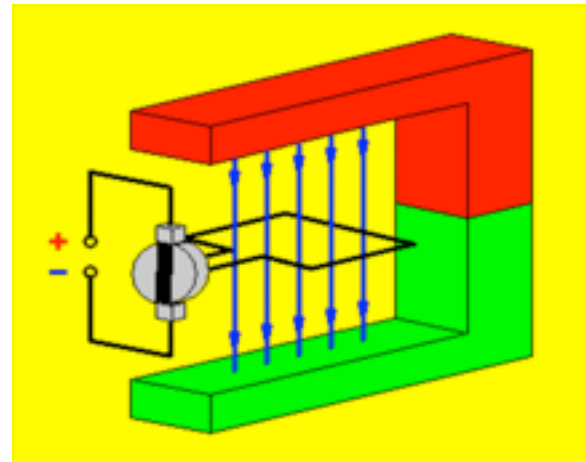


# *Electricity & Magnetism*

## *Lecture 13*

Today's Concept:

Torques

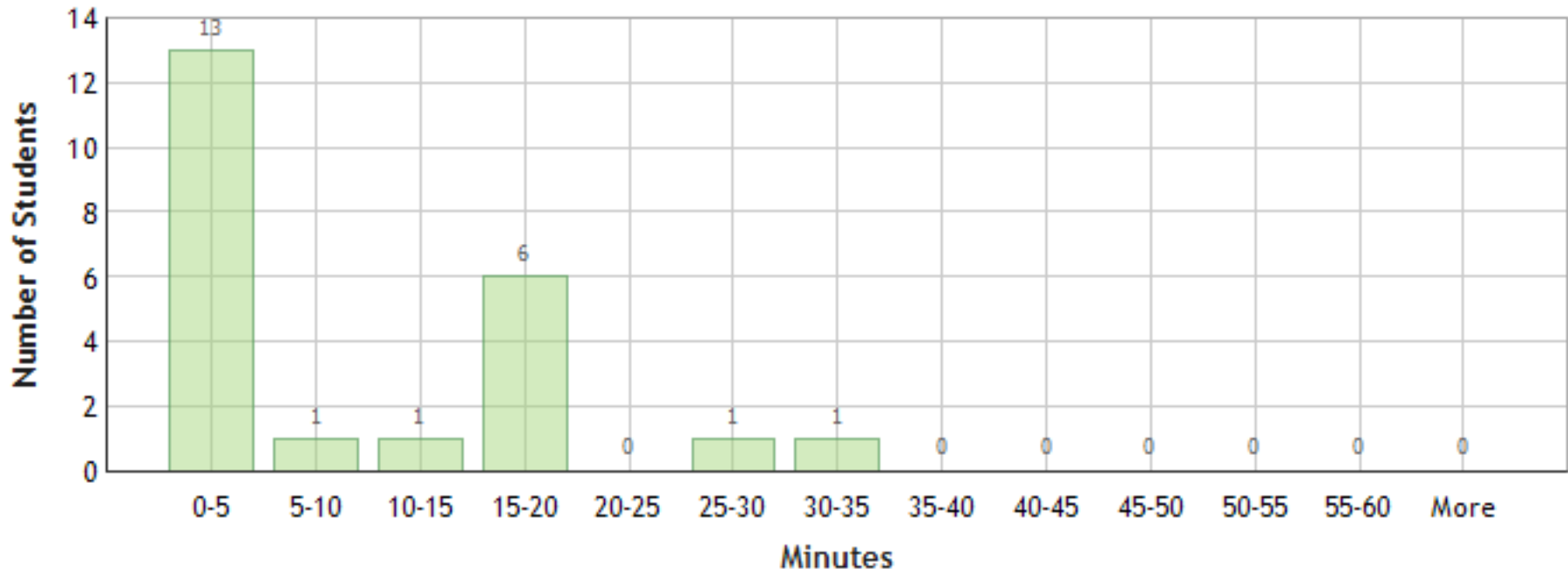


# Extra Deadlines

★ Extra deadlines have been set up for Prelectures and Checkpoints that happened last week. Please do them if you haven't.

but...

Time Spent Viewing Prelecture (N = 23)



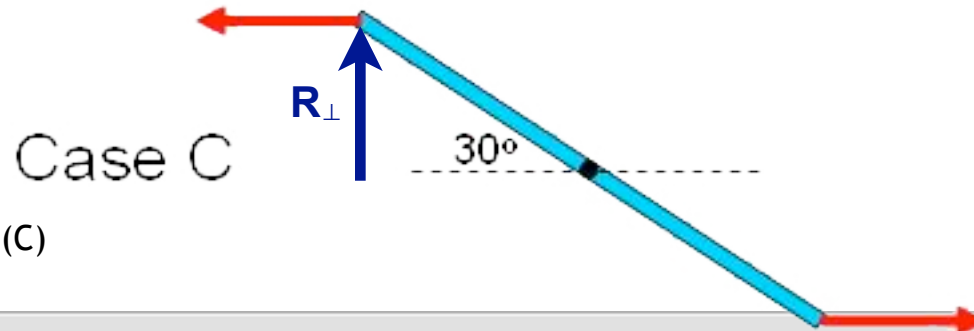
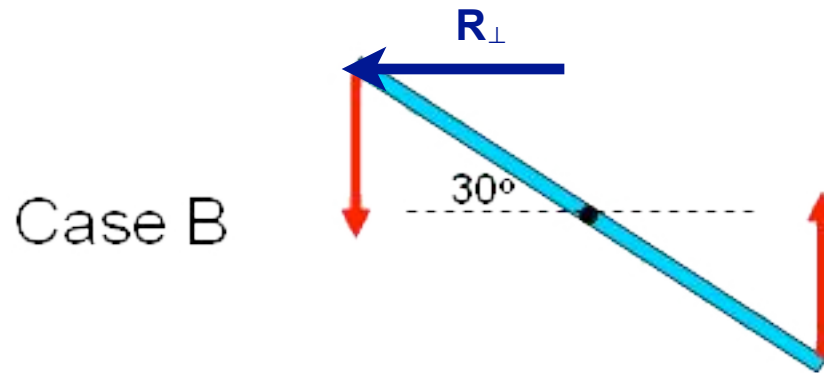
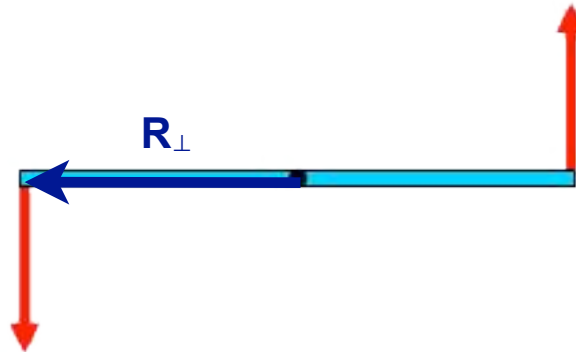
# Torque Review

