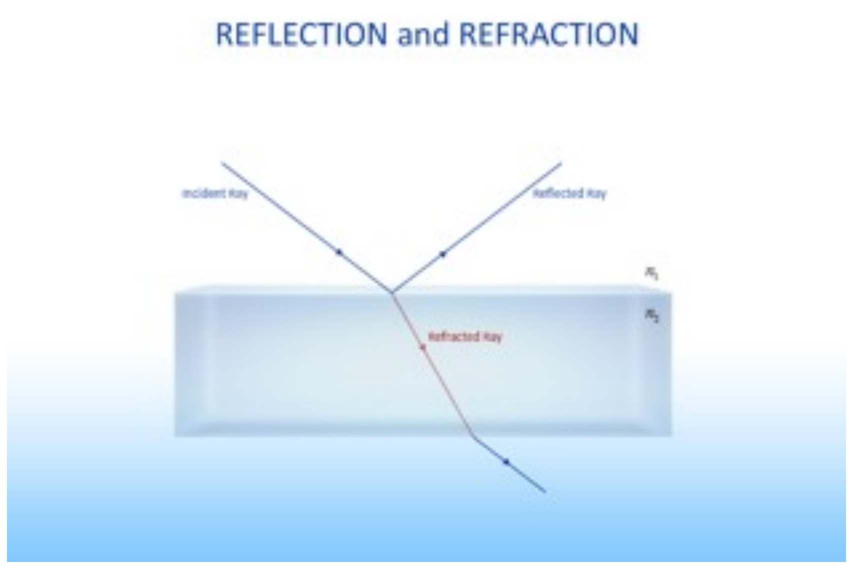
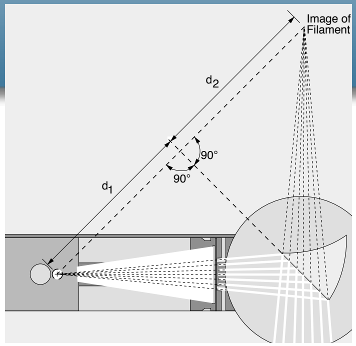
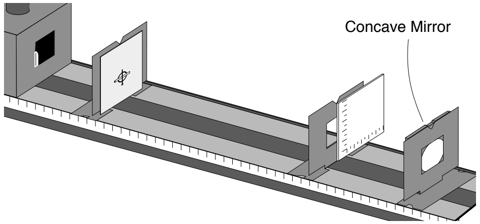
Optics
Lecture 25







Unit 29: One day only

Suggested activities:

*Activity 29-5: Ray Tracing to Locate the Virtual Image

*Activity 29-7: Forming an Image with a Concave Mirror

No grades for this unit, attendance only.
We'll move on to Unit 30 on Wed

Unit 29 is not on the Midterm, but may be on the Final Exam.

