Electricity & Magnetism
Lecture 3: Electric Flux and Field Lines

Today’s Concepts:

A) Electric Flux
B) Field Lines

Gauss’ Law
What the heck is epsilon 0?

IT’S JUST A CONSTANT

\[ k = \frac{1}{4\pi\varepsilon_0} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2 \]

\[ \vec{E} = k \frac{q}{r^2} \hat{r} \]

I don't understand electric flux, how it's derived and the formula. I also need someone to explain Ε₀ or ε not. Cause I don't know what that is.

“Calculating Electric Field from Arc of Charge!”  [hint]

“Why is gauss' law so important? Why is flux a useful value?”
I find the idea of representing electric field as a finite number of lines N a bit absurd.

Coulomb was a lazy Frenchman, he puts me to sleep. Tesla, Weber, Faraday, those guys would help me build a flux capacitor and go back to the 60s to party with Jerry Garcia...

im having troubles in written homeworks , and integrals.. and can you please explain more about flux and gauss' law please~!

To be honest, the prelecture and homework are talking 2 different things
What part of
\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \]
\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]
\[ \oint \mathbf{E} \cdot d\mathbf{S} = -\frac{d\Phi_B}{dt} \]
\[ \oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 \mathbf{j} + \mu_1 \varepsilon_0 \frac{d\Phi_E}{dt} \]
don't you understand?
Review of calculations + practical examples please! How to solve these problems! + Could we have more time for the activity guides? We're only given 50 minutes to do them, but for them to be done properly it would be nice to be given at least 70 minutes for the proper completion of the activity guides!

We’ll skip checkpoint questions most ppl get right. Many examples will have symbols, not numbers.

Read activity guide before class—know your mission.

Get busy right when you come in.
- You will need to understand integrals in this course!!!
- Forces and Fields are Vectors
- Always Draw a Picture First. What do the Forces/Fields Look Like?

Prelecture

Activity Guide

Exam?

\[ \vec{E} = \int k \frac{dq}{r^2} \hat{r} \]

WORKS FOR ALL!
Electric Field Lines

Point Charge:
  Direction is radial
  Density \( \propto \frac{1}{R^2} \)

Direction & Density of Lines represent Direction & Magnitude of \( E \)
Dipole Charge Distribution: Direction & Density much more interesting.
Compare the magnitudes of the two charges

- $|Q_1| > |Q_2|$
- $|Q_1| = |Q_2|$
- $|Q_1| < |Q_2|$
- There is not enough information to determine the relative magnitudes of the charges.
What do we know about the signs of the charges from looking at the picture?

- $Q_1$ and $Q_2$ have the same sign
- $Q_1$ and $Q_2$ have opposite signs
- There is not enough information in the picture to determine the relative signs of the charges
Compare the magnitudes of the electric fields at points A and B.

- $|E_A| > |E_B|$
- $|E_A| = |E_B|$
- $|E_A| < |E_B|$

There is not enough information to determine the relative magnitudes of the fields at A and B.
“Telling the difference between positive and negative charges while looking at field lines. Does field line density from a certain charge give information about the sign of the charge?”

What charges are inside the red circle?

A: +Q

B: -Q

C: -Q, +2Q

D: -2Q, +Q

E: -Q
Which of the following field line pictures best represents the electric field from two charges that have the same sign but different magnitudes?

A

B

C

D

Equipotential lines not Field LInes
Electric Flux “Counts Field Lines”

“I’m very confused by the general concepts of flux through surface areas. Please help”

\[ \Phi_S = \int_S \vec{E} \cdot d\vec{A} \]

Flux through surface \( S \)

Integral of \( \vec{E} \cdot d\vec{A} \) on surface \( S \)
CheckPoint: Flux from Uniformly Charged Rod

An infinitely long charged rod has uniform charge density of $\lambda$, and passes through a cylinder (gray). The cylinder in case 2 has twice the radius and half the length compared to the cylinder in case 1.

Compare the magnitude of the flux through the surface of the cylinder in both cases.

A. $\Phi_1 = 2 \Phi_2$
B. $\Phi_1 = \Phi_2$
C. $\Phi_1 = 1/2 \Phi_2$
D. None of these
CheckPoint Results: Flux Unif. Charged Rod

Compare the magnitude of the flux through the surface of the cylinder in both cases.

A. $\Phi_1 = 2 \Phi_2$

B. $\Phi_1 = \Phi_2$

C. $\Phi_1 = 1/2 \Phi_2$

D. None of these

The first cylinder encloses twice the amount of charge as the second. The flux doesn’t depend on length or radius because case 1 has more surface area that is parallel to the electric field line than case 2, so case 2 is half case 1.