Compare the torques?

$$\vec{\tau} = \vec{R} \times \vec{F}$$

Case A

$$|\vec{\tau}| = R_{\perp} F$$



Torque(A) > Torque(B) > Torque(C)

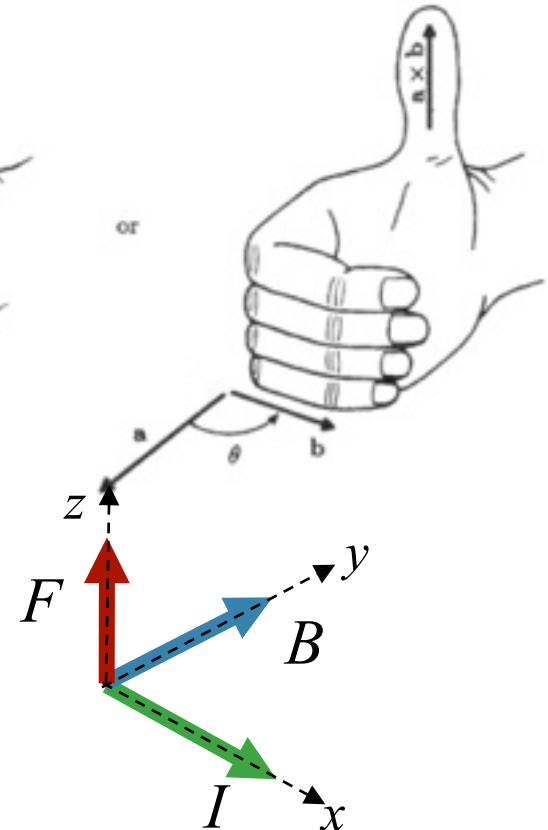
Last Time:

$$\vec{F} = q\vec{v} \times \vec{B}$$

This Time:

$$\vec{F} = q \sum_i \vec{v}_i \times \vec{B}$$

*qv and IL have same  
Dimensions!*



$$\vec{F} = qN\vec{v}_{avg} \times \vec{B} \quad \longrightarrow \quad \vec{F} = I\vec{L} \times \vec{B}$$

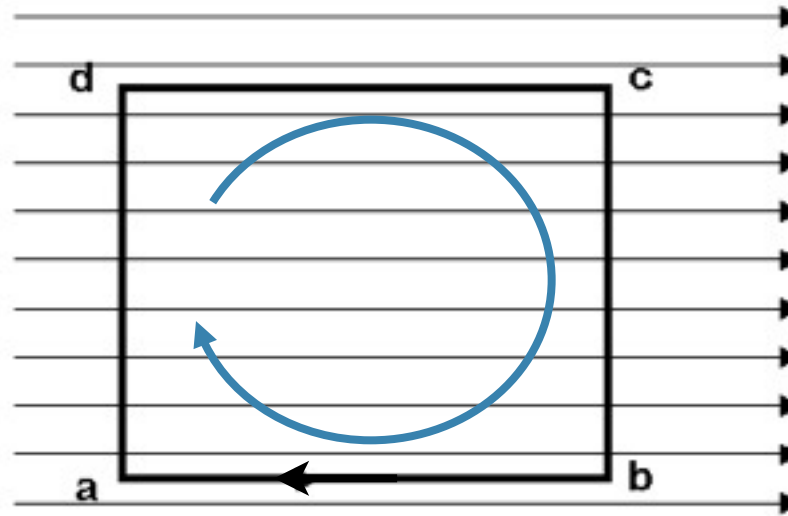
$N = nAL$   
 $I = qnAv_{avg}$

# Clicker Question



A square loop of wire is carrying current in the clockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$



What is the force on section a-b on the loop?

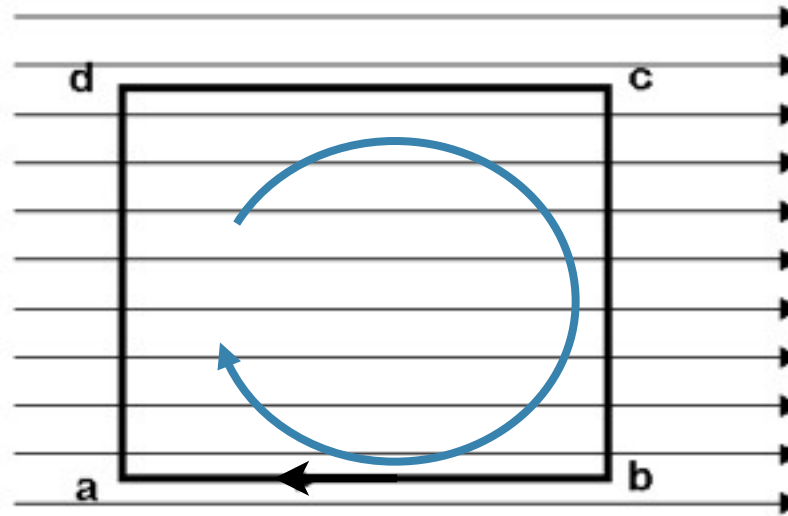
- A. Zero
- B. Out of page
- C. Into the Page

# Clicker Question



A square loop of wire is carrying current in the clockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$



What is the force on section **b-c** on the loop?

