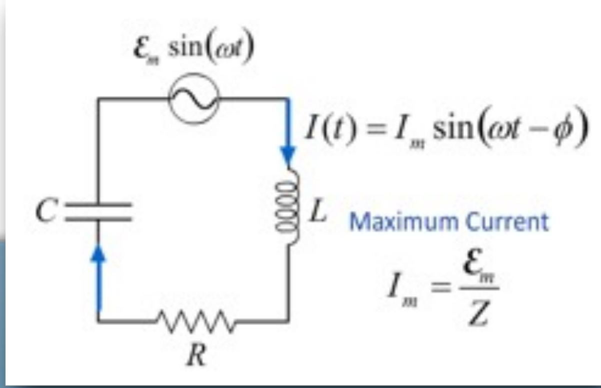


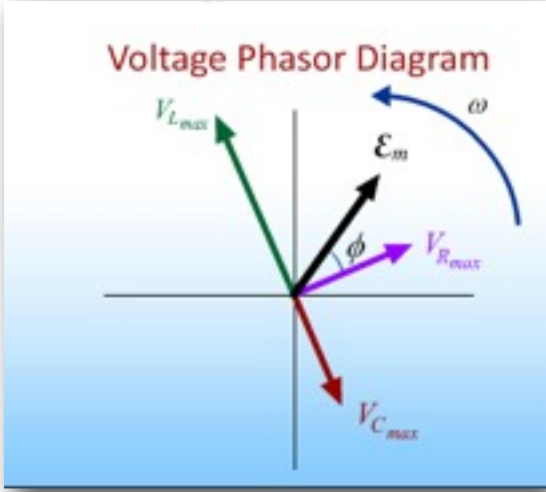
Electricity & Magnetism
Lecture 20

Today's Concept:

AC Circuits

Maximum currents & voltages

Phasors: A Simple Tool



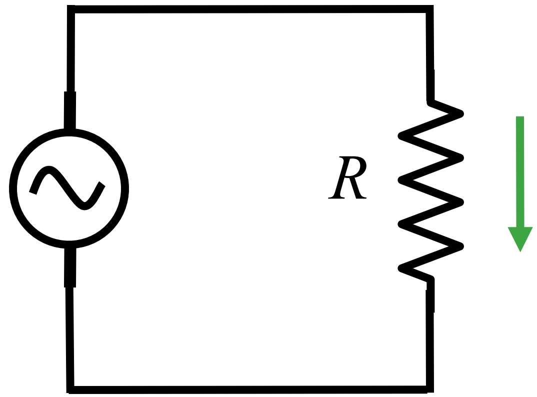
Other videos:

Prof. W. Lewin, MIT Open Courseware

Mechanical Universe, Driven LRC circuits

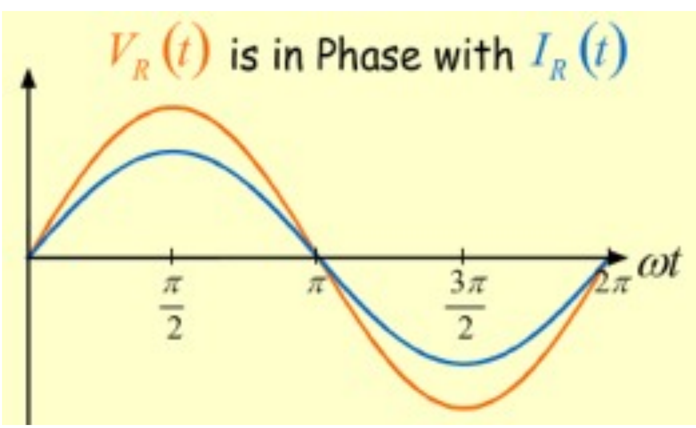
Resistors

$\mathcal{E} = V_{max}\sin(\omega t)$



$I = V_R/R = \boxed{V_{max}/R}\sin(\omega t)$

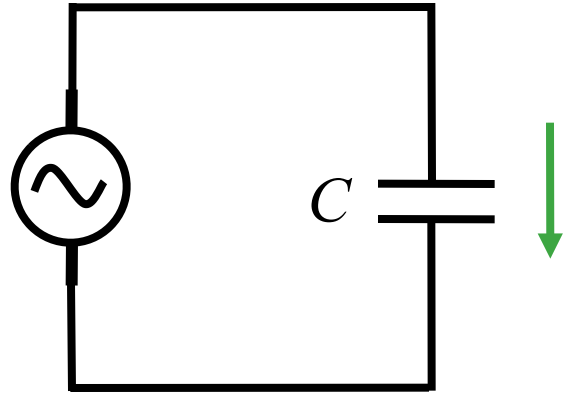
Amplitude = V_{max}/R



$$Q = CV = CV_{max}\sin(\omega t)$$

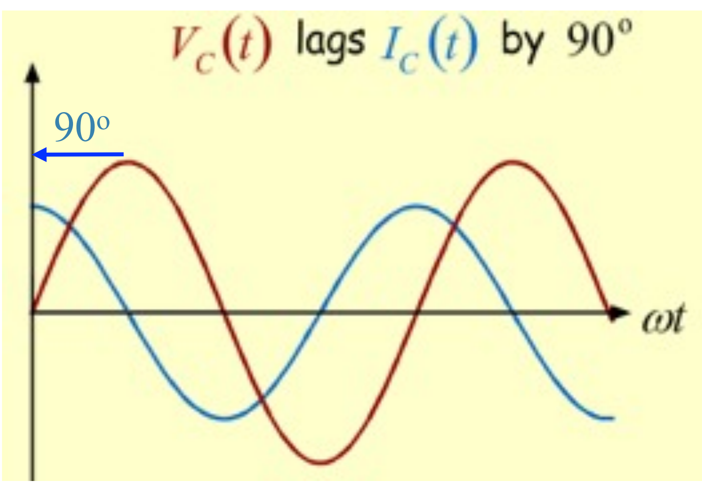
$$I = dQ/dt$$

$\mathcal{E} = V_{max}\sin(\omega t)$



$$I = \boxed{V_{max}\omega C}\cos(\omega t)$$

$$\boxed{\text{Amplitude} = V_{max}/X_C}$$

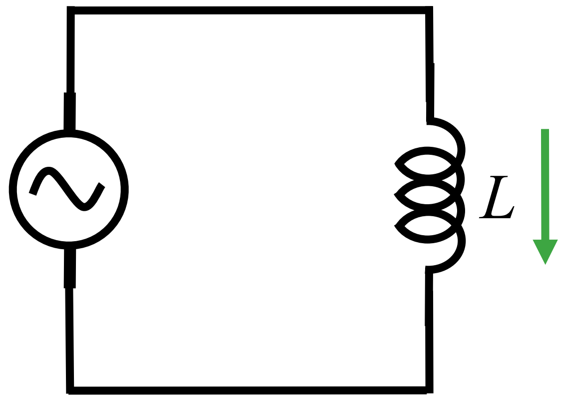


where $X_C = 1/\omega C$
is like the "resistance"
of the capacitor
X_C depends on ω

