

# *Electricity & Magnetism*

## *Lecture 8: Capacitors*

Today's Concept:

Capacitors

(Capacitors in a circuits, Dielectrics, Energy in capacitors)

Alternate terms: “condensors”, “capacitators”, ....

# Stuff you asked about:

- “the formula for calculating capacitance in parallel and series are the opposite of the formula for resistance “
- “Please go over the Capacitors with and without a dielectric from checkpoint.”
- “this stuff is a lot harder than what we’ve been doing before, one thing i don’t understand is when you disconnect a battery from a capacitor what exactly happens, how come it still has a charge?? ”

# Capacitors, connected and unconnected

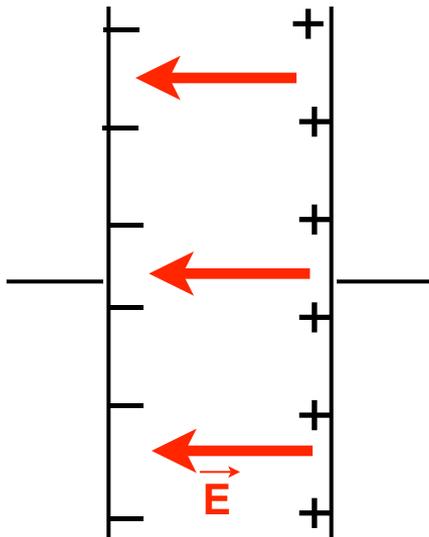
- ★ A capacitor that is **un**connected to a battery has constant charge:  $V = Q/C$  ( $V$  is determined by  $Q$ )
- ★ Capacitors in parallel have the same voltage. Charge may redistribute among them:  $Q_1/C_1 = Q_2/C_2 = Q_2/C_2 \dots$
- ★ A capacitor connected to a battery has a constant voltage.  $Q = CV$  ( $Q$  is determined by  $V$ )

# Dielectric

- ★ Charged capacitor, not connected to battery
- ★ Dielectric makes  $\Delta V$  smaller

air capacitor

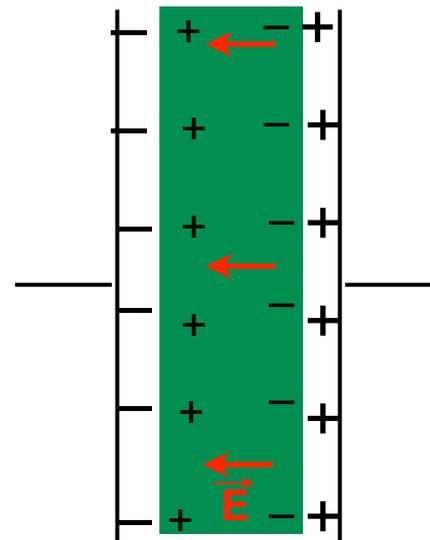
$$\kappa = 1$$



$$C = \epsilon_0 A/L$$

dielectric capacitor

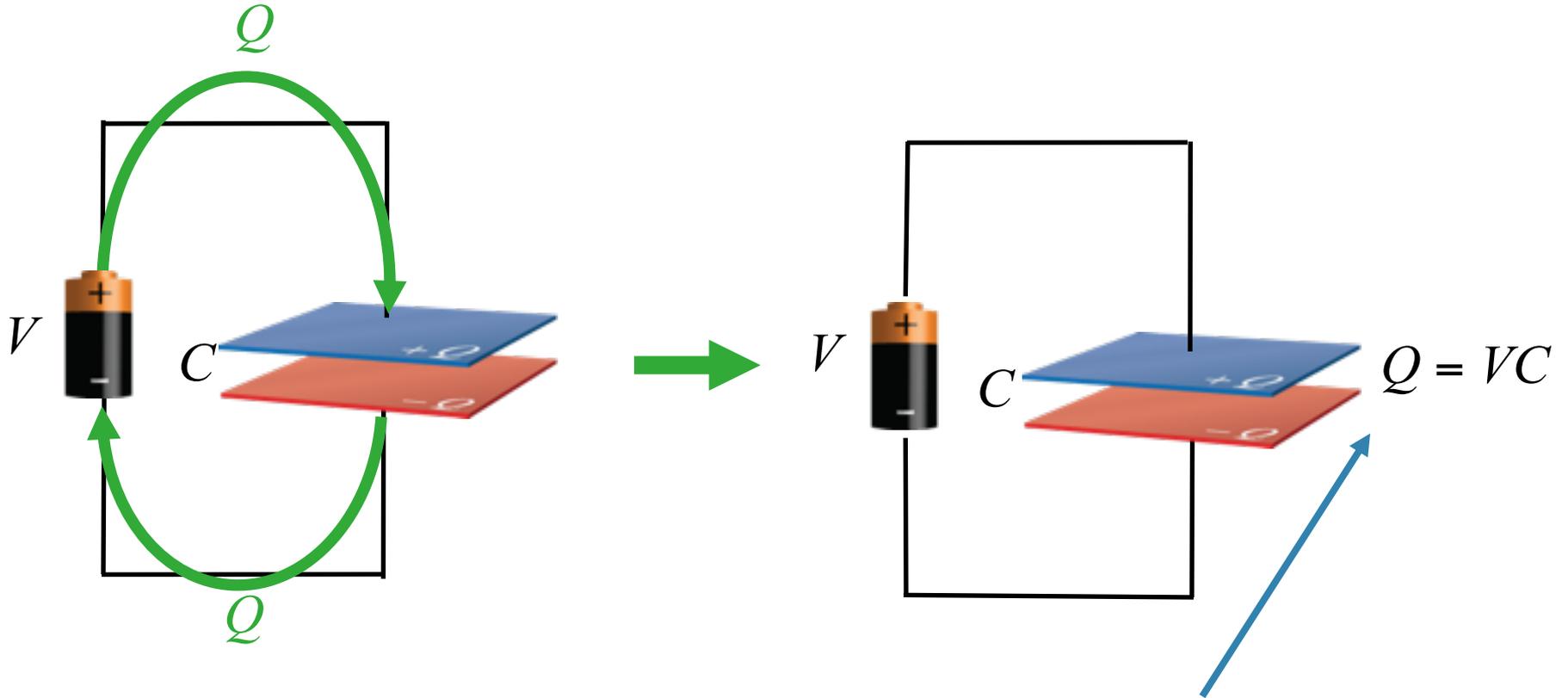
$$\kappa > 1$$



$$C = \kappa \epsilon_0 A/L$$

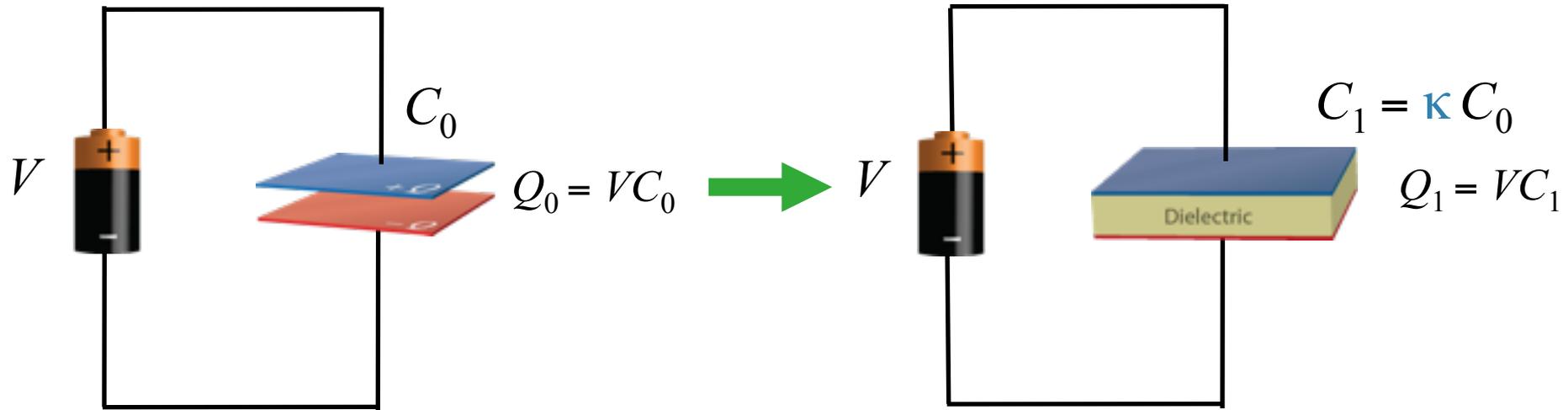
$E$  inside dielectric is smaller than  $E$  outside.  
The charge on the surface of the dielectric partially cancels the  $E$  field from the charge on the plates.  
Therefore the  $\Delta V$  between the plates is less.

