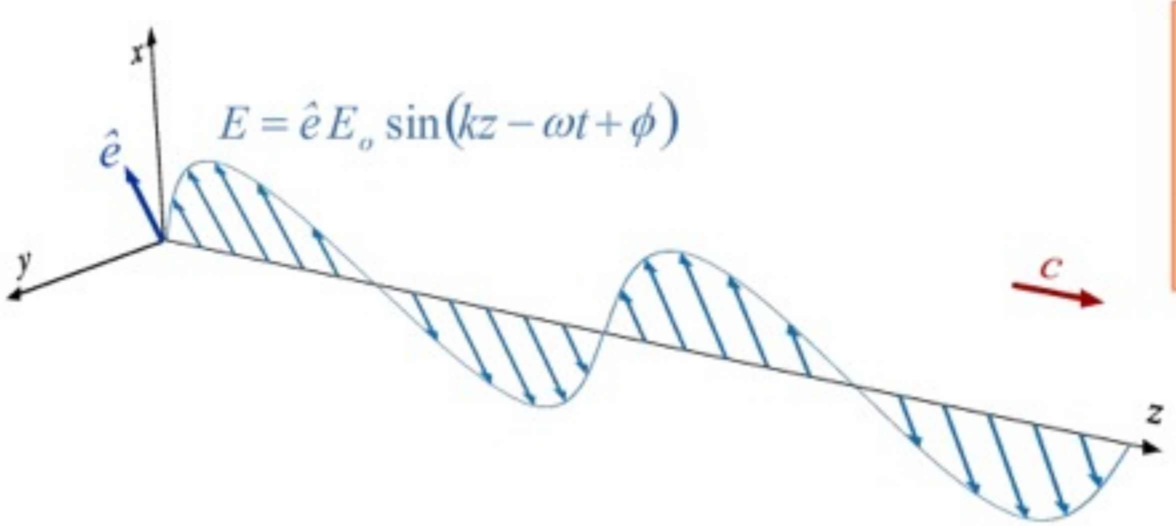
$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

From now on just draw E and remember that B is still there:

$\vec{E}$ Field determines Polarization



Linear Polarization



$$\omega = kc$$

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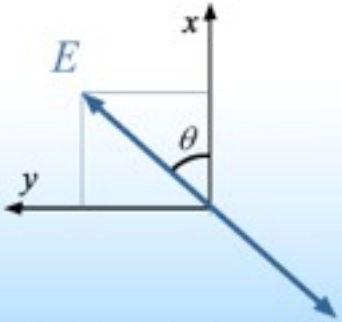
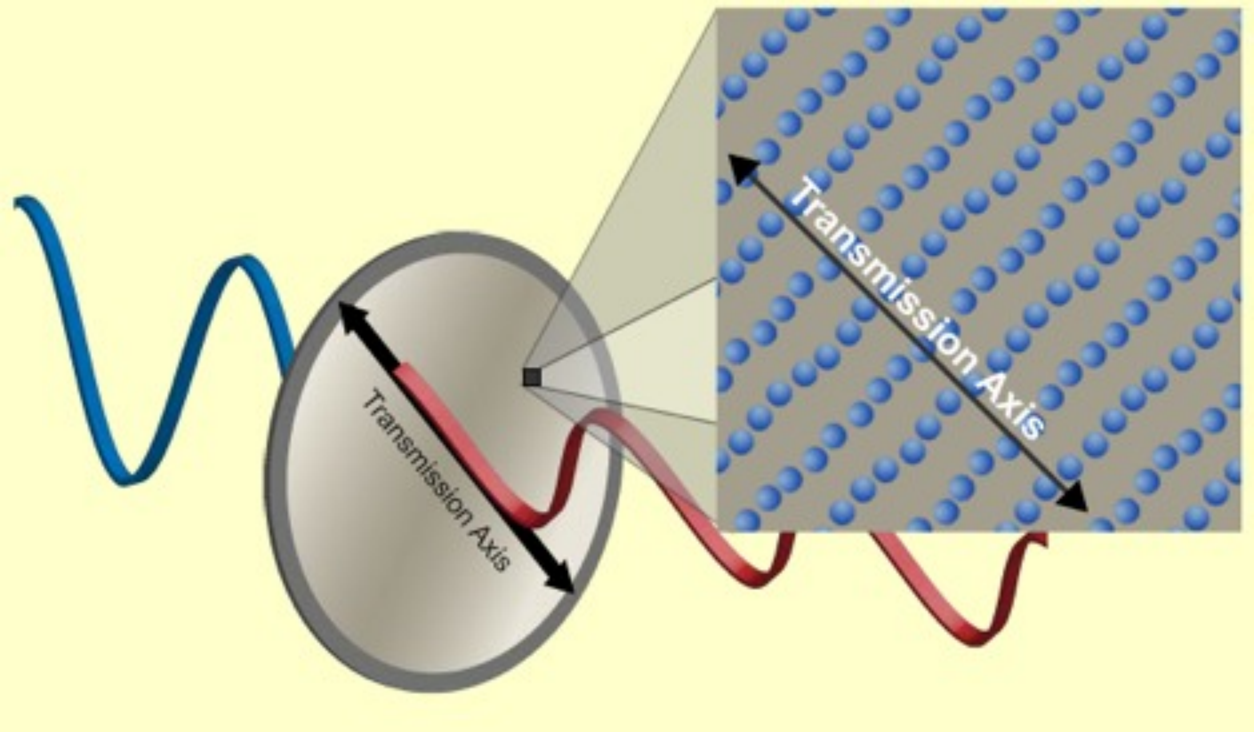


Wave Coming Out of the Screen

$$E_x = E_o \cos\theta \sin(kz - \omega t + \phi)$$

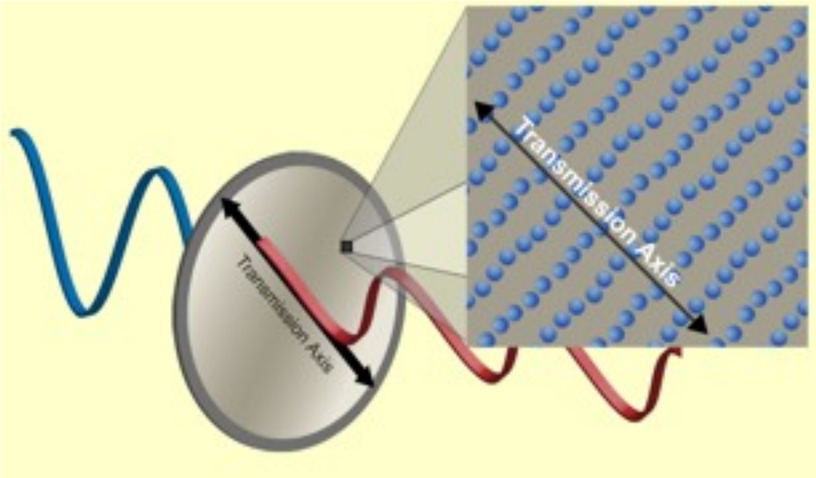
$$E_y = E_o \sin\theta \sin(kz - \omega t + \phi)$$

Polarizer

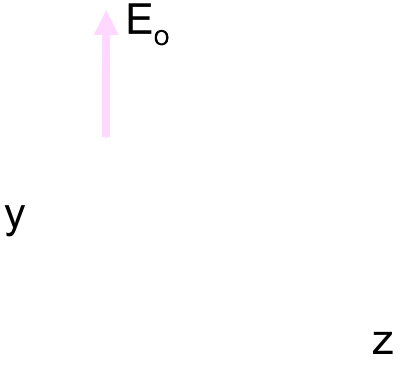


The molecular structure of a polarizer causes the component of the E field perpendicular to the Transmission Axis to be absorbed.

Clicker Question



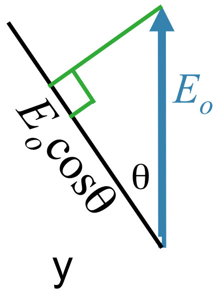
The molecular structure of a polarizer causes the component of the E field perpendicular to the Transmission Axis to be absorbed.

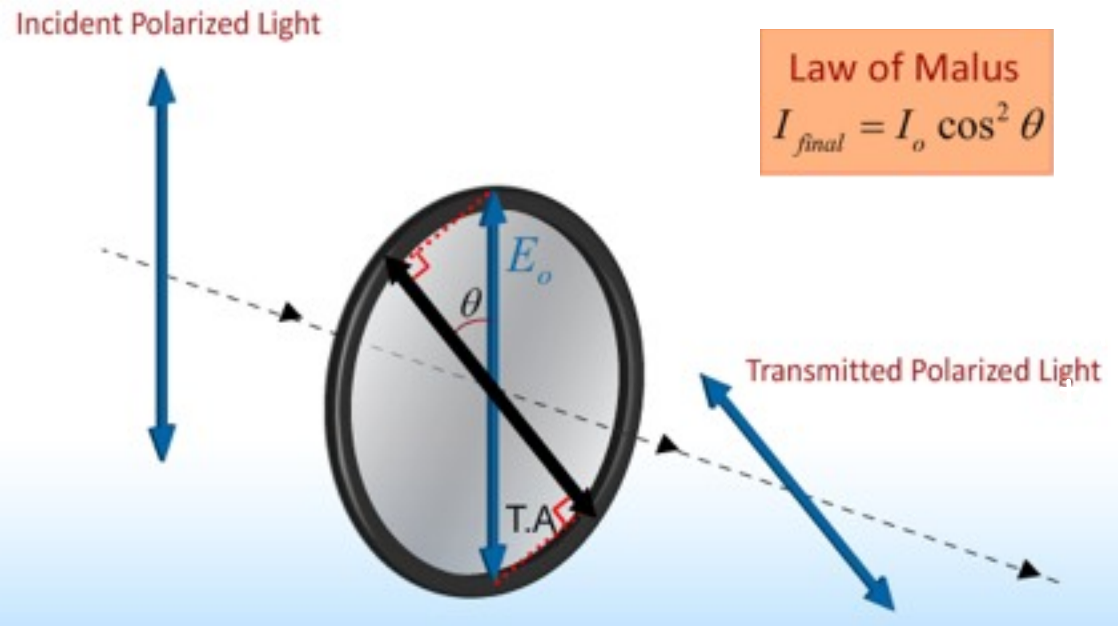


Suppose we have a beam traveling in the $+z$ direction.
At $t = 0$ and $z = 0$, the electric field is aligned along the positive x axis and has a magnitude equal to E_o

What is the component of E_o along a direction in the $x - y$ plane that makes an angle of θ with respect to the x – axis?