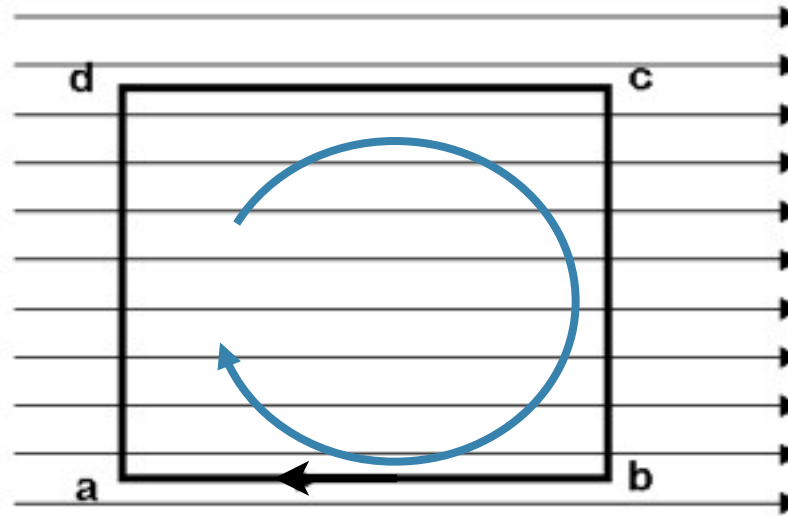
- A. Zero
- B. Out of page
- C. Into the Page

# Clicker Question



A square loop of wire is carrying current in the clockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$



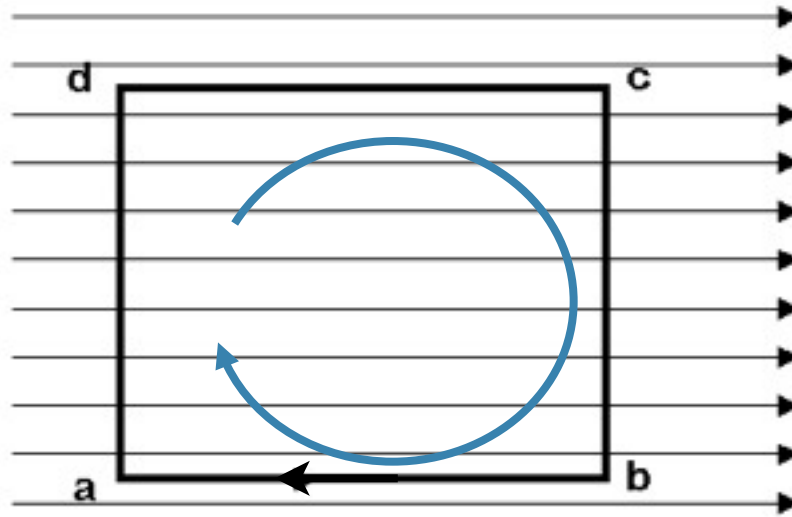
What is the force on section **c-d** on the loop?

- A. Zero
- B. Out of page
- C. Into the Page

## CheckPoint 2

A square loop of wire is carrying current in the clockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$



What is the direction of the net force on the loop?

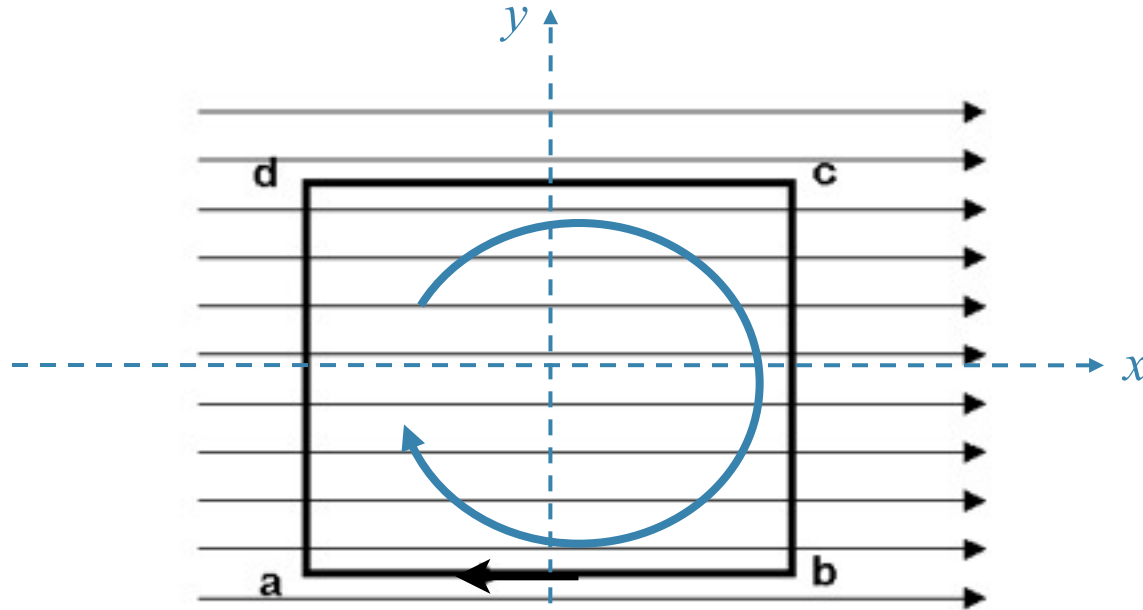
- A. Zero
- B. Out of page
- C. Into the Page



# CheckPoint 4



A square loop of wire is carrying current in the clockwise direction. There is a horizontal uniform magnetic field pointing to the right.



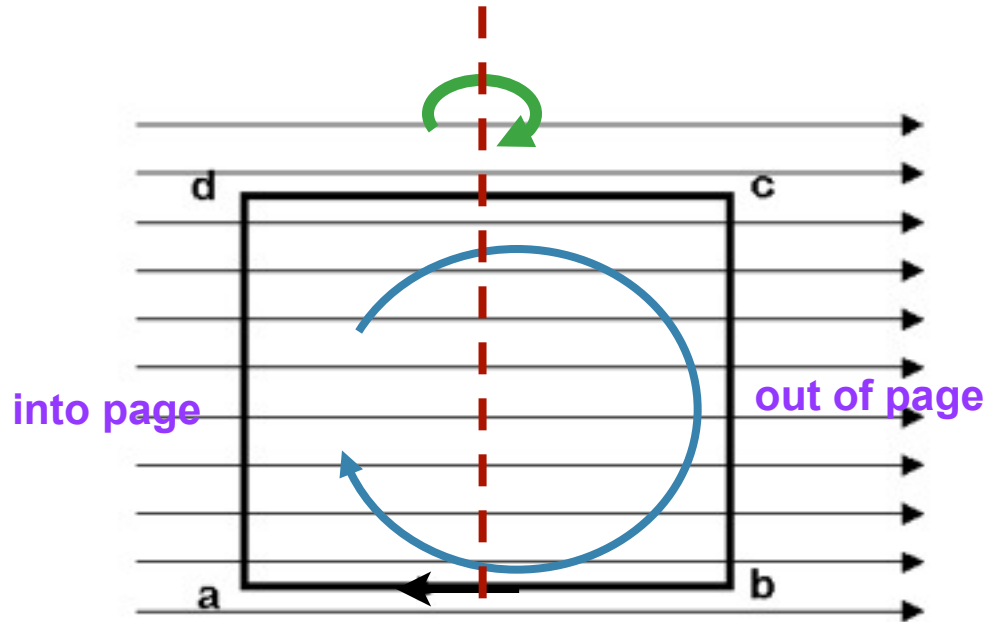
In which direction will the loop rotate?  
(assume the  $z$  axis is out of the page)

- A) Around the  $x$  axis
- ☒ B) Around the  $y$  axis
- C) Around the  $z$  axis
- D) It will not rotate

# CheckPoint 6



A square loop of wire is carrying current in the clockwise direction. There is a horizontal uniform magnetic field pointing to the right.