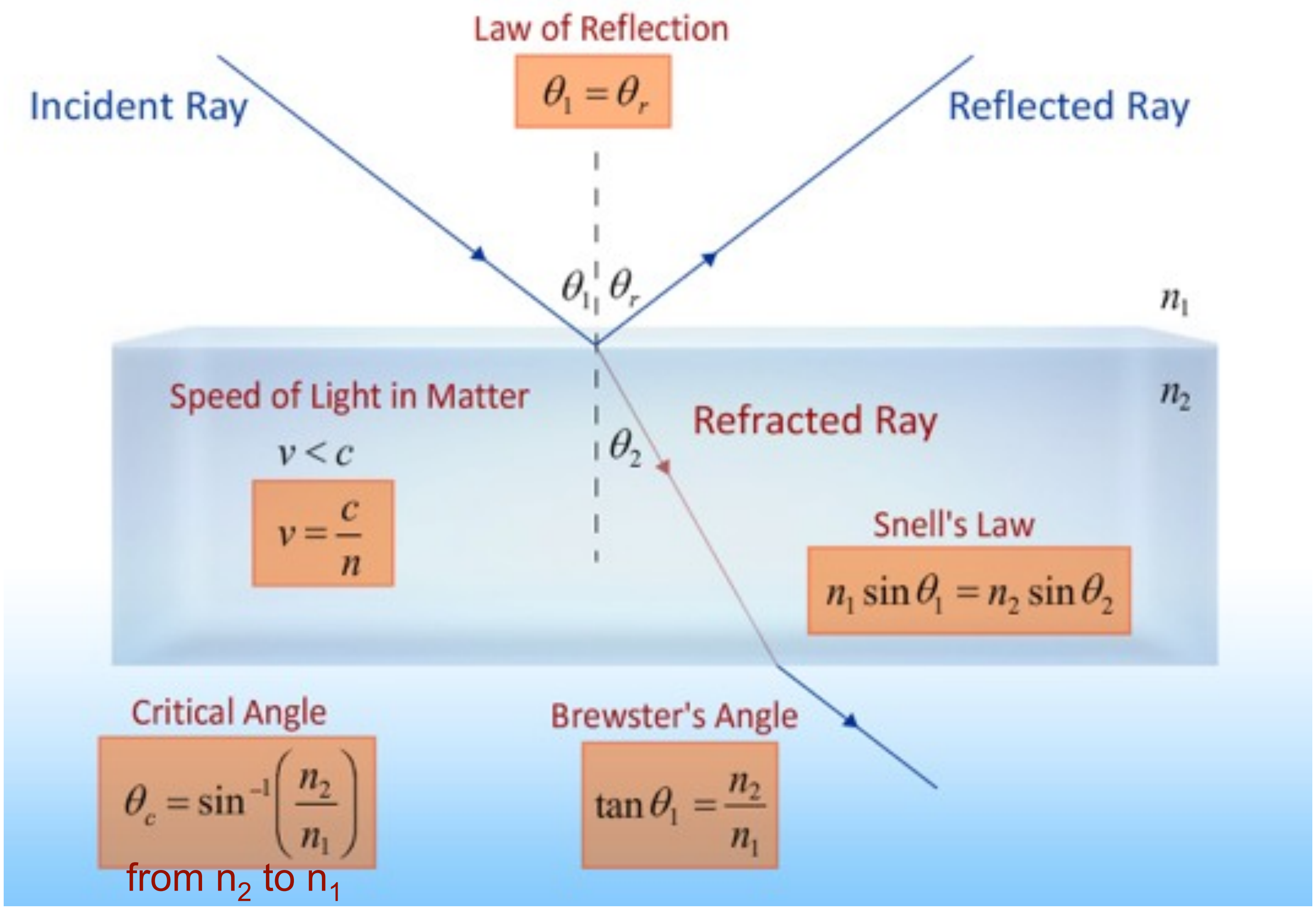
Geometrical Optics

Father of Geometrical Optics

- “Al Hazen”: Abū ʿAlī al-Ḥasan ibn al-Ḥasan ibn al-Haytham (Arabic : ابن الهيثم)
- Kitab al-Manazir كتاب المناظر (Book of Optics), written from 1011 to 1021
- Translated to latin in 1200’s.
- Electromagnetic and Quantum Theories of optics are consistent with geometrical optics in its domain of relevance.

Picture of how Archimedes burnt Roman ships using parabolic mirrors. From the 1572 Latin edition.

Let's Start with a Summary:



The speed of light in a medium is slower than in empty space:

Speed of Light

$$v = \frac{1}{\sqrt{\mu\varepsilon}} < \frac{1}{\sqrt{\mu_o\varepsilon_o}}$$

Index of Refraction

$$n \equiv \frac{c}{v} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_o\varepsilon_o}} \approx \sqrt{\frac{\varepsilon}{\varepsilon_o}} \approx \sqrt{\kappa}$$