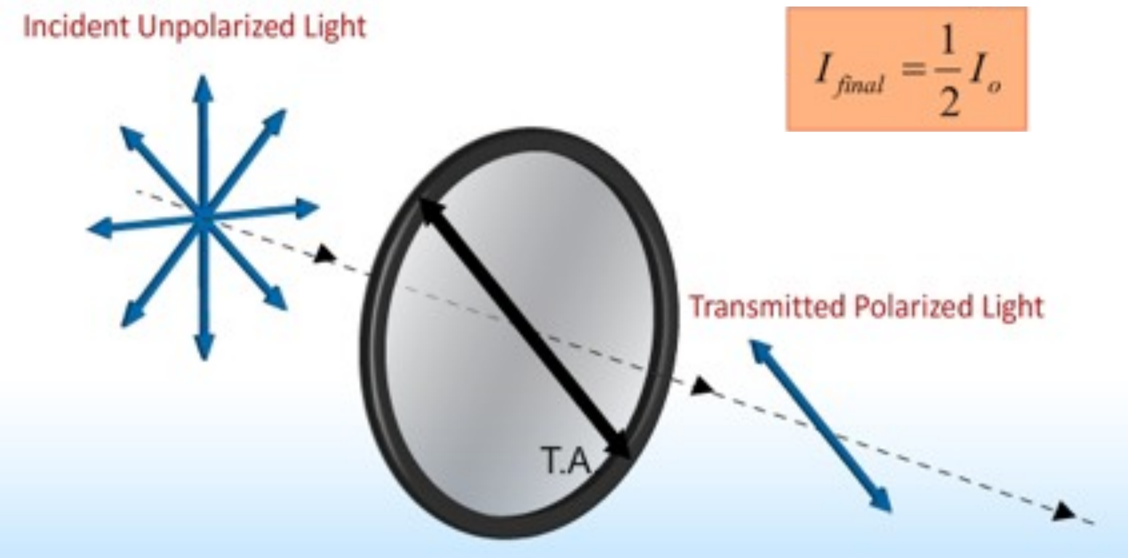
A) $E_o\sin\theta$ $\boxed{\text{B)}\ E_o\cos\theta}$ C) 0 D) $E_o/\sin\theta$ E) $E_o/\cos\theta$





Law of Malus

$$I_{final} = I_o \cos^2 \theta$$

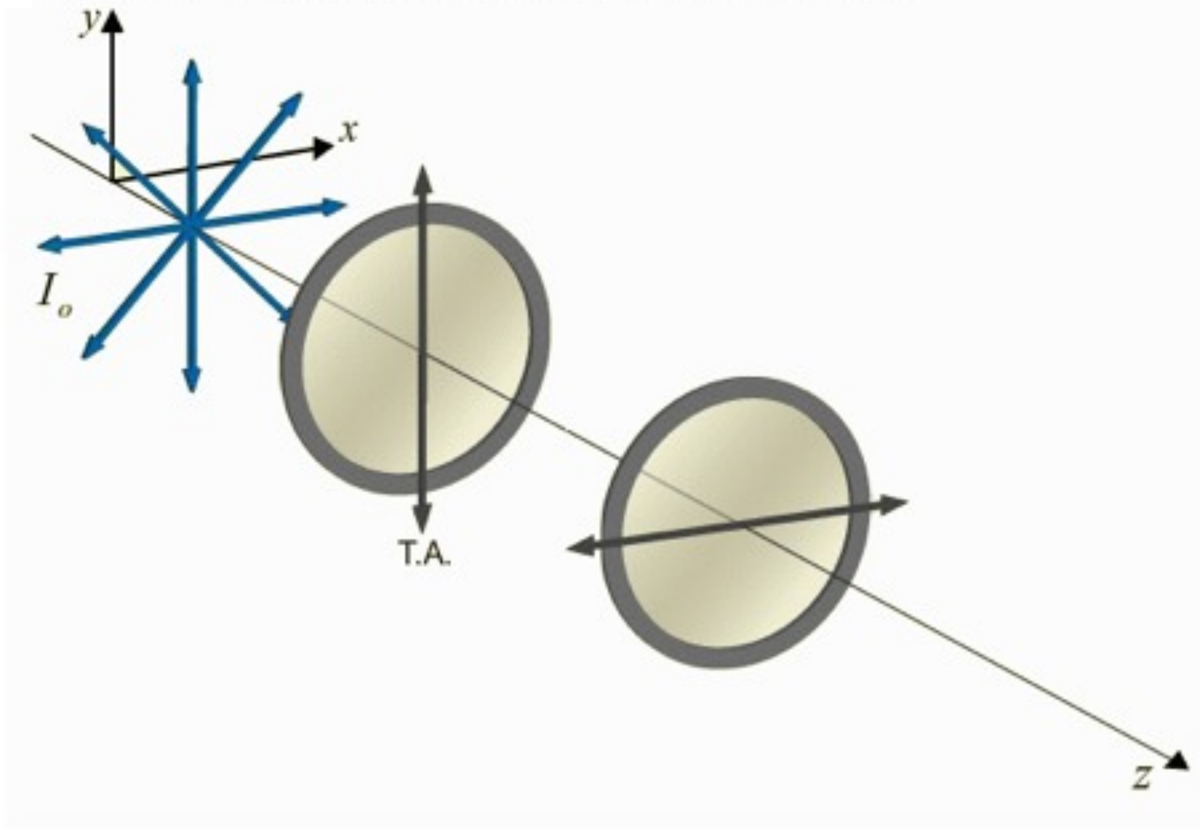


$$I_{final} = \frac{1}{2} I_o$$

CheckPoint 2

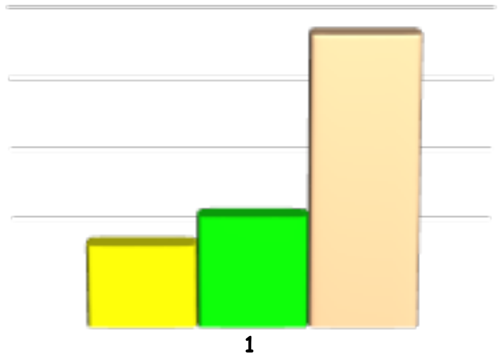
Two Polarizers

An unpolarized EM wave is incident on two orthogonal polarizers.



What percentage of the intensity gets through both polarizers?

- ○ 50%
- ○ 25%
- [✓ ○ 0%]



The second polarizer is orthogonal to the first

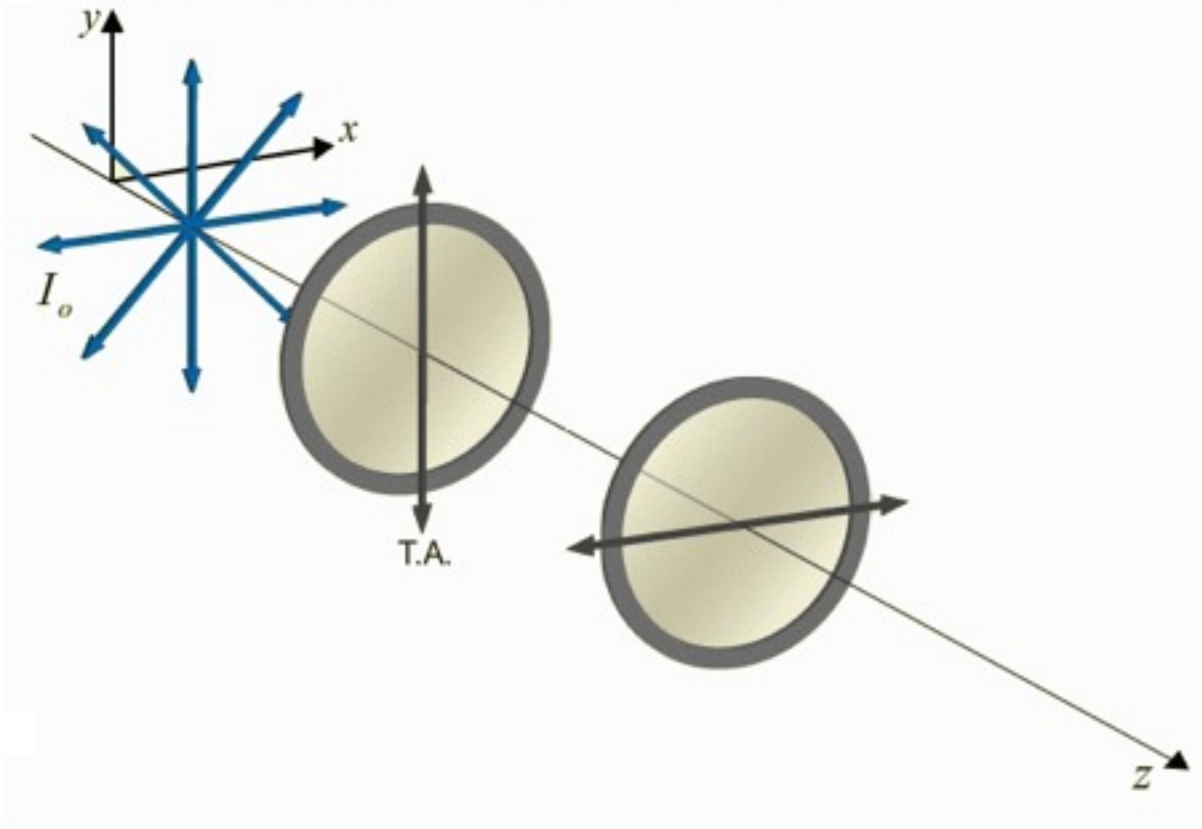
↓

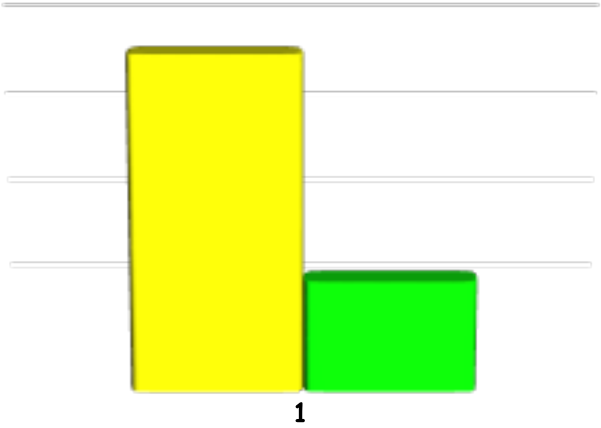
No light will come through. $\cos(90^o) = 0$

CheckPoint 4

Two Polarizers

An unpolarized EM wave is incident on two orthogonal polarizers.





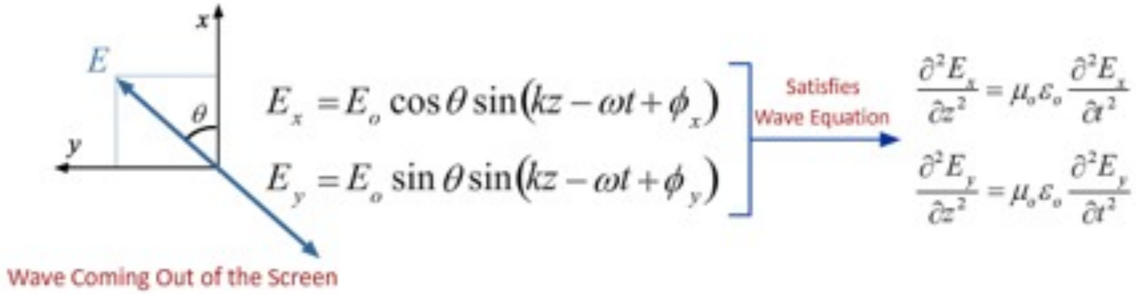
4) Is it possible to increase this percentage by inserting another polarizer between the original two?

