Discussion Question 9A
P141, Unit 26
Ampere's Law in a Nutshell

Ampere's Law is the magnetic analogue of Gauss' Law.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

- Gauss' Law relates the integral of the electric field through a closed "Gaussian surface" to the total charge enclosed by that surface.
- Ampere's Law relates the integral of the magnetic field around a closed "Amperian loop" to the total current enclosed by that loop.

Just like Gauss' Law, Ampere's Law provides the simplest method for determining the magnetic field of a known current distribution … but it can *only* be used in a practical way if the problem has **enough symmetry**. It all boils down to choosing a suitable **Amperian loop**. The mathematical curve we choose has to have these properties:

- The magnetic field must have a constant magnitude on our curve.
- The magnetic field must make a constant angle with our curve (or portions thereof).

Otherwise, we'll never be able to extract the ***B*** field we want from that line integral!
The other part of the procedure is finding the **current enclosed** by our Amperian loop:

- In your mind, imagine that the Amperian curve is a loop of coat-hanger wire, and visualize stretching a rubber sheet across it. The enclosed current is simply the total amount of current punching through that rubber sheet.

Procedure

1. Visualize and **sketch the magnetic field**, using all the symmetry that the problem offers. Without knowing the direction and spatial behavior of the field *beforehand*, it is ***impossible*** to solve for the field using Ampere's Law!
2. Based on the field geometry, choose a suitable **Amperian loop** that passes through the field point of interest.
3. Determine the **total current enclosed** by your Amperian loop.
4. Finally, evaluate the line integral over the loop to **determine *B***.

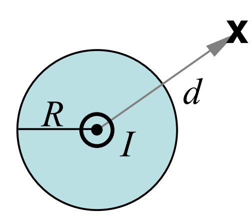
Problem Classes

There are four classes of current distribution that can be analyzed easily with Ampere's Law:

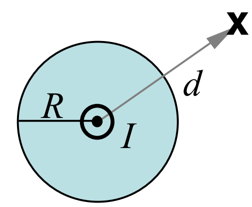
1. Straight wires and cylinders of infinite length
2. Solenoids of infinite length
3. Sheets or slabs of infinite area
4. Toroids

That's it! All the Ampere's Law problems you will encounter will be constructed from these basic systems. So let's go through them! Follow the steps outlined above to find the magnetic field ***B*** (both magnitude and *direction*!) due to the four current-carrying objects shown below.

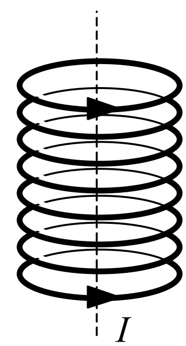
Basic Object #1: A straight wire of radius R and infinite length, carrying a uniformly-distributed current I coming out of the page ⇒ find ***B*** at a distance $d > R$ from the axis of the wire
(i.e. outside the wire … we'll do the interior in the next discussion question).



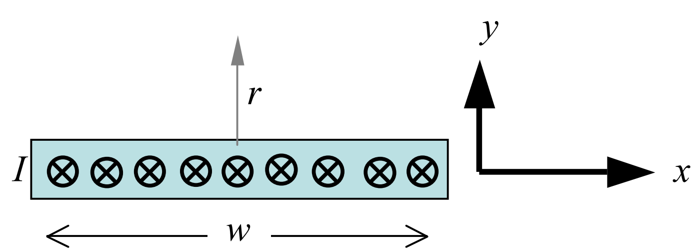
Now find $\boldsymbol{B}$ at a distance $d < R$ from the axis of the wire (i.e. Inside the wire).



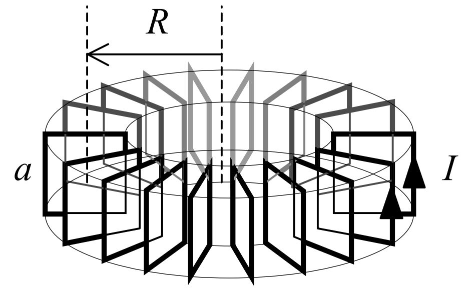
<u>Basic Object #2</u>: An infinitely-long solenoid with n turns of wire per meter. The turns of the solenoid are circular with radius R, and the wrapped wire carries a current $I \Rightarrow$ find $\boldsymbol{B}$ at a distance d from the axis of the solenoid, considering points both *inside* and *outside* the coils.



<u>Basic Object #3</u>: A sheet of infinite area carrying a total current I into the page. The current is distributed uniformly across the very-large width w of the sheet. $\Rightarrow$ find $\boldsymbol{B}$ at a distance d both *above* and *below* the sheet.



Basic Object #4: A toroid with *N* square windings of side *a*. The radius of the toroid (from the axis of the toroid to the center of the coils) is *R*, and the wrapped wire carries a current *I* in the direction shown on the figure. ⇒ find ***B*** at a distance *d* from the axis of the toroid, considering points both *inside* and *outside* the coils.



Note: The **shape** of the coils in the solenoid and the toroid actually makes *no difference*, as long as it's *constant*. Can you explain why?