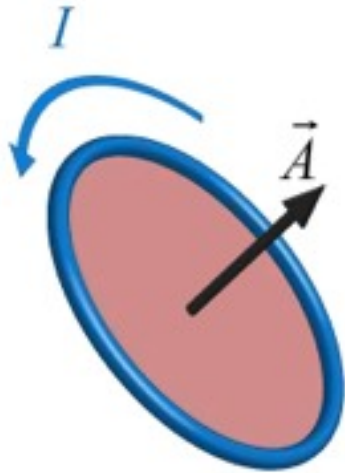
What is the direction of the **net torque** on the loop?

$$\vec{\tau} = \vec{R} \times \vec{F}$$

- A) up (on page)
- ☒ B) down (on page)
- C) out of page
- D) into page
- E) net torque is zero



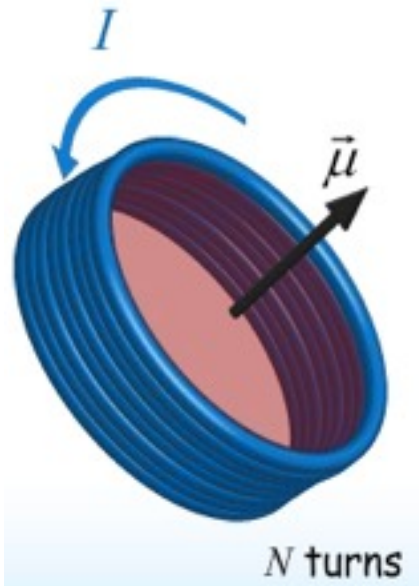
# Magnetic Dipole Moment



Area vector

Magnitude = Area

Direction uses R.H.R.



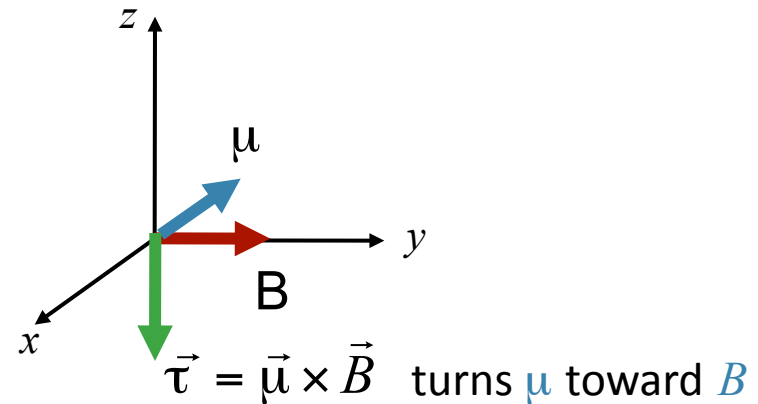
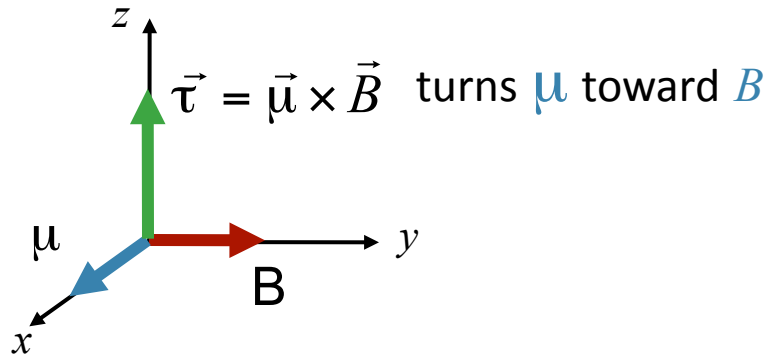
Magnetic Dipole moment

$$\vec{\mu} \equiv N I \vec{A}$$

# $\mu$ Makes Torque Easy!

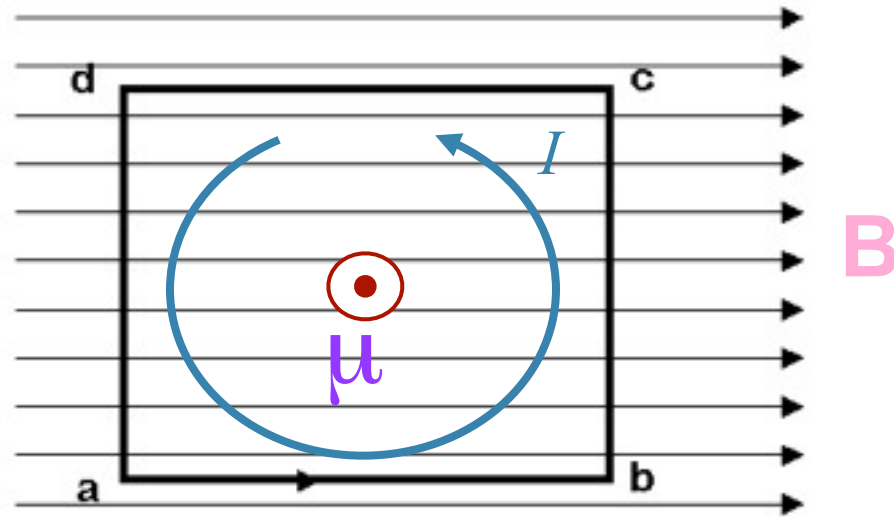
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The torque always wants to line  $\mu$  up with  $B$ !