○ yes
○ no

Any non-horizontal polarizer after the first polarizer will produce polarized light at that angle

Part of that light will make it through the horizontal polarizer

There is no reason that ϕ has to be the same for E_x and E_y:



Making ϕ_x different from ϕ_y causes circular or elliptical polarization:

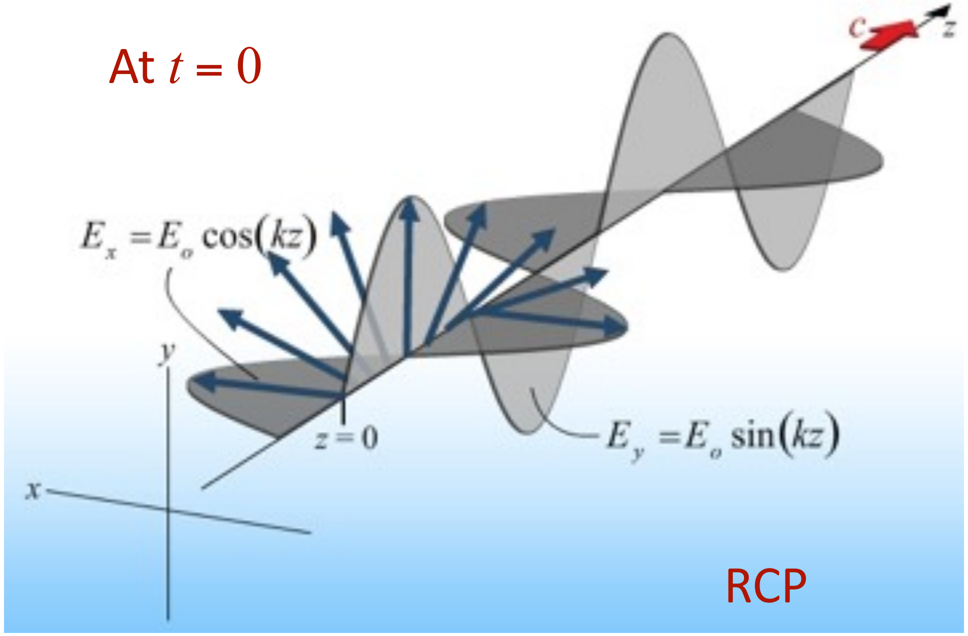
Example:

$$\phi_x - \phi_y = 90^\circ = \frac{\pi}{2}$$

$$\theta = 45^\circ = \pi / 4$$

$$E_x = \frac{E_0}{\sqrt{2}} \cos(kz - \omega t)$$

$$E_x = \frac{E_0}{\sqrt{2}} \sin(kz - \omega t)$$

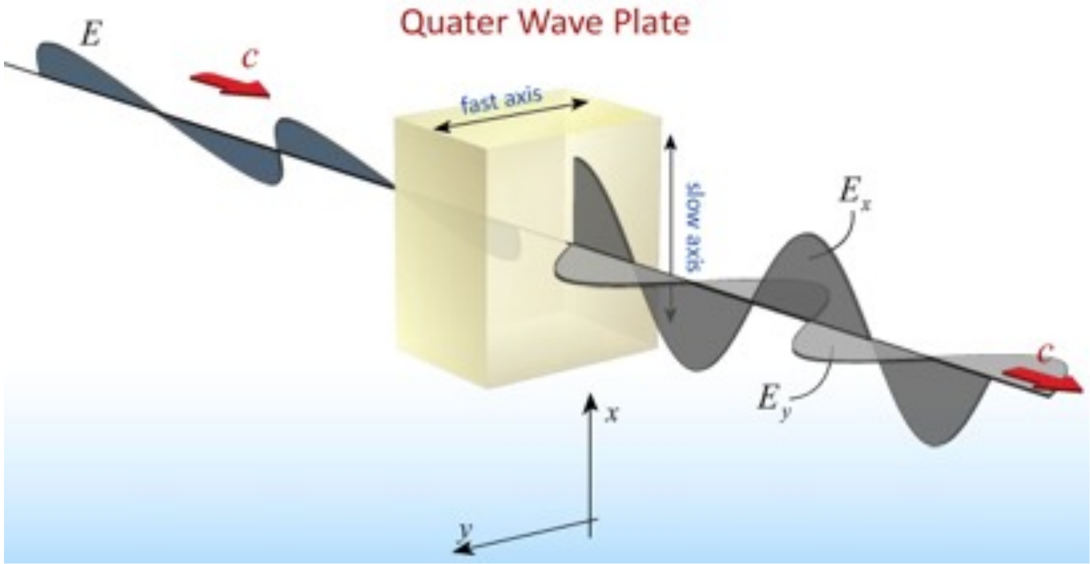


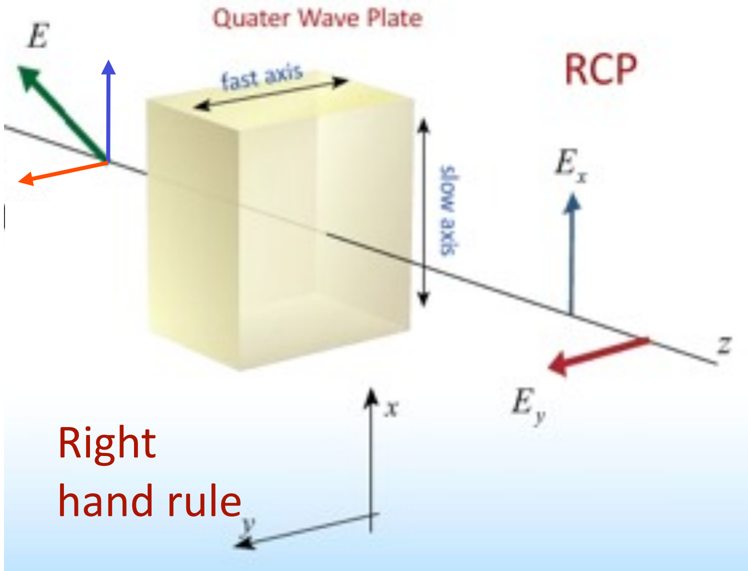
At $t = 0$

RCP

Q: How do we change the relative phase between E_x and E_y?

A: Birefringence

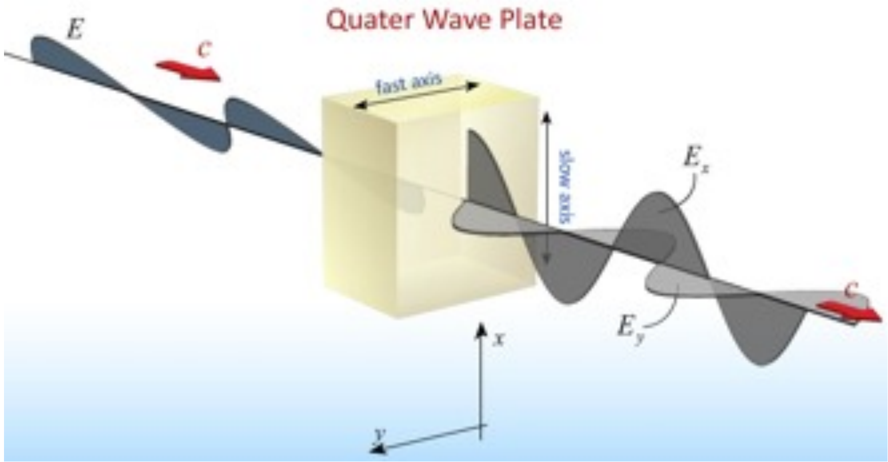




By picking the right thickness we can change the relative phase by exactly 90^o.

This changes linear to circular polarization and is called a *quarter wave plate*

“talk something about intensity”



NOTE: No Intensity is lost passing through the QWP !

BEFORE QWP:

$$E = E_o\sin(kz-\omega t)\left[\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right] \rightarrow I = c\varepsilon_o\langle E^2\rangle = c\varepsilon_o\langle E_x^2 + E_y^2\rangle$$

$$= c\varepsilon_o\left(\frac{E_o^2}{2}+\frac{E_o^2}{2}\right)\langle\sin^2(kz-\omega t)\rangle = \boxed{c\varepsilon_o E_o^2\frac{1}{2}}$$

AFTER

$$E = \frac{E_o}{\sqrt{2}}\left[\hat{i}\cos(kz-\omega t)+\hat{j}\sin(kz-\omega t)\right] \rightarrow I = c\varepsilon_o\langle E^2\rangle = c\varepsilon_o\langle E_x^2 + E_y^2\rangle$$

$$= c\varepsilon_o\frac{E_o^2}{2}\langle\cos^2(kz-\omega t)+\sin^2(kz-\omega t)\rangle$$

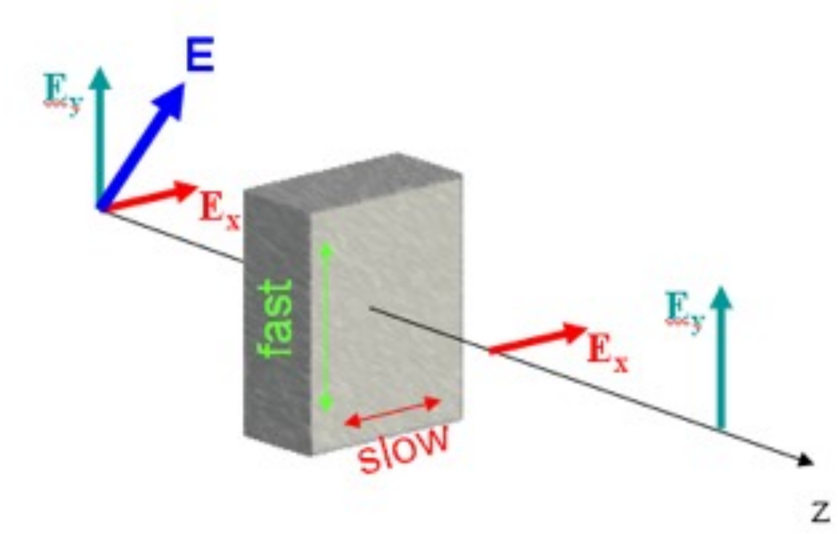
$$= c\varepsilon_o\frac{E_o^2}{2}\langle 1\rangle = \boxed{c\varepsilon_o\frac{E_o^2}{2}}$$

THE SAME!

Right or Left?

“red fox”

got it?



Right circularly polarized

Do right hand rule

Fingers along slow direction

Cross into fast direction

If thumb points in direction of propagation: RCP

Circular Light on Linear Polarizer

Q: What happens when circularly polarized light is put through a polarizer along the y (or x) axis ?

