

# *Electricity & Magnetism*

## *Lecture 12*

Today's Concept:

Magnetic Force on Moving Charges

$$\vec{F} = q\vec{v} \times \vec{B}$$

# Comments

this stuff gets fun now :)

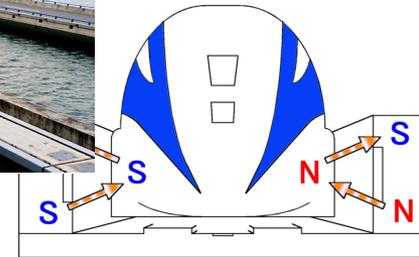
Will a magnet car move??????

Please go over cross product.

What is the "sense" in these magnetic concepts? Is it the same as a vector "direction" ? Also, can you demonstrate the right-hand rule. It is a bit abstract to "see" on a flat screen, a visual demo would be nice.

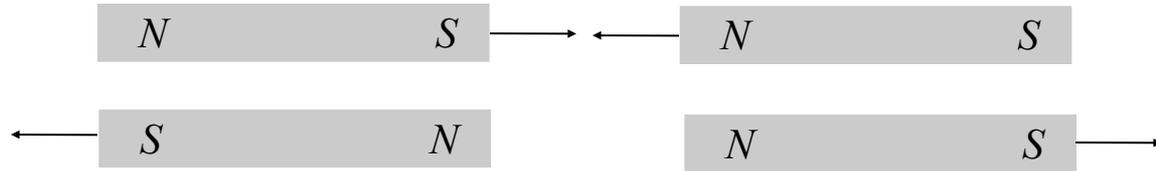
etc., etc., ...

**Will do!**



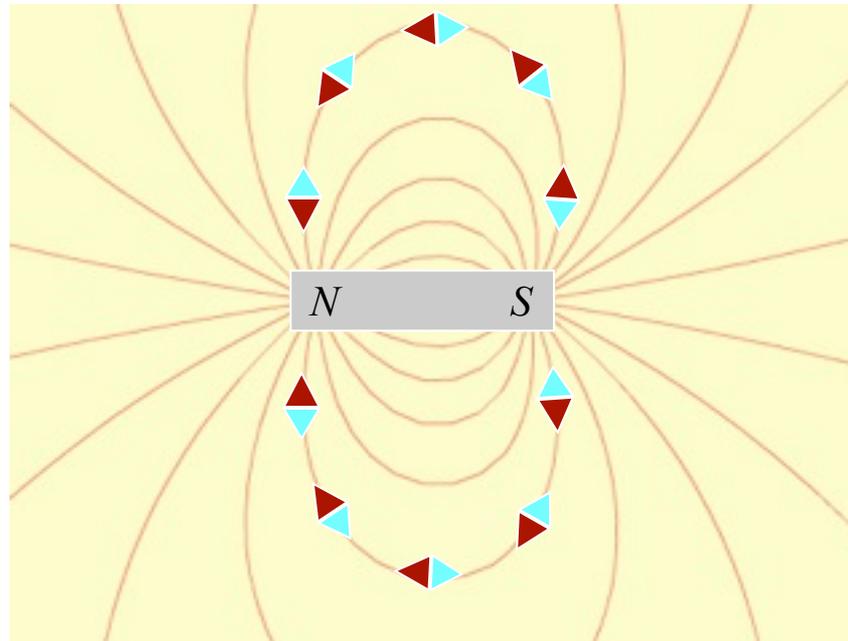
# Magnetic Observations

## Bar Magnets



## Compass Needles

These are “magnetic dipoles” and behave similarly to “electric dipoles”