TAKE $s$ TO BE RADIUS!
"The flux is proportional to the Area that the field is passing through. Although the radius is twice as long in the second case, its length is half as long. Both cases have the same surface area that the field passes through, so the fluxes are equal."

\[ \Phi_S = \int_S \vec{E} \cdot d\vec{A} \]

\[ \Phi = E \int d\vec{A} = EA_{\text{barrel}} \]

**Case 1**

\[ E_1 = \frac{\lambda}{2\pi\varepsilon_0 s} \]

\[ \Phi_1 = \frac{\lambda L}{\varepsilon_0} \]

**Case 2**

\[ E_2 = \frac{\lambda}{2\pi\varepsilon_0 (2s)} \]  

\[ A_2 = (2\pi (2s))L/2 = 2\pi sL \]

\[ \Phi_2 = \frac{\lambda (L/2)}{\varepsilon_0} \]

**RESULT: GAUSS’ LAW**

\[ \Phi \text{ proportional to charge enclosed} ! \]

CheckPoint Results: Flux Unif. Charged Rod

A. \( \Phi_1 = 2 \Phi_2 \)
B. \( \Phi_1 = \Phi_2 \)
C. \( \Phi_1 = 1/2 \Phi_2 \)
D. None of these

TAKE \( s \) TO BE RADIUS!
Direction Matters:

For a closed surface, \( \vec{dA} \) points outward

\[
\Phi_S = \int \vec{E} \cdot \vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
\]
For a closed surface, \( \vec{dA} \) points outward

\[
\Phi_S = \int_S \vec{E} \cdot \vec{dA} < 0
\]
Clicker Question: Trapezoid in Constant Field

Label faces:
1: \( x = 0 \)
2: \( z = +a \)
3: \( x = +a \)
4: slanted

Define \( \Phi_n = \) Flux through Face \( n \)

A) \( \Phi_1 < 0 \)
B) \( \Phi_1 = 0 \)
C) \( \Phi_1 > 0 \)

A) \( \Phi_2 < 0 \)
B) \( \Phi_2 = 0 \)
C) \( \Phi_2 > 0 \)

A) \( \Phi_3 < 0 \)
B) \( \Phi_3 = 0 \)
C) \( \Phi_3 > 0 \)

A) \( \Phi_4 < 0 \)
B) \( \Phi_4 = 0 \)
C) \( \Phi_4 > 0 \)
Add a charge $+Q$ at $(-a, a/2, a/2)$

How does Flux change? ("more negative" is "decreases", "less negative" is "increases")

A) $\Phi_1$ increases
B) $\Phi_1$ decreases
C) $\Phi_1$ remains same

A) $\Phi_3$ increases
B) $\Phi_3$ decreases
C) $\Phi_3$ remains same

A) $\Phi$ increases
B) $\Phi$ decreases
C) $\Phi$ remains same
Gauss Law

\[ \Phi_S = \oint \mathbf{E} \cdot \mathbf{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} \]

Closed surface
A positive charge (blue) is contained inside a spherical shell (black).

How does the electric flux through the two surface elements, $d\Phi_A$ and $d\Phi_B$ change when the charge is moved from position 1 to position 2?

- $d\Phi_A$ increases and $d\Phi_B$ decreases
- $d\Phi_A$ decreases and $d\Phi_B$ increases
- Both $d\Phi_A$ and $d\Phi_B$ do not change
A positive charge (blue) is contained inside a spherical shell (black).

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- $d\Phi_A$ increases and $d\Phi_B$ decreases
- $d\Phi_A$ decreases and $d\Phi_B$ increases
- Both $d\Phi_A$ and $d\Phi_B$ do not change
A positive charge (blue) is contained inside a spherical shell (black).

How does the flux $\Phi_E$ through the entire surface change when the charge is moved from position 1 to position 2?

- $\Phi_E$ increases
- $\Phi_E$ decreases
- $\Phi_E$ does not change
A positive charge (blue) is contained inside a spherical shell (black).

How does the flux $\Phi_E$ through the entire surface change when the charge is moved from position 1 to position 2?

- $\Phi_E$ increases
- $\Phi_E$ decreases
- $\Phi_E$ does not change

Gauss law refers to the total charge enclosed, regardless of where it is.
Think of it this way:

The total flux is the same in both cases (just the total number of lines)
The flux through the right (left) hemisphere is smaller (bigger) for case 2.
Things to notice about Gauss Law

If $Q_{\text{enclosed}}$ is the same, the flux has to be the same, which means that the integral must yield the same result for any surface.

$$
\Phi_S = \int \vec{E} \cdot \hat{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
$$
In cases of high symmetry it may be possible to bring $E$ outside the integral. In these cases we can solve Gauss Law for $E$

$$\Phi_S = \int_{\text{closed surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

$$E \int dA = EA = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

$$E = \frac{Q_{\text{enclosed}}}{A \varepsilon_0}$$

So - if we can figure out $Q_{\text{enclosed}}$ and the area of the surface $A$, then we know $E$!

This is the topic of the next lecture.
One more thing...

- Is it *Gauss’ Law* or *Gauss’s Law*?
- Either, but never
  ... *Gausses Law.*