

Electricity & Magnetism

Lecture 22

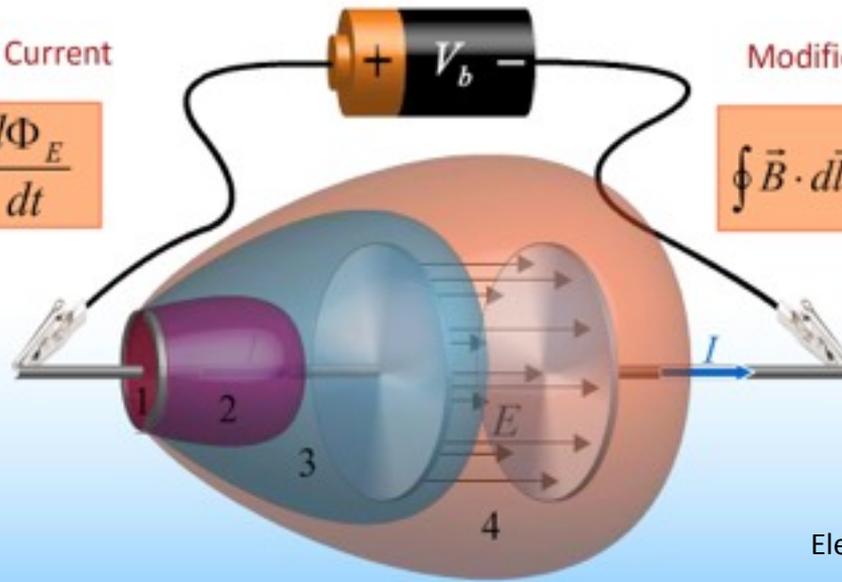
DISPLACEMENT CURRENT and EM WAVES

Displacement Current

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

Modified Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I + I_D)$$



Your Comments

“You LIED about electric potential being the hardest part of the course! Please go through everything in detail because this is a confusing prelecture and I don't understand most of it at all.

“I feel like I don't understand this well enough to even ask questions about it. Little bit shell-shocked right now.”

“THAT WAS AWESOME!! DISCUSS ALL PLEASE!!!! “

“wave equations, this is my first time seeing them (aside from oscillators)”

“B field looks like the shadow of E field, or the other way around”

“Do we have to know how to solve differential equations or will the problems be limited to these specific types of examples?”

“More concerned about the midterm.”

ALL TRUE
We will try to
make clear, at
least the BIG
IDEAS

We will discuss waves
You will not have to solve
new diff eqns

What part of

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \epsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\phi_{\text{el}} / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 I + \mu_0 \epsilon_0 d\phi_{\text{em}} / dt$$

don't you understand?

And God Said

$$\oiint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\oiint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

$$\oint_{\partial S} \vec{E} \cdot d\vec{l} = - \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

$$\oint_{\partial S} \vec{B} \cdot d\vec{l} = \mu_0 I_s + \mu_0 \epsilon_0 \iint_S \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

and *then* there was light.

Order online.

smart**Physics**

What We Knew Before Prelecture 22

MAXWELL'S EQUATIONS

Gauss' Law for E Fields

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Gauss' Law for B Fields

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

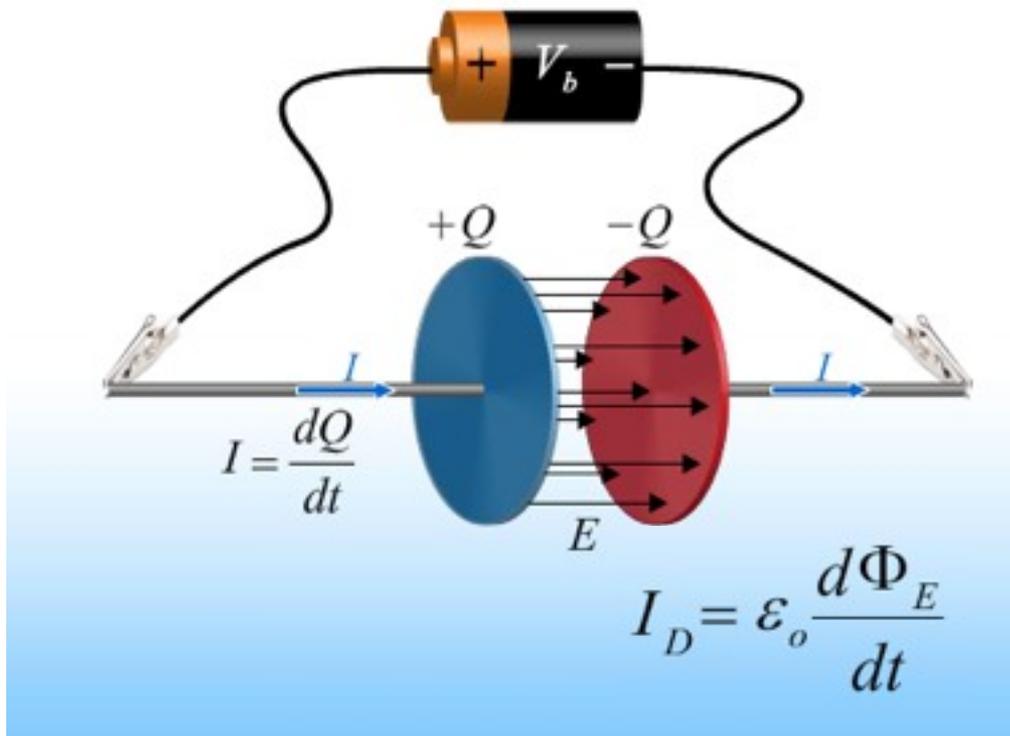
Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

After Prelecture 21: Modify Ampere's Law

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}} = \mu_0 (I + I_D)$$



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$



$$\Phi = EA = \frac{Q}{\epsilon_0}$$



$$Q = \epsilon_0 \Phi$$



$$\frac{dQ}{dt} = \epsilon_0 \frac{d\Phi}{dt} \equiv I_D$$

Displacement Current

Real Current:

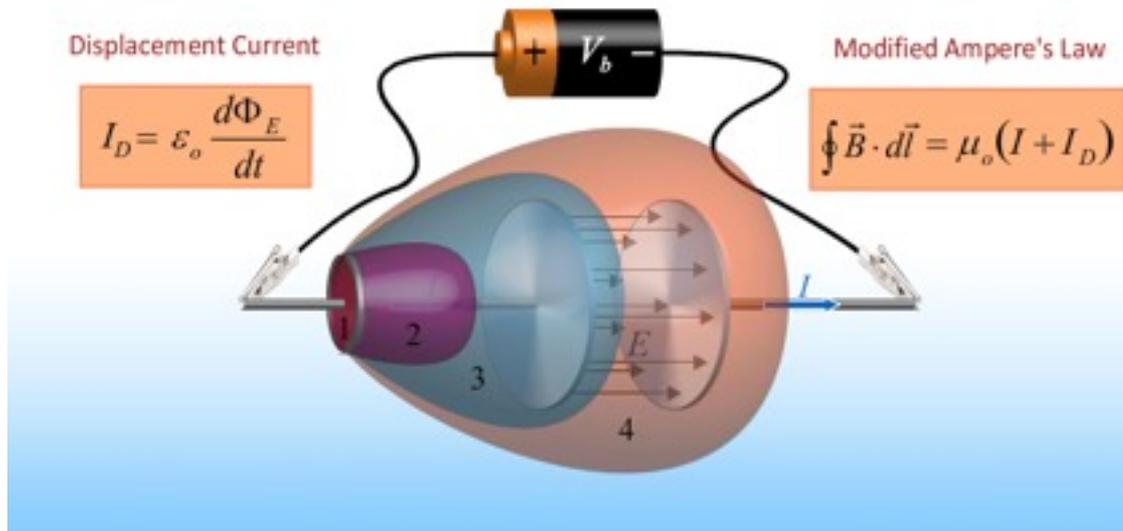
Charge Q passes through area A in time t :

$$I = \frac{dQ}{dt}$$

Displacement Current: Electric flux through area A changes in time

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

DISPLACEMENT CURRENT and EM WAVES



Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$



Modified Ampere's Law

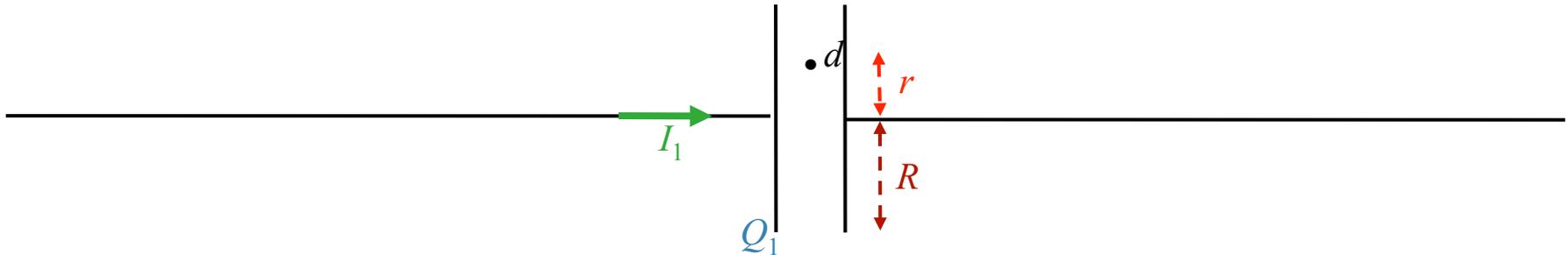
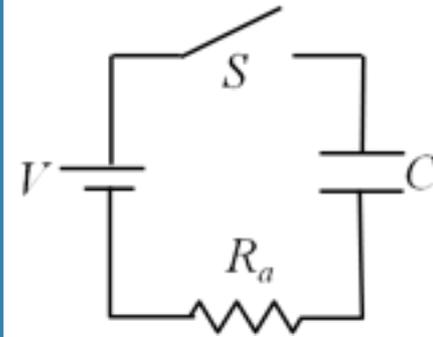
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

Free space

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .

At time t_1 , what is the magnetic field B_1 at a radius r (point d) in between the plates of the capacitor?



Conceptual and Strategic Analysis

Charge Q_1 creates electric field between the plates of C

Charge Q_1 changing in time gives rise to a changing electric flux between the plates

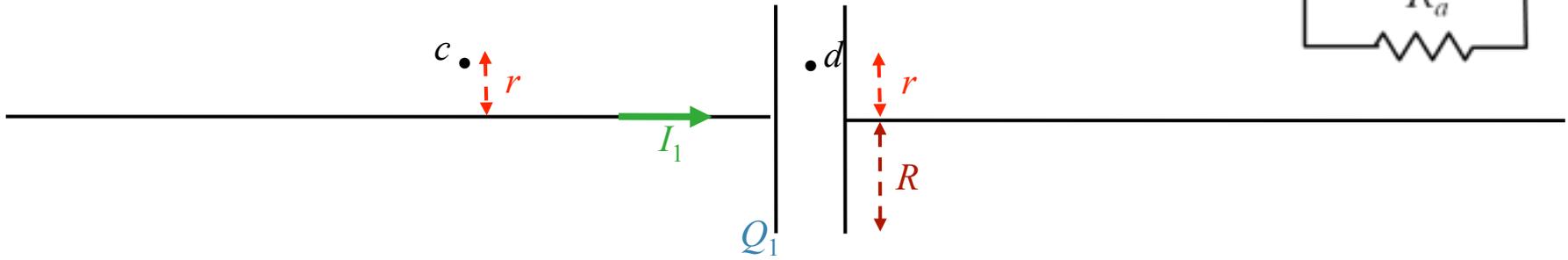
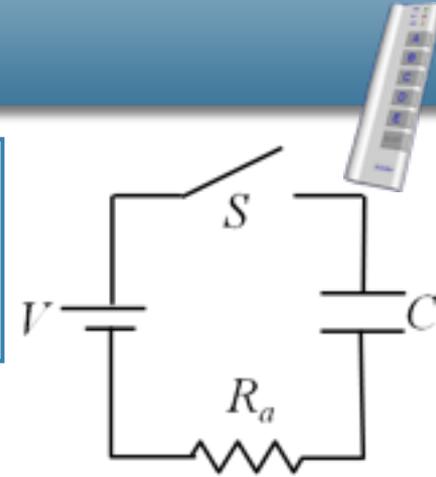
Changing electric flux gives rise to a displacement current I_D in between the plates

Apply (modified) Ampere's law using I_D to determine B

Calculation

Switch S has been open a long time when at $t = 0$, it is closed.

Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



Compare the magnitudes of the B fields at points c and d .

A) $B_c < B_d$

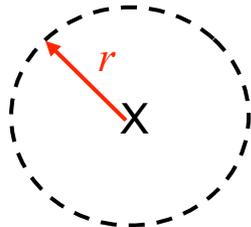
B) $B_c = B_d$

C) $B_c > B_d$

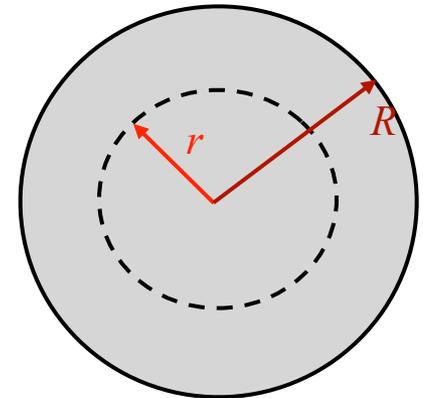
What is the difference?