Inductors

$$dI/dt = V_L = V_{max}\sin(\omega t)$$

$$\mathcal{E} = V_{max}\sin(\omega t)$$



$$I = -\boxed{(V_{max}/\omega L)}\cos(\omega t)$$

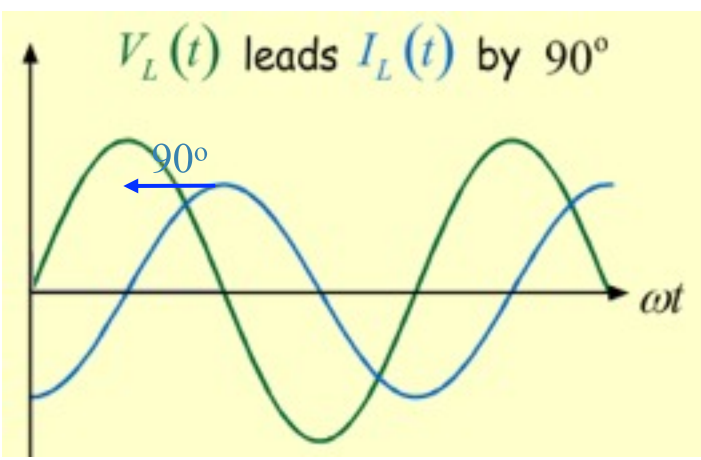
$$\boxed{\text{Amplitude} = V_{max}/X_L}$$

where $X_L = \omega L$

is like the "resistance"

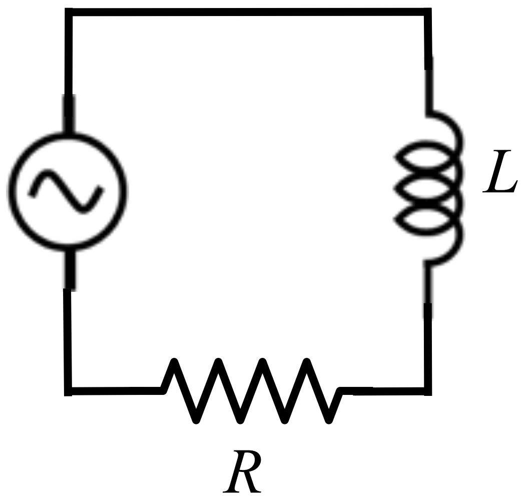
of the inductor

X_L depends on ω



RL Clicker Question

An RL circuit is driven by an AC generator as shown in the figure.



$X_L = \omega L$

As $\omega \to 0$, so does X_L

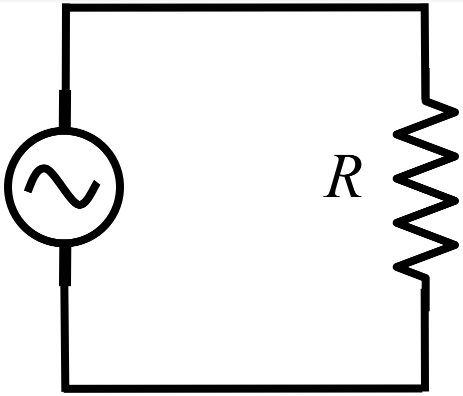
↓

As $\omega \to 0$,
impedance ~~resistance~~ of circuit $\to R$
current gets bigger

For what driving frequency ω of the generator will the current through the resistor be largest

A) ω large
B) Current through R doesn't depend on ω
C) ω small

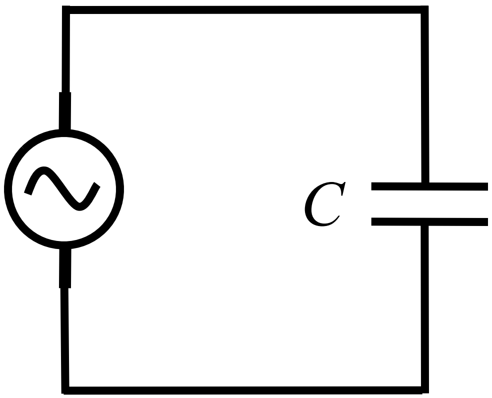
Summary



$I_{max} = V_{max}/R$

V_R in phase with I

Because resistors are simple



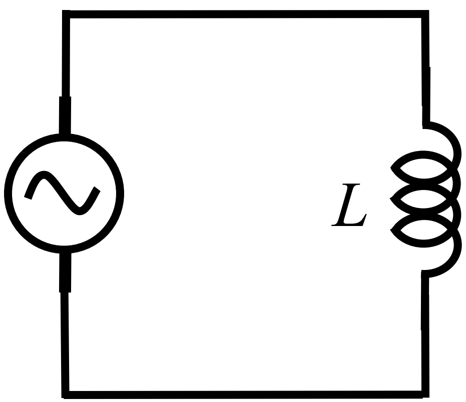
$I_{max} = V_{max}/X_C$

$X_C = 1/\omega C$

V_C 90^o behind I

Current comes first since it charges capacitor

Like a wire at high ω



$I_{max} = V_{max}/X_L$

$X_L = \omega L$

V_L 90^o ahead of I

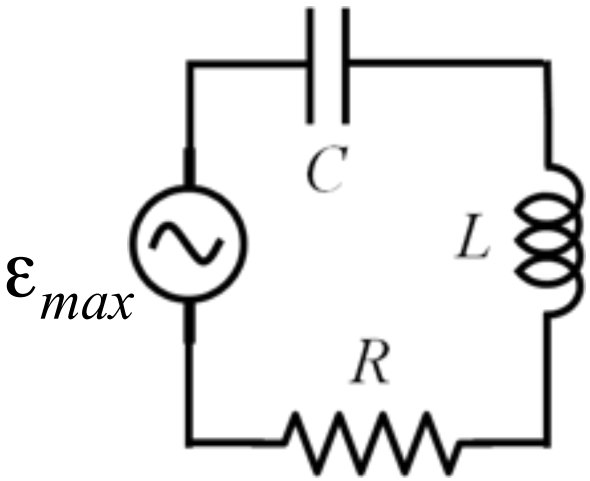
Opposite of capacitor

Like a wire at low ω

Makes sense to write everything in terms of I since this is the same everywhere in a one-loop circuit:

$$V_{max} = I_{max} X_C$$

V 90^o behind I



$$V_{max} = I_{max} X_L$$

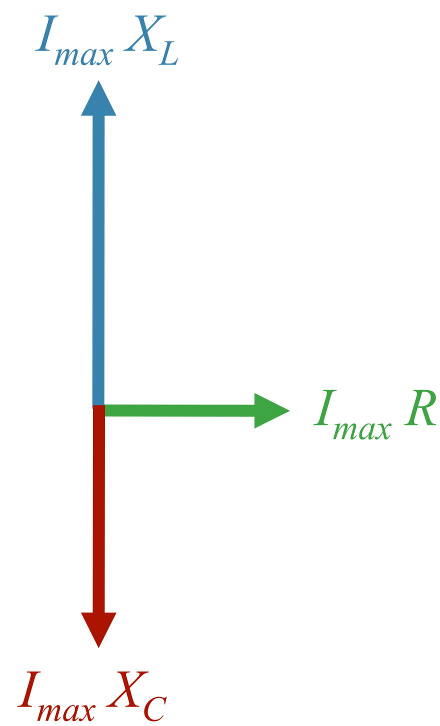
V 90^o ahead of I

$$V_{max} = I_{max} R$$

V in phase with I

"Do you have any fancy-schmancy simulations for to show me?"