A) $I = 0$

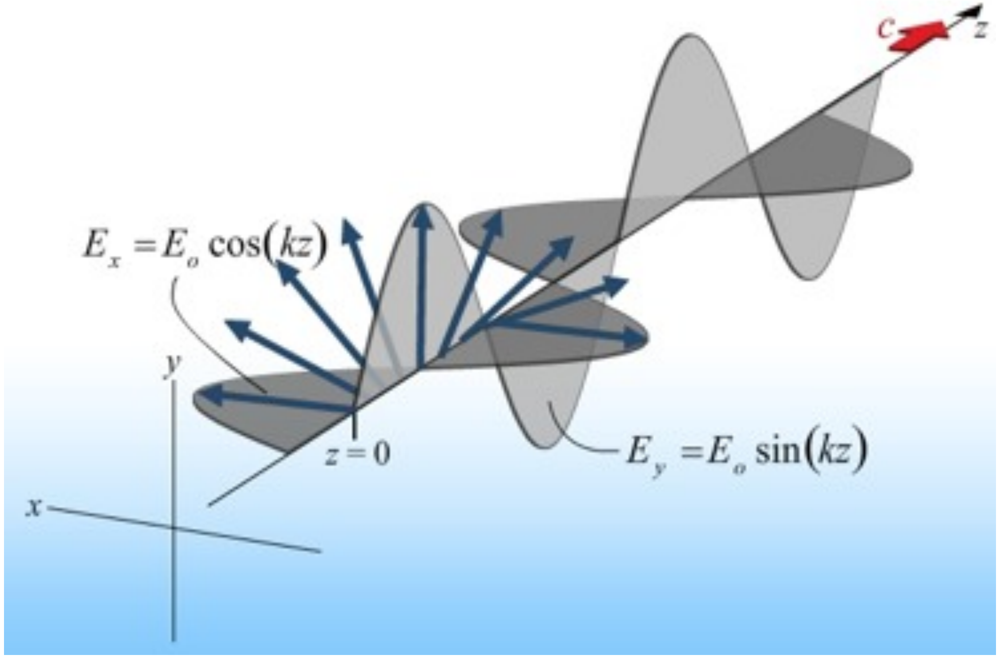
[✓ B) $I = \frac{1}{2} I_0$]

C) $I = I_0$

$$I = \varepsilon_0 c \langle E^2 \rangle$$

$$= \varepsilon_0 c \langle \cancel{E_x^2} + E_y^2 \rangle$$

$$= \varepsilon_0 c \frac{E_0^2}{2} \underbrace{\langle \cos^2(kz - \omega t) \rangle}_{1/2}$$

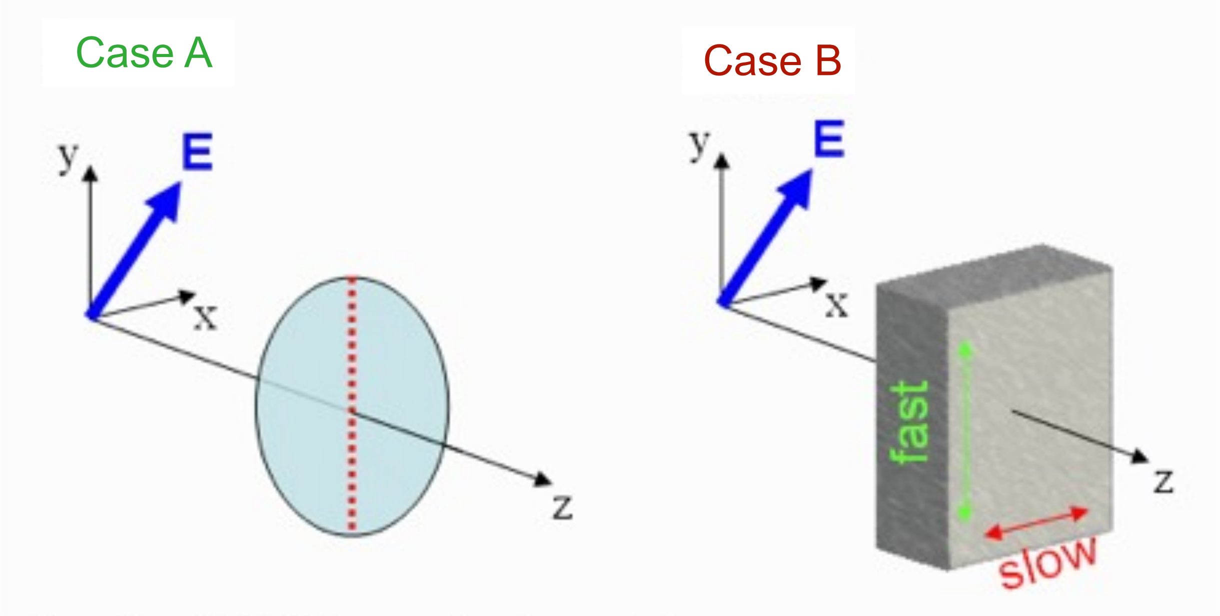


$$= \frac{1}{2} \times \frac{1}{2} \varepsilon_0 c E_0^2$$

Half of before

CheckPoint 6

Identical linearly polarized light at 45° from the y-axis and propagating along the z-axis is incident on two different objects. In case A the light intercepts a linear polarizer with polarization along the y-axis. In case B the light intercepts a quarter wave plate with fast axis along the y-axis



Compare the intensities of the light waves after transmission.

○ [✓ $I_A < I_B$]
○ $I_A = I_B$
○ $I_A > I_B$

Case A:

E_x is absorbed

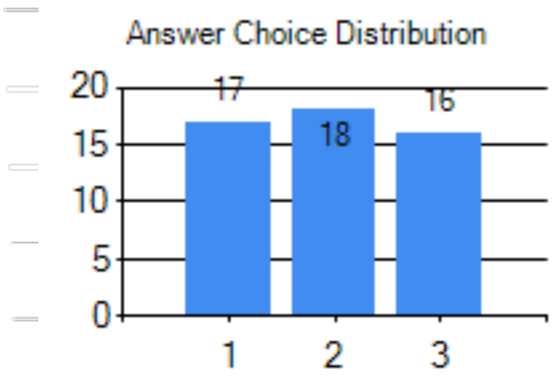
$$I_A = I_0 \cos^2(45^o)$$

$$I_A = \tfrac{1}{2} I_0$$

Case B:

(E_x, E_y) phase changed

$$I_B = I_0$$



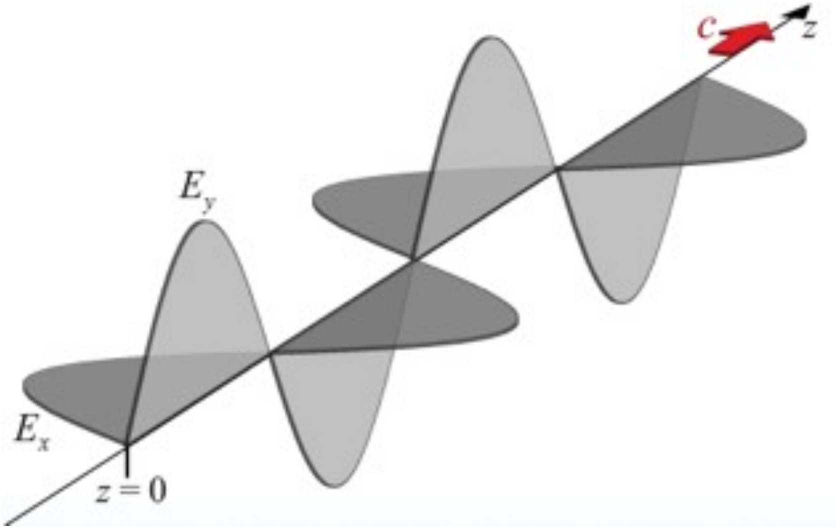
Intensity:

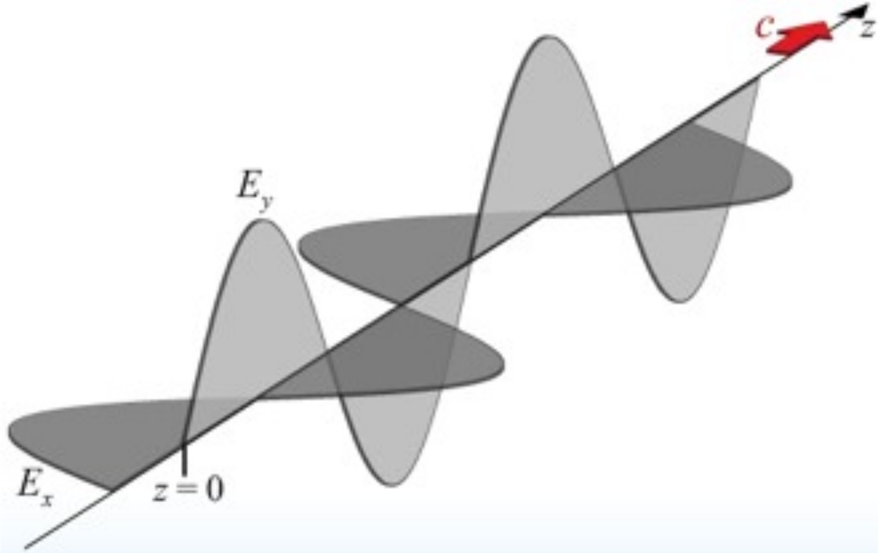
$$I = \epsilon_0 c \left[\langle E_x^2 \rangle + \langle E_y^2 \rangle \right]$$

PHASE

QW Plate

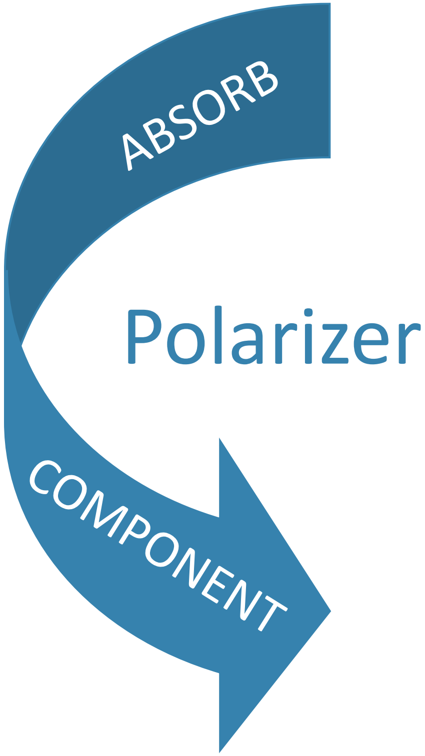
CHANGE





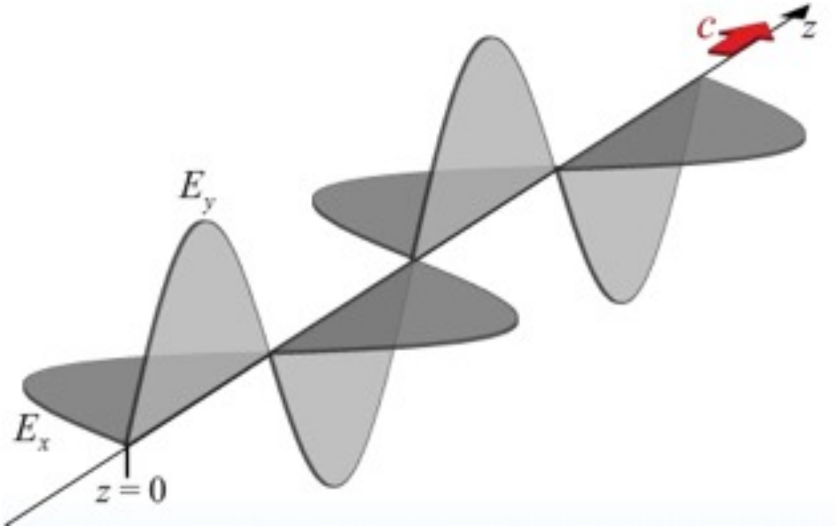
Both E_x and E_y
are still there, so
intensity is the same