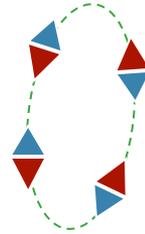


## Magnetic Charge?



# Magnetic Observations

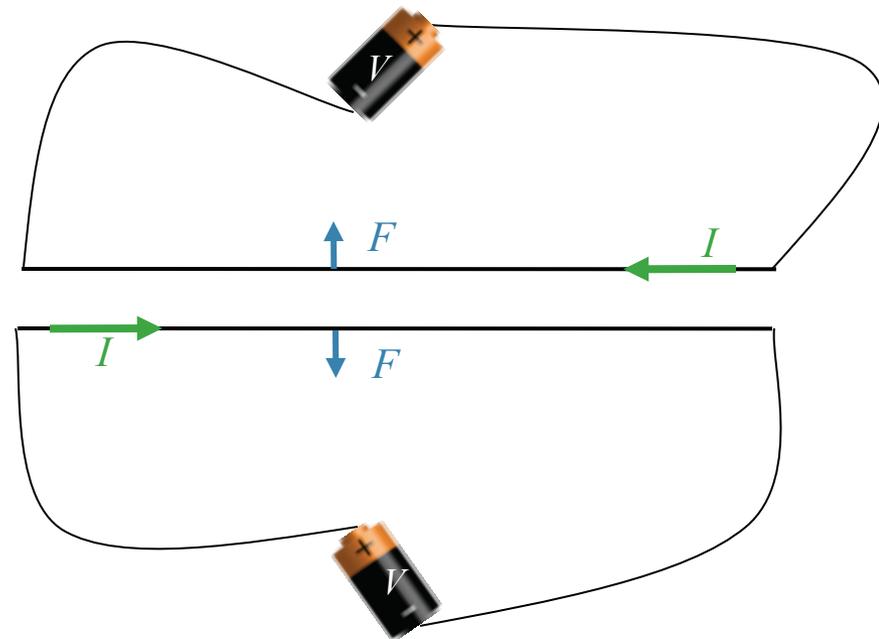
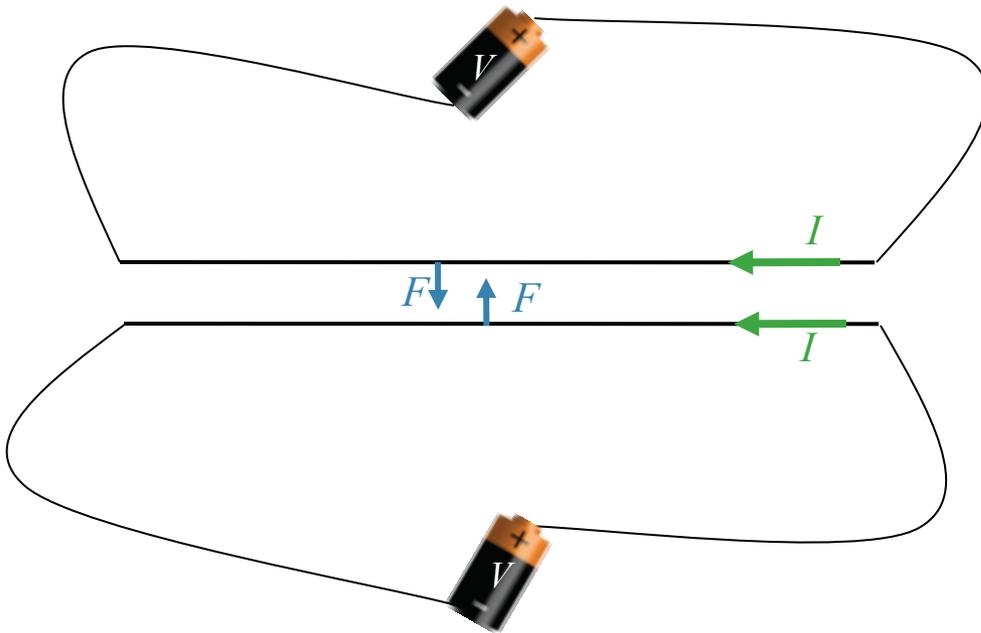
Compass needle deflected by electric current



I

Magnetic fields created by electric currents

Magnetic fields exert forces on electric currents (charges in motion)



# Magnetism & Moving Charges

All observations are explained by two equations:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Today

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

Next Week

# Cross Product Review

## Cross Product different from Dot Product

$\mathbf{A} \cdot \mathbf{B}$  is a scalar;  $\mathbf{A} \times \mathbf{B}$  is a vector

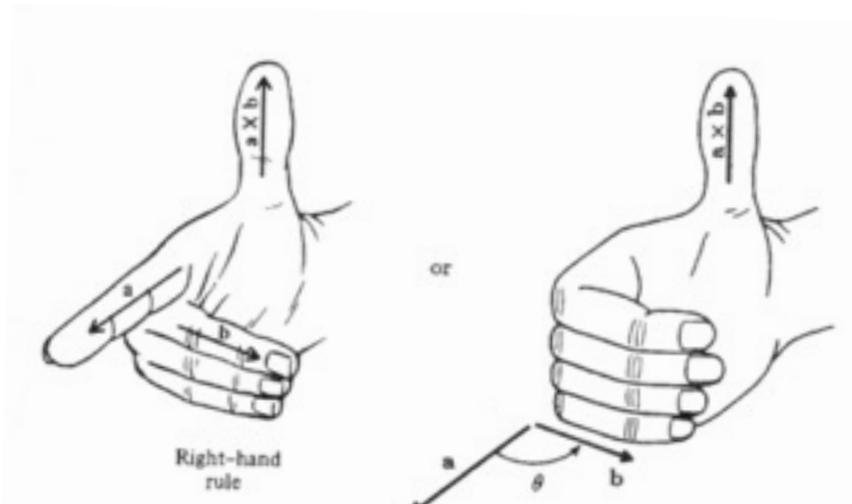
$\mathbf{A} \cdot \mathbf{B}$  proportional to the component of  $\mathbf{B}$  parallel to  $\mathbf{A}$

$\mathbf{A} \times \mathbf{B}$  proportional to the component of  $\mathbf{B}$  perpendicular to  $\mathbf{A}$

## Definition of $\mathbf{A} \times \mathbf{B}$

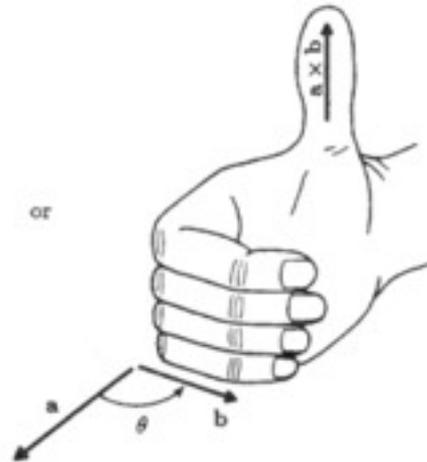
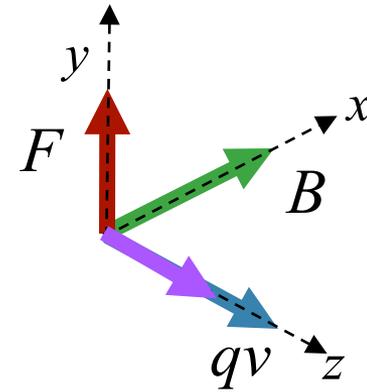
Magnitude:  $|\mathbf{A}||\mathbf{B}| \sin \theta$