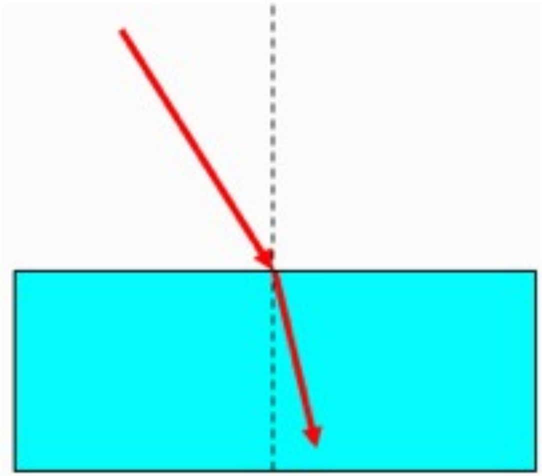
Examples for Visible Light

$n_{air} = 1.0$ $\quad n_{glass} = 1.5$ $\quad n_{diamond} = 2.4$

$$v_{\text{medium}} = c \,/\, n_{\text{medium}}$$

CheckPoint 2

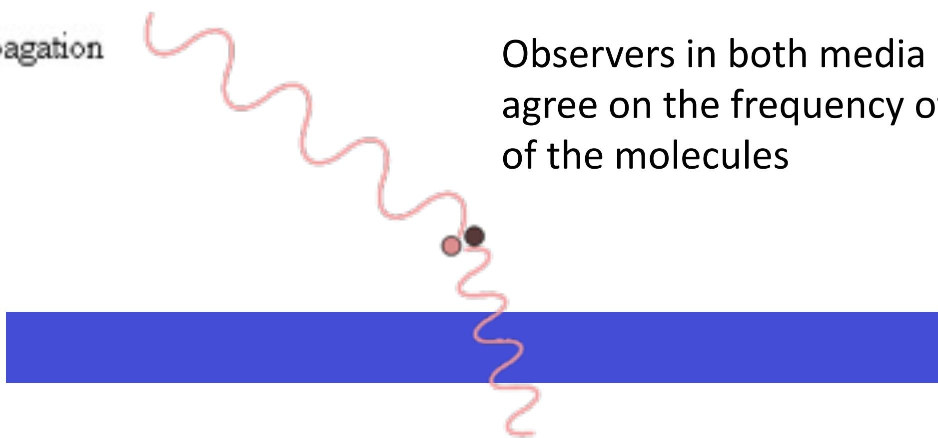
2) A ray of light passes from air into water with an angle of incidence of 30 degrees.



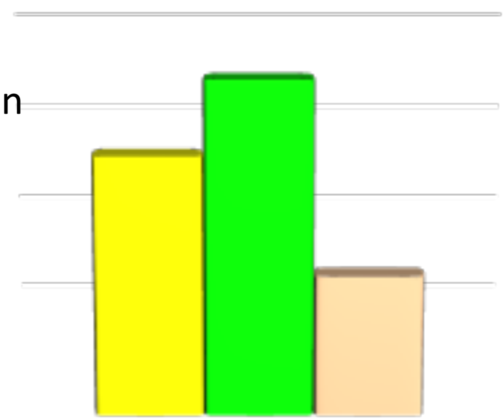
Which of the following quantities does not change as the light enters the water. Mark all correct answers.

