

Name: _____ Student number: _____

Academic Honesty: Cheating in an examination includes the following:

1. the unauthorized sharing of material such as textbooks during an open book' examination;
2. concealing information pertaining to the examination in the examination room, or in washrooms or other places in the vicinity of the examination room;
3. using course notes or any other aids not approved by an Instructor during an examination; or,
4. the unauthorized possession or use of an examination question sheet, an examination answer book, or a completed examination or assignment.

Question #	Mark	Maximum Mark
Multiple Choice		20
11		10
12		10
13		10
14		10

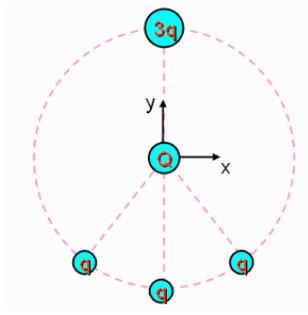
Total= _____ /60

Enter your answers to the multiple choice questions here by blackening in the circle corresponding to the best answer. There is only one answer per question.

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)

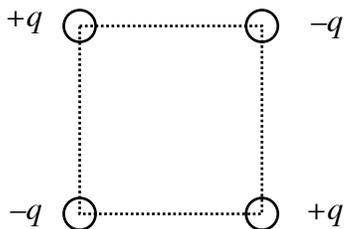
There are 10 multiple choice questions. Select the Correct answer for each one and mark it on the bubble form. Each question has only one Correct answer. (2 marks each) $k = 1/4\pi\epsilon_0$

1. A positive charge Q is placed at the origin. Four other positive charges are placed on a circle in the x - y plane centred on Q as shown. The top charge is located on the y axis and has a magnitude of $3q$. The three bottom charges are symmetric about the y axis as shown and each has magnitude q .



- (a) $F_y < 0$ Correct
 (b) $F_y = 0$
 (c) $F_y > 0$

2. Four charges are fixed in place at the corners of a square as shown in the figure. No other charges are nearby. Which of the following statements is true?

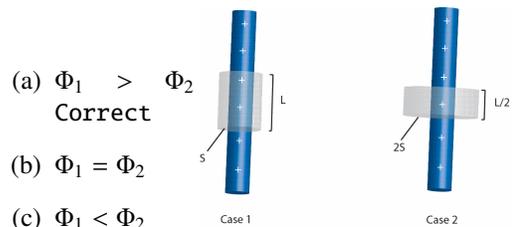


- (a) \vec{E} is zero at the midpoints of all four sides of the square.
 (b) \vec{E} is zero at the centre of the square. Correct
 (c) \vec{E} is zero midway between the top two charges and midway between the bottom two charges.

3. Equipotential surfaces

- (a) may intersect each other
 (b) are always spheres
 (c) are perpendicular to the conducting surfaces
 (d) are normal to electric field lines that they intersect Correct

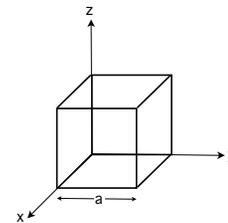
4. An infinitely long charged rod has uniform charge density of λ , and passes through a cylinder (gray). The cylinder in case 2 has twice the radius and half the length compared to the cylinder in case 1. Compare the magnitude of the flux through the surface of the cylinder in both cases.



- (a) $\Phi_1 > \Phi_2$ Correct
 (b) $\Phi_1 = \Phi_2$
 (c) $\Phi_1 < \Phi_2$

5. A cube is positioned as shown where the electric field is given by $\vec{E} = 2.0\hat{i}$ V/m. What is the net electric flux through the surface of the cube with side $a = 2$ m with its corner at the origin.

- (a) zero Correct
 (b) 4 V-m
 (c) 8 V-m
 (d) 24 V-m



- (e) None of the above

6. Two resistors are connected in parallel. One resistor has resistance R_1 and the other R_2 . If $R_1 \ll R_2$ the equivalent resistance of the combination is approximately

- (a) R_1 Correct
 (b) R_2
 (c) zero
 (d) infinity

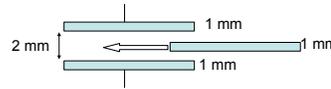
7. An ideal battery maintains a constant voltage across its terminals regardless of what is connected to it or how much it has been used. A real battery deviates from the ideal one and is modelled by

- (a) an ideal battery in series with an internal resistance Correct
 (b) an ideal battery in parallel with an internal resistance
 (c) an ideal battery in parallel with another battery

- (d) an ideal battery in series with another battery
8. Two conducting spheres have the same potential respect to infinity. Sphere 1 has twice the radius of sphere 2. How do the surface charge densities, σ_1 and σ_2 , compare?
- (a) $\sigma_1 < \sigma_2$ Correct
 (b) $\sigma_1 = \sigma_2$
 (c) $\sigma_1 > \sigma_2$
 (d) cannot be computed with this information
9. A conductor with total charge Q on it has a cavity inside it with an arbitrary shape. What is the electric potential inside the cavity?.
- (a) kQ/r where r is the distance from origin.
 (b) zero
 (c) kQ/r where r is the distance from the surface of the conductor.