Direction: perpendicular to plane defined by  $\mathbf{A}$  and  $\mathbf{B}$  with sense given by right-hand-rule



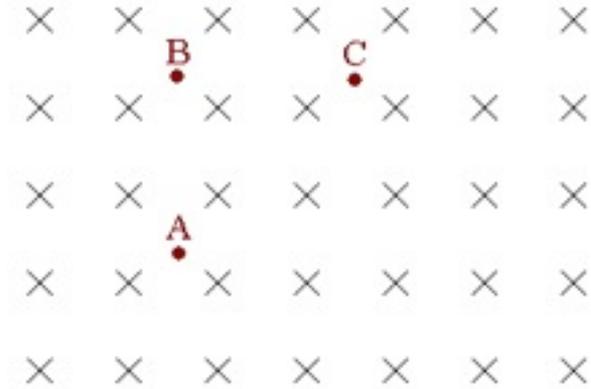
# Remembering Directions: The Right Hand Rule

$$\vec{F} = q\vec{v} \times \vec{B}$$



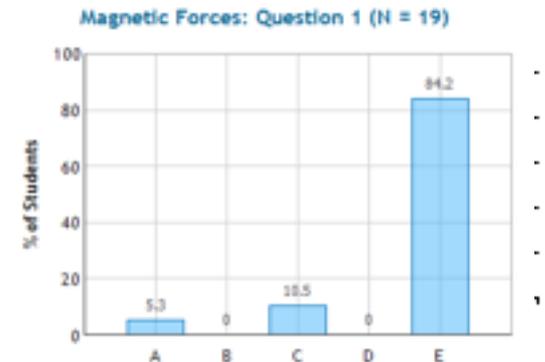
# Checkpoint 2

Three points are arranged in a uniform magnetic field. The B field points into the screen.



A positively charged particle is at point A and is stationary. The direction of the magnetic force on the particle is

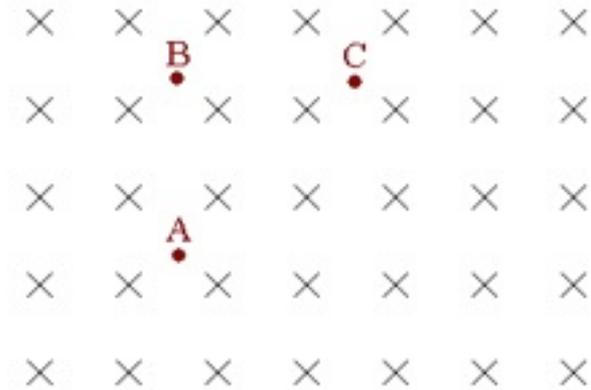
- A. right
- B. left
- C. into screen
- D. out of screen
- E. zero



# Checkpoint 4

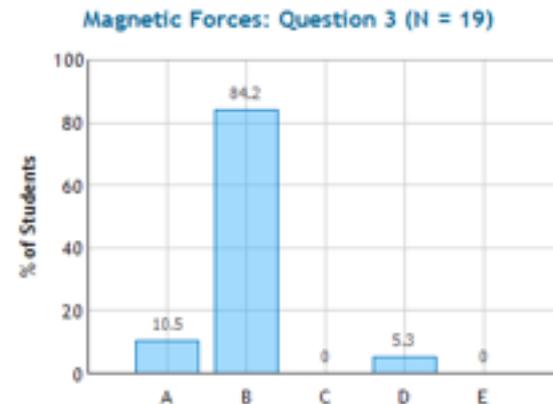


Three points are arranged in a uniform magnetic field. The **B** field points into the screen.



The positive charge moves from A to B. The direction of the magnetic force on it is