# Simple Capacitor Circuit



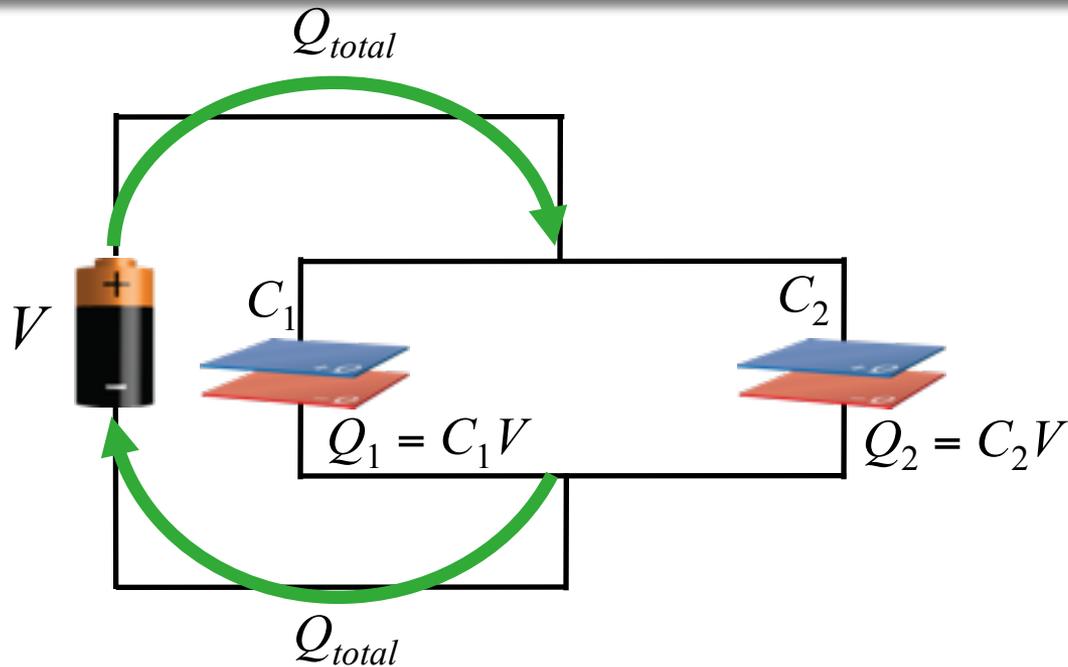
This “ $Q$ ” really means that the battery has moved charge  $Q$  from one plate to the other, so that one plate holds  $+Q$  and the other  $-Q$ .

# Dielectrics



By adding a dielectric you are just making a new capacitor with larger capacitance (factor of  $\kappa$ )

# Parallel Capacitor Circuit

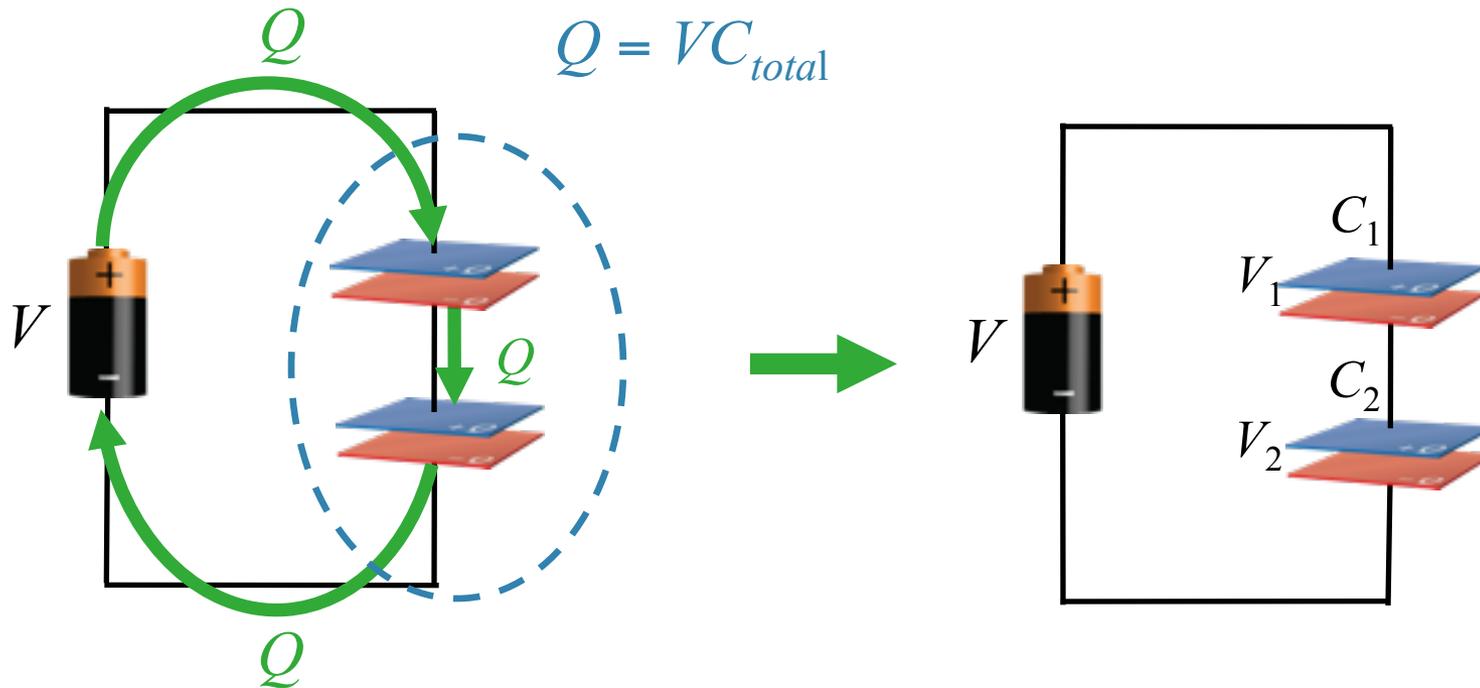


**Key point:**  $V$  is the same for both capacitors

**Key Point:**  $Q_{total} = Q_1 + Q_2 = VC_1 + VC_2 = V(C_1 + C_2)$

$$C_{total} = C_1 + C_2$$

# Series Capacitor Circuit



**Key point:**  $Q$  is the same for both capacitors

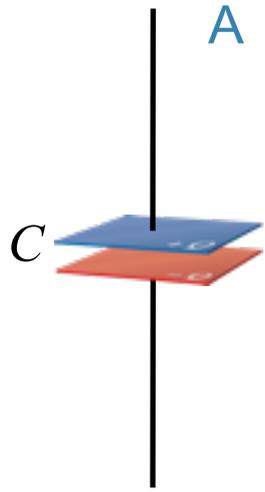
**Key point:**  $Q = VC_{total} = V_1C_1 = V_2C_2$

**Also:**  $V = V_1 + V_2$   $\longrightarrow$   $Q/C_{total} = Q/C_1 + Q/C_2$

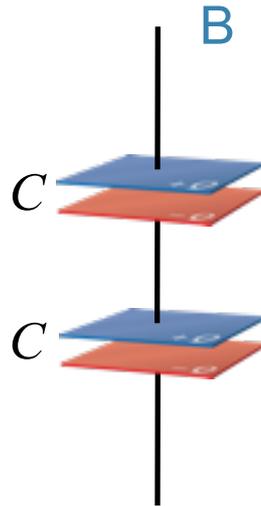
$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$

# CheckPoint: Three Capacitor Configurations

The three configurations shown below are constructed using identical capacitors. Which of these configurations has lowest total capacitance?