- (d) constant and equals the potential on the surface of the conductor. Correct
 (e) none of the above

10. A capacitor is formed of two 1 mm thick parallel plates that are separated by 2mm. An identical metal plate is now carefully inserted between the two plates so that a space of 0.5mm is left on either side of the new plate, as shown below.



When the metal plate is inserted the capacitance

- (a) increases Correct
 (b) decreases
 (c) remains the same

To get full credit for the *written problems* you must clearly show all your steps and explain well.

11. In the Coulomb's law movie analysis, the hanging ball is displaced 1 cm to the left of its equilibrium position when the approaching ball is about 3.5 cm away from it, centre-to-centre, at equilibrium. The distance from the point of suspension to the centre of the ball is 230 mm.

- (a) Calculate the magnitude of charge Q on the balls assuming equal charge on both. Show your work.[5]



Draw Free Body Diagram! Show that $\tan \theta = F_e/mg$ where $F_e = kq^2/r^2$ or $q^2/(4\pi\epsilon_0 r^2)$ (use either form of Coulomb's law). Here r is distance between the charged balls. Because the angle is small $\theta = \tan \theta = \sin \theta = 1/23$ (in radians). Don't waste your time calculating in this case.

Therefore

$$\theta = \frac{kq^2}{r^2 mg} = \frac{q^2}{4\pi\epsilon_0 r^2 mg}$$

$$q = r \sqrt{\frac{mg\theta}{k}} = r \sqrt{4\pi\epsilon_0 mg\theta}$$

Plug in your numbers to get

$$q = 2.4 \times 10^{-9} \text{ C}$$

- (b) Predict the position of the approaching ball when the hanging ball is displaced 2 cm horizontal distance to the left of equilibrium, i.e., at the "3 cm" position. [5]

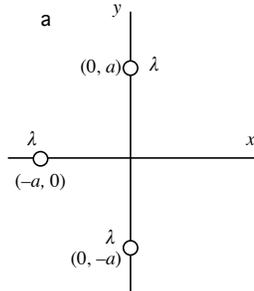
Just use ratios. Don't retrace your steps:

$$\frac{r_2^2}{r_1^2} = \frac{\theta_1}{\theta_2} = \frac{1}{2}$$

$$r_2 = r_1 / \sqrt{2} = 3.5 \text{ cm} / 1.414 = 2.5 \text{ cm}$$

12. Three infinitely long wires each with linear charge density λ are oriented parallel to the z axis and pass through the x - y plane at $(0, a)$, $(0, -a)$ and $(-a, 0)$.

- (a) Calculate the magnitude and direction of the electric field at the origin. Express in rectangular coordinates. [4]



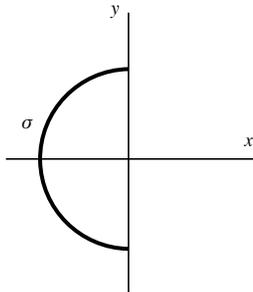
The two lines on the y axis pull symmetrically on the origin so their effect cancels out: $E_y = 0$. For the line on the $-x$ axis the electric field is given by

$$E_x = \frac{\lambda}{2\pi\epsilon_0 a}$$

Therefore

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 a} \hat{i} + 0\hat{j}$$

- (b) Now replace the wires with an infinitely long half cylinder which is also oriented parallel to the z with radius a and centred at the origin. Let the total charge from the three wires be distributed on the half cylinder. What is the surface charge density σ in terms of λ ? [2]



Consider a length of the 3 wires ℓ . The charge on that length is $Q = 3\ell\lambda$. The area of an equivalent length of the surface of the half cylinder is $A = \pi a\ell$ therefore

$$\sigma = Q/A = \frac{3\ell\lambda}{\pi a\ell} = \frac{3\lambda}{\pi a}$$

- (c) Calculate the magnitude and direction of the electric field at the origin produced by the charge on the half cylinder. [4]