- A. right
- B. left
- C. into screen
- D. out of screen
- E. zero



# Cross Product Practice

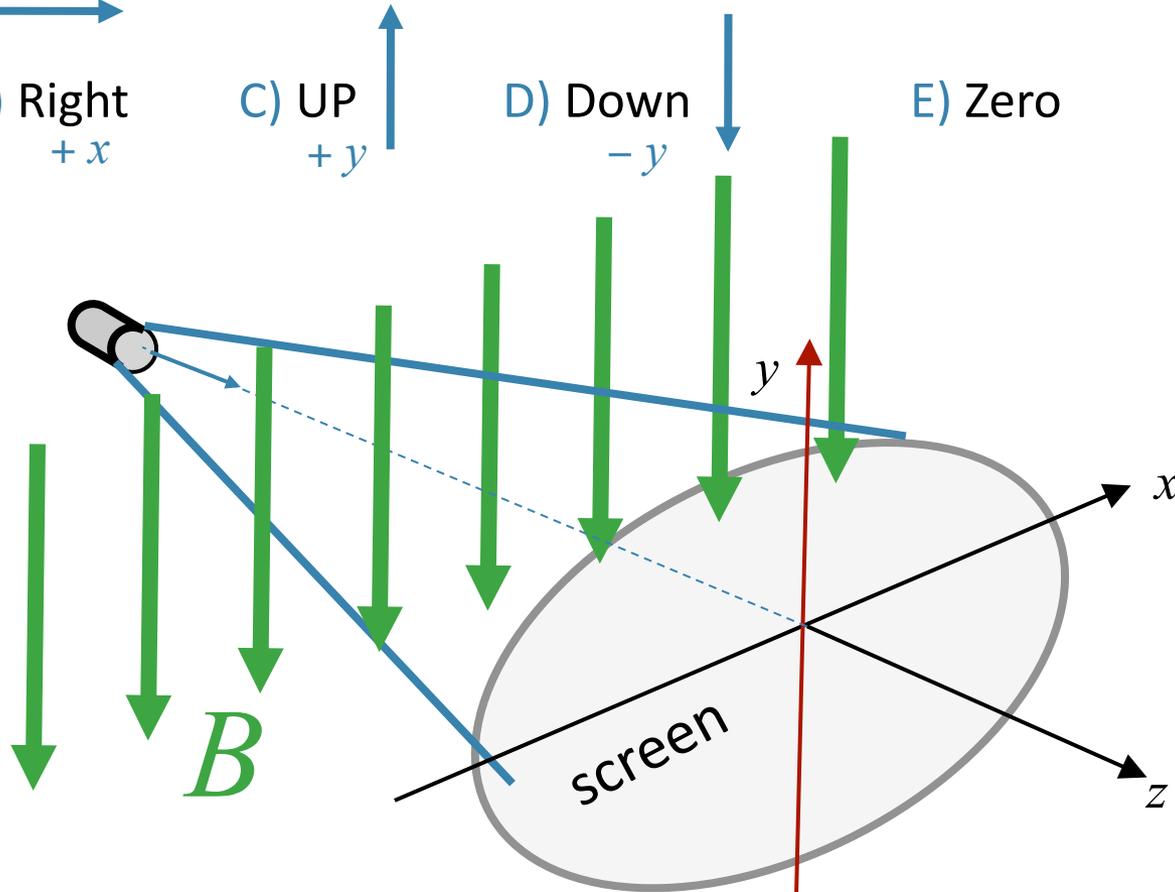
Protons (positive charge) coming out of screen

Magnetic field pointing down

What is direction of force on POSITIVE charge?

$$\vec{F} = q\vec{v} \times \vec{B}$$

- ←      →
- A) Left      B) Right      C) UP      D) Down      E) Zero
- $-x$        $+x$        $+y$        $-y$



# Motion of Charge $q$ in Uniform $B$ Field

Force is perpendicular to  $\mathbf{v}$

Speed does not change

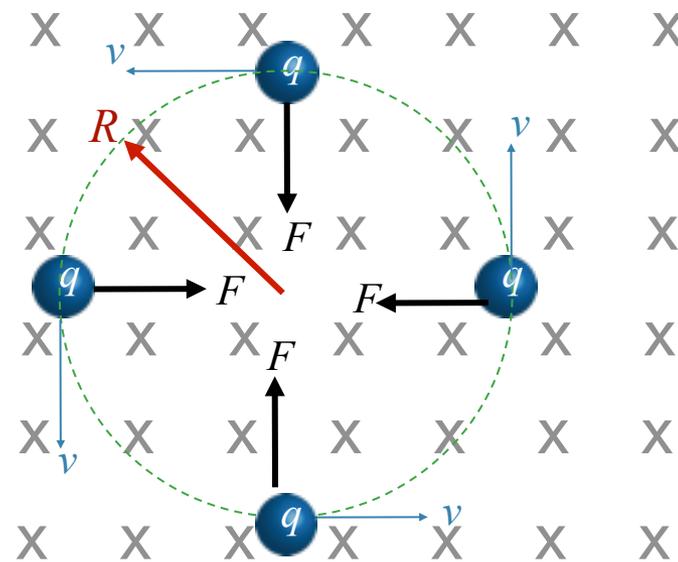
Uniform Circular Motion

Solve for  $R$ :

$$\vec{F} = q\vec{v} \times \vec{B} \Rightarrow F = qvB$$

$$a = \frac{v^2}{R}$$

$$qvB = m \frac{v^2}{R} \quad \longrightarrow \quad R = \frac{mv}{qB}$$

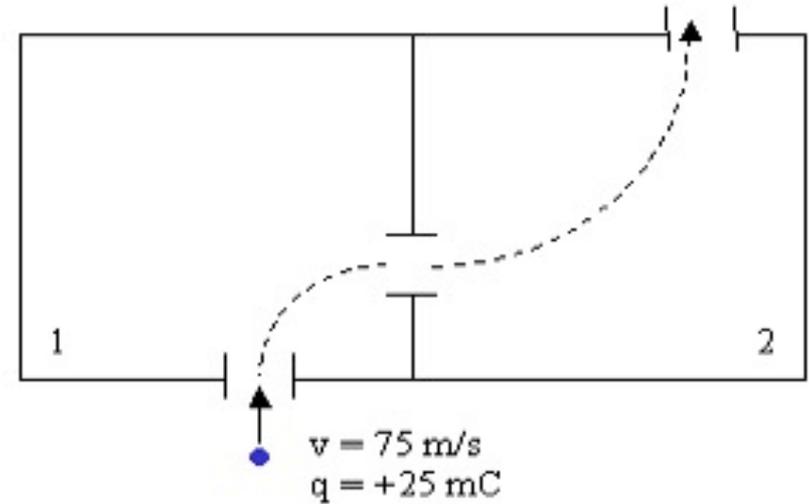


Uniform  $\mathbf{B}$  into page

# Checkpoint 6



The drawing shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.