Apply (modified) Ampere's Law

point c :
 $I(\text{enclosed}) = I_1$

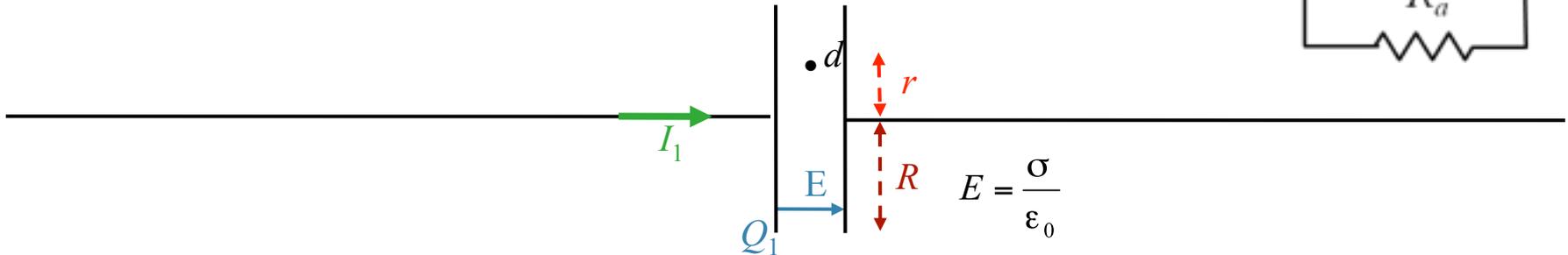
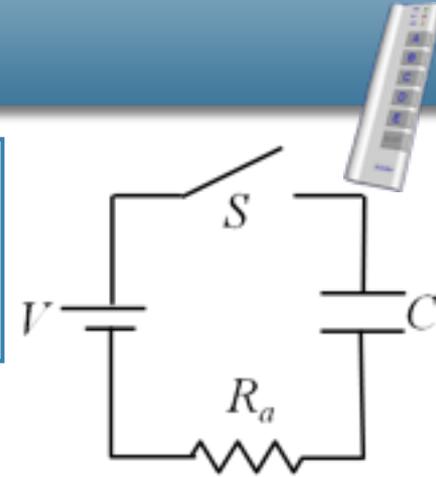


point d :
 $I_D(\text{enclosed}) < I_1$



Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



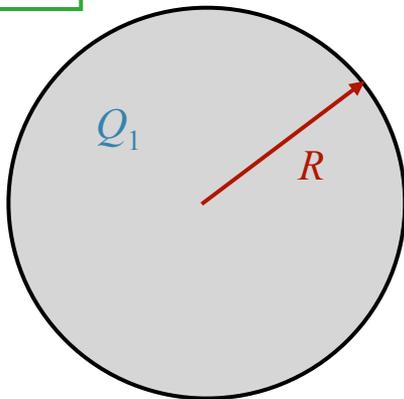
What is the magnitude of the electric field between the plates?

A) $E = \frac{Q_1}{\pi R^2 \epsilon_0}$

B) $E = \frac{Q_1 \pi R^2}{\epsilon_0}$

C) $E = \frac{Q_1}{\epsilon_0}$

D) $E = \frac{Q_1}{r}$

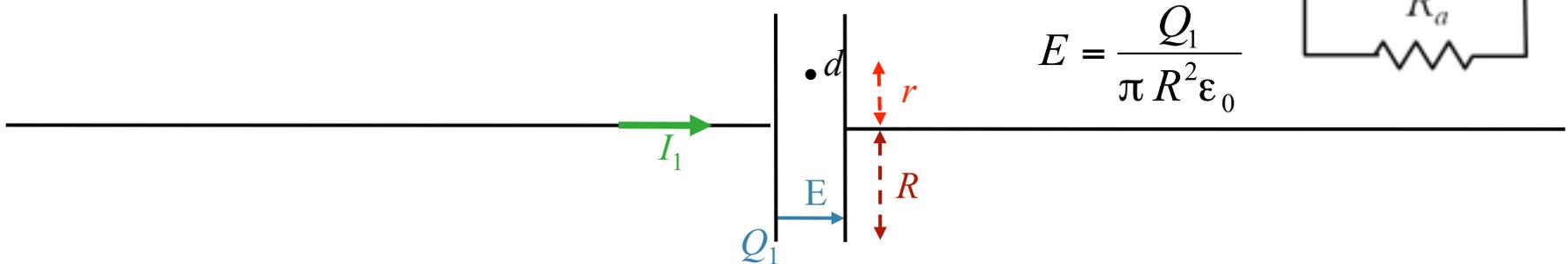
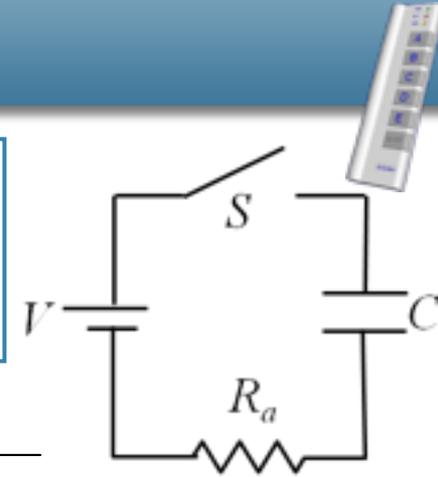


$$E = \frac{\sigma}{\epsilon_0} \rightarrow \sigma = \frac{Q_1}{A} = \frac{Q_1}{\pi R^2} \rightarrow E = \frac{Q_1}{\epsilon_0 \pi R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed.

Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



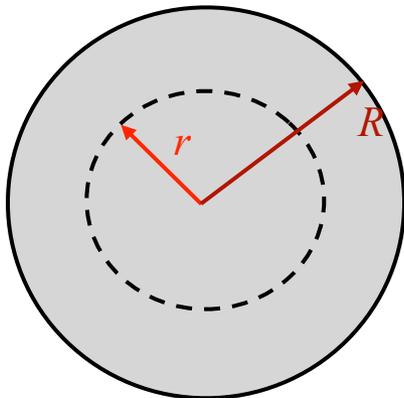
What is the electric flux through a circle of radius r in between the plates?