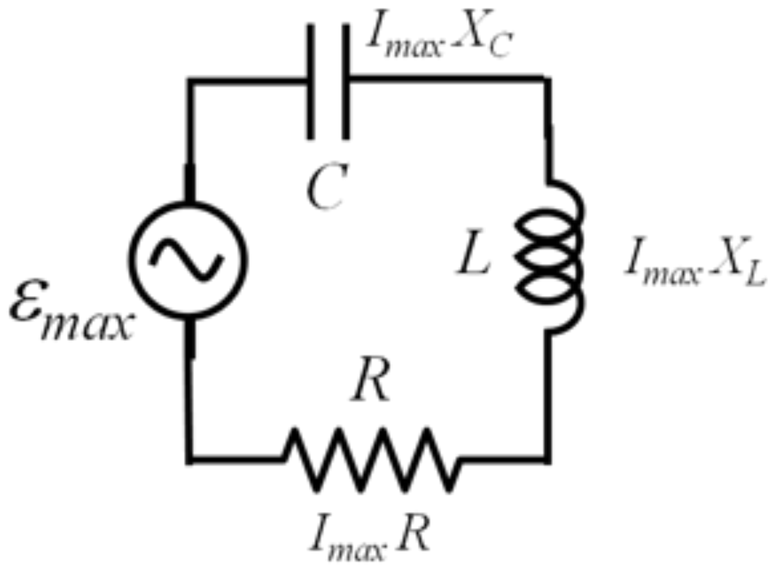
Phasors make this simple to see

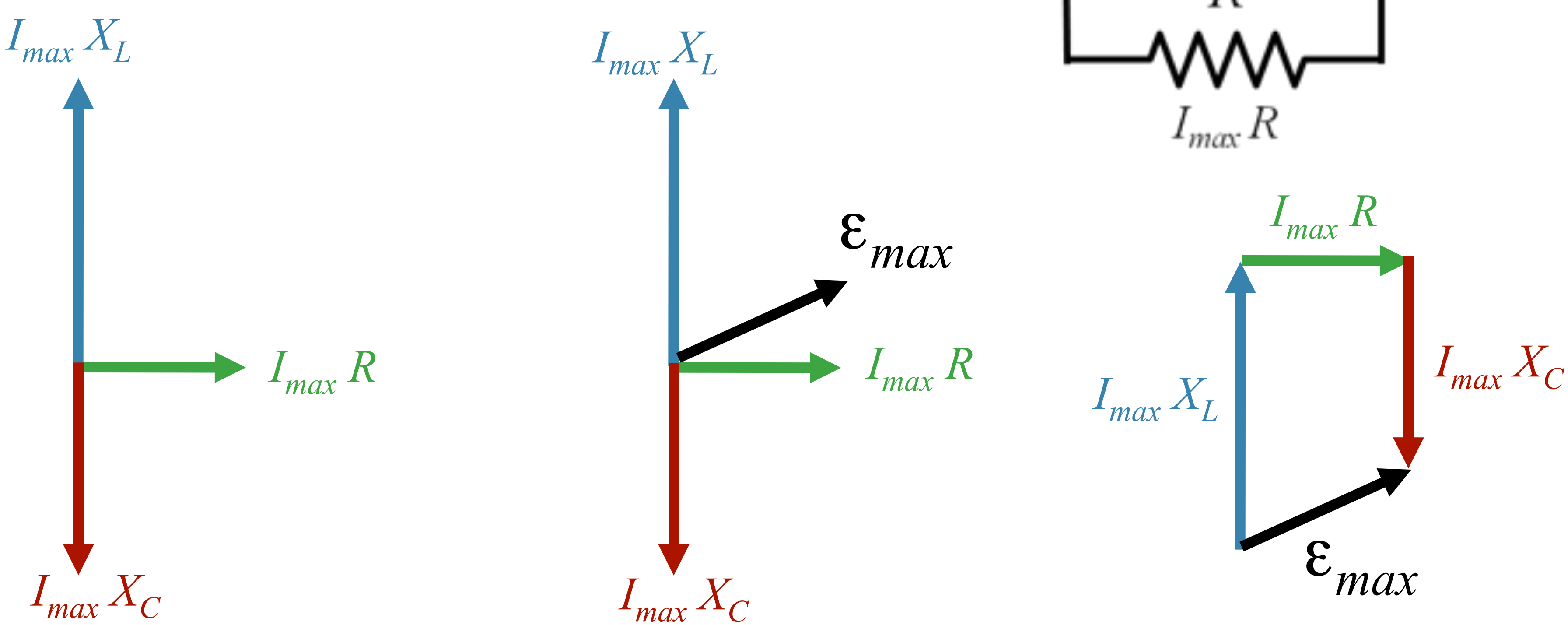


Always looks the same.
Only the lengths will change

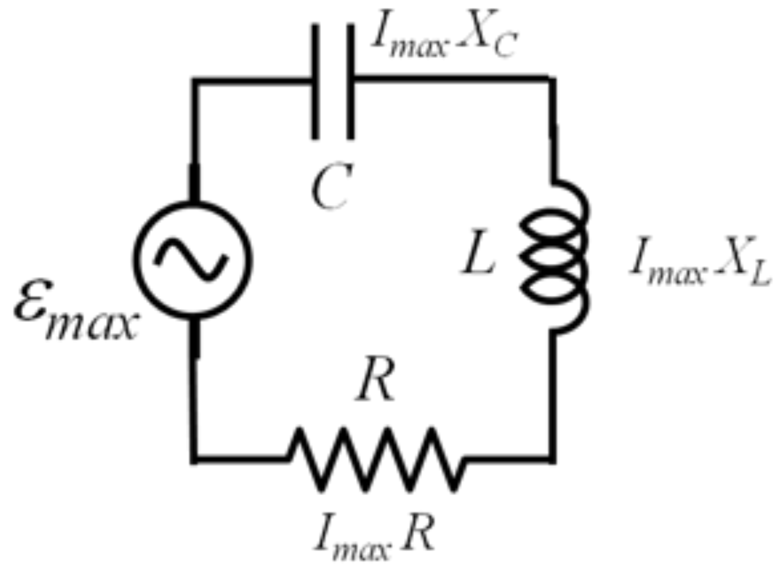
The Voltages still Add Up

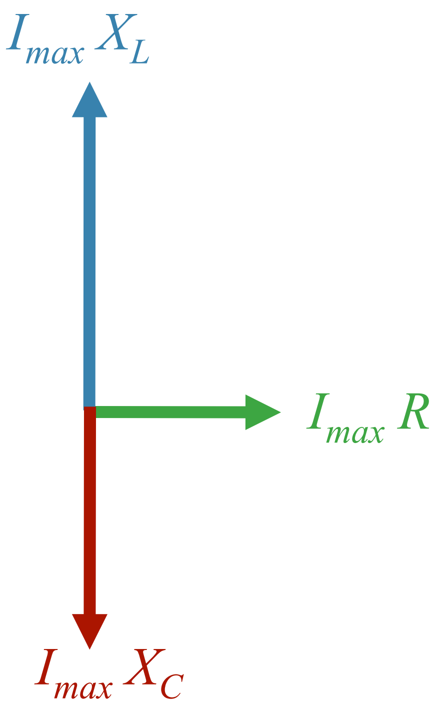
But now we are adding vectors:

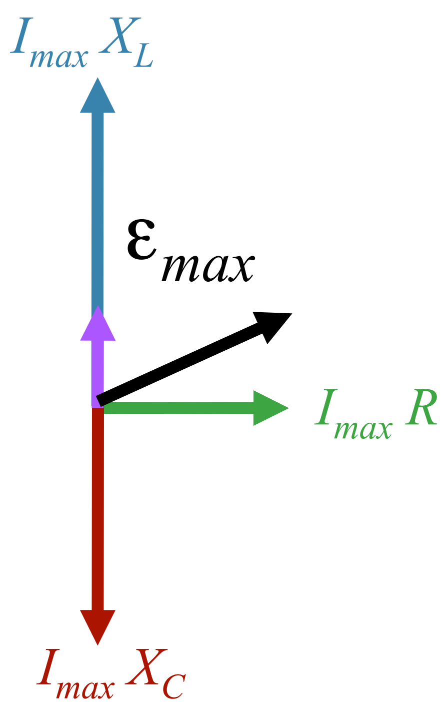




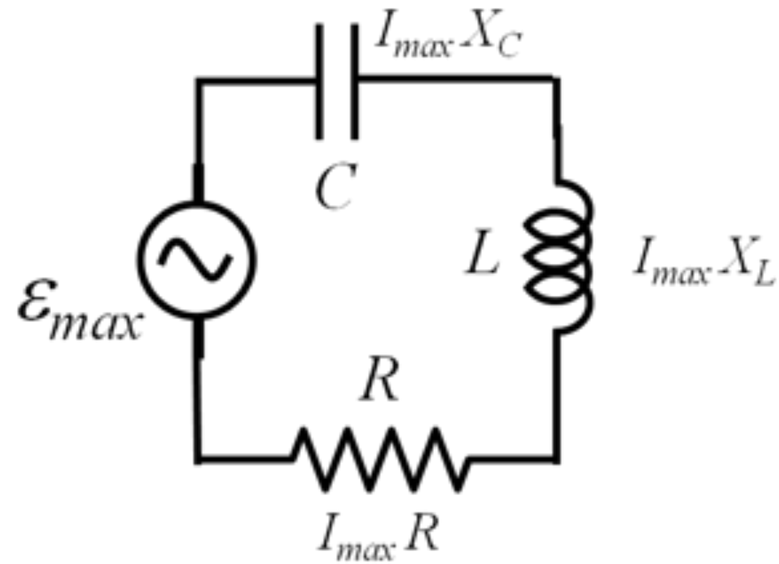
Make this Simpler

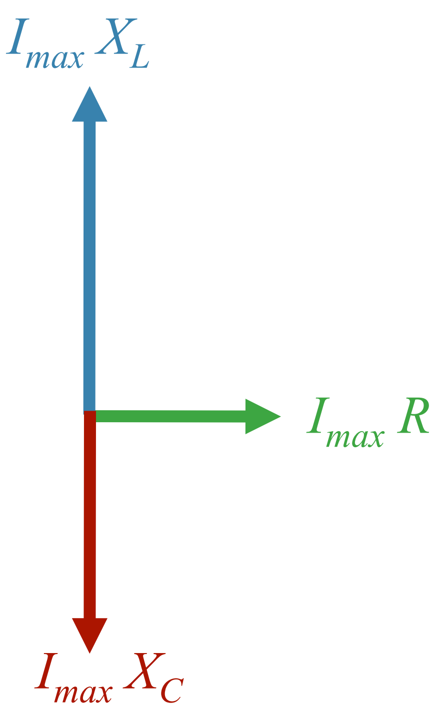




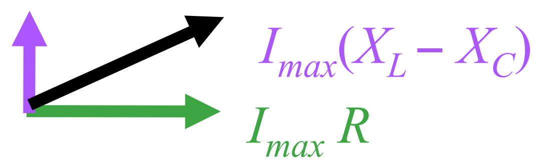


Make this Simpler

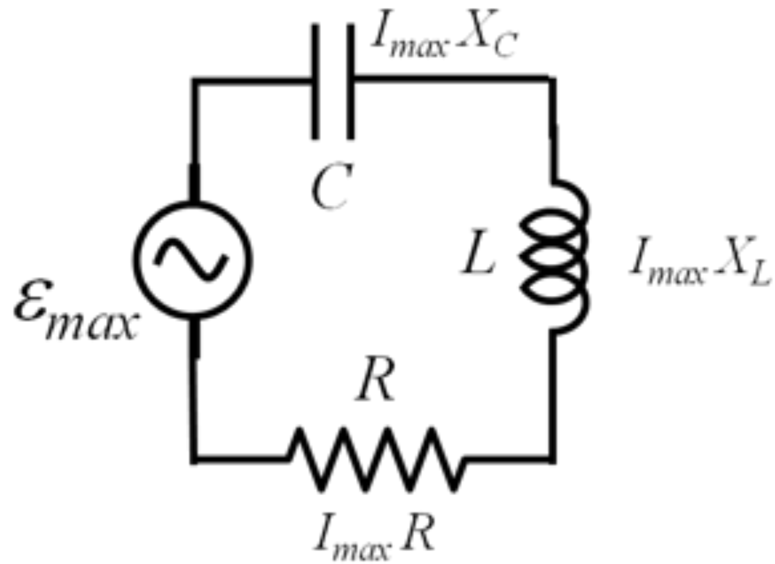




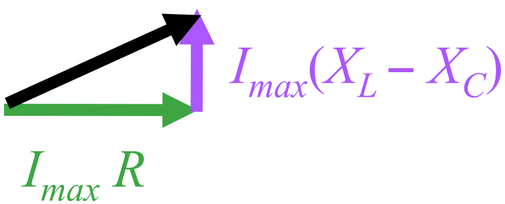
$$\varepsilon_{max} = I_{max} Z$$



Make this Simpler

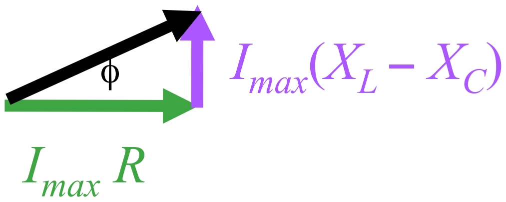


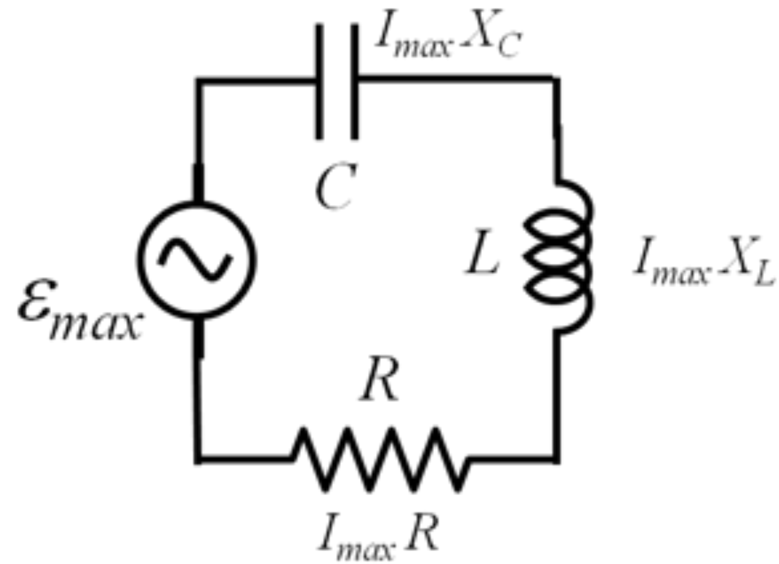
$$\varepsilon_{max} = I_{max}Z$$



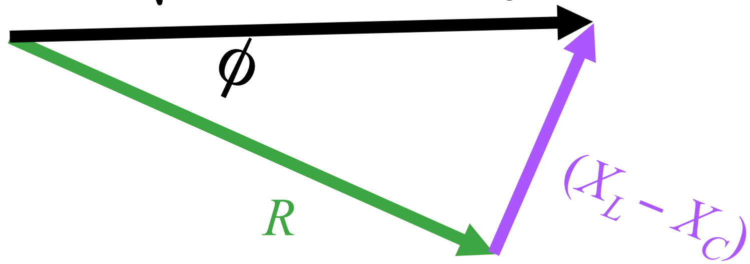
Make this Simpler

$$\varepsilon_{max} = I_{max} Z$$





$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



Impedance Triangle

$$\tan(\phi) = \frac{X_L - X_C}{R}$$

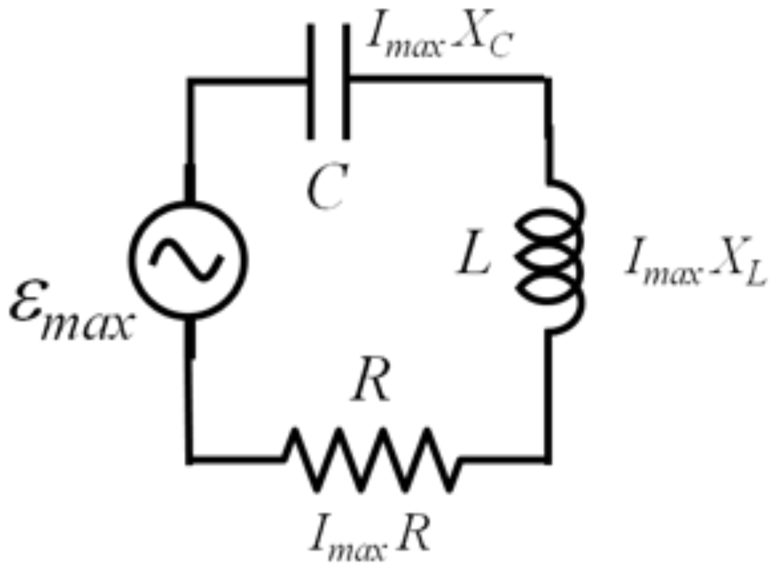
