Electricity & Magnetism
Lecture 7: Conductors and Capacitance

Today’s Concept:
A) Conductors
B) Capacitance

Today’s Experiment:
Battery and Bulb Circuits
(we return to capacitance soon)
“They never said what capacitance is or what its used for”

“How do I "charge" a capacitance? if I don't put a wire in between, the circuit is not complete. If I put a wire in between, I won't be able to build up a charge difference. Also, if I charge them as is (with no wire in between), won't they discharge immediately? Don't understand how this works.”

“What is the difference between capacitors and batteries, although they both stores energy, can we say that capacitors are a kind a batteries or vice versa?”

“I'm not looking forward to the midterm.”

Below is a list of topics I would like to discuss in explicit detail during the lecture: 1) Capacitors 2) Capacitors 3) Capacitors ...
A battery uses an electrochemical reaction to separate charges. $\Delta V$ between terminals is called “EMF”, symbol: $\mathcal{E}$ or $\mathcal{E}$.

- Capacitors store charge on its plates.
Some capacitors
WE BELIEVE THERE ARE ONLY THREE THINGS YOU NEED TO KNOW TO DO ALL OF HOMEWORK!

1. \( E = 0 \) within the material of a conductor: (at static equilibrium)

Charges move inside a conductor in order to cancel out the fields that would be there in the absence of the conductor. This principle determines the induced charge densities on the surfaces of conductors.

2. Gauss’ Law:

\[
\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
\]

If charge distributions have sufficient symmetry (spherical, cylindrical, planar), then Gauss’ law can be used to determine the electric field everywhere.

3. Definition of Potential:

\[
\Delta V_{a\rightarrow b} \equiv \frac{\Delta U_{a\rightarrow b}}{q} = -\int_{a}^{b} \vec{E} \cdot d\vec{l}
\]

CONCEPTS DETERMINE THE CALCULATION!
The Main Points

- Charges free to move
- $E = 0$ in a conductor
- Surface = Equipotential
- $E$ at surface perpendicular to surface
Two spherical conductors are separated by a large distance. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B.

Compare the potential at the surface of conductor A with the potential at the surface of conductor B.

A. $V_A > V_B$
B. $V_A = V_B$
C. $V_A < V_B$

“The potential is the same as from a point charge at the center of A or B. So, following $V = kQ/r$, A would have a smaller potential.”
Two spherical conductors are separated by a large distance. They each carry the same positive charge $Q$. Conductor A has a larger radius than conductor B.

The two conductors are now connected by a wire. How do the potentials at the conductor surfaces compare now?

A. $V_A > V_B$
B. $V_A = V_B$
C. $V_A < V_B$

“The potentials will become equal since the charges will want to go to places of lower potential, until it balances out.”
Two spherical conductors are separated by a large distance. They each carry the same positive charge $Q$. Conductor $A$ has a larger radius than conductor $B$.

What happens to the charge on conductor $A$ after it is connected to conductor $B$ by the wire?

A. $Q_A$ increases
B. $Q_A$ decreases
C. $Q_A$ does not change

“Since $B$ initially has a higher potential, charges move from $B$ to $A$. “
Two parallel plates of equal area carry equal and opposite charge \( Q_0 \). The potential difference between the two plates is measured to be \( V_0 \). An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value \( Q_1 \) such that the potential difference between the plates remains the same.

Compare \( Q_1 \) and \( Q_0 \).

- \( Q_1 < Q_0 \)
- \( Q_1 = Q_0 \)
- \( Q_1 > Q_0 \)

THE CAPACITOR QUESTIONS WERE TOUGH!

THE PLAN:

We’ll work through the example in the PreLecture and then do the preflight questions.
Capacitance is defined for any pair of spatially separated conductors.

\[ C \equiv \frac{Q}{V} \]

How do we understand this definition?

- Consider two conductors, one with excess charge = \(+Q\) and the other with excess charge = \(-Q\)

These charges create an electric field in the space between them.

We can integrate the electric field between them to find the potential difference between the conductor.

This potential difference should be proportional to \(Q\)!