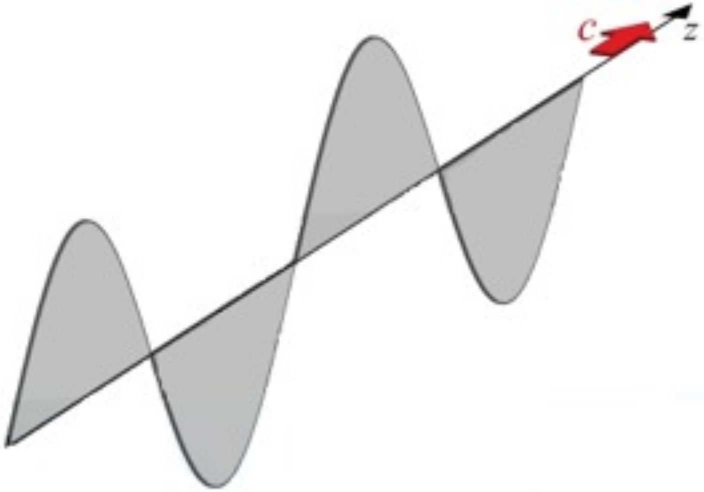
$$I = \epsilon_0 c \left[\langle \xcancel{E_x^2} \rangle + \langle E_y^2 \rangle \right]$$


ABSORB

Polarizer

COMPONENT

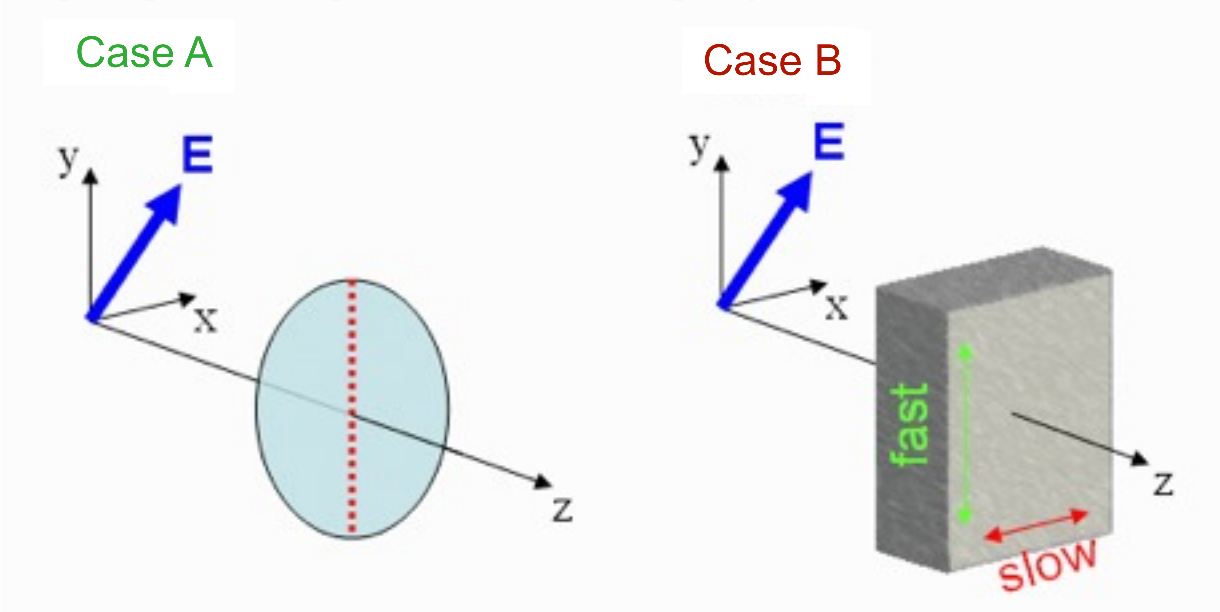




E_x is missing, so intensity is lower

CheckPoint 8

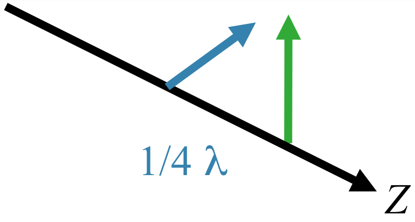
Identical linearly polarized light at 45° from the y-axis and propagating along the z-axis is incident on two different objects. In case A the light intercepts a linear polarizer with polarization along the y-axis. In case B the light intercepts a quarter wave plate with fast axis along the y-axis



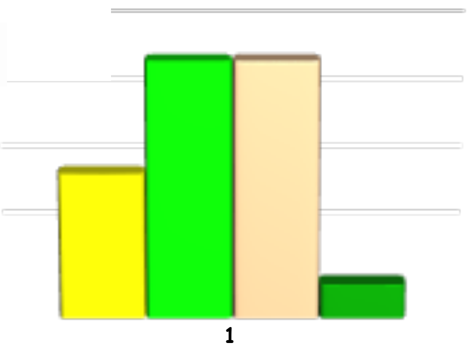
8) What is the polarization of the light wave in case B after it passed through the quarter wave plate?

○ linearly polarized
○ left circularly polarized
○ right circularly polarized
○ undefined

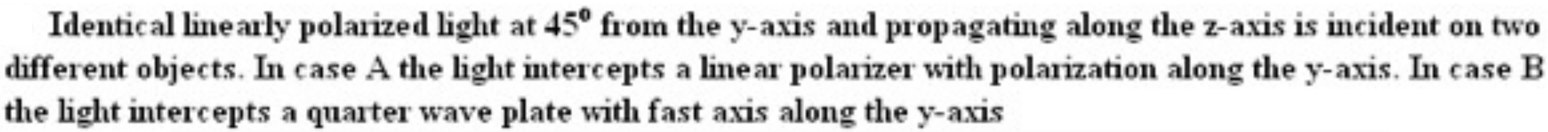
ate?



RCP



CheckPoint 10



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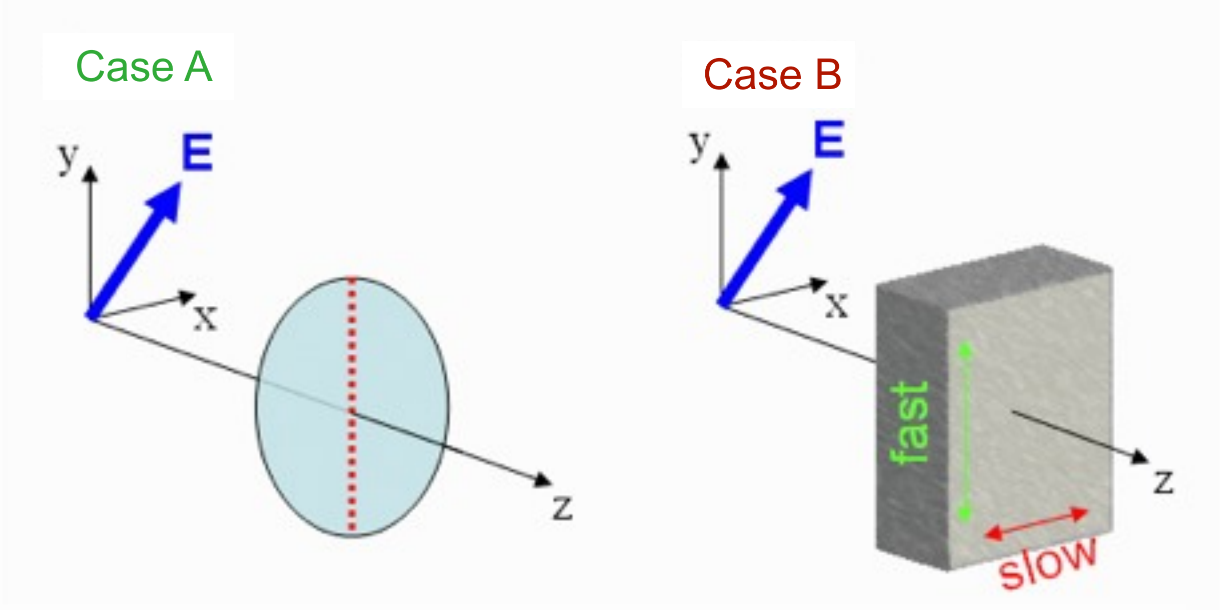
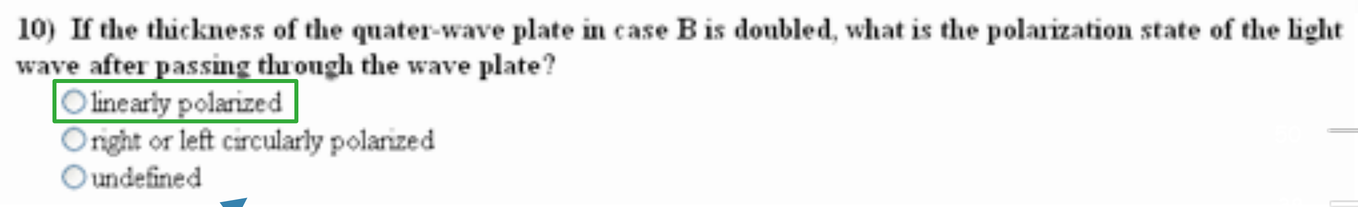
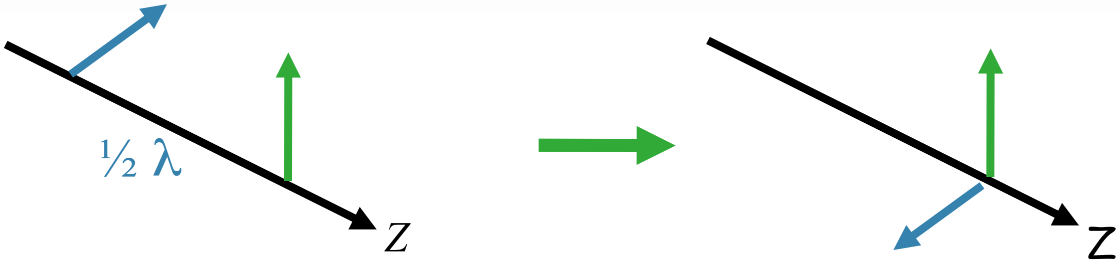
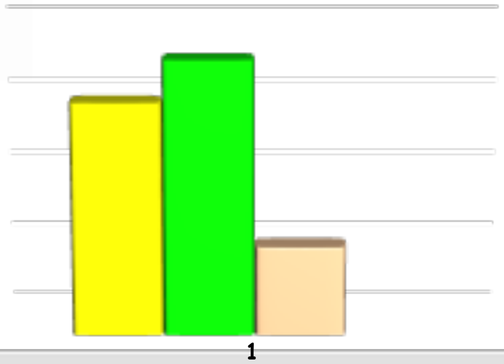




10) If the thickness of the quater-wave plate in case B is doubled, what is the polarization state of the light wave after passing through the wave plate?