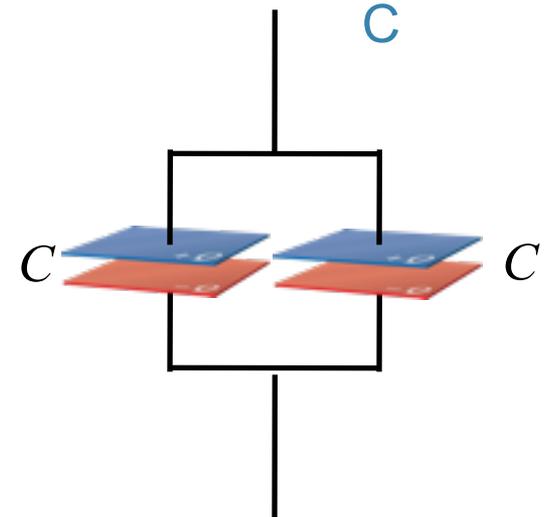


$$C_{total} = C$$



$$\begin{aligned} 1/C_{total} &= 1/C + 1/C \\ &= 2/C \end{aligned}$$

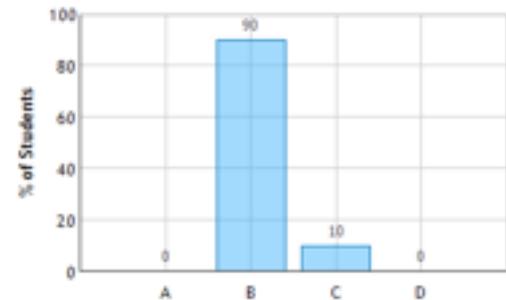
$$C_{total} = C/2$$



$$C_{total} = 2C$$

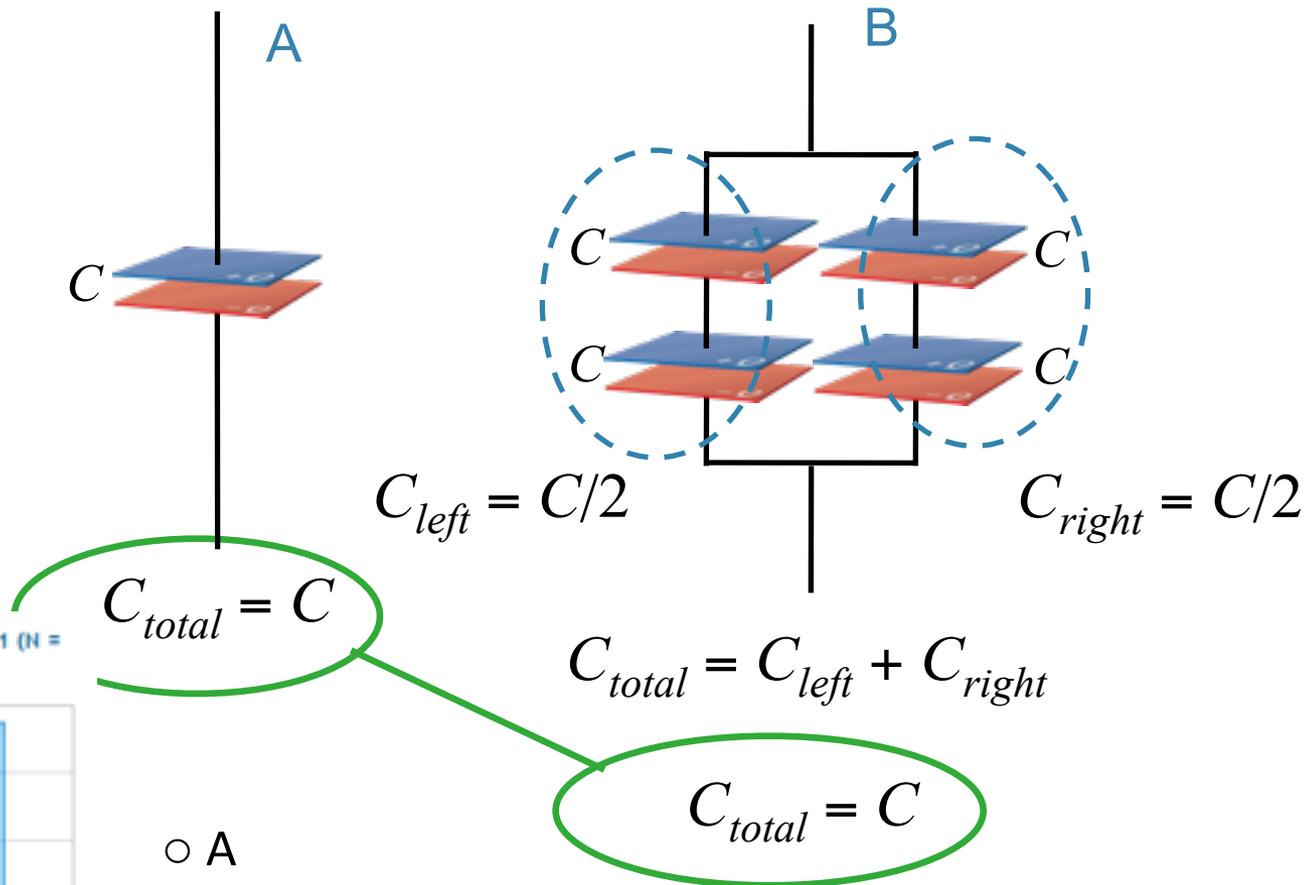
D: All 3 are the same

Three Capacitor Configurations: Question 1 (N = 20)

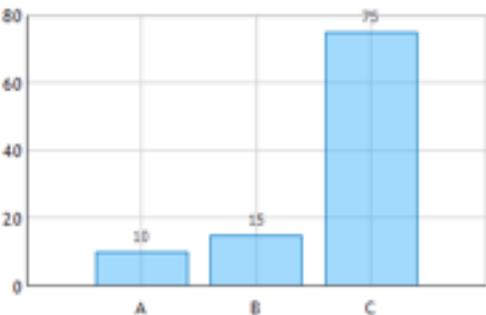


# CheckPoint: Two Capacitor Configurations

The two configurations shown below are constructed using identical capacitors. Which of these configurations has the lowest overall capacitance?