Summary

$$V_{Cmax} = I_{max} X_C$$

$$V_{Lmax} = I_{max} X_L$$

$$V_{Rmax} = I_{max} R$$

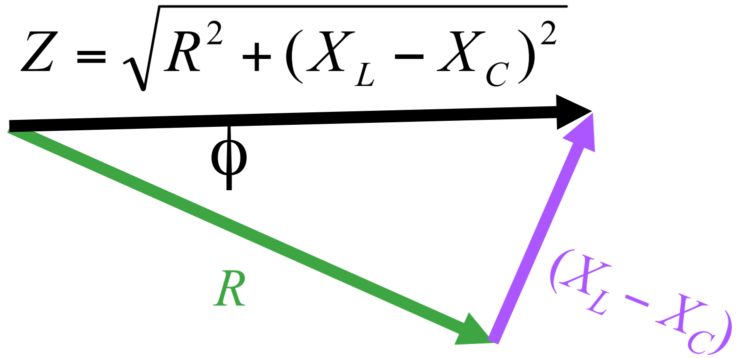
$$\varepsilon_{max} = I_{max} Z$$



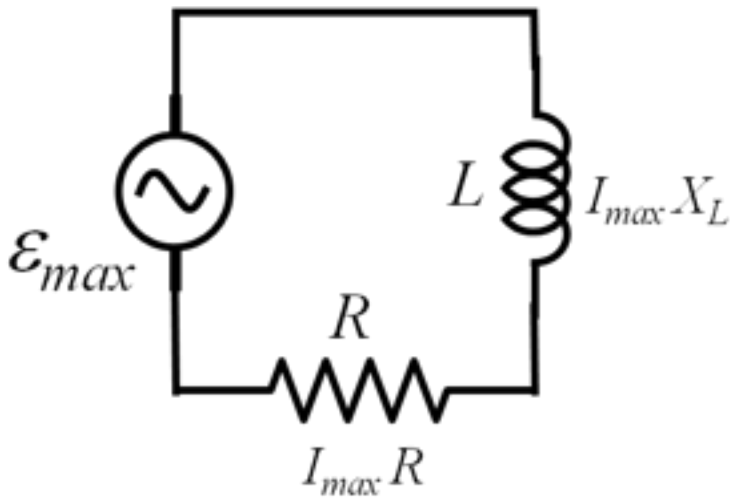
$$I_{max} = \varepsilon_{max} / Z$$

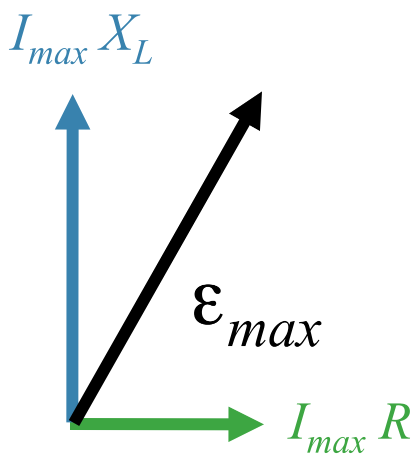
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan(\phi) = \frac{X_L - X_C}{R}$$



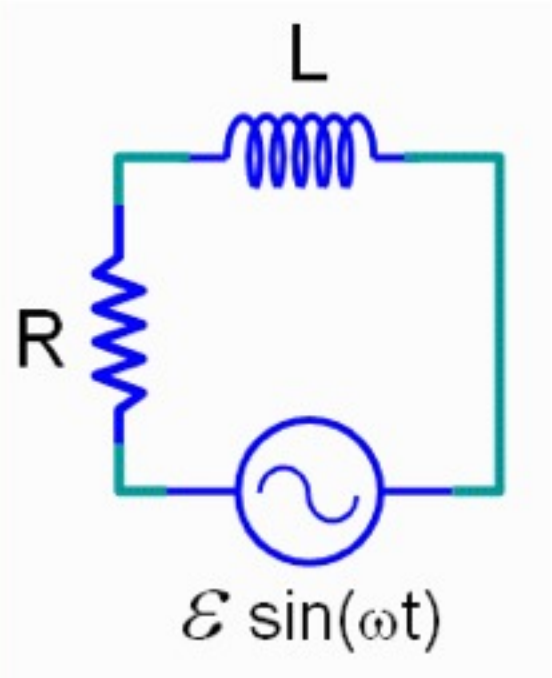
Example: RL Circuit $X_c = 0$



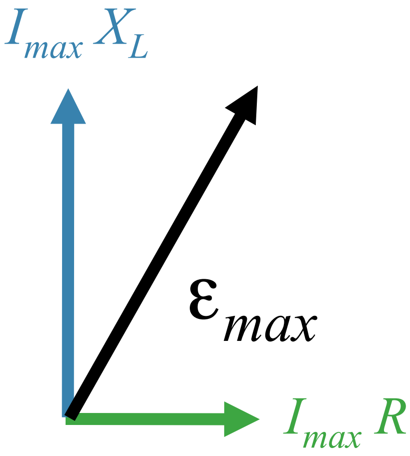


CheckPoint 2

2) A RL circuit is driven by an AC generator as shown in the figure.



Draw Voltage Phasors

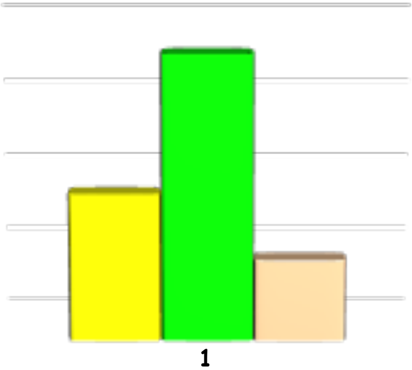


The voltages across the resistor and generator are ________________.