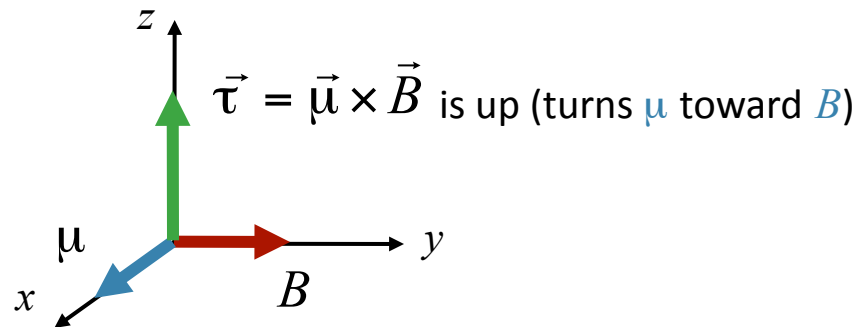


# Practice with $\mu$ and $\tau$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

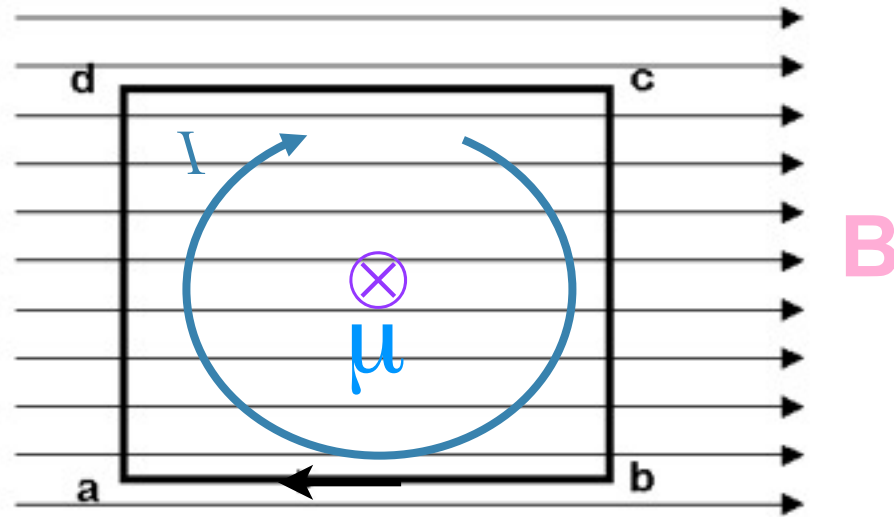


In this case  $\mu$  is out of the page (using right hand rule)

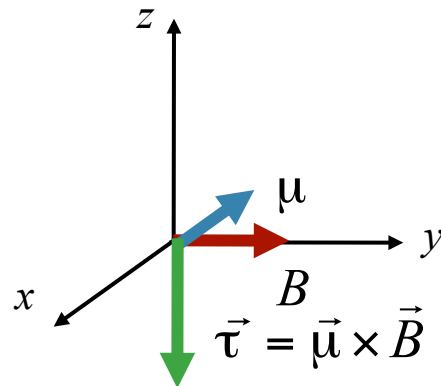


# Practice with $\mu$ and $\tau$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

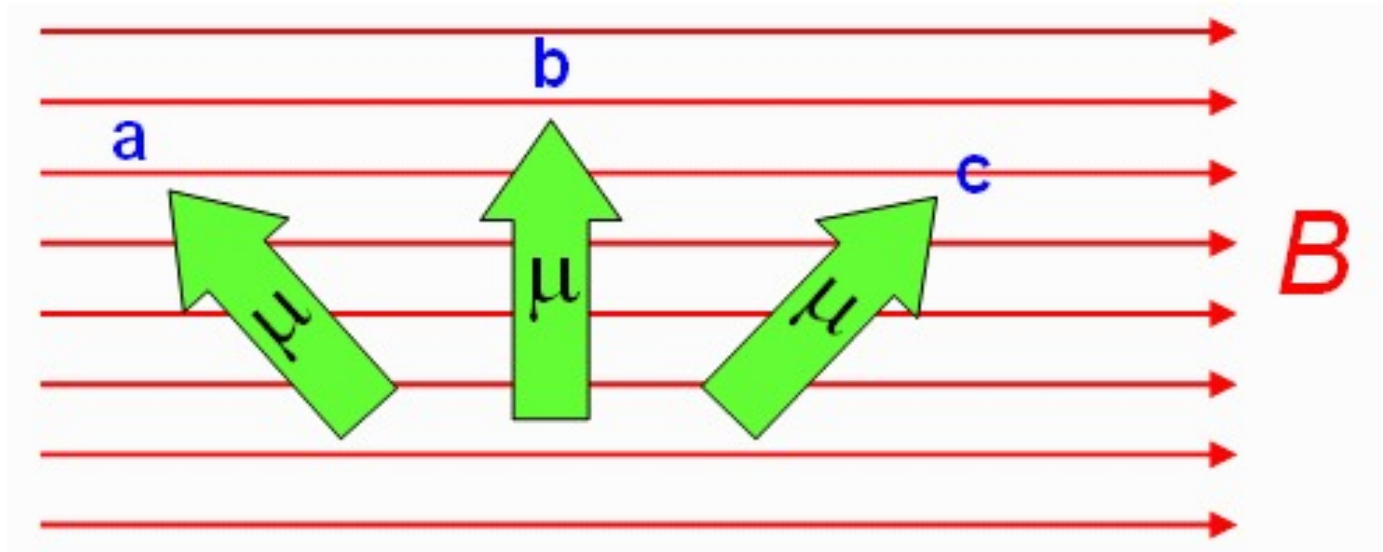


In this case  $\mu$  is IN to the page (using right hand rule)

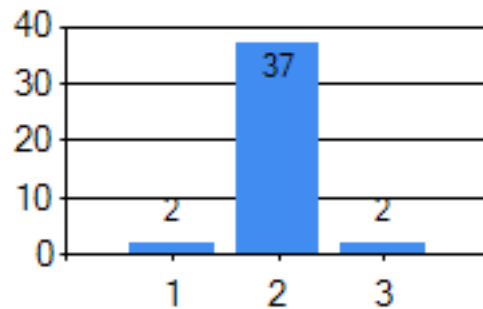


# CheckPoint 8

Three different orientations of a magnetic dipole moment in a constant magnetic field are shown below. Which orientation results in the largest magnetic torque on the dipole ?