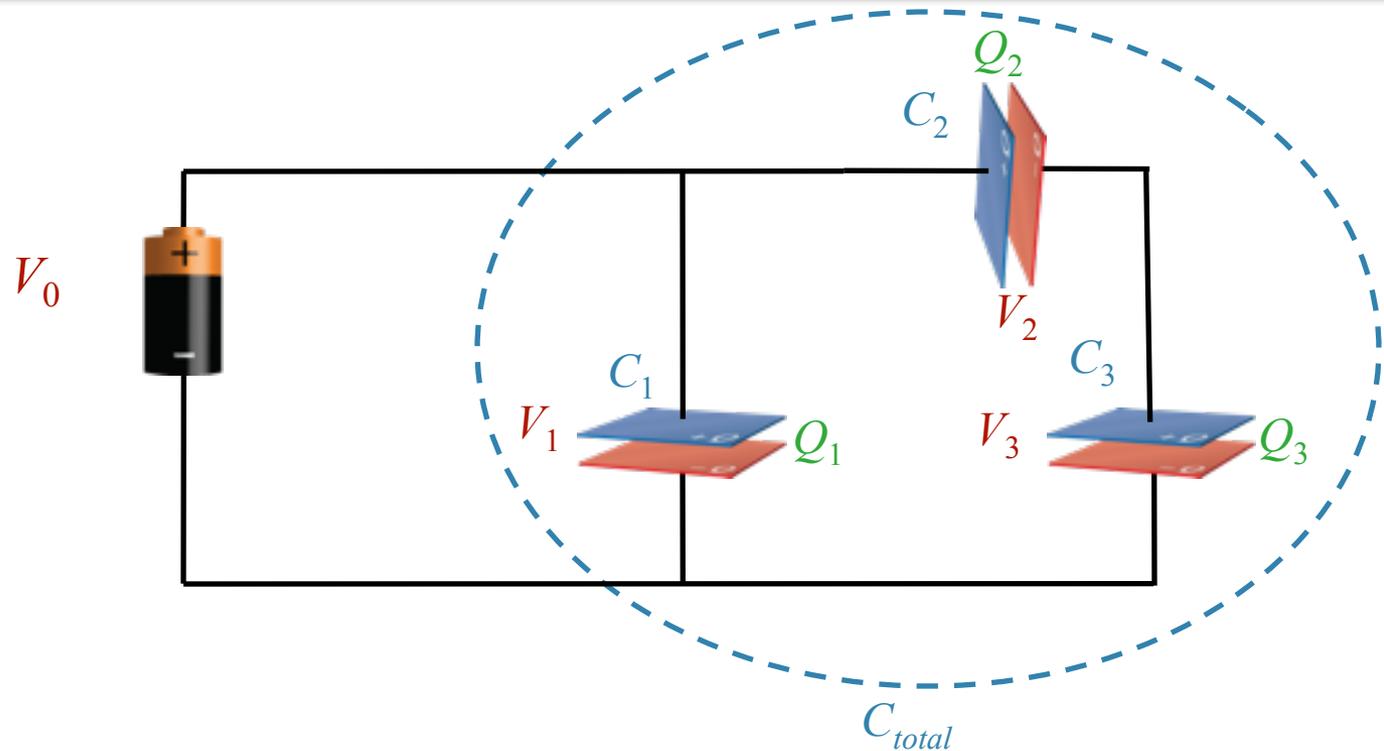


Two Capacitor Configurations: Question 1 (N = 20)



- A
- B
- Both configurations have the same capacitance

# Clicker Question: Capacitor Network



Which of the following is **NOT** necessarily true:

- A)  $V_0 = V_1$
- B)  $C_{total} > C_1$
- C)  $V_2 = V_3$**
- D)  $Q_2 = Q_3$
- E)  $V_1 = V_2 + V_3$

# CheckPoint: Capacitor Network

A circuit consists of three unequal capacitors  $C_1$ ,  $C_2$ , and  $C_3$  which are connected to a battery of voltage  $V_0$ . The capacitance of  $C_2$  is twice that of  $C_1$ . The capacitance of  $C_3$  is three times that of  $C_1$ . The capacitors obtain charges  $Q_1$ ,  $Q_2$ , and  $Q_3$ .

Compare  $Q_1$ ,  $Q_2$ , and  $Q_3$ .

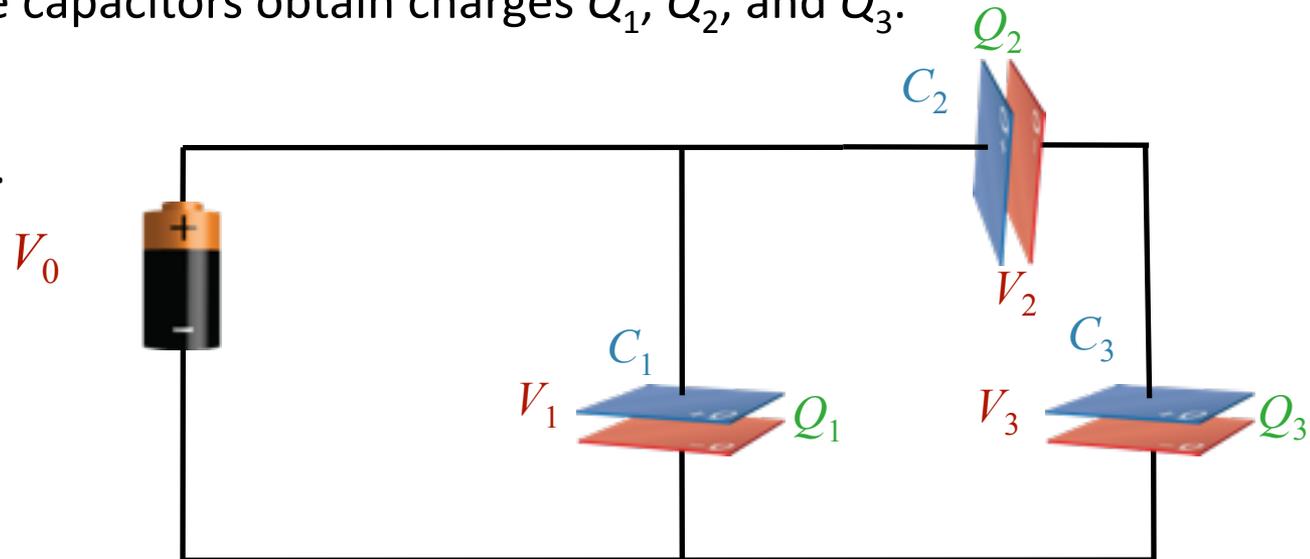
A.  $Q_1 > Q_3 > Q_2$

B.  $Q_1 > Q_2 > Q_3$

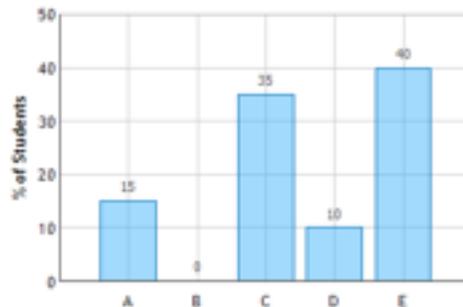
C.  $Q_1 > Q_2 = Q_3$

D.  $Q_1 = Q_2 = Q_3$

E.  $Q_1 < Q_2 = Q_3$

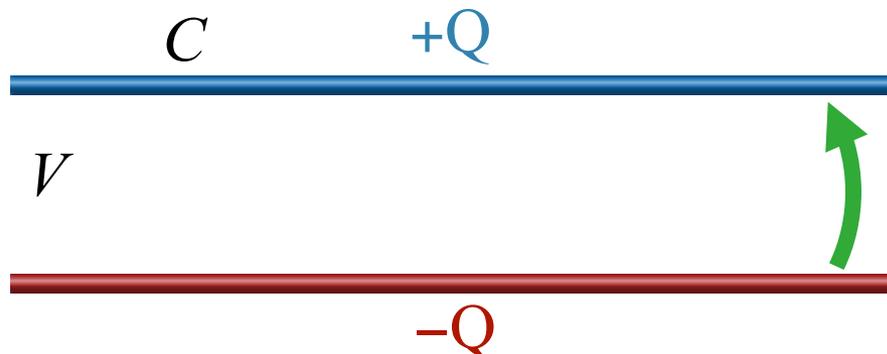


Capacitor Network: Question 1 (N = 20)



# Energy in a Capacitor

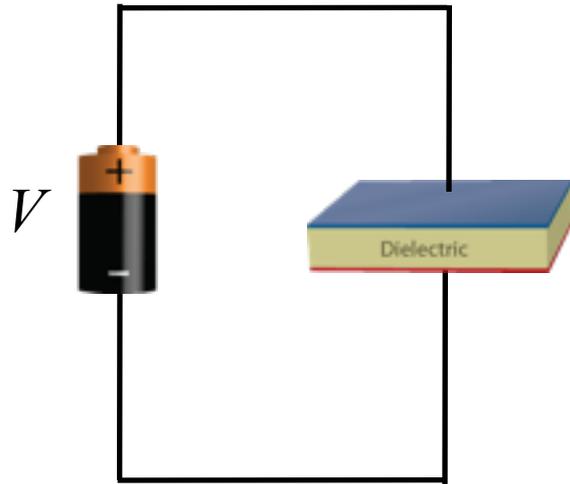
In Prelecture 7 we calculated the work done to move charge  $Q$  from one plate to another:


$$\begin{aligned} U &= \frac{1}{2}QV \\ &= \frac{1}{2}CV^2 \\ &= \frac{1}{2}Q^2/C \end{aligned} \quad \left. \vphantom{\begin{aligned} U &= \frac{1}{2}QV \\ &= \frac{1}{2}CV^2 \\ &= \frac{1}{2}Q^2/C \end{aligned}} \right\} \text{Since } Q = VC$$

This is potential energy waiting to be used...