By symmetry $E_y = 0$. To find the x component, integrate over the half-cylinder from $-\pi/2$ to $\pi/2$. First

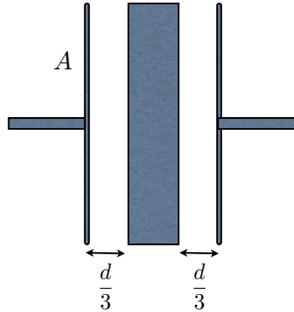
$$dE_x = \frac{\sigma a \cos \theta d\theta}{2\pi\epsilon_0 a}$$

$$E_x = \frac{\sigma}{2\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta$$

$$E_x = \frac{3\lambda}{2\pi^2 a \epsilon_0} [\sin \theta]_{-\pi/2}^{\pi/2} = \frac{3\lambda}{\pi^2 a \epsilon_0}$$

13. There is a parallel-plate capacitor with area $A = 20. \text{ cm}^2$ and plate separation $d = 3\text{mm}$. A dielectric plate with the same cross-sectional area, A , and thickness $d/3$, is between the two plates such that it has $d/3$ distance from both plates of the capacitor. Assume that the magnitude of the E field is 2 V/m in the air space between the plates and is 1 V/m in the dielectric material. The capacitor plates are isolated.

- (a) What is the surface charge density on the capacitor plates? (Fig. a).[3]



The charge on the parallel plates depends on the E field between the plates. The dielectric is neutral; it develops a charge on the opposite faces but their effect in the air spaces will cancel out because the magnitude of E doesn't depend on distance in this case.

$$E = \frac{\sigma}{\epsilon_0}$$

Remember that there is no 2 in the denominator because the field from both plates contributes the same amount in the same direction.

So the charge on the plate is given by

$$Q = \sigma A = EA\epsilon_0 = 2 \times 0.002 \times 8.8 \times 10^{-12} = 3.5 \times 10^{-14} \text{ C}$$

- (b) What are the surface charge densities on the left and right surfaces of the dielectric?[3]

Imagine a Gaussian pill-box-shaped surface enclosing an area of the interface between dielectric and air. The net flux through the surface area would be $\Phi_E = A(E_{\text{air}} - E_{\text{dielectric}}) = \sigma A / \epsilon_0$

$$\sigma = (E_{\text{air}} - E_{\text{dielectric}})\epsilon_0 = 1 \times 8.8 \times 10^{-12}$$

- (c) What is the potential difference between the plates? [2]

Just sum up the potential differences through the three regions:

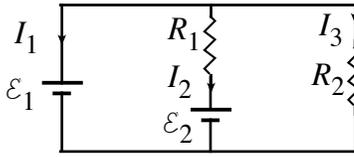
$$\sum E_i \Delta x_i = (2 + 1 + 2)(0.001) = 0.005 \text{ V}$$

- (d) What is the capacitance? [2]

From the definition of capacitance:

$$C = \frac{Q}{\Delta V} = \frac{3.5 \times 10^{-14}}{0.005} = 7 \times 10^{-12} \text{ F}$$

14. Analyse the circuit below using Kirchhoff's Rules.



(a) Write out the equations derived from applying Kirchhoff's rules to the circuit.[3]

$$I_1 + I_2 + I_3 = 0$$

taking the left-hand inside loop

$$\mathcal{E}_1 - I_2 R_1 - \mathcal{E}_2 = 0$$

taking the outside loop:

$$\mathcal{E}_1 - I_3 R_2 = 0$$

(b) Find the equation for the current I_1 in terms of the \mathcal{E} s and R s.[4]

$$I_2 = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1}$$

$$I_3 = \frac{\mathcal{E}_1}{R_2}$$

$$I_1 = -(I_2 + I_3) = -\left(\frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1} + \frac{\mathcal{E}_1}{R_2}\right)$$

(c) If $I_1 = I_2$ and $R_2 = 2R_1$ what is the ratio $\mathcal{E}_1/\mathcal{E}_2$? [3]

$$-\left(\frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1} + \frac{\mathcal{E}_1}{R_2}\right) = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1}$$

$$-\left(\frac{\mathcal{E}_1 - \mathcal{E}_2}{R} + \frac{\mathcal{E}_1}{2R}\right) = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R}$$

$$5\mathcal{E}_1/2 = 2\mathcal{E}_2$$

$$\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{4}{5}$$