Answer Choice Distribution



# Magnetic Field does Work on Current-carrying wire

From Physics 140:  $W = \int \tau d\theta$

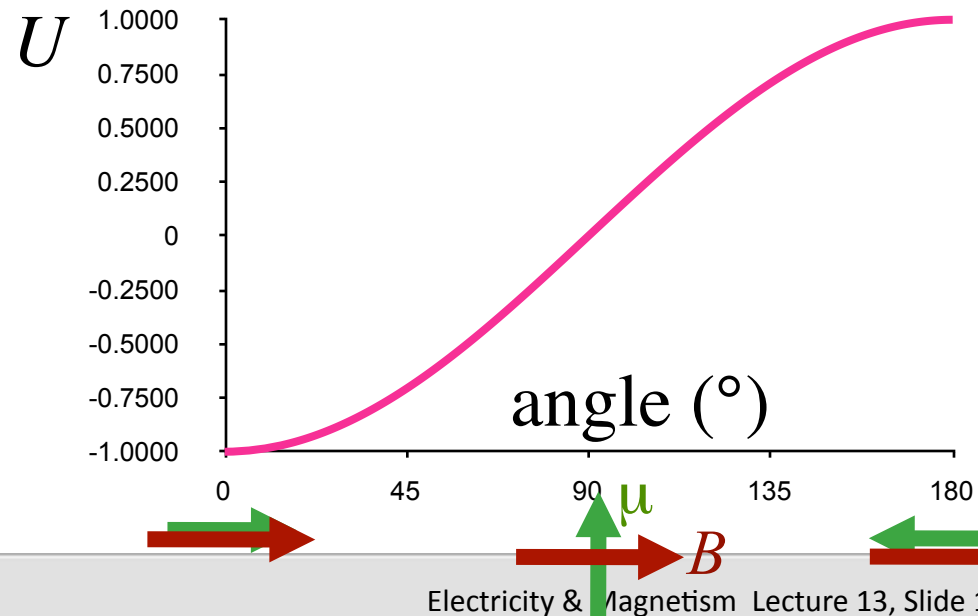
From Physics 141:  $\vec{\tau} = \vec{\mu} \times \vec{B} = \mu B \sin(\theta)$

$$W = \int_{\theta}^{\pi/2} \mu B \sin \theta' d\theta' = \mu B \cos \theta = \vec{\mu} \cdot \vec{B}$$

$$\Delta U = -W$$

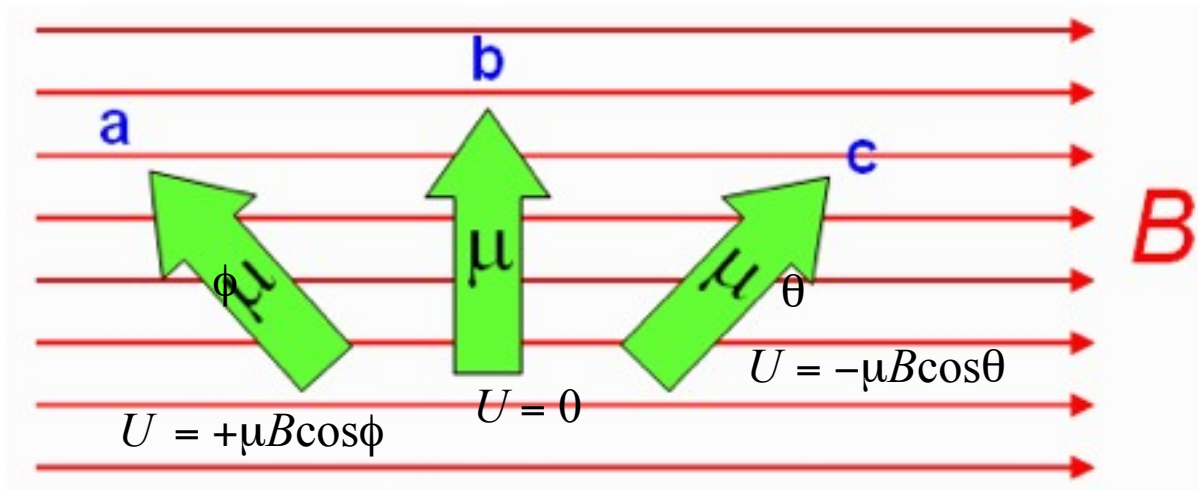
Define  $U = 0$  at position of maximum torque

$$U = -\vec{\mu} \cdot \vec{B}$$

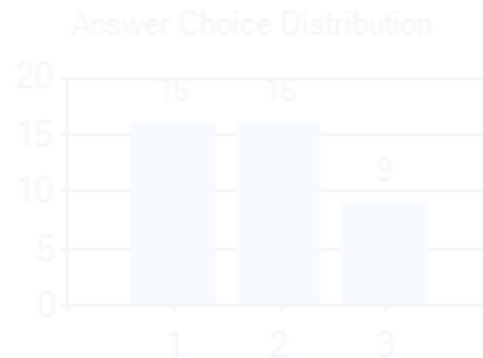




# CheckPoint 10

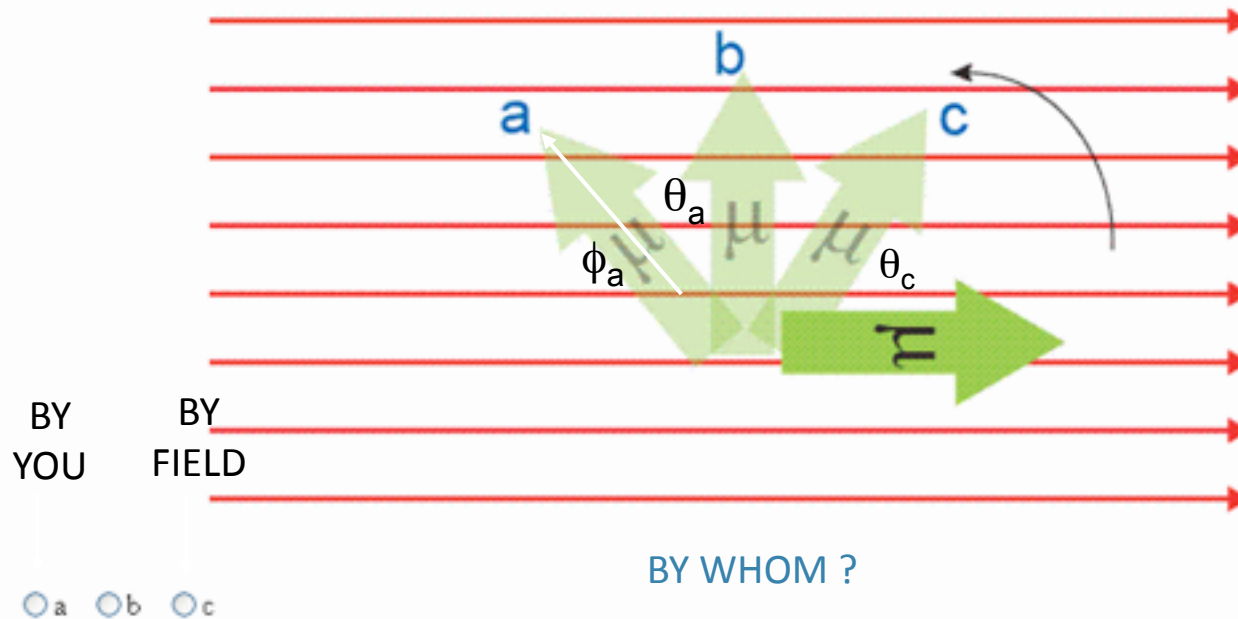


Which orientation has the lowest potential energy?