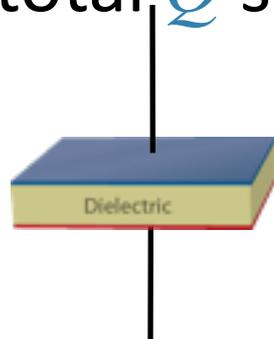
# Messing with Capacitors

If connected to a battery  $V$  stays constant



$$\begin{array}{l} V_1 = V \\ C_1 = \kappa C \end{array} \left. \vphantom{\begin{array}{l} V_1 = V \\ C_1 = \kappa C \end{array}} \right\} \rightarrow \begin{array}{l} Q_1 = C_1 V_1 \\ = \kappa C V = \kappa Q \end{array}$$

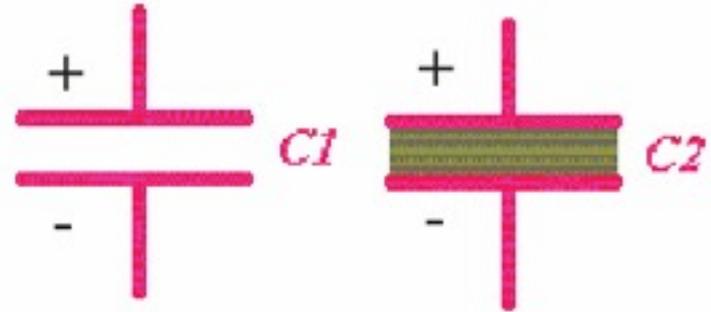
If isolated then total  $Q$  stays constant



$$\begin{array}{l} Q_1 = Q \\ C_1 = \kappa C \end{array} \left. \vphantom{\begin{array}{l} Q_1 = Q \\ C_1 = \kappa C \end{array}} \right\} \rightarrow \begin{array}{l} V_1 = Q_1 / C_1 \\ = Q / \kappa C = V / \kappa \end{array}$$

# CheckPoint: Capacitors and Dielectrics 1

Two identical parallel plate capacitors are given the same charge  $Q$ , after which they are disconnected from the battery. After  $C_2$  has been charged and disconnected, it is filled with a dielectric.



Compare the voltages of the two capacitors.

A.  $V_1 > V_2$

B.  $V_1 = V_2$

C.  $V_1 < V_2$

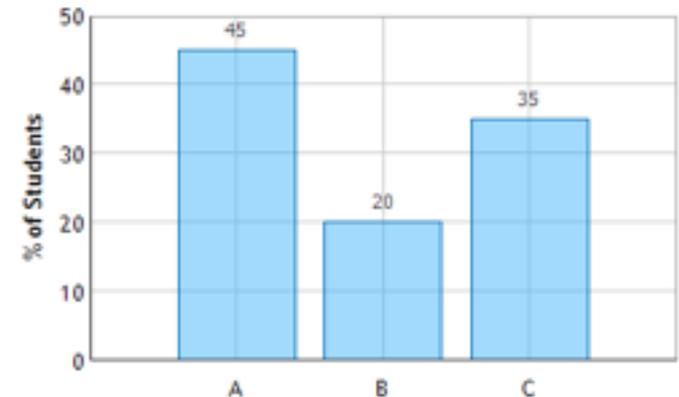
“The electric field decreases on the second due to the dielectric, so its voltage must decrease.”

“The little brown thing is stopping the pikachus from reaching the other side.”

“ Dielectrics don't change voltages. ”

“Capacitance lowers in case 2, meaning  $V$  increases in case 2. ’

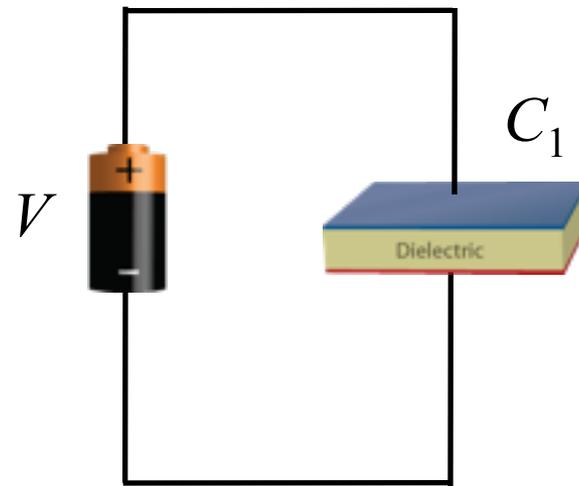
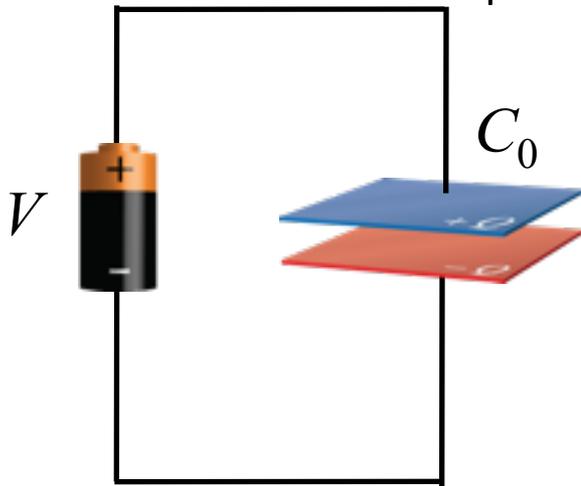
Capacitors with and without a Dielectric:  
Question 1 (N = 20)



# Clicker Question: Messing with Capacitors



Two identical parallel plate capacitors are connected to identical batteries. Then a dielectric is inserted between the plates of capacitor  $C_1$ . Compare the energy stored in the two capacitors.



A)  $U_1 < U_0$

B)  $U_0 = U_1$

C)  $U_1 > U_0$

Compare using  $U = \frac{1}{2}CV^2$

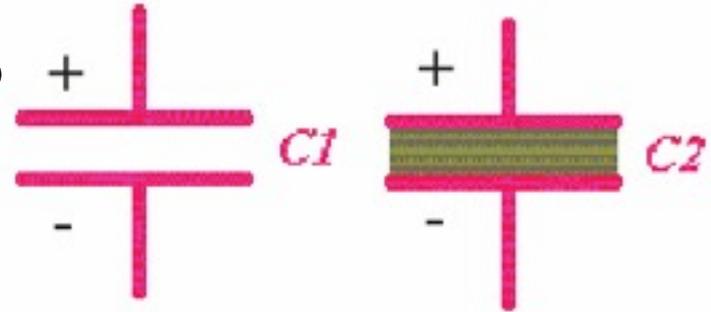
$$U_1/U_0 = \kappa$$

→ Potential Energy goes UP

# CheckPoint: Capacitors and Dielectrics 2

Two identical parallel plate capacitors are given the same charge  $Q$ , after which they are disconnected from the battery. After  $C_2$  has been charged and disconnected, it is filled with a dielectric.

Compare the potential energy stored by the two capacitors.



A.  $U_1 > U_2$

B.  $U_1 = U_2$

C.  $U_1 < U_2$

“If the electric field goes down, then so does the potential energy of the second system..”

“Dielectrics don't affect potential.”