A ○ always out of phase

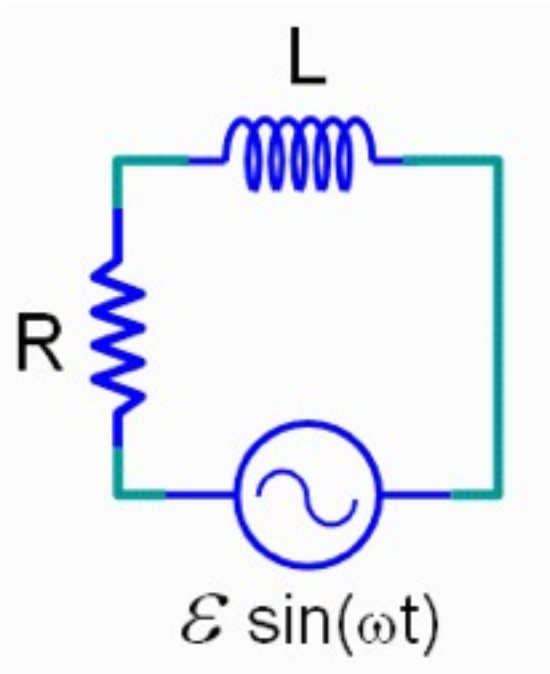
B ○ always in phase

C ○ sometimes in phase and sometimes out of phase

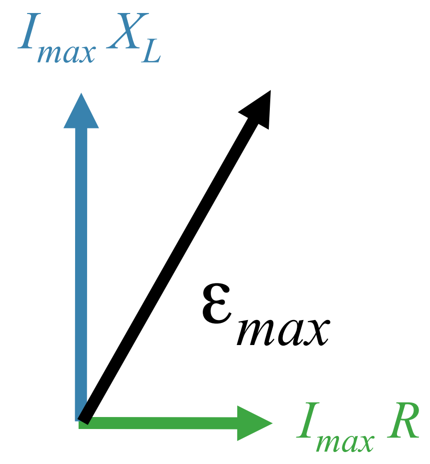


CheckPoint 4

A RL circuit is driven by an AC generator as shown in the figure.



Draw Voltage Phasors

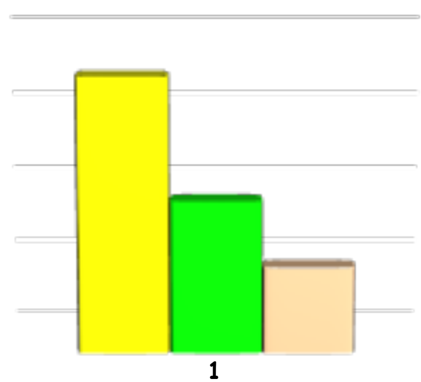


The voltages across the resistor and the inductor are ________________.

A ○ always out of phase

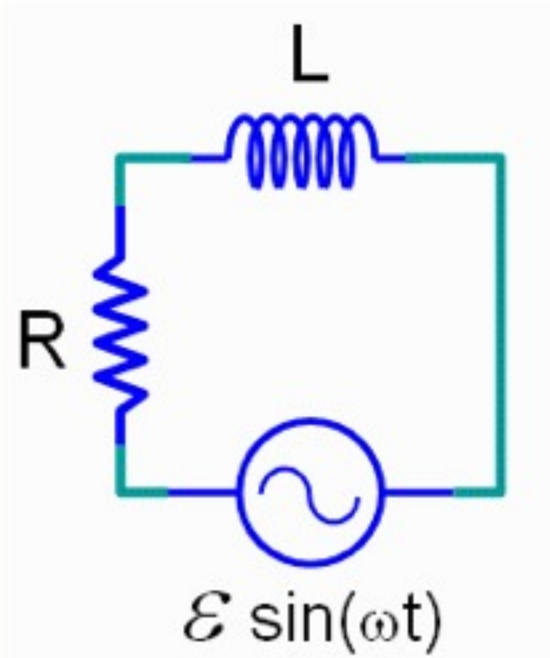
B ○ always in phase

C ○ sometimes in phase and sometimes out of phase

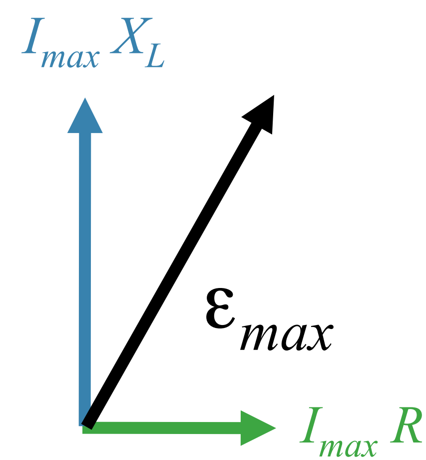


CheckPoint 6

A RL circuit is driven by an AC generator as shown in the figure.



The CURRENT is THE CURRENT



ϕ is the phase between generator and current

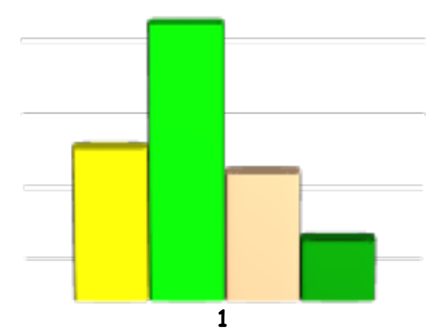
6) The phase difference between the CURRENT through the resistor and inductor ____

A ○ is always zero

B ○ is always 90°

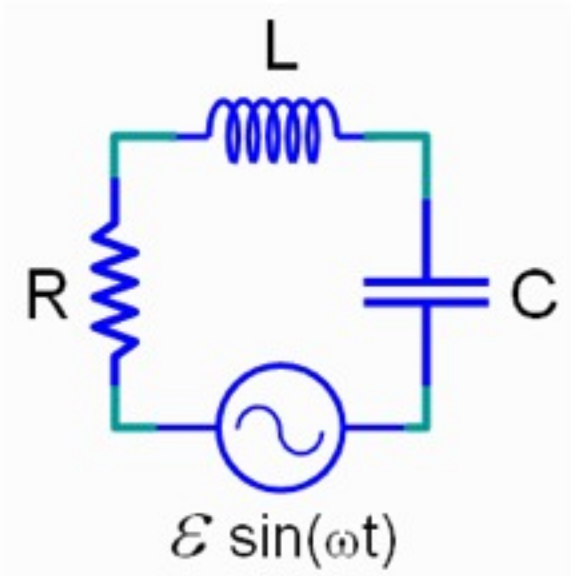
C ○ depends on the value of L and R

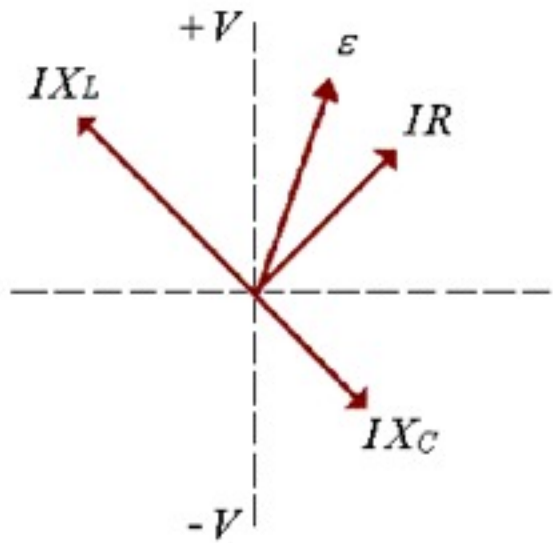
D ○ depends on L, R and the generator voltage



CheckPoint 8

A driven RLC circuit is represented by the phasor diagram below.