A ☐ wavelength
B ☐ frequency
C ☐ speed of propagation

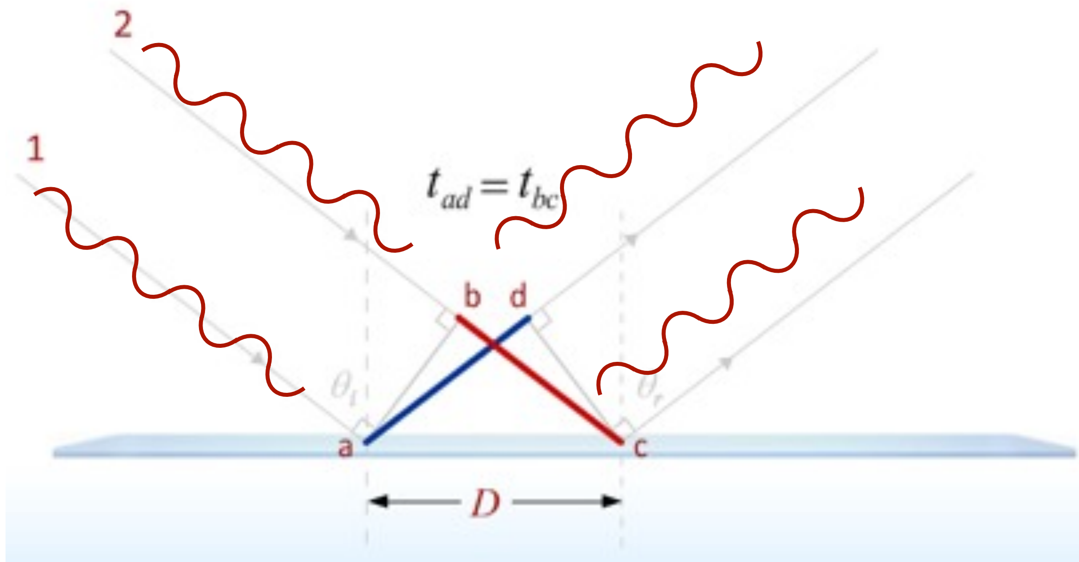
What about the wave must be the same on either side?

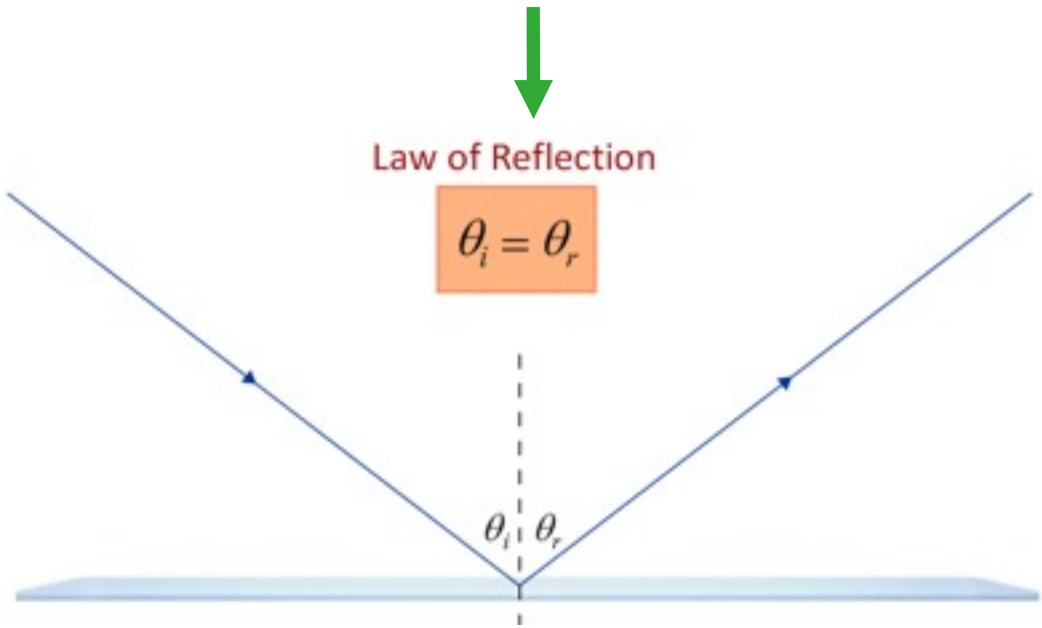


Observers in both media must agree on the frequency of vibration of the molecules