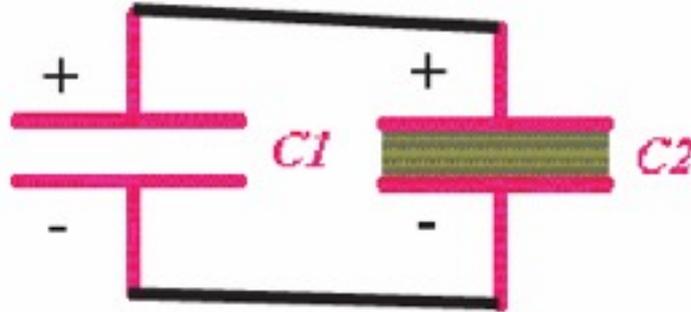
“ $U = QV$ . Since voltage remains constant and charge increases, potential energy must also increase.”

Capacitors with and without a Dielectric:  
Question 3 (N = 20)



# CheckPoint: Capacitors and Dielectrics 3

The two capacitors are now connected to each other by wires as shown. How will the charge redistribute itself, if at all?



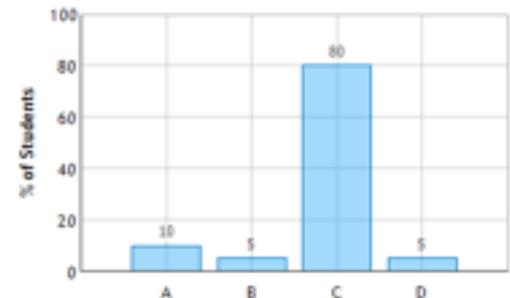
$V$  must be the same !

- The charges will flow so that the charge on C1 will become equal to the charge on C2.
- The charges will flow so that the energy stored in C1 will become equal to the energy stored in C2
- The charges will flow so that the potential difference across C1 will become the same as the potential difference across C2.
- No charges will flow. The charge on the capacitors will remain what it was before they were connected.

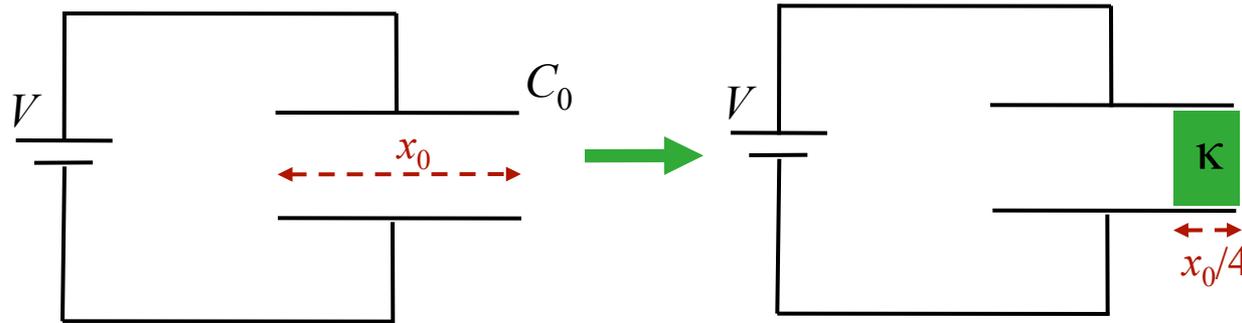
$$Q: \quad \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \quad \longrightarrow \quad Q_1 = \frac{C_1}{C_2} Q_2$$

$$U: \quad \begin{aligned} U_1 &= \frac{1}{2} C_1 V^2 \\ U_2 &= \frac{1}{2} C_2 V^2 \end{aligned} \quad \longrightarrow \quad U_1 = \frac{C_1}{C_2} U_2$$

Capacitors with and without a Dielectric:  
Question 5 (N = 20)



# Calculation



An air-gap capacitor, having capacitance  $C_0$  and width  $x_0$  is connected to a battery of voltage  $V$ .

A dielectric ( $\kappa$ ) of width  $x_0/4$  is inserted into the gap as shown.

Conceptual Analysis:

$$C \equiv \frac{Q}{V}$$

What changes when the dielectric added?

- A) Only  $C$     B) only  $Q$     C) only  $V$     **D)  $C$  and  $Q$**     E)  $V$  and  $Q$

Adding dielectric changes the physical capacitor



$C$  changes

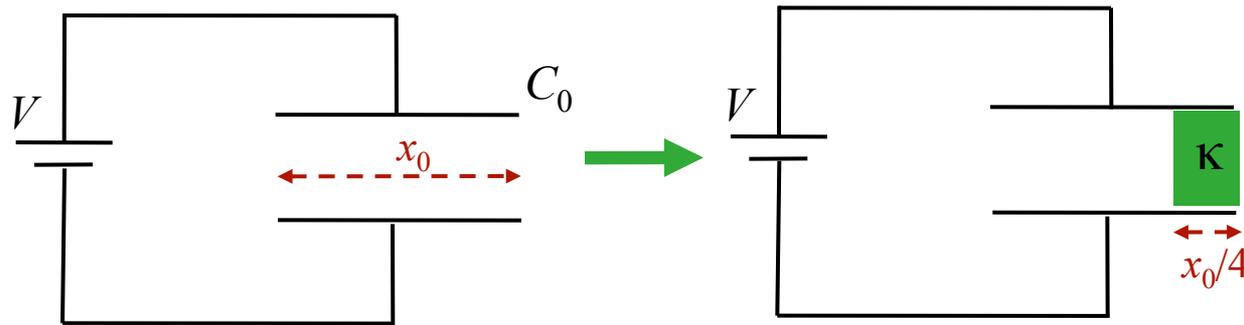
$V$  does not change and  $C$  changes



$Q$  changes

What is  $Q_f$ , the final charge on the capacitor?

# Calculation



An air-gap capacitor, having capacitance  $C_0$  and width  $x_0$  is connected to a battery of voltage  $V$ .

A dielectric ( $\kappa$ ) of width  $x_0/4$  is inserted into the gap as shown.

## Strategic Analysis:

- Calculate new capacitance  $C$
- Apply definition of capacitance to determine  $Q$

To calculate  $C$ , let's first look at:



A)  $V_{left} < V_{right}$

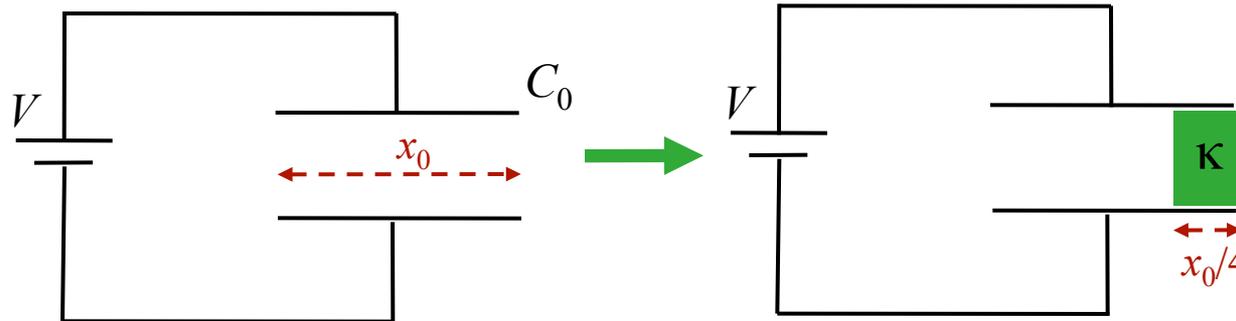
B)  $V_{left} = V_{right}$

C)  $V_{left} > V_{right}$

What is  $Q_f$ , the final charge on the capacitor?

The conducting plate is an equipotential !