A) $\Phi_E = \frac{Q_1}{\epsilon_0} \pi r^2$

B) $\Phi_E = \frac{Q_1}{\epsilon_0} \pi R^2$

C) $\Phi_E = \frac{Q_1 r^2}{\epsilon_0 R^2}$

D) $\Phi_E = \frac{Q_1 \pi r^2}{\epsilon_0 R^2}$

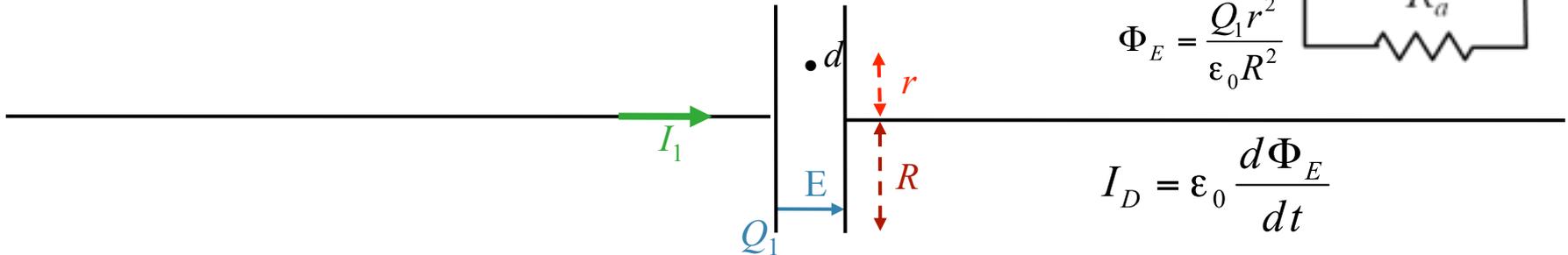
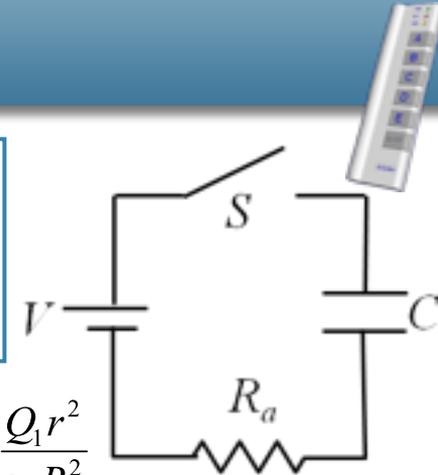


$$\Phi_E = \vec{E} \cdot \vec{A} \rightarrow \Phi_E = \frac{Q_1}{\epsilon_0 \pi R^2} \pi r^2 \rightarrow \Phi_E = \frac{Q_1}{\epsilon_0} \frac{r^2}{R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed.

Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



$$\Phi_E = \frac{Q_1 r^2}{\epsilon_0 R^2}$$

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

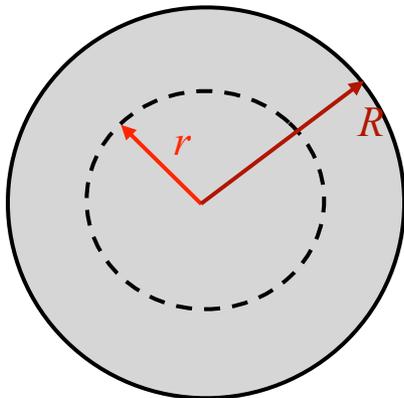
What is the displacement current enclosed by circle of radius r ?

A) $I_D = I_1 \frac{R^2}{r^2}$

B) $I_D = I_1 \frac{r}{R}$

C) $I_D = I_1 \frac{r^2}{R^2}$

D) $I_D = I_1 \frac{R}{r}$



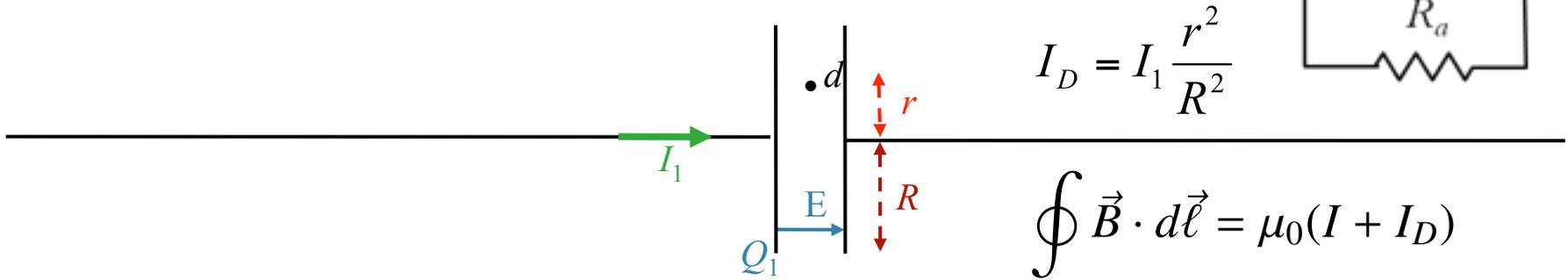
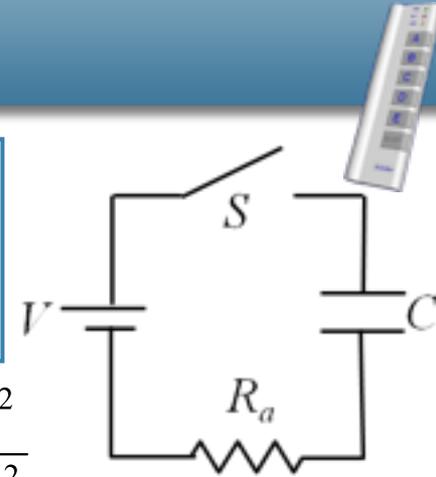
$$I_D = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ_1}{dt} \frac{r^2}{R^2} = I_1 \frac{r^2}{R^2}$$

→ $I_D = I_1 \frac{r^2}{R^2}$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed.

Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



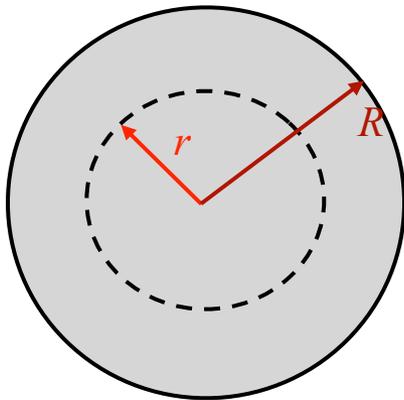
What is the magnitude of the B field at radius r ?