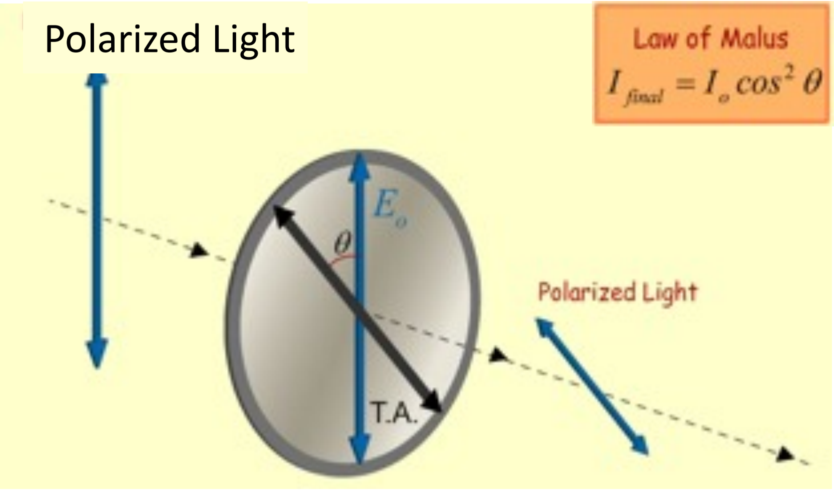
- ○ linearly polarized
- ○ right or left circularly polarized
- ○ undefined

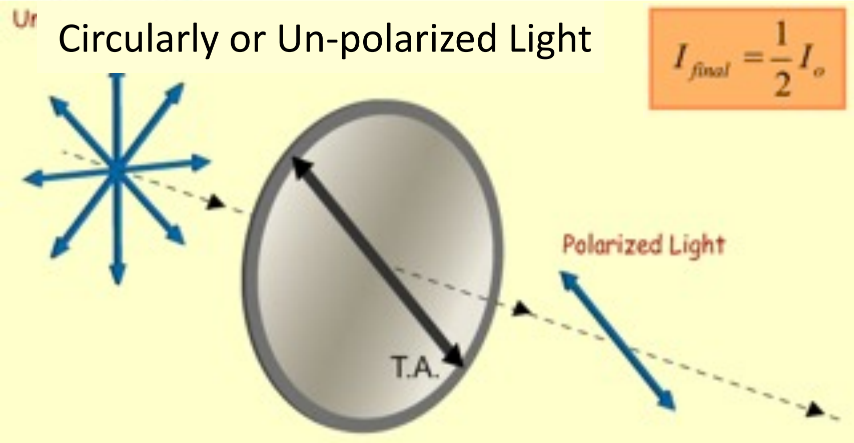




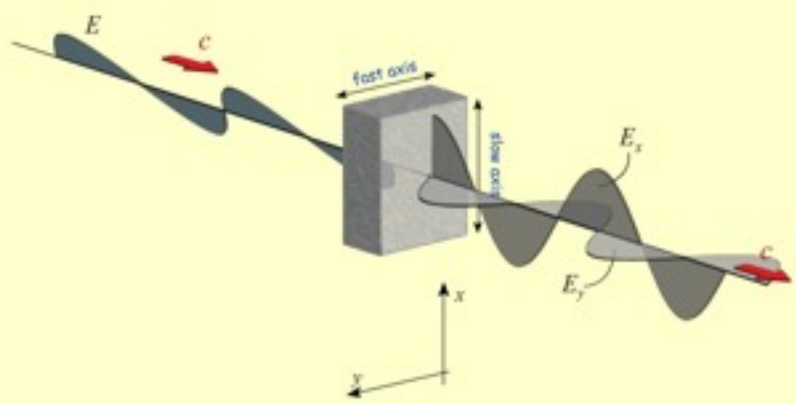
Executive Summary:

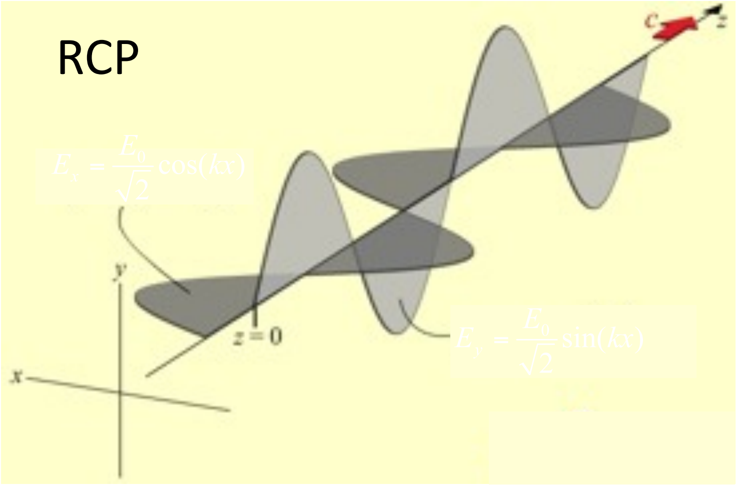
Polarizers & QW Plates:


Polarized Light

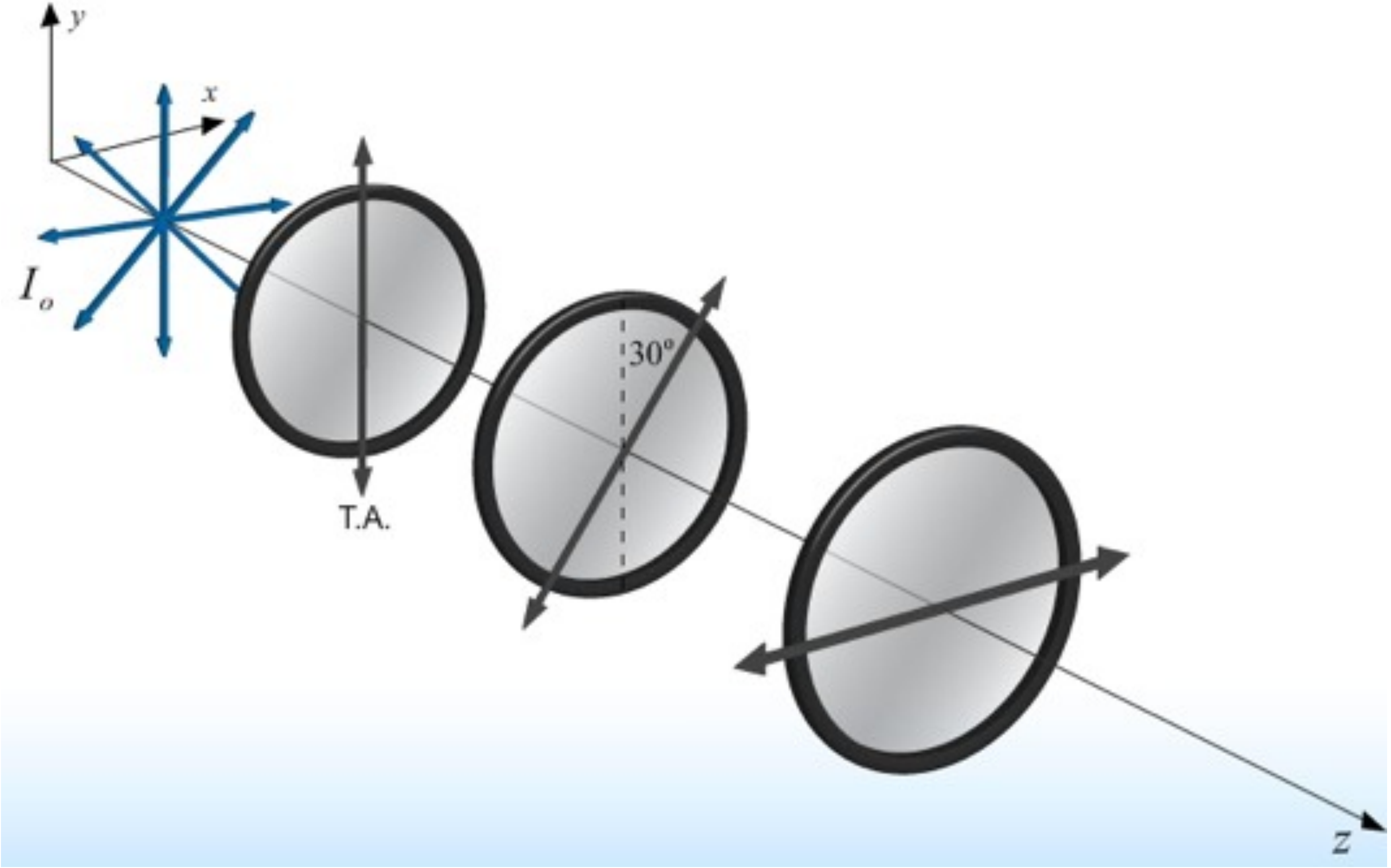

Circularly or Un-polarized Light

Birefringence




RCP

Demo

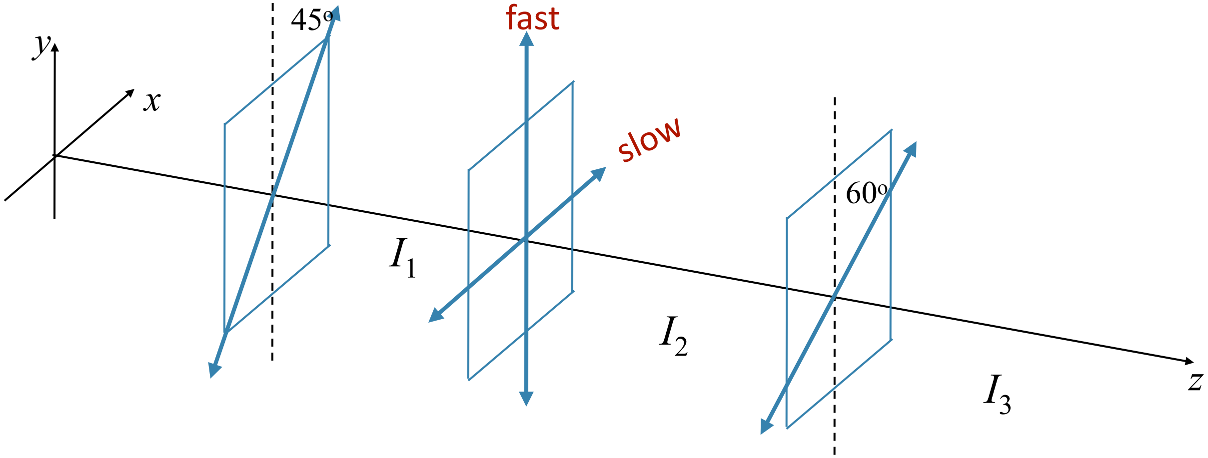


What else can we put in there to change the polarization?