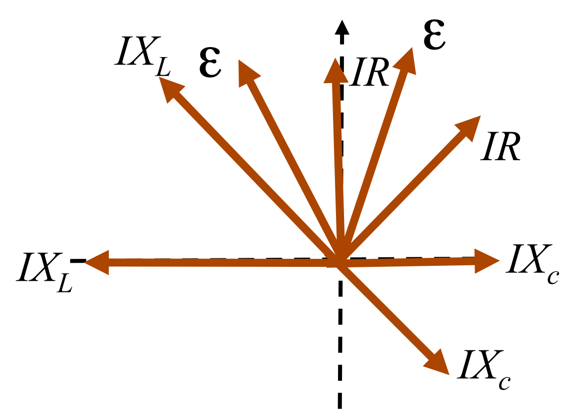


The vertical axis of the phasor diagram represents voltage. When the current through the circuit is maximum, what is the potential difference across the inductor?

A ○ $V_L = 0$

B ○ $V_L = V_{Lmax}/2$

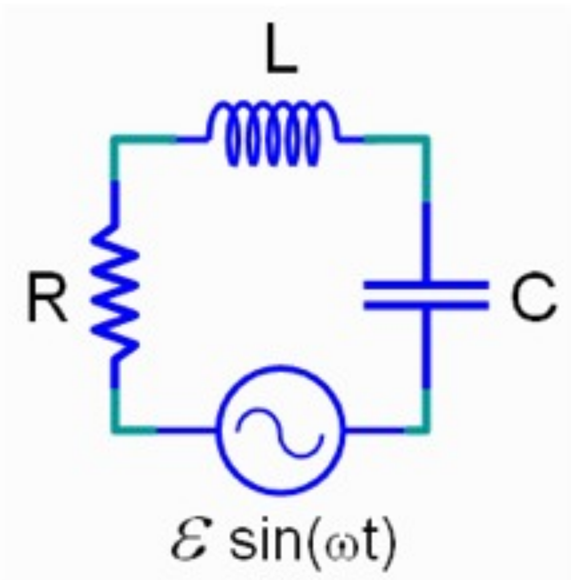
C ○ $V_L = V_{Lmax}$

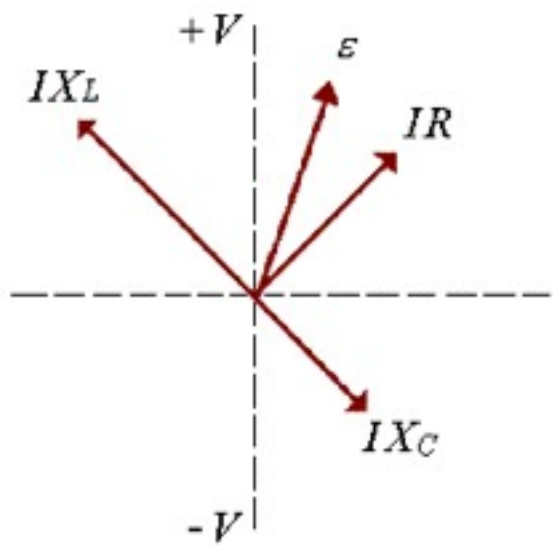


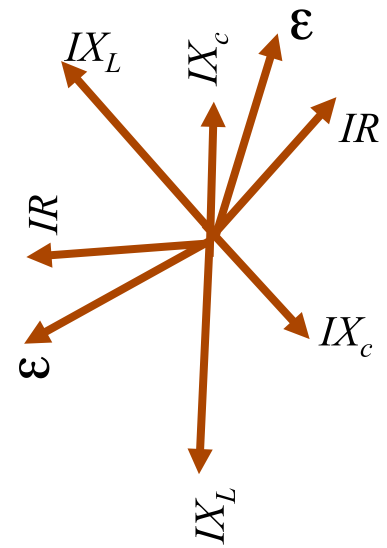
What does the voltage phasor diagram look like when the current is a maximum?

CheckPoint 10

A driven RLC circuit is represented by the phasor diagram below.







When the capacitor is fully charged, what is the magnitude of the voltage across the inductor?

A ○ $V_L = 0$

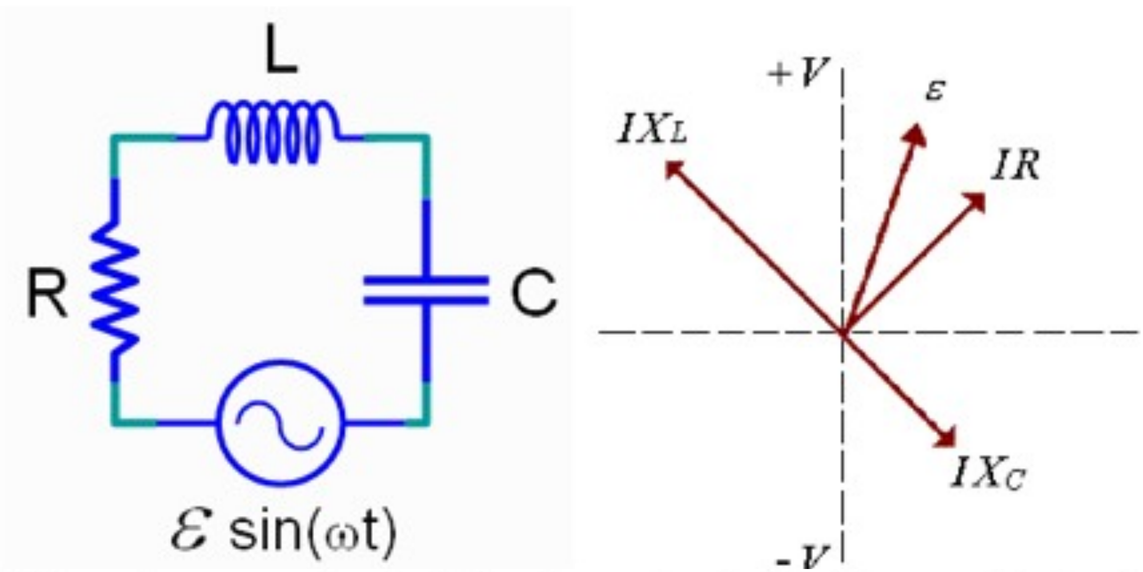
B ○ $V_L = V_{Lmax}/2$

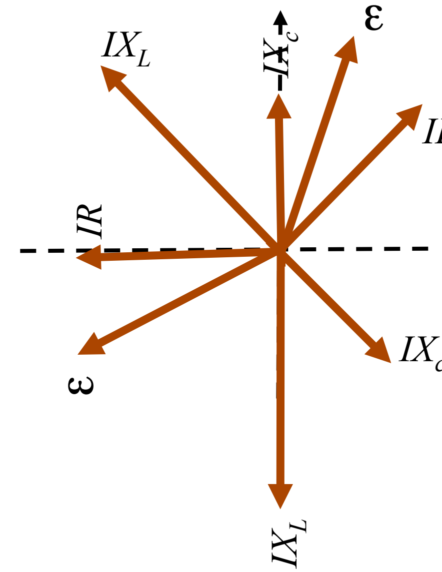
C ○ $V_L = V_{Lmax}$

What does the voltage phasor diagram look like when the capacitor is fully charged?

CheckPoint 12

A driven RLC circuit is represented by the phasor diagram below.





12) When the voltage across the capacitor is at its positive maximum, $V_C=+V_{Cmax}$, what is the voltage across the inductor?

A ○ $V_L = 0$

B ○ $V_L = V_{Lmax}$

C ○ $V_L = -V_{Lmax}$

1

What does the voltage phasor diagram look like when the voltage across capacitor is at its positive maximum?