A) $B = \frac{\mu_0 I_1}{2\pi R}$

B) $B = \frac{\mu_0 I_1}{2\pi r}$

C) $B = \frac{\mu_0 I_1}{2\pi} \frac{R}{r^2}$

D) $B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$



Ampere's Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I + I_D)$

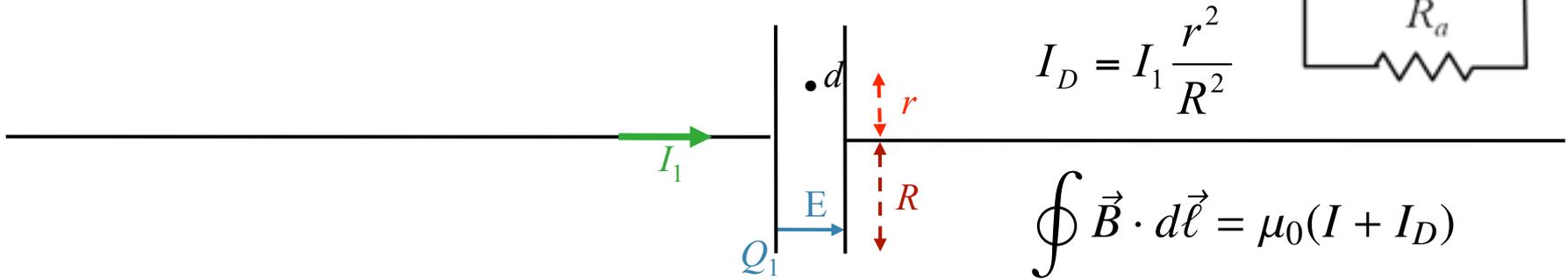
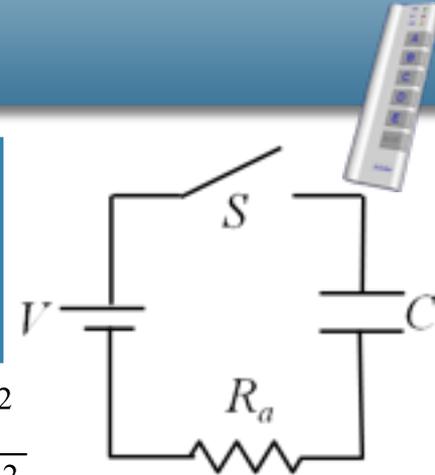
$\rightarrow B(2\pi r) = \mu_0 \left(0 + I_1 \frac{r^2}{R^2} \right)$

$\rightarrow B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$

Calculate

Switch S has been open a long time when at $t = 0$, it is closed.

Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I + I_D)$$

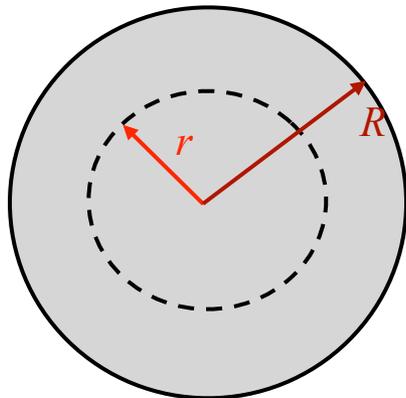
What is the magnitude of the B field at radius r ?

$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$

Let:

$$I_1 = 1 \text{ A}$$

$$R = 1 \text{ m}$$



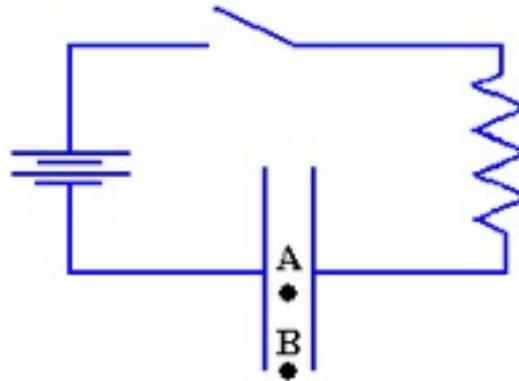
What is B at $r = 0.5 \text{ m}$?
(answer on next page)

$$B = 1 \times 10^{-7} \text{ T}$$

Checkpoint 2



At time $t = 0$ the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; A is at the center and B is toward the outer edge.



After the switch is closed, there will be a magnetic field at point A which is proportional to the current in the circuit.

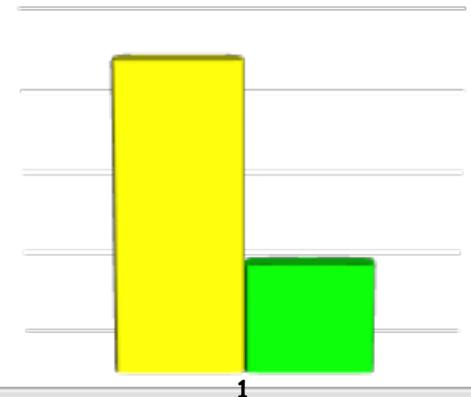
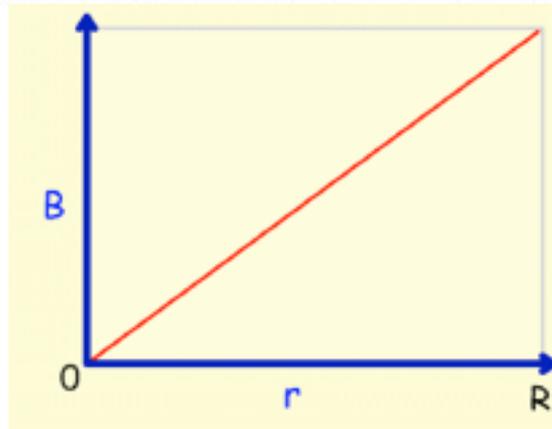
True False

$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$

B is proportional to I

but

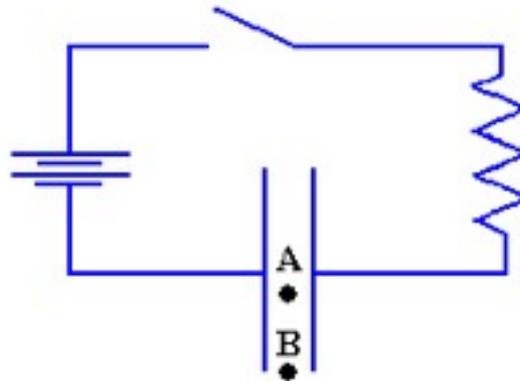
At A, $B = 0$!!



Checkpoint 4



At time $t = 0$ the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; A is at the center and B is toward the outer edge.

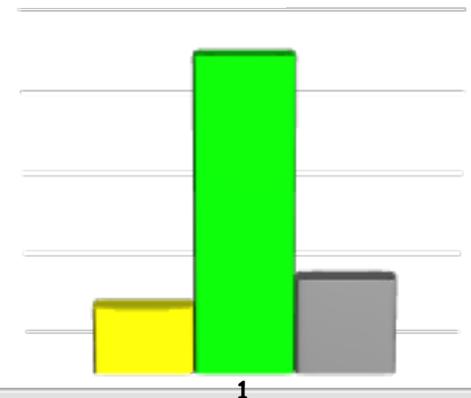
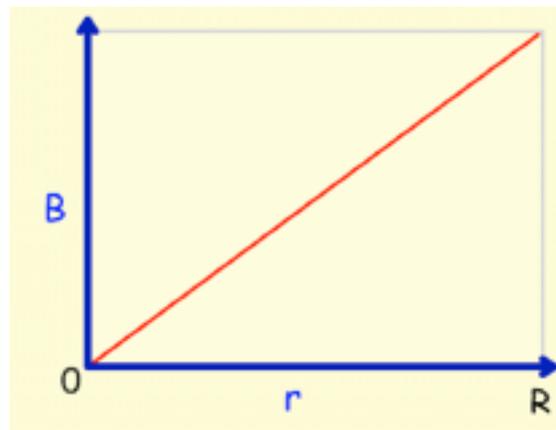


Compare the magnitudes of the magnetic fields at points A and B just after the switch is closed:

- A $B_A < B_B$
- B $B_A = B_B$
- C $B_A > B_B$

From the calculation we just did:

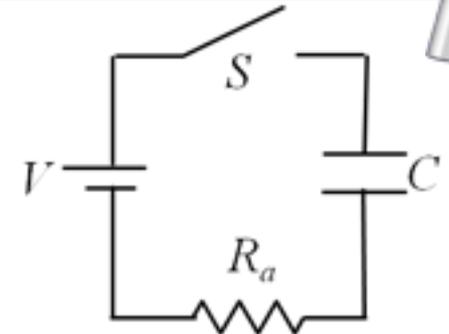
$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$



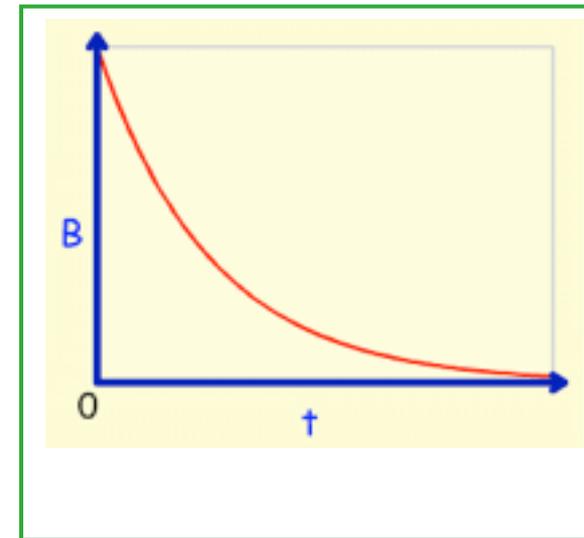
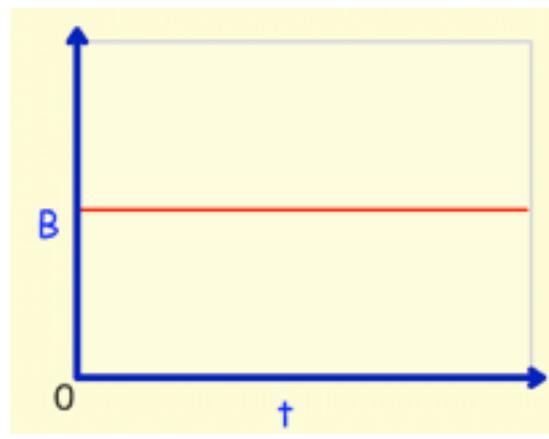
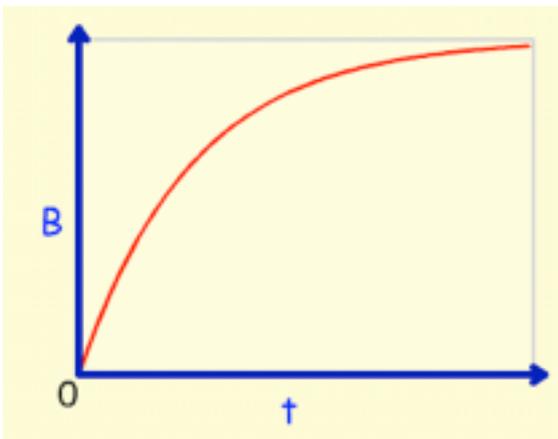
Follow-Up

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .

What is the time dependence of the magnetic field B at a radius r between the plates of the capacitor?



$$B_1 = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$



B at fixed r is proportional to the current I

Close switch: $V_C = 0 \Rightarrow I = V/R_a$ (maximum)

I exponentially decays with time constant $\tau = R_a C$

Follow-Up 2

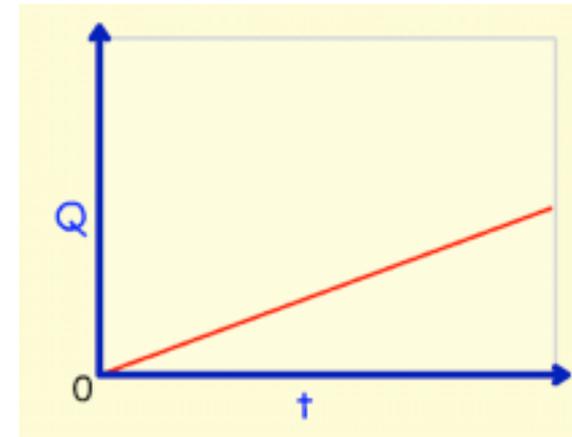
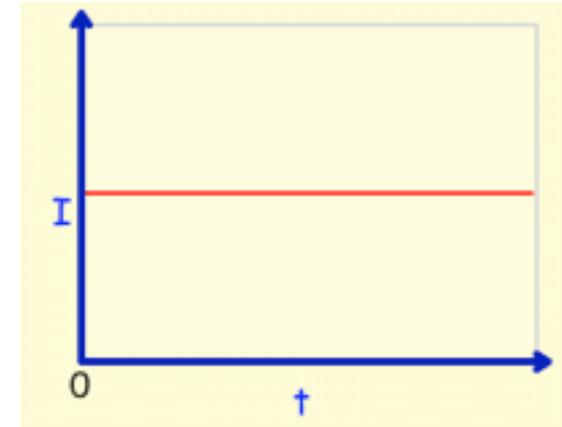


Suppose you were able to charge a capacitor with constant current (does not change in time).

Does a B field exist in between the plates of the capacitor?

A) YES

B) NO



Constant current $\Rightarrow Q$ increases linearly with time

Therefore E increases linearly with time, $E = Q/(A\epsilon_0)$

dE/dt is not zero \Rightarrow Displacement current is not zero
 $\Rightarrow B$ is not zero !

We learned about waves in Physics 140

1-D Wave Equation

$$\frac{d^2 h}{dx^2} = \frac{1}{v^2} \frac{d^2 h}{dt^2}$$

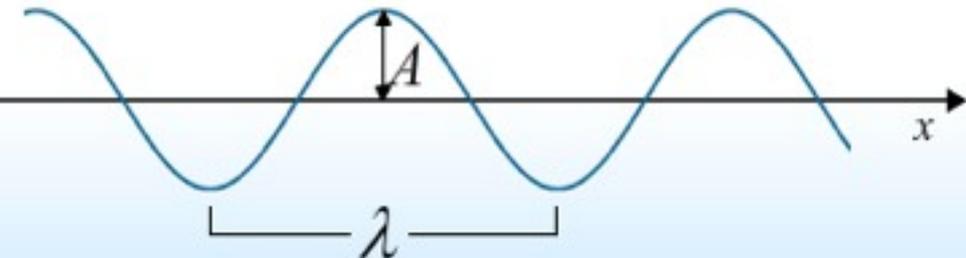


Solution

$$h(x,t) = h_1(x - vt) + h_2(x + vt)$$

Common Example: Harmonic Plane Wave

$$h(x,t) = A \cos(kx - \omega t)$$



Variable Definitions

Amplitude: A

Wave Number: $k = \frac{2\pi}{\lambda}$

Wavelength: λ

Angular Frequency: $\omega = \frac{2\pi}{T}$

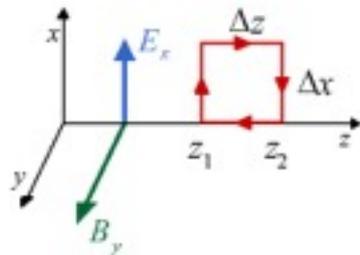
Period: T

Frequency: $f = \frac{1}{T}$

Velocity: $v = \lambda f = \frac{\omega}{k}$

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$



$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z} \frac{\partial B_y}{\partial t}$$