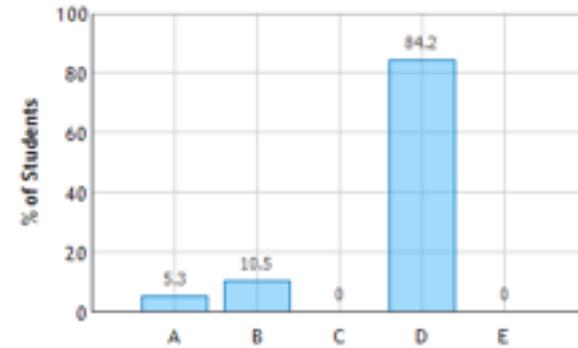


What is the direction of the magnetic field in chamber 1?

- A. up
- B. down
- C. into page
- D. out of page

Confusion?

Motion in a Magnetic Field: Question 1 (N = 19)

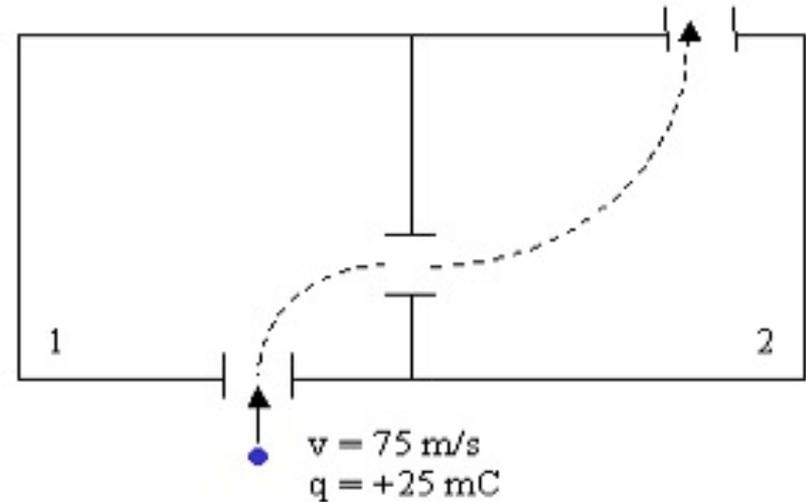


# Checkpoint 8

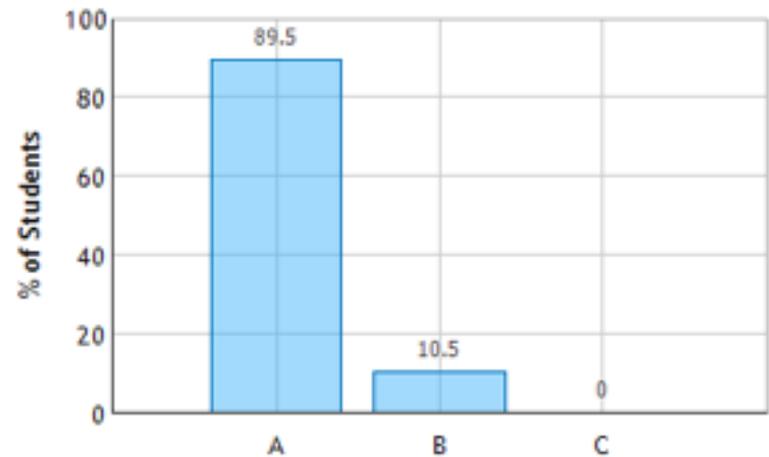
The drawing shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.

Compare the magnitude of the magnetic field in chamber 1 to the magnitude of the magnetic field in chamber 2.

- A.  $|B_1| > |B_2|$
- B.  $|B_1| = |B_2|$
- C.  $|B_1| < |B_2|$



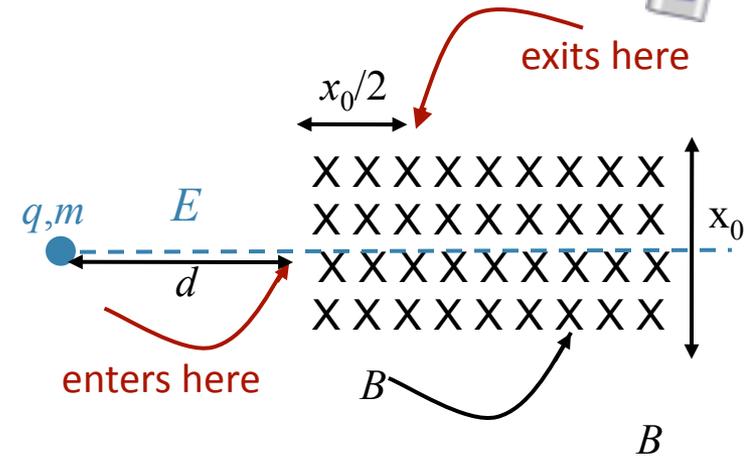
Motion in a Magnetic Field: Question 3 (N = 19)



# Calculation

A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?