Calculation

Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.

What is the intensity I_3 in terms of I_1?



Conceptual Analysis

Linear Polarizers: absorbs E field component perpendicular to Transmission Axis (TA)

Quarter Wave Plate: Shifts phase of E field components in fast-slow directions

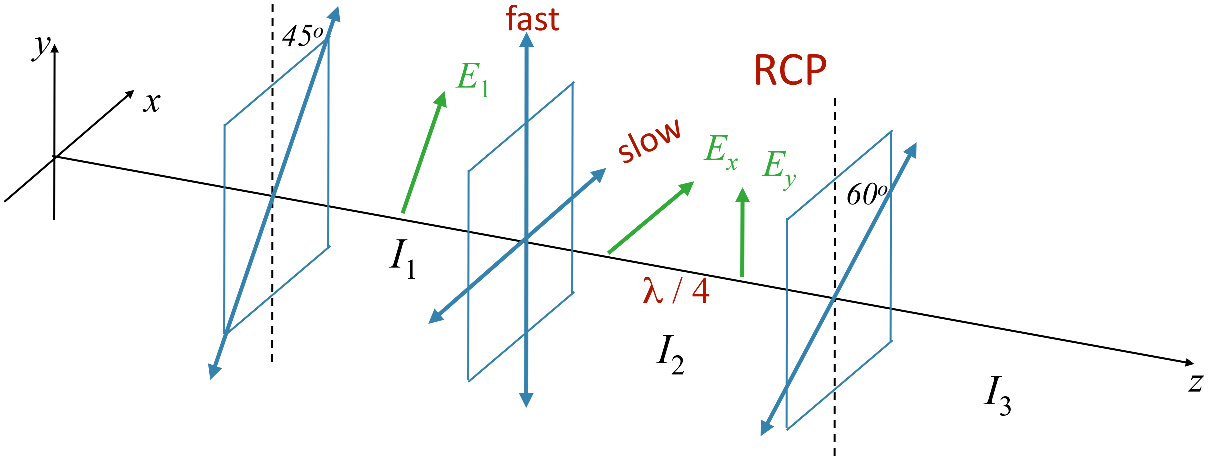
Strategic Analysis

Determine state of polarization and intensity reduction after each object

Multiply individual intensity reductions to get final reduction.

Calculation

Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.

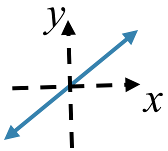


What is the polarization of the light after the QWP?

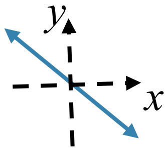
A) LCP

B) RCP

C)



D)



E) un-polarized

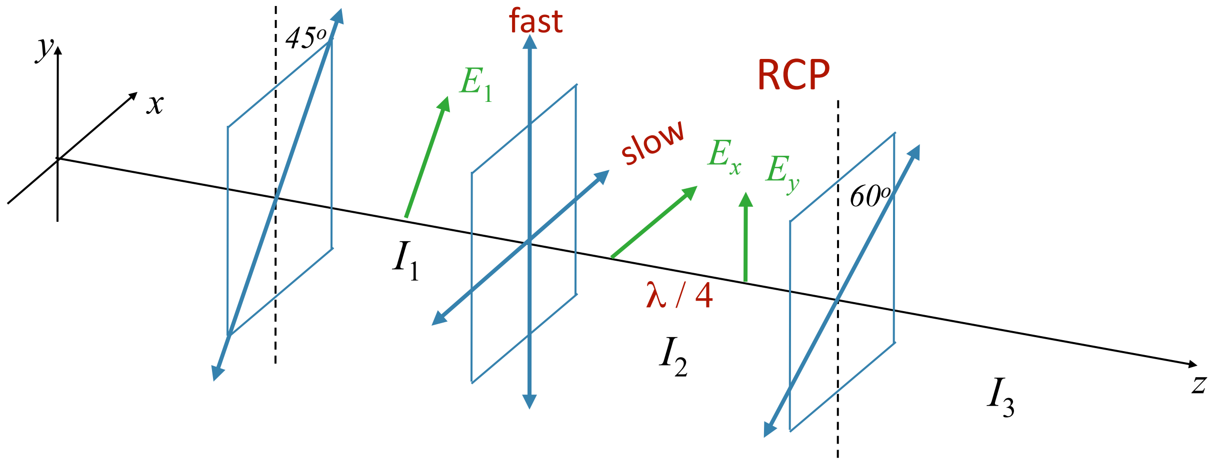
Light incident on QWP is linearly polarized at 45° to fast axis → Light will be circularly polarized after QWP

LCP or RCP? Easiest way:
Right Hand Rule:

Curl fingers of RH back to front
Thumb points in dir of propagation
if right hand polarized. → RCP

Calculation

Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.



What is the intensity I_2 of the light after the QWP?

A) $I_2 = I_1$ B) $I_2 = \frac{1}{2} I_1$ C) $I_2 = \frac{1}{4} I_1$

Before:

$$E_x = \frac{E_1}{\sqrt{2}}\sin(kz-\omega t)$$

$$E_y = \frac{E_1}{\sqrt{2}}\sin(kz-\omega t)$$

No absorption: Just a phase change!

$$I = \epsilon_0 c\left[\langle E_x^2\rangle + \langle E_y^2\rangle\right]$$

Same before & after!

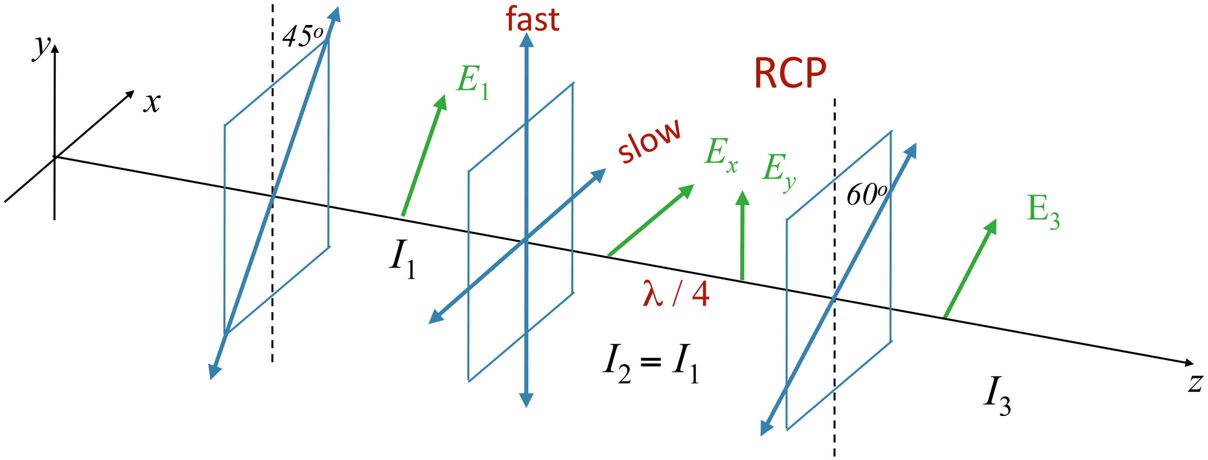
After:

$$E_x = \frac{E_1}{\sqrt{2}}\cos(kz-\omega t)$$

$$E_y = \frac{E_1}{\sqrt{2}}\cos(kz-\omega t)$$

Calculation

Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.

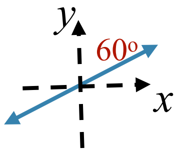


What is the polarization of the light after the 60° polarizer?

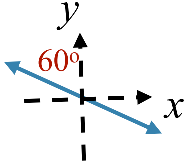
A) LCP

B) RCP

C)

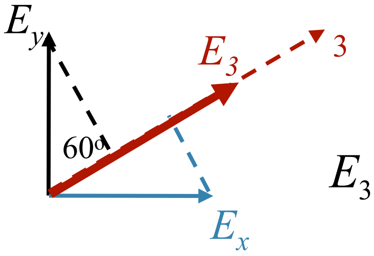


D)



E) un-polarized

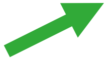
Absorption: only passes components of E parallel to TA $(\theta = 60^o)$



$$E_3 = E_x \sin\theta + E_y \cos\theta$$

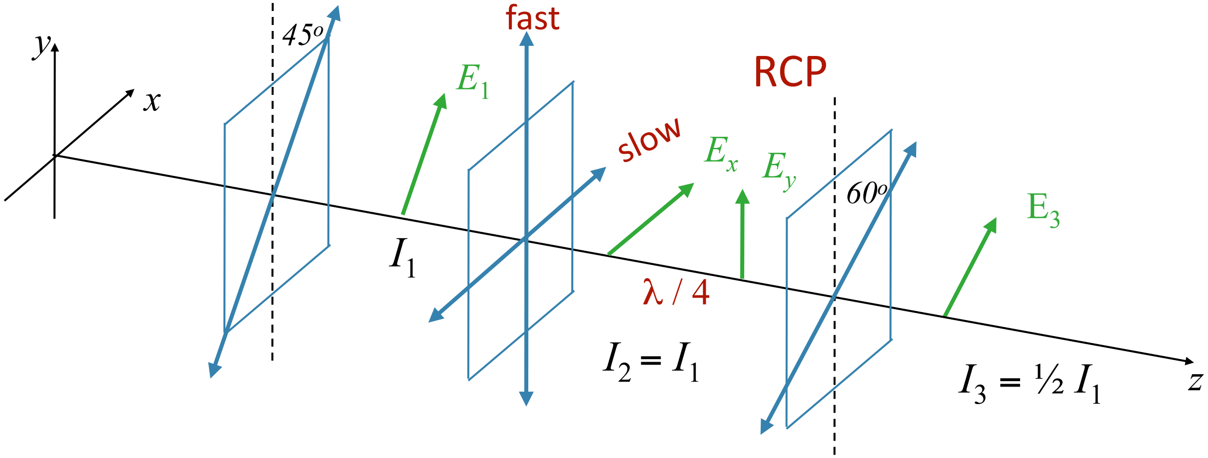
$$E_3 = \frac{E_1}{\sqrt{2}}\left(\sin(kz - \omega t + \theta)\right)$$

$$E_3 = \frac{E_1}{\sqrt{2}}\left(\cos(kz - \omega t)\sin\theta + \sin(kz - \omega t)\cos\theta\right)$$



Calculation

Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.



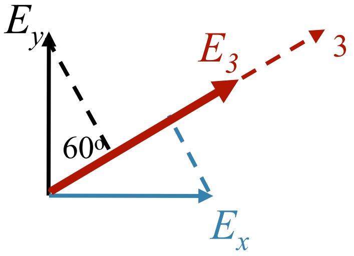
What is the intensity I_3 of the light after the 60° polarizer?