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100$ V

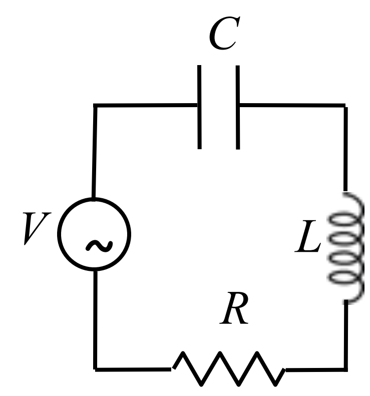
$I_{max} = 2$ mA

$V_{Cmax} = 113$ V

The current leads generator voltage by 45^{o}

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



Conceptual Analysis

The maximum voltage for each component is related to its reactance and to the maximum current.

The impedance triangle determines the relationship between the maximum voltages for the components

Strategic Analysis

Use V_{max} and I_{max} to determine Z

Use impedance triangle to determine R

Use V_{Cmax} and impedance triangle to determine X_L

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100$ V

$I_{max} = 2$ mA

$V_{Cmax} = 113$ V

The current leads generator voltage by 45^o

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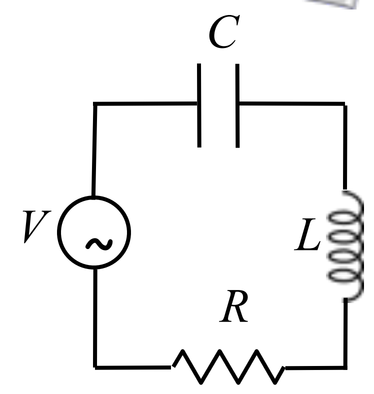


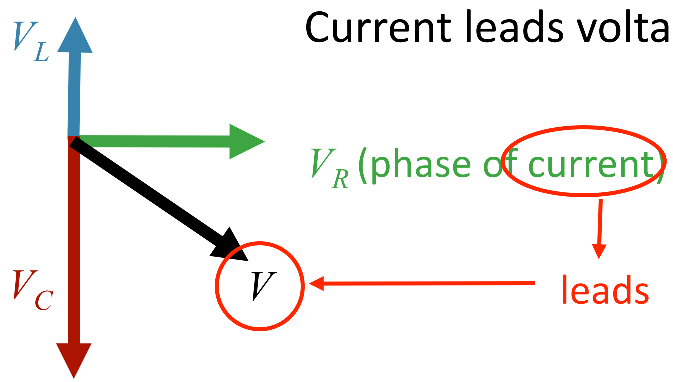
Compare X_L and X_C at this frequency:

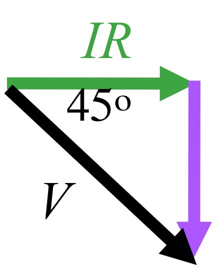
A) $X_L < X_C$ B) $X_L = X_C$ C) $X_L > X_C$ D) Not enough information

This information is determined from the phase

Current leads voltage

$$V_L = I_{\max} X_L$$
$$V_C = I_{\max} X_C$$





Calculation

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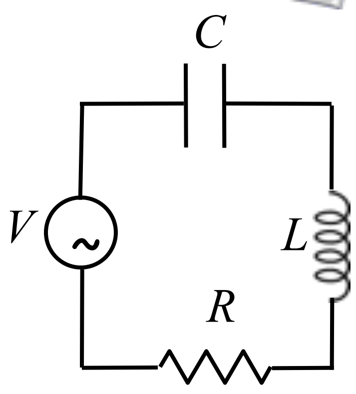


What is Z, the total impedance of the circuit?

A) 70.7 kΩ

B) $\boxed{50 \text{ k}\Omega}$

C) 35.4 kΩ

D) 21.1 kΩ

$$Z = \frac{V_{\max}}{I_{\max}} = \frac{100V}{2mA} = 50k\Omega$$

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ \mathrm{V}$

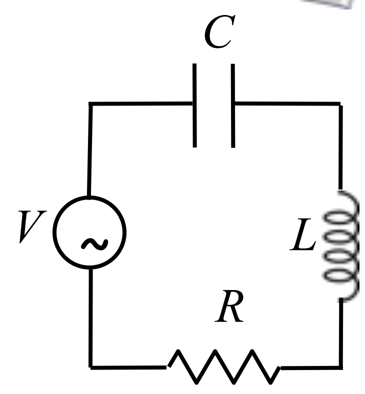
$I_{max} = 2\ \mathrm{mA}$

$V_{Cmax} = 113\ \mathrm{V}$

The current leads generator voltage by 45^{o}

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



$Z = 50\ \mathrm{k\Omega}$

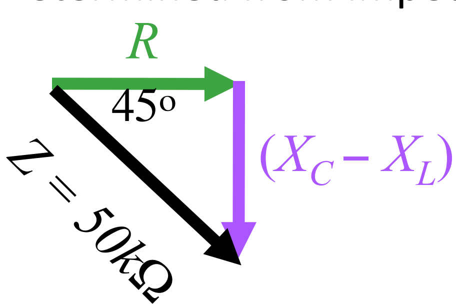
$\sin(45°) = 0.707$

$\cos(45°) = 0.707$

What is R?

A) $70.7\ \mathrm{k\Omega}$ B) $50\ \mathrm{k\Omega}$ C) $35.4\ \mathrm{k\Omega}$ D) $21.1\ \mathrm{k\Omega}$

Determined from impedance triangle



$$\cos(45) = \frac{R}{Z} \rightarrow R = Z\cos(45^{o})$$

$$= 50\ \mathrm{k\Omega} \times 0.707$$

$$= 35.4\ \mathrm{k\Omega}$$

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100$ V

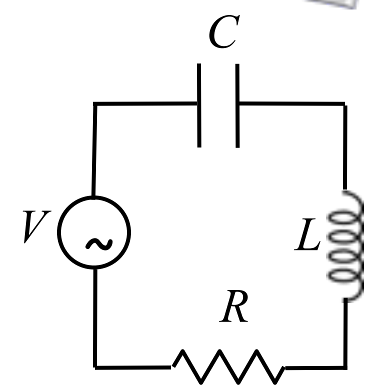
$I_{max} = 2$ mA

$V_{Cmax} = 113$ V

The current leads generator voltage by 45^{o}

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?

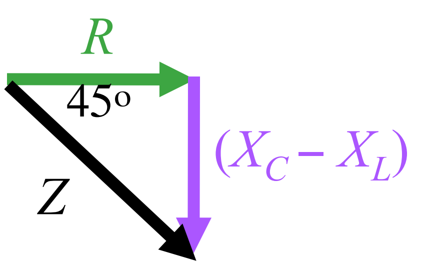


$Z = 50$ kΩ

$R = 35.4$kΩ

A) 70.7 kΩ B) 50 kΩ C) 35.4 kΩ $\boxed{\text{D) } 21.1 \text{ k}\Omega}$

We start with the impedance triangle:



$$\frac{X_C - X_L}{R} = \tan 45° = 1 \rightarrow X_L = X_C - R$$

What is X_C?

$$V_{Cmax} = I_{\max} X_C$$

$$X_C = \frac{113}{2} = 56.5k\Omega$$

$$X_L = 56.5 \text{ k}\Omega - 35.4 \text{ k}\Omega$$

Practical Test Hints

You will have

- 4 banana plug wires, ~4 alligator clips
- 1 Scope Probe (x1/x10)
- 1 BNC wire
- adaptors: Tee, BNC-Male Banana, BNC-Female Banana
- proto-board (Use it!)
- scope, function generator, DMM

We will have two 60 min sessions:

- 12:30 — 13:30
- 13:40 — 14:30