## Conceptual Analysis

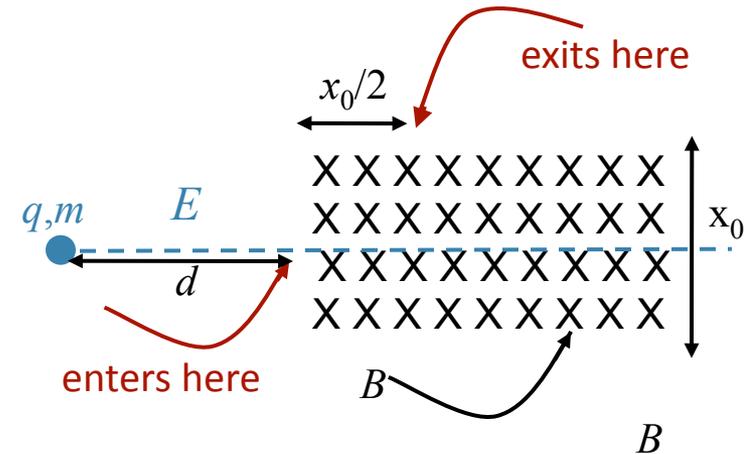
What do we need to know to solve this problem?

- A) Lorentz Force Law      B)  $E$  field definition      C)  $V$  definition  
D) Conservation of Energy/Newton's Laws      **E) All of the above**

# Calculation

A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?



## Strategic Analysis

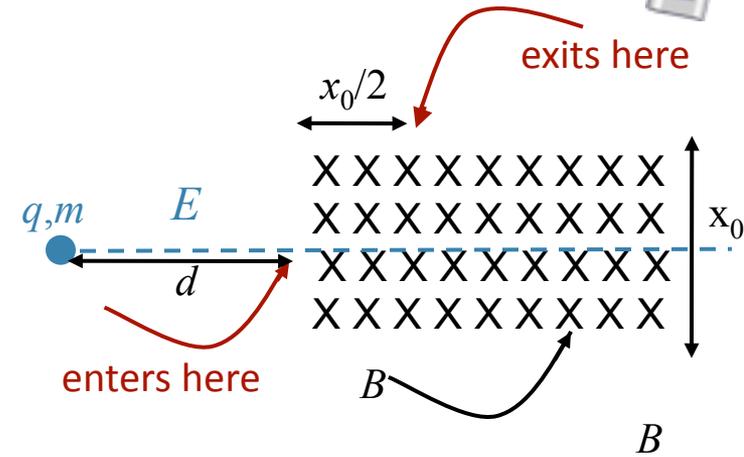
Calculate  $v$ , the velocity of the particle as it enters the magnetic field

Use Lorentz Force equation to determine the path in the field as a function of  $B$

Apply the entrance-exit information to determine  $B$

# Calculation

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What is  $B$ ?

What is  $v_0$ , the speed of the particle as it enters the magnetic field ?

$$v_o = \sqrt{\frac{2E}{m}}$$

A

$$v_o = \sqrt{\frac{2qEd}{m}}$$

B

$$v_o = \sqrt{2ad}$$

C

$$v_o = \sqrt{\frac{2qE}{md}}$$

D

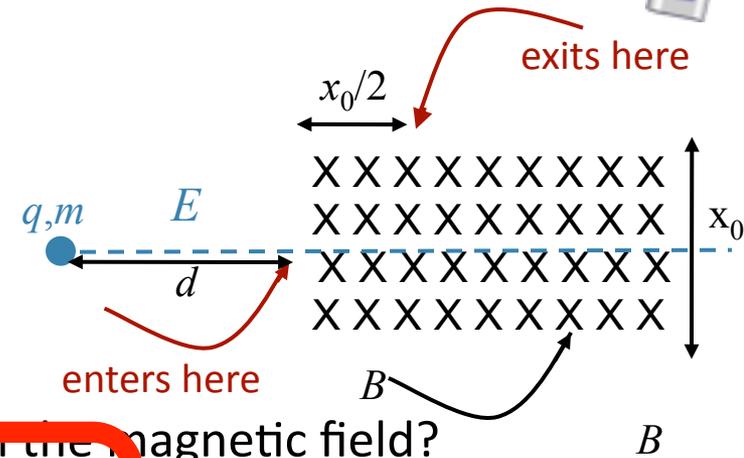
$$v_o = \sqrt{\frac{qEd}{m}}$$

E

# Calculation

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What is  $B$ ?  $v_o = \sqrt{\frac{2qEd}{m}}$



What is the path of the particle as it moves through the magnetic field?

**A**

**B**

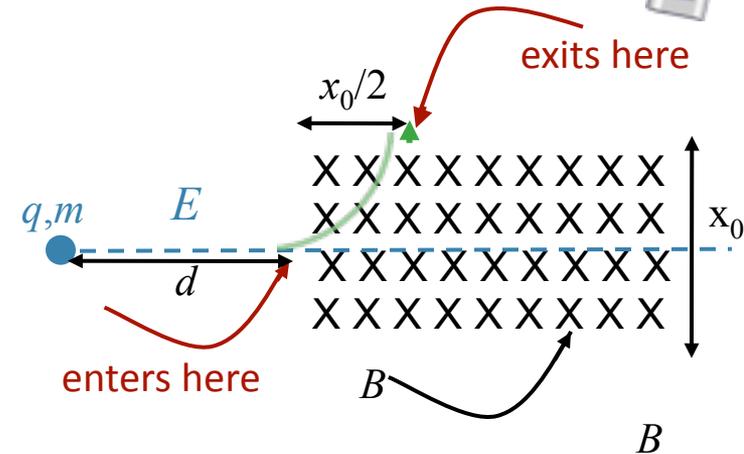
**C**

# Calculation



A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?  $v_o = \sqrt{\frac{2qEd}{m}}$



What is the radius of path of particle?