- The ratio of \(Q\) to the potential difference is the capacitance and only depends on the geometry of the conductors.
First determine $E$ field produced by charged conductors:

Second, integrate $E$ to find the potential difference $V$

$$V = -\int_0^d \vec{E} \cdot d\vec{y} \quad \Rightarrow \quad V = -\int_0^d (-Edy) = E\int_0^d dy = \frac{Q}{\varepsilon_o A} d$$

As promised, $V$ is proportional to $Q$!

$$C \equiv \frac{Q}{V} = \frac{Q}{\frac{Qd}{\varepsilon_o A}} \quad \Rightarrow \quad C = \frac{\varepsilon_o A}{d}$$

What is $\sigma$?

$$\sigma = \frac{Q}{A}$$

$A = \text{area of plate}$

$C$ determined by geometry!
Initial charge on capacitor \( = Q_0 \)

Insert uncharged conductor

Charge on capacitor now \( = Q_1 \)

How is \( Q_1 \) related to \( Q_0 \)?

A) \( Q_1 < Q_0 \)
B) \( Q_1 = Q_0 \)
C) \( Q_1 > Q_0 \)

Plates not connected to anything

CHARGE CANNOT CHANGE!
What is the total charge induced on the bottom surface of the conductor?

A) $+Q_0$
B) $-Q_0$
C) 0
D) Positive but the magnitude unknown
E) Negative but the magnitude unknown
WHAT DO WE KNOW?

\[ E = 0 \]

\( E \) must be = 0 in conductor!

Charges inside conductor move to cancel \( E \) field from top & bottom plates.
Now calculate $V$ as a function of distance from the bottom conductor.

$$V(y) = - \int_{0}^{y} \vec{E} \cdot d\vec{y}$$

What is $\Delta V = V(d)$?

A) $\Delta V = E_0 d$

B) $\Delta V = E_0 (d - t)$

C) $\Delta V = E_0 (d + t)$

The integral = area under the curve
Two parallel plates of equal area carry equal and opposite charge $Q_0$. The potential difference between the two plates is measured to be $V_0$. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value $Q_1$ such that the potential difference between the plates remains the same.

Compare $Q_1$ and $Q_0$.

A. $Q_1 < Q_0$

B. $Q_1 = Q_0$

C. $Q_1 > Q_0$

“The distance for $Q_1$ is smaller therefore the charge must decrease to compensate for the change in distance “

“Since the potential remains the same, there is no change in the charge of $Q$ “

“the field through the conductor is zero, so it has constant potential. Because of this it must have greater charge so the total $V$ is that same “
Two parallel plates of equal area carry equal and opposite charge $Q_0$. The potential difference between the two plates is measured to be $V_0$. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value $Q_1$ such that the potential difference between the plates remains the same.

Compare the capacitance of the two configurations in the above problem.

A. $C_1 > C_0$
B. $C_1 = C_0$
C. $C_1 < C_0$

“The distance for $Q_1$ is smaller therefore the charge must decrease to compensate for the change in distance “

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Compare the capacitance of the two configurations in the above problem.

A. $C_1 > C_0$
B. $C_1 = C_0$
C. $C_1 < C_0$

We can determine $C$ from either case

- Same $V$ (preflight)
- Same $Q$ (lecture)

$C$ depends only on geometry!

$E_0 = \frac{Q_0}{\varepsilon_0 A}$

$C_0 = \frac{Q_0}{E_0 d}$

$C_1 = \frac{Q_0}{[E_0(d - t)]}$

$C_0 = \frac{\varepsilon_0 A}{d}$

$C_1 = \frac{\varepsilon_0 A}{(d - t)}$
Energy in Capacitors

Energy Stored in Capacitors

\[ U = \frac{1}{2} QV \]  
\[ U = \frac{1}{2} \frac{Q^2}{C} \]  
\[ U = \frac{1}{2} CV^2 \]

Energy Density

\[ u = \frac{1}{2} \varepsilon_0 E^2 \]

BANG
A capacitor is constructed from two conducting cylindrical shells of radii $a_1$, $a_2$, $a_3$, and $a_4$ and length $L$ ($L \gg a_i$).

What is the capacitance $C$ of this capacitor?

**Conceptual Analysis:**

$$C \equiv \frac{Q}{V}$$

But what is $Q$ and what is $V$? They are not given?

**Important Point:** $C$ is a property of the object! (concentric cylinders here)

- Assume some $Q$ (i.e., $+Q$ on one conductor and $-Q$ on the other)
- These charges create $E$ field in region between conductors
- This $E$ field determines a potential difference $V$ between the conductors
- $V$ should be proportional to $Q$; the ratio $Q/V$ is the capacitance.
A capacitor is constructed from two conducting cylindrical shells of radii $a_1$, $a_2$, $a_3$, and $a_4$ and length $L$ ($L \gg a_i$).