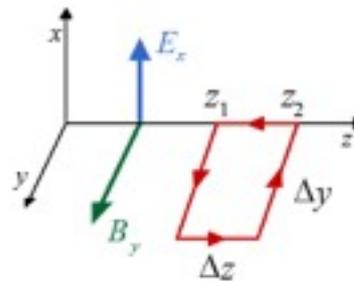
Plane Wave Solution

$$\vec{E} \rightarrow \vec{E}(z,t)$$

$$\vec{B} \rightarrow \vec{B}(z,t)$$

Modified Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$



$$\frac{\partial B_y}{\partial z} = -\mu_0 \epsilon_0 \frac{\partial E_x}{\partial t}$$

$$\frac{\partial}{\partial t} \frac{\partial B_y}{\partial z} = -\mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

Wave Equation

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

Speed of Electromagnetic Wave

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = 3.00 \times 10^8 \text{ m/s}$$

Speed of Light !



Special Relativity (1905)

Speed of Light is Constant

Albert Einstein



“How can light move at the same velocity in any inertial frame of reference? That's really trippy.”

see PHYS 285

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

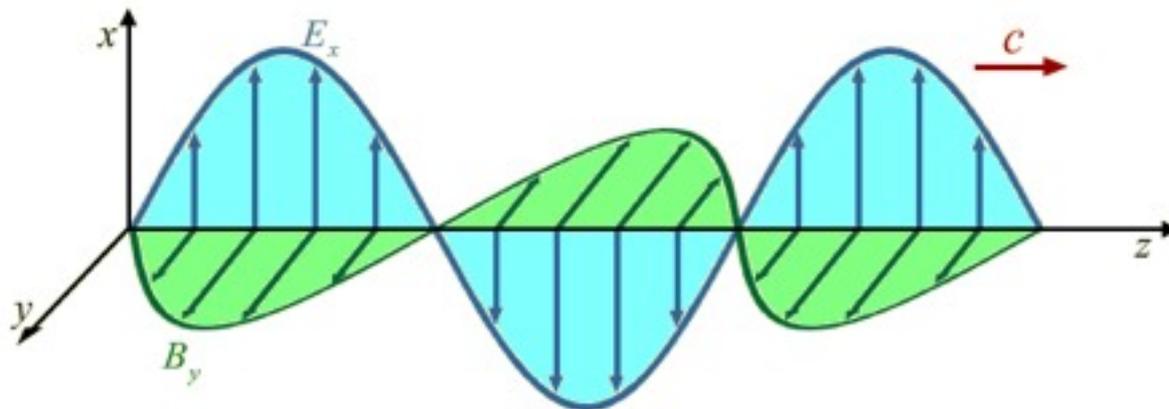
$$\frac{\partial^2 B_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

Example: A Harmonic Solution

$$E_x = E_o \cos(kz - \omega t) \xrightarrow{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}} B_y = \frac{k}{\omega} E_o \cos(kz - \omega t)$$

Two Important Features

1. B_y is in phase with E_x
2. $B_o = \frac{E_o}{c}$



THE ELECTROMAGNETIC SPECTRUM

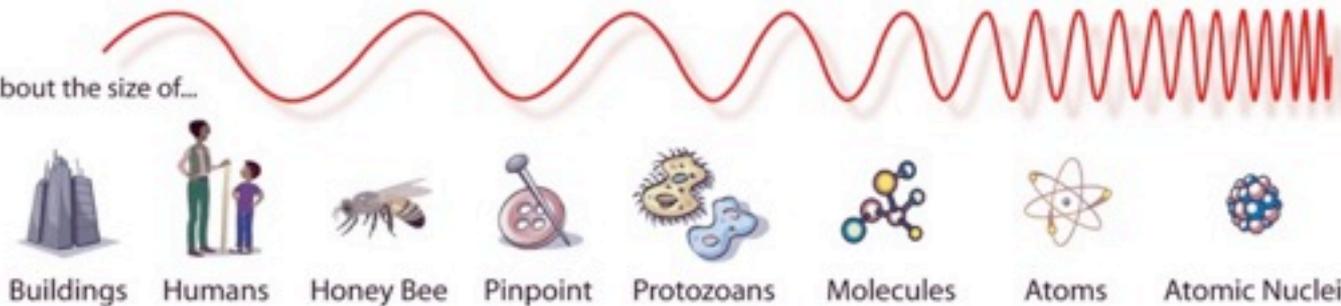
Penetrates Earth Atmosphere?



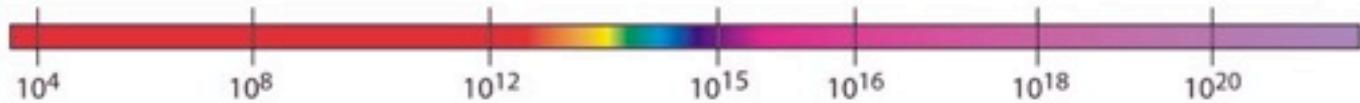
Wavelength (meters)



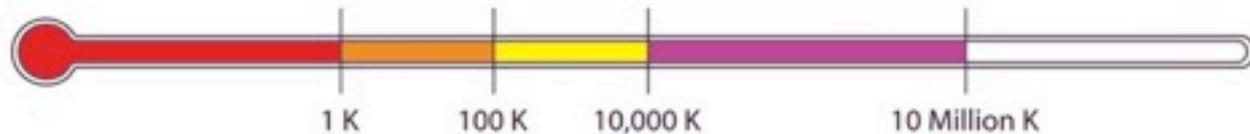
About the size of...