$R = x_o$        $R = 2x_o$        $R = \frac{1}{2}x_o$

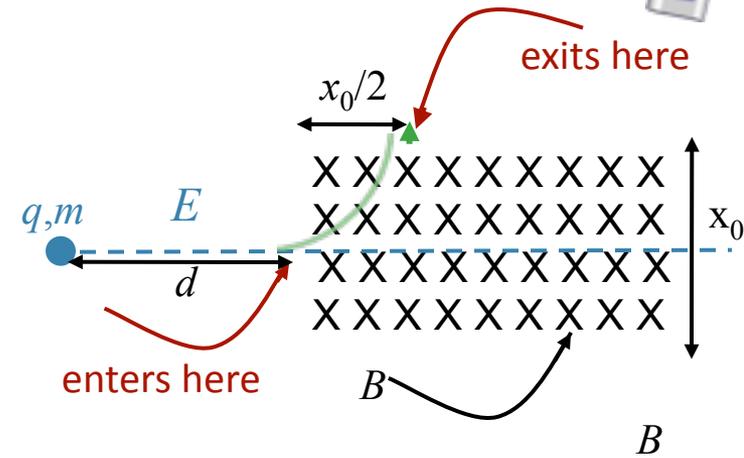
A
B
C

$R = \frac{mv_o}{qB}$        $R = \frac{v_o^2}{a}$

D
E

# Calculation

A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.



What is  $B$ ?

$$v_o = \sqrt{\frac{2qEd}{m}} \quad R = \frac{1}{2}x_0$$

$$B = \frac{2}{x_o} \sqrt{\frac{2mEd}{q}}$$

A

$$B = \frac{E}{v}$$

B

$$B = E \sqrt{\frac{m}{2qEd}}$$

C

$$B = \frac{1}{x_o} \sqrt{\frac{2mEd}{q}}$$

D

$$B = \frac{mv_o}{qx_o}$$

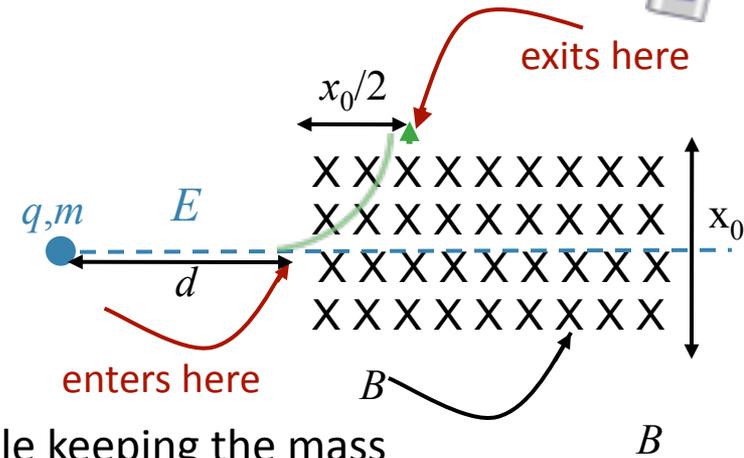
E

# Follow-Up

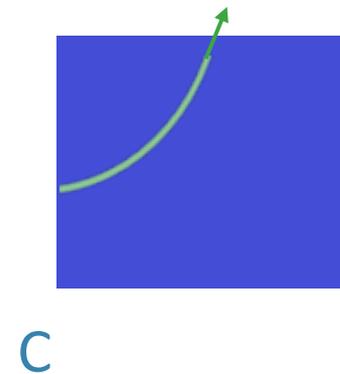
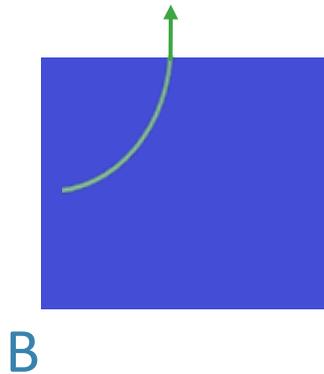
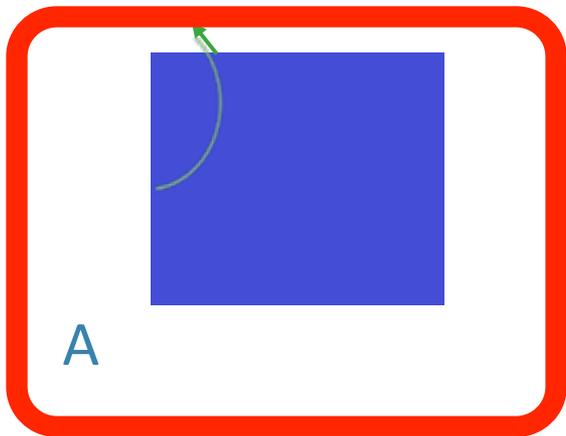
A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?

$$B = \frac{2}{x_0} \sqrt{\frac{2mEd}{q}}$$



Suppose the charge of the particle is doubled ( $Q = 2q$ ), while keeping the mass constant. How does the path of the particle change?