# Calculation

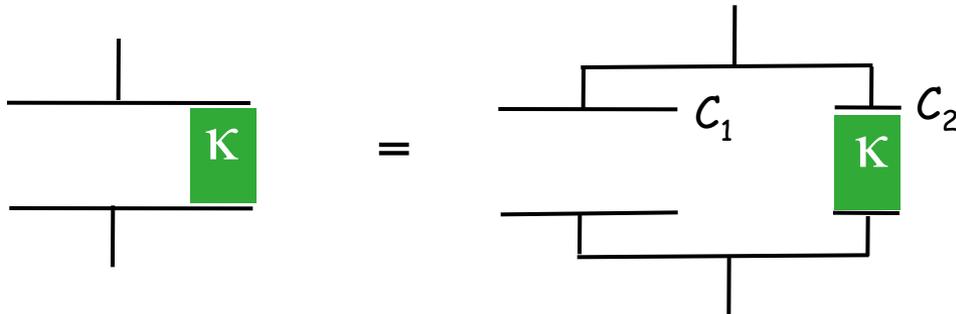


An air-gap capacitor, having capacitance  $C_0$  and width  $x_0$  is connected to a battery of voltage  $V$ .

A dielectric ( $\kappa$ ) of width  $x_0/4$  is inserted into the gap as shown.

What is  $Q_f$ , the final charge on the capacitor?

Can consider capacitor to be two capacitances,  $C_1$  and  $C_2$ , in parallel



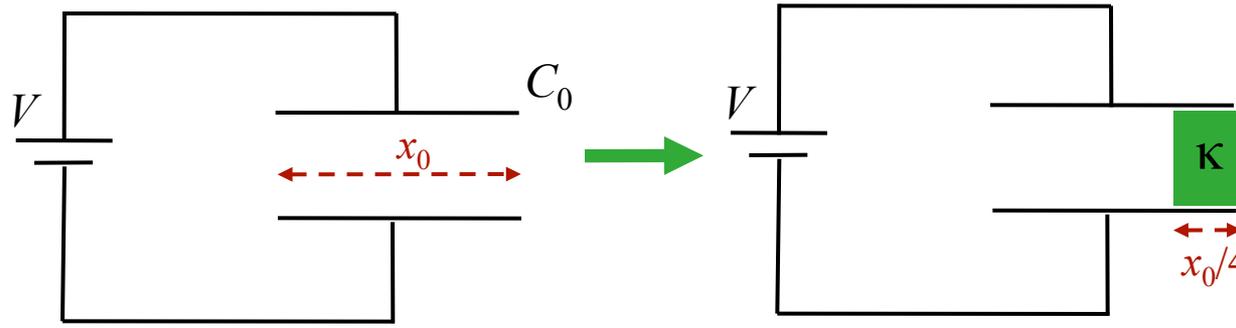
What is  $C_1$  ?

- A)  $C_1 = C_0$    **B)  $C_1 = \frac{3}{4}C_0$**    C)  $C_1 = \frac{4}{3}C_0$    D)  $C_1 = \frac{1}{4}C_0$

In general. For parallel plate capacitor:  $C = \epsilon_0 A/d$

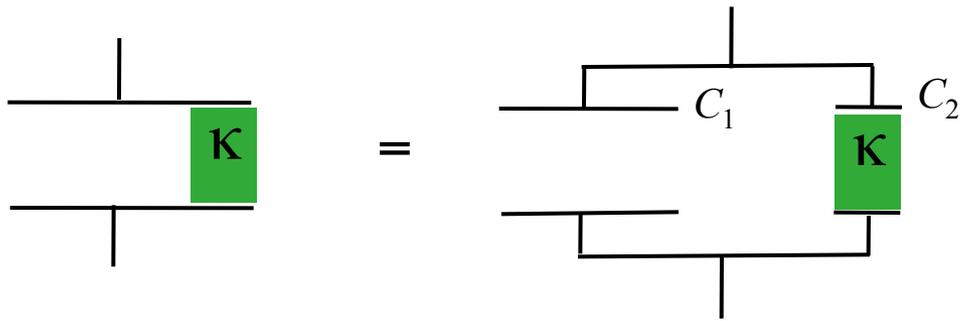
$$\begin{matrix} A = \frac{3}{4}A_0 \\ d = d_0 \end{matrix} \quad \square \quad \longrightarrow \quad C_1 = \frac{3}{4}(\epsilon_0 A_0/d_0) \quad \longrightarrow \quad \boxed{C_1 = \frac{3}{4}C_0}$$

# Calculation



An air-gap capacitor, having capacitance  $C_0$  and width  $x_0$  is connected to a battery of voltage  $V$ .

A dielectric ( $\kappa$ ) of width  $x_0/4$  is inserted into the gap as shown.



What is  $Q_f$ , the final charge on the capacitor?  
 $C_1 = \frac{3}{4}C_0$

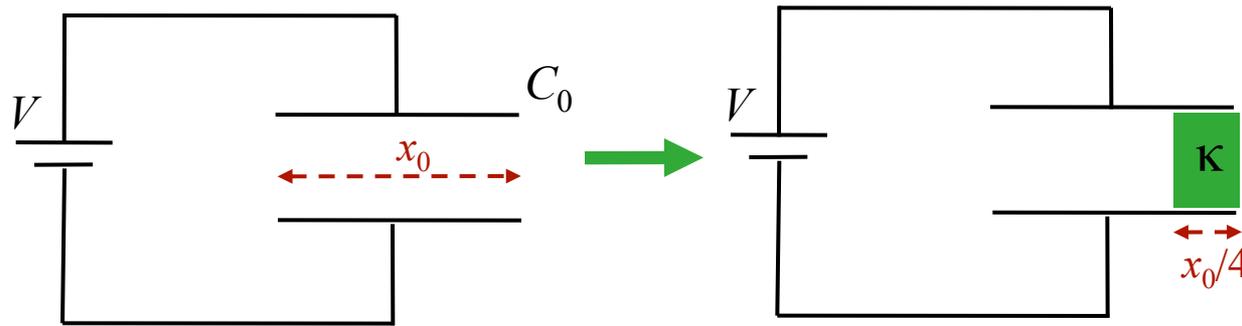
What is  $C_2$  ?

- A)  $C_2 = \kappa C_0$     B)  $C_2 = \frac{3}{4} \kappa C_0$     C)  $C_2 = \frac{4}{3} \kappa C_0$     **D)  $C_2 = \frac{1}{4} \kappa C_0$**

In general. For parallel plate capacitor filled with dielectric:  $C = \kappa \epsilon_0 A/d$

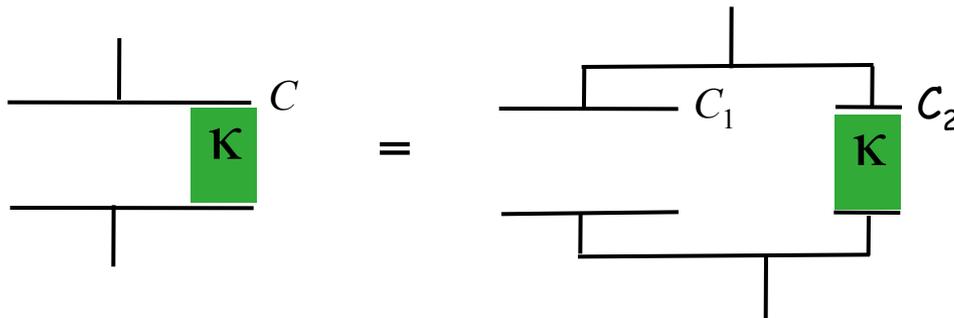
$$\begin{matrix} A = \frac{1}{4}A_0 \\ d = d_0 \end{matrix} \quad \left. \vphantom{\begin{matrix} A = \frac{1}{4}A_0 \\ d = d_0 \end{matrix}} \right\} \longrightarrow C = \frac{1}{4}(\kappa \epsilon_0 A_0/d_0) \longrightarrow C_2 = \frac{1}{4} \kappa C_0$$

# Calculation



An air-gap capacitor, having capacitance  $C_0$  and width  $x_0$  is connected to a battery of voltage  $V$ .

A dielectric ( $\kappa$ ) of width  $x_0/4$  is inserted into the gap as shown.



What is  $Q_f$ , the final charge on the capacitor?  
 $C_1 = \frac{3}{4}C_0$       $C_2 = \frac{1}{4}\kappa C_0$

What is  $C$ ?