Frequency (Hz)

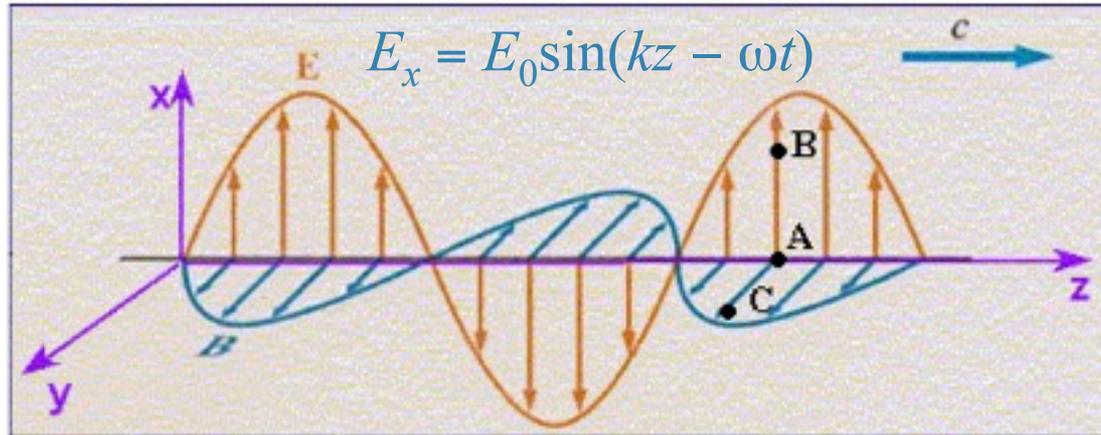


Temperature of bodies emitting the wavelength (K)



CheckPoint 6

6) An electromagnetic plane-wave is traveling in the $+z$ direction. The illustration below shows this wave at some instant in time. Points A, B, and C have the same z coordinate.



Compare the magnitudes of the electric field at points A and B.

$E_a < E_b$

$E_a = E_b$

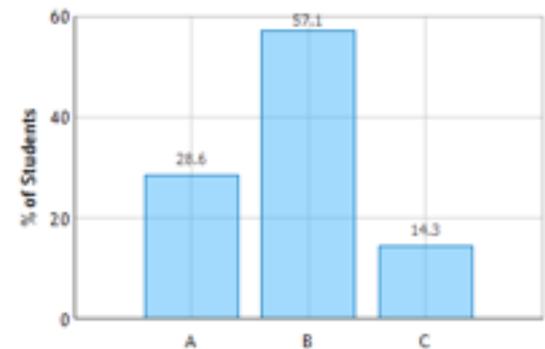
$E_a > E_b$

$$E = E_0 \sin(kz - \omega t):$$

E depends only on z coordinate for constant t .

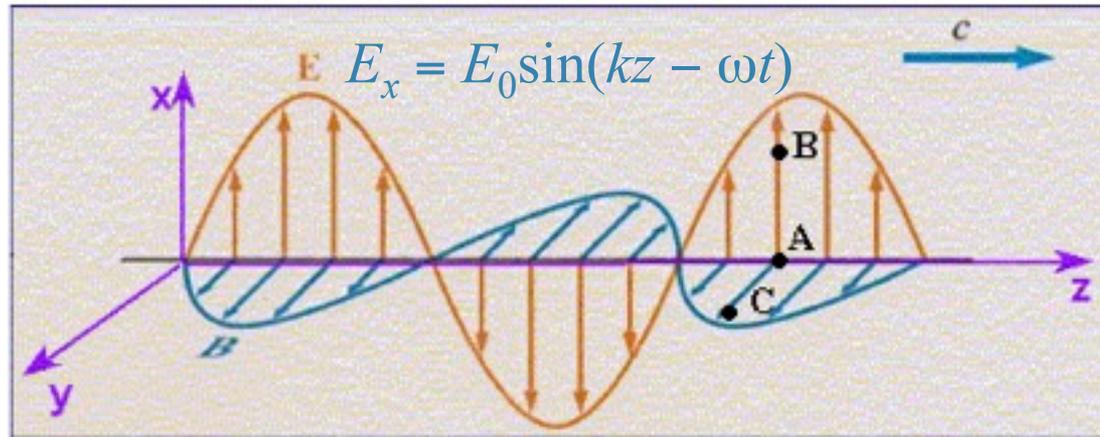
z coordinate is same for A, B, C.

Electromagnetic Waves: Question 1 (N = 21)



CheckPoint 7

An electromagnetic plane-wave is traveling in the $+z$ direction. The illustration below shows this wave at some instant in time. Points A, B, and C have the same z coordinate.



Compare the magnitudes of the electric field at points A and C.

$E_a < E_c$

$E_a = E_c$

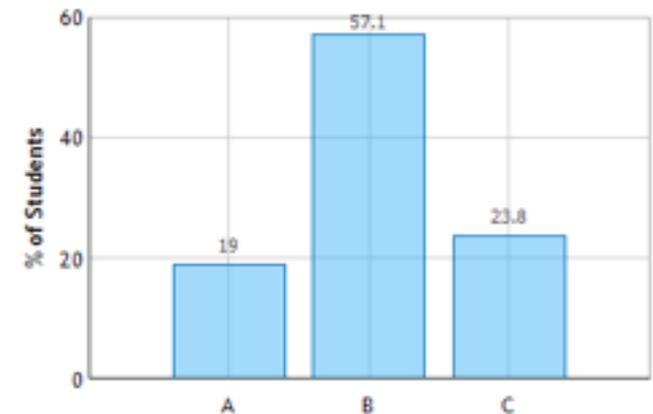
$E_a > E_c$

$$E = E_0 \sin(kz - \omega t):$$

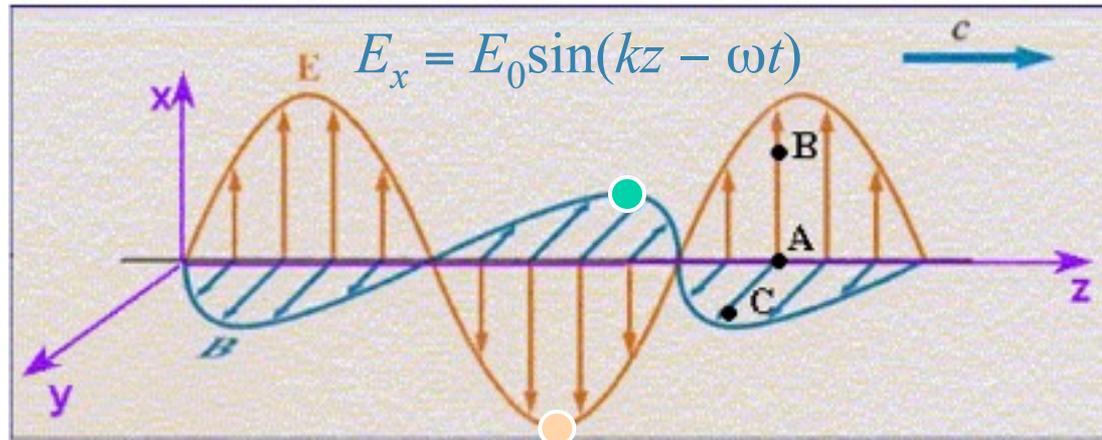
E depends only on z coordinate for constant t .

z coordinate is same for A, B, C.

Electromagnetic Waves: Question 2 (N = 21)



Clicker Question



Consider a point (x,y,z) at time t when E_x is negative and has its maximum value.

At (x,y,z) at time t , what is B_y ?

- A) B_y is positive and has its maximum value
- B) B_y is negative and has its maximum value
- C) B_y is zero
- D) We do not have enough information