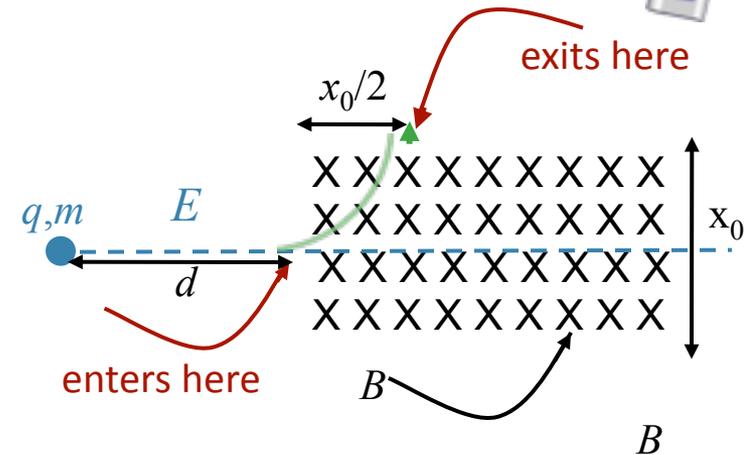


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What is  $B$ ?

$$B = \frac{2}{x_0} \sqrt{\frac{2mEd}{q}}$$



Suppose the charge of the particle is doubled ( $Q = 2q$ ), while keeping the mass constant. How does the path of the particle change?

How does  $v$ , the new velocity at the entrance, compare to the original velocity  $v_0$ ?

A  $v = \frac{v_0}{2}$

B  $v = \frac{v_0}{\sqrt{2}}$

C  $v = v_0$

D  $v = \sqrt{2}v_0$

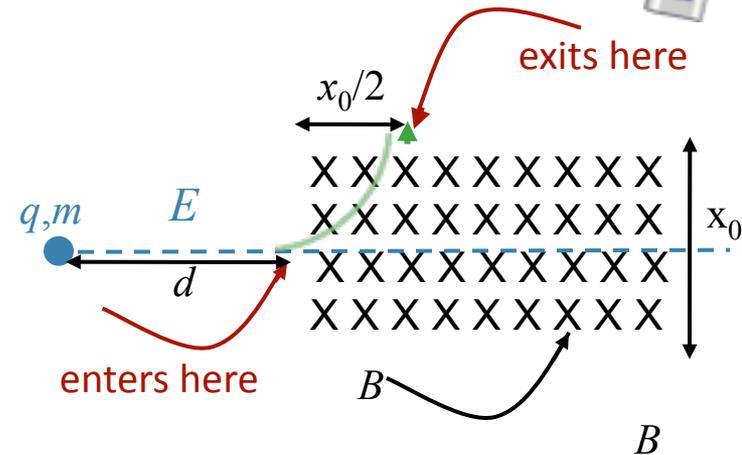
E  $v = 2v_0$

# Follow-Up

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What is  $B$ ?

$$B = \frac{2}{x_0} \sqrt{\frac{2mEd}{q}}$$



Suppose the charge of the particle is doubled ( $Q = 2q$ ), while keeping the mass constant. How does the path of the particle change?

$$v = \sqrt{2}v_0$$

How does  $F$ , the magnitude of the new force at the entrance, compare to  $F_0$ , the magnitude of the original force?

A  $F = \frac{F_0}{\sqrt{2}}$

B  $F = F_0$

C  $F = \sqrt{2}F_0$

D  $F = 2F_0$

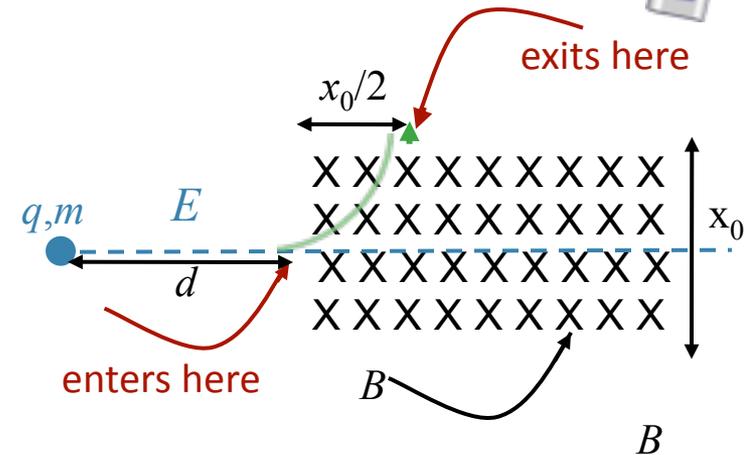
E  $F = 2\sqrt{2}F_0$

# Follow-Up

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What is  $B$ ?

$$B = \frac{2}{x_0} \sqrt{\frac{2mEd}{q}}$$



Suppose the charge of the particle is doubled ( $Q = 2q$ ), while keeping the mass constant. How does the path of the particle change?

$$v = \sqrt{2}v_0 \quad F = 2\sqrt{2}F_0$$

How does  $R$ , the radius of curvature of the path, compare to  $R_0$ , the radius of curvature of the original path?

A  $R = \frac{R_0}{2}$

**B  $R = \frac{R_0}{\sqrt{2}}$**

C  $R = R_0$

D  $R = \sqrt{2}R_0$

E  $R = 2R_0$

$$\frac{mv^2}{R} = F$$

$$R = \frac{mv^2}{F}$$

$$R = \frac{m2v_0^2}{2\sqrt{2}F_0} = \frac{mv_0^2}{\sqrt{2}F_0} = \frac{1}{\sqrt{2}}R_0$$