# CheckPoint 12

12) In order to rotate an horizontal magnetic dipole to the three positions shown, which one requires the most work done?



$$W_{by\ field} = -\Delta U = U_i - U_f$$

$$U = \vec{\mu} \cdot \vec{B}$$

C):  $\longrightarrow W_{by\_field} = -\mu B - (-\mu B \cos \theta_c) = -\mu B(1 - \cos \theta_c)$

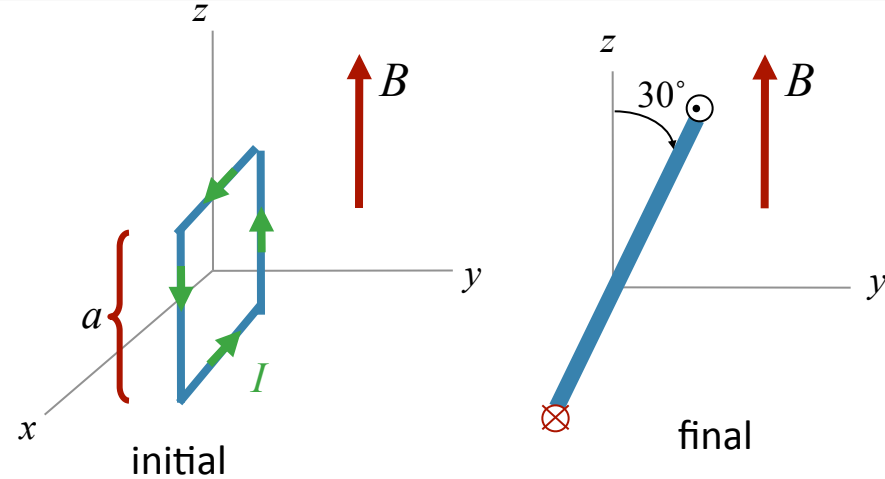
B):  $\longrightarrow W_{by\_field} = -\mu B - 0 = -\mu B$

A):  $\longrightarrow W_{by\_field} = -\mu B - (-\mu B \cos \theta_a) = -\mu B(1 + \cos \theta_a)$

# Calculation

A square loop of side  $a$  lies in the  $x$ - $z$  plane with current  $I$  as shown. The loop can rotate about  $x$  axis without friction. A uniform field  $B$  points along the  $+z$  axis. Assume  $a$ ,  $I$ , and  $B$  are known.

How much does the potential energy of the system change as the coil moves from its initial position to its final position.



## Conceptual Analysis

A current loop may experience a torque in a constant magnetic field

$$\tau = \mu \times B$$

We can associate a potential energy with the orientation of loop

$$U = -\mu \cdot B$$

## Strategic Analysis

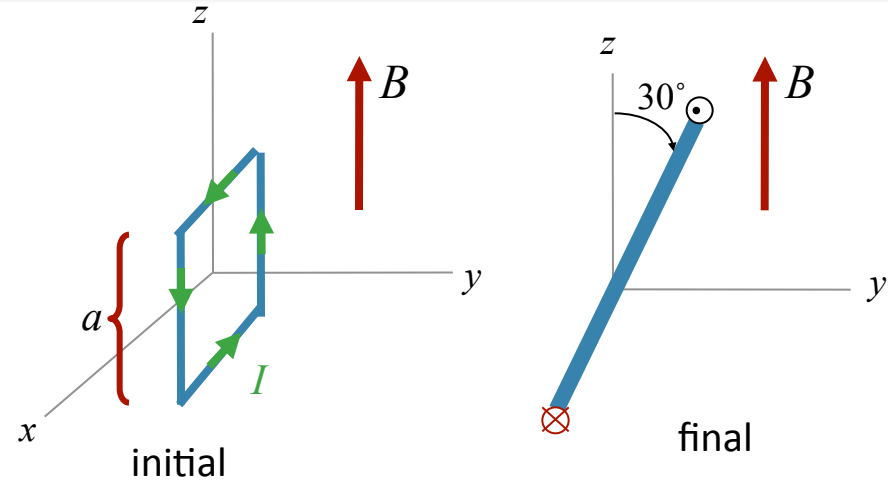
Find  $\mu$

Calculate the change in potential energy from initial to final

# Calculation

A square loop of side  $a$  lies in the  $x$ - $z$  plane with current  $I$  as shown. The loop can rotate about  $x$  axis without friction. A uniform field  $B$  points along the  $+z$  axis. Assume  $a$ ,  $I$ , and  $B$  are known.

$$\vec{\mu} = L\vec{A}$$

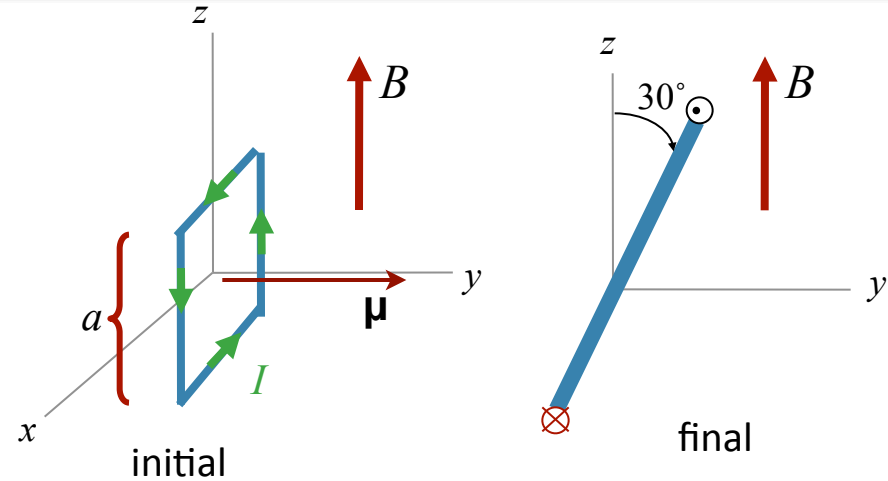


What is the direction of the magnetic moment of this current loop in its initial position?