**A)  $C = C_1 + C_2$**

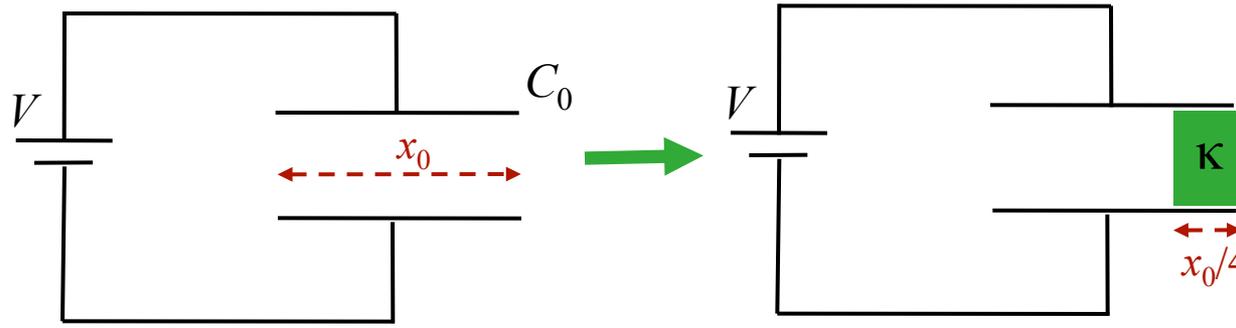
B)  $C = C_1 + \kappa C_2$

C)  $C = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$

$C =$  parallel combination of  $C_1$  and  $C_2$ :  $C = C_1 + C_2$

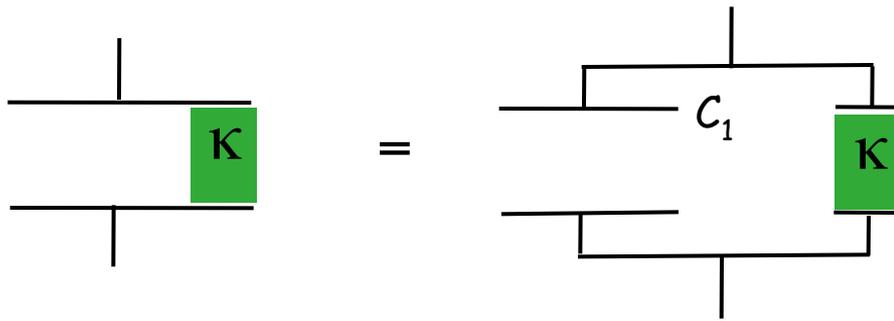
$\rightarrow C = C_0 \left( \frac{3}{4} + \frac{1}{4} \kappa \right)$

# Calculation



An air-gap capacitor, having capacitance  $C_0$  and width  $x_0$  is connected to a battery of voltage  $V$ .

A dielectric ( $\kappa$ ) of width  $x_0/4$  is inserted into the gap as shown.



What is  $Q_f$ , the final charge on the capacitor?

$$C_1 = \frac{3}{4}C_0$$

$$C_2 = \frac{1}{4}\kappa C_0$$

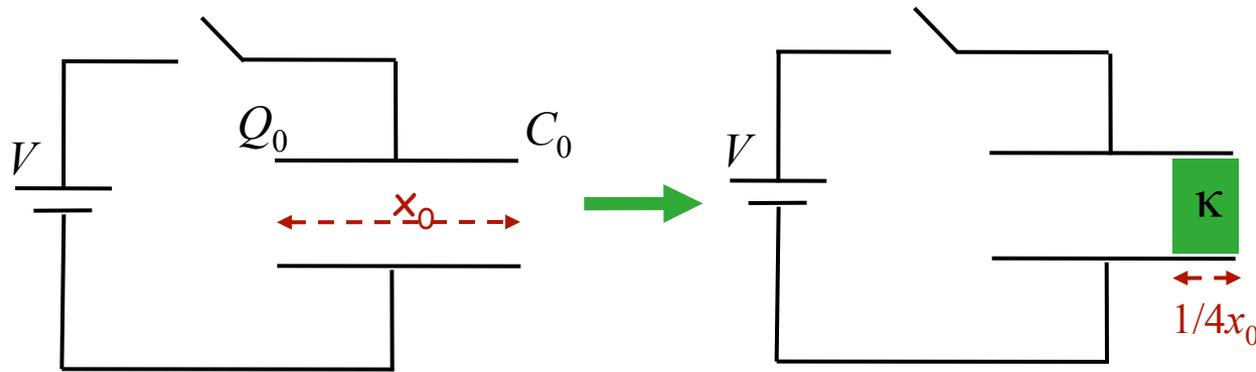
$$\rightarrow C = C_0 \left( \frac{3}{4} + \frac{1}{4}\kappa \right)$$

What is  $Q$ ?

$$C \equiv \frac{Q}{V} \rightarrow Q = VC$$

$$Q_f = VC_0 \left( \frac{3}{4} + \frac{1}{4}\kappa \right)$$

# Different Problem



An air-gap capacitor, having capacitance  $C_0$  and width  $x_0$  is connected to a battery of voltage  $V$  and then **battery is disconnected**.

A dielectric ( $\kappa$ ) of width  $1/4x_0$  is inserted into the gap as shown.

What is  $V_f$ , the final voltage on the capacitor?

A)  $V_f < V$

B)  $V_f = V$

C)  $V_f > V$

$Q$  stays same: no way to add or subtract

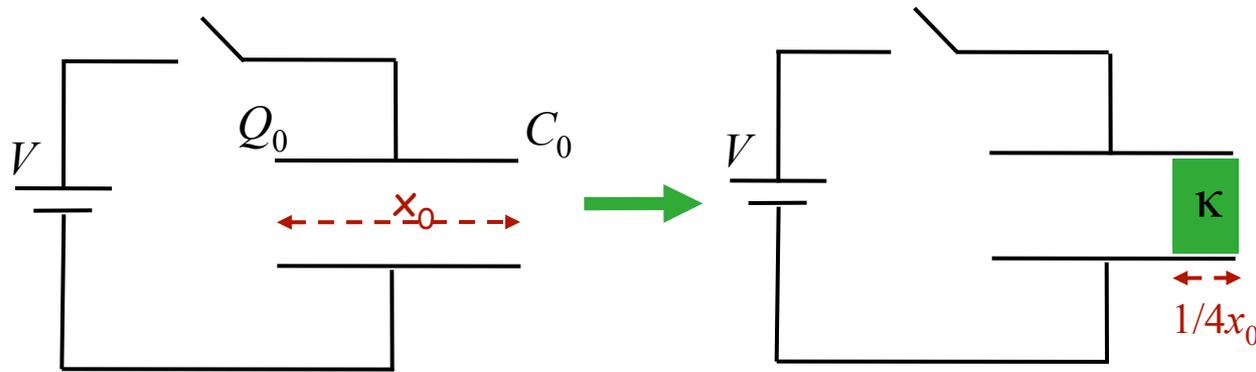
$$\longrightarrow Q = Q_0 = C_0 V$$

We know  $C$ : (property of capacitor)

$$\longrightarrow C = C_0 \left( \frac{3}{4} + \frac{1}{4} \kappa \right)$$

$$\longrightarrow V_f = Q/C = V / \left( \frac{3}{4} + \frac{1}{4} \kappa \right)$$

# Different Problem



An air-gap capacitor, having capacitance  $C_0$  and width  $x_0$  is connected to a battery of voltage  $V$  and then **battery is disconnected**.

A dielectric ( $\kappa$ ) of width  $1/4x_0$  is inserted into the gap as shown.

What is  $V_f$ , the final voltage on the capacitor?

$$V_f = Q/C = V/(3/4 + 1/4 \kappa)$$

How did energy stored in capacitor change when dielectric inserted?

- A)  $U$  increased    B)  $U$  stayed same    **C)  $U$  decreased**

$$U = \frac{1}{2} Q^2/C$$

$Q$  remained same

$C$  increased



$U$  decreased