A) $I_3 = I_1$

$\boxed{\text{B) } I_3 = \frac{1}{2} I_1}$

C) $I_3 = \frac{1}{4} I_1$

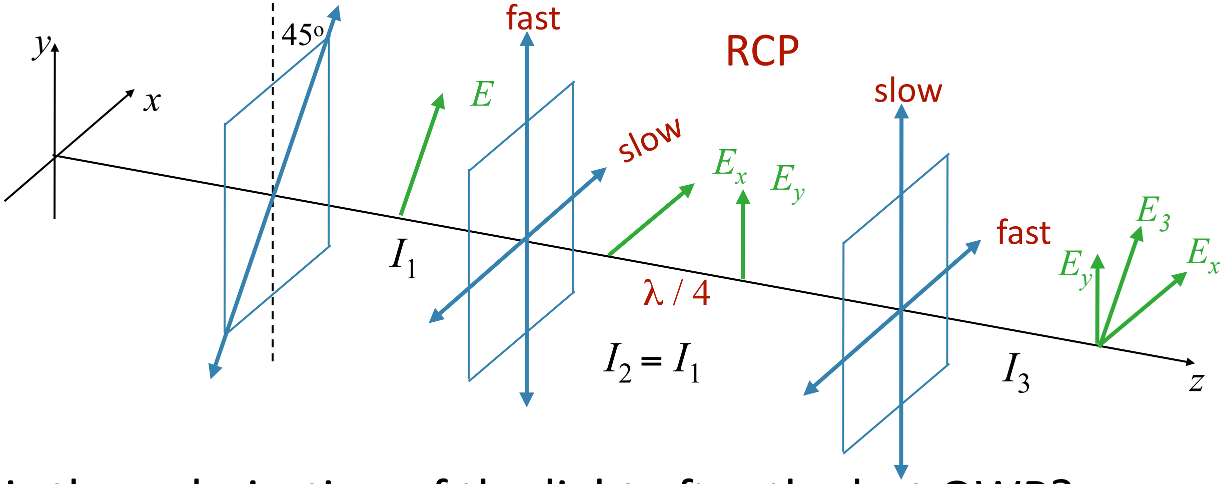
$$E_3 = \frac{E_1}{\sqrt{2}} \qquad I \propto E^2 \quad \Rightarrow \quad I_3 = \frac{1}{2} I_1$$



NOTE: This does not depend on θ !

Follow-Up 1

Replace the 60^o polarizer with another QWP as shown.

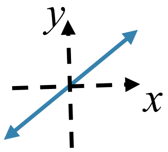


What is the polarization of the light after the last QWP?

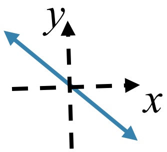
A) LCP

B) RCP

C)



D)



E) un-polarized

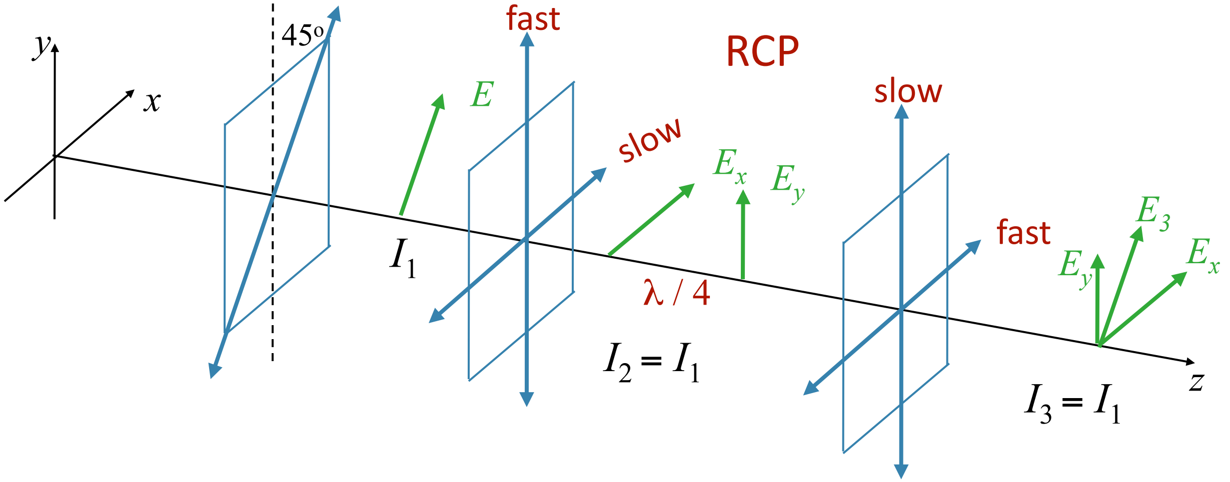
Easiest way:
E_{fast} is $\lambda/4$ ahead of E_{slow}

→

Brings E_x and E_y back in phase!

Follow-Up 2

Replace the 60^o polarizer with another QWP as shown.



What is the intensity I_3 of the light after the last QWP?

A) I_1

B) ½ I_1

C) ¼ I_1

Before:

$$E_x = \frac{E_1}{\sqrt{2}}\cos(kz - \omega t)$$

$$E_y = \frac{E_1}{\sqrt{2}}\sin(kz - \omega t)$$

No absorption: Just a phase change!

Intensity $= \langle E^2 \rangle$

$$I_{before} = \frac{E_1^2}{2} \longleftrightarrow I_{after} = \frac{E_1^2}{2}$$

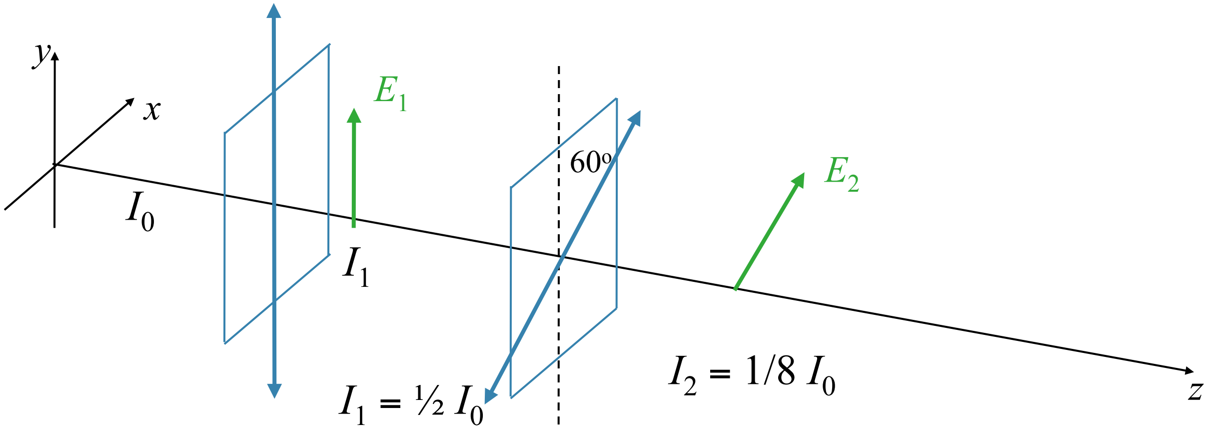
After:

$$E_x = \frac{E_1}{\sqrt{2}}\cos(kz - \omega t)$$

$$E_y = \frac{E_1}{\sqrt{2}}\cos(kz - \omega t)$$

Follow-Up 3

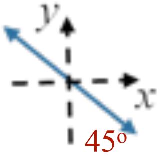
Consider light incident on two linear polarizers as shown. Suppose $I_2 = 1/8\ I_0$



What is the possible polarization of the input light?

A) LCP

B)



C) un-polarized

D) all of above

E) none of above

After first polarizer: LP along y–axis with intensity I_1

After second polarizer: LP at 60^o wrt y–axis

Intensity: $I_2 = I_1\cos^2(60^o) = ¼\ I_1$

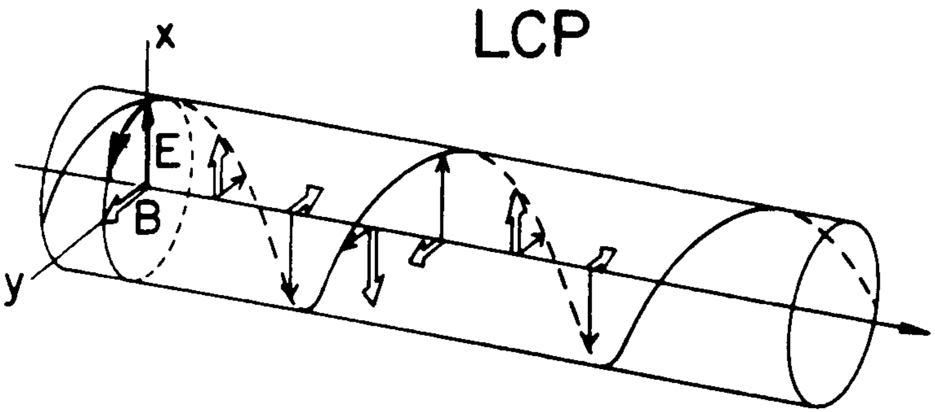
$I_2 = 1/8\ I_0 \Rightarrow I_1 = ½\ I_0$

Question is: What kind of light loses ½ of its intensity after passing through vertical polarizer?

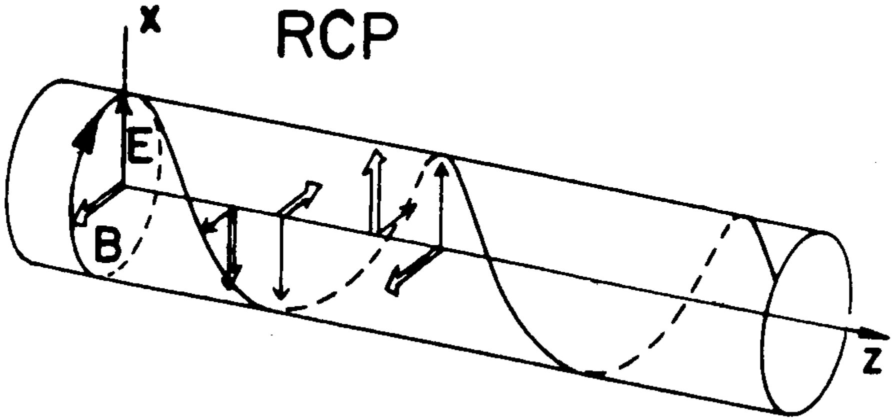
Answer: Everything except LP at θ other than 45^o

Real Quarter Wave Plate

- Only shifts light exactly λ/4 for one wavelength.
- Typically calibrated for 598 nm, Na light
- Circular Polarizers only work perfectly at 598 nm.
- Other wavelengths produce *elliptical* polarization.
- If you look at white light, you see colour-changes when you rotate circular polarizers.
- Use a Green filter, to reduce this effect. (not perfect)
- Real-life λ/? material, birefringent material: cellophane, clear scotch tape.



- **E** rotates counterclockwise as a function of time
- **E** spirals clockwise as a function of z



- **E** rotates clockwise as a function of time
- **E** spirals counterclockwise as a function of z

Radio link 3D glasses