- A)  $+x$                       B)  $-x$                       C)  $+y$                       D)  $-y$

# Calculation

A square loop of side  $a$  lies in the  $x$ - $z$  plane with current  $I$  as shown. The loop can rotate about  $x$  axis without friction. A uniform field  $B$  points along the  $+z$  axis. Assume  $a$ ,  $I$ , and  $B$  are known.



What is the direction of the torque on this current loop in the initial position?

A)  $+x$

B)  $-x$

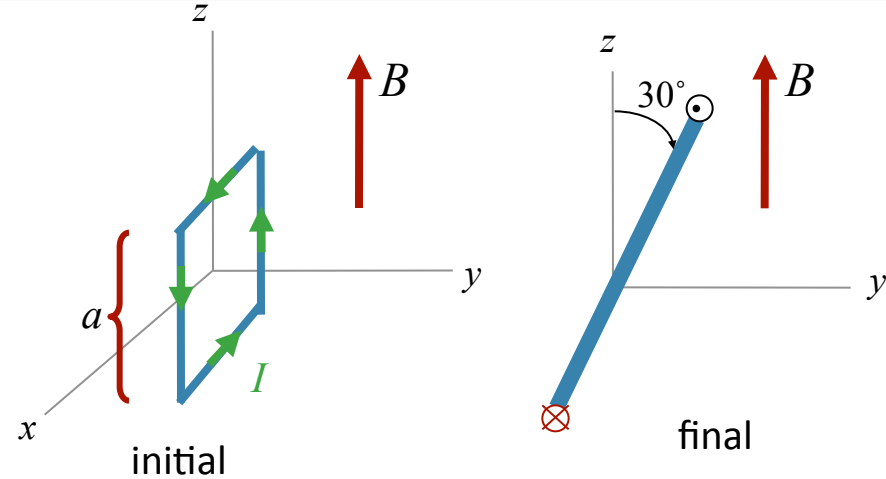
C)  $+y$

D)  $-y$

# Calculation

A square loop of side  $a$  lies in the  $x$ - $z$  plane with current  $I$  as shown. The loop can rotate about  $x$  axis without friction. A uniform field  $B$  points along the  $+z$  axis. Assume  $a$ ,  $I$ , and  $B$  are known.

$$U = -\vec{\mu} \cdot \vec{B}$$



What is the potential energy of the initial state?

A)  $U_{initial} < 0$

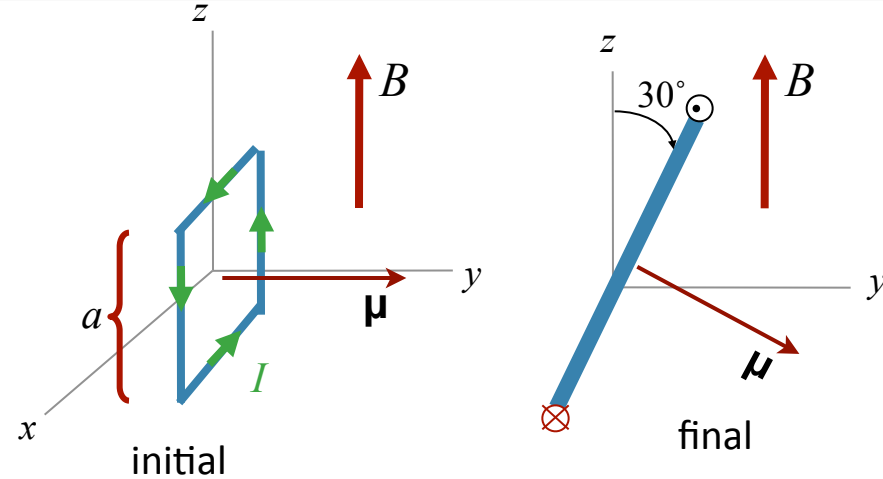
B)  $U_{initial} = 0$

C)  $U_{initial} > 0$

# Calculation

A square loop of side  $a$  lies in the  $x$ - $z$  plane with current  $I$  as shown. The loop can rotate about  $x$  axis without friction. A uniform field  $B$  points along the  $+z$  axis. Assume  $a$ ,  $I$ , and  $B$  are known.

$$U = -\vec{\mu} \cdot \vec{B}$$



What is the potential energy of the final state?

A)  $U_{final} < 0$

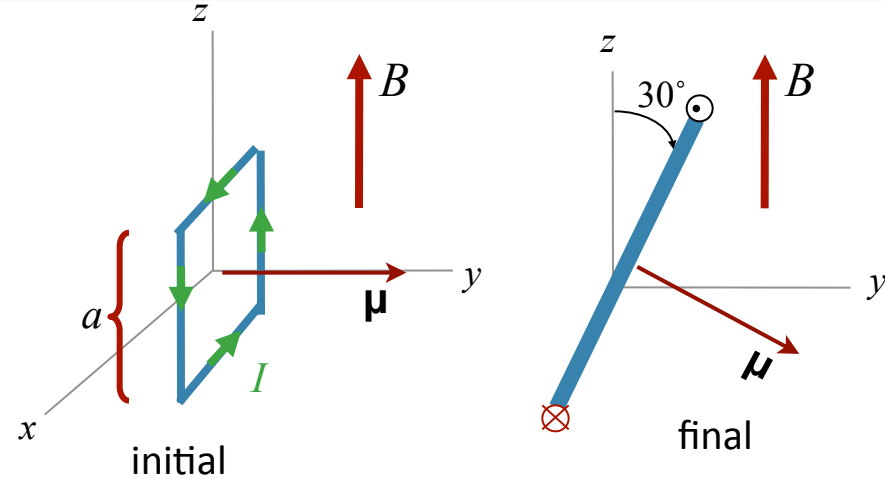
B)  $U_{final} = 0$

C)  $U_{final} > 0$

# Calculation

A square loop of side  $a$  lies in the  $x$ - $z$  plane with current  $I$  as shown. The loop can rotate about  $x$  axis without friction. A uniform field  $B$  points along the  $+z$  axis. Assume  $a$ ,  $I$ , and  $B$  are known.

$$U = -\vec{\mu} \cdot \vec{B}$$



What is the potential energy of the final state?

A)  $U = Ia^2 B$

B)  $U = \frac{\sqrt{3}}{2} Ia^2 B$

C)  $U = \frac{1}{2} Ia^2 B$