What is the capacitance $C$ of this capacitor?

$$C \equiv \frac{Q}{V}$$

**Strategic Analysis:**

- Put $+Q$ on outer shell and $-Q$ on inner shell
- Cylindrical symmetry: Use Gauss’ Law to calculate $E$ everywhere
- Integrate $E$ to get $V$
- Take ratio $Q/V$: should get expression only using geometric parameters $(a_i, L)$
A capacitor is constructed from two conducting cylindrical shells of radii $a_1$, $a_2$, $a_3$, and $a_4$ and length $L$ ($L >> a_i$).

What is the capacitance $C$ of this capacitor?

$$C \equiv \frac{Q}{V}$$

Where is $+Q$ on outer conductor located?

A) at $r = a_4$  
B) at $r = a_3$  
C) both surfaces  
D) throughout shell

Why?

Gauss’ law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

We know that $E = 0$ in conductor (between $a_3$ and $a_4$)

$$Q_{\text{enclosed}} = 0$$

$+Q$ must be on inside surface ($a_3$), so that $Q_{\text{enclosed}} = +Q - Q = 0$
A capacitor is constructed from two conducting cylindrical shells of radii \( a_1, a_2, a_3, \) and \( a_4 \) and length \( L \) \((L \gg a_i)\).

What is the capacitance \( C \) of this capacitor?

\[
C \equiv \frac{Q}{V}
\]

Where is \(-Q\) on inner conductor located?

A) at \( r = a_2 \)  
B) at \( r = a_1 \)  
C) both surfaces  
D) throughout shell

Why?

Gauss' law:

\[
\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
\]

We know that \( E = 0 \) in conductor (between \( a_1 \) and \( a_2 \))

\[
Q_{\text{enclosed}} = 0
\]

\( +Q \) must be on outer surface \((a_2)\), so that \( Q_{\text{enclosed}} = 0 \)
Calculation

A capacitor is constructed from two conducting cylindrical shells of radii \(a_1, a_2, a_3,\) and \(a_4\) and length \(L\) (\(L >> a_i\)).

What is the capacitance \(C\) of this capacitor?

\[
C \equiv \frac{Q}{V}
\]

\(a_2 < r < a_3:\) What is \(E(r)\)?

A) 0  
B) \(\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\)  
C) \(\frac{1}{2\pi\varepsilon_0} \frac{Q}{Lr}\)  
D) \(\frac{1}{2\pi\varepsilon_0} \frac{2Q}{Lr}\)  
E) \(\frac{1}{4\pi\varepsilon_0} \frac{2Q}{r^2}\)

Why?

Gauss’ law:

\[
\int_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} 
\rightarrow \quad E(2\pi rL) = \frac{Q}{\varepsilon_0} \quad \rightarrow \quad E = \frac{1}{2\pi\varepsilon_0} \frac{Q}{Lr}
\]

Direction: Radially In
A capacitor is constructed from two conducting cylindrical shells of radii $a_1$, $a_2$, $a_3$, and $a_4$ and length $L$ ($L \gg a_i$).

What is the capacitance $C$ of this capacitor?

$$C \equiv \frac{Q}{V} \quad a_2 < r < a_3: \quad E = \frac{1}{2\pi\varepsilon_0} \frac{Q}{Lr}$$

$r < a_2$: $E(r) = 0$

since $Q_{enclosed} = 0$

What is $V$?

The potential difference between the conductors.

What is the sign of $V = V_{outer} - V_{inner}$?

A) $V_{outer} - V_{inner} < 0$  
B) $V_{outer} - V_{inner} = 0$  
C) $V_{outer} - V_{inner} > 0$
A capacitor is constructed from two conducting cylindrical shells of radii $a_1, a_2, a_3,$ and $a_4$ and length $L$ ($L >> a_i$).

What is the capacitance $C$ of this capacitor?

$$C \equiv \frac{Q}{V}$$

For $a_2 < r < a_3$: 
$$E = \frac{1}{2\pi \varepsilon_0 L} \frac{Q}{r}$$

What is $V \equiv V_{outer} - V_{inner}$?

$$V = -\int_{a_2}^{a_3} \frac{Q}{2\pi \varepsilon_0 L} \frac{dr}{r}$$

$V$ proportional to $Q$, as promised

$$C \equiv \frac{Q}{V} = \frac{2\pi \varepsilon_0 L}{ln(a_3 / a_2)}$$