Reflection


2
1
$t_{ad} = t_{bc}$
b
d
θ_i
θ_r
a
c
D


Law of Reflection
$\theta_i = \theta_r$
θ_i
θ_r

Mirrors in Movies

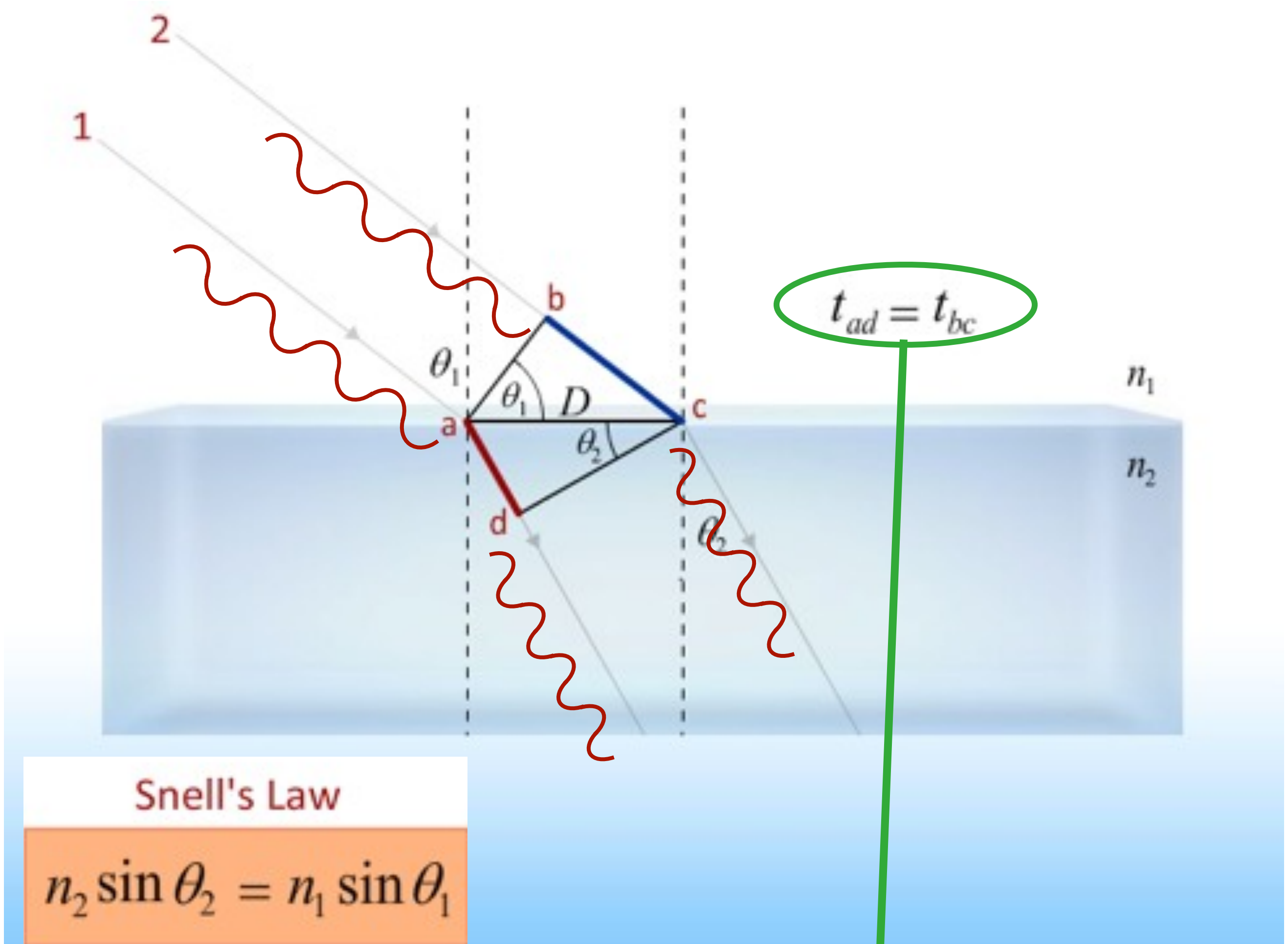
Duck Soup (1933)

The Lady from Shanghai (1947)

Enter the Dragon (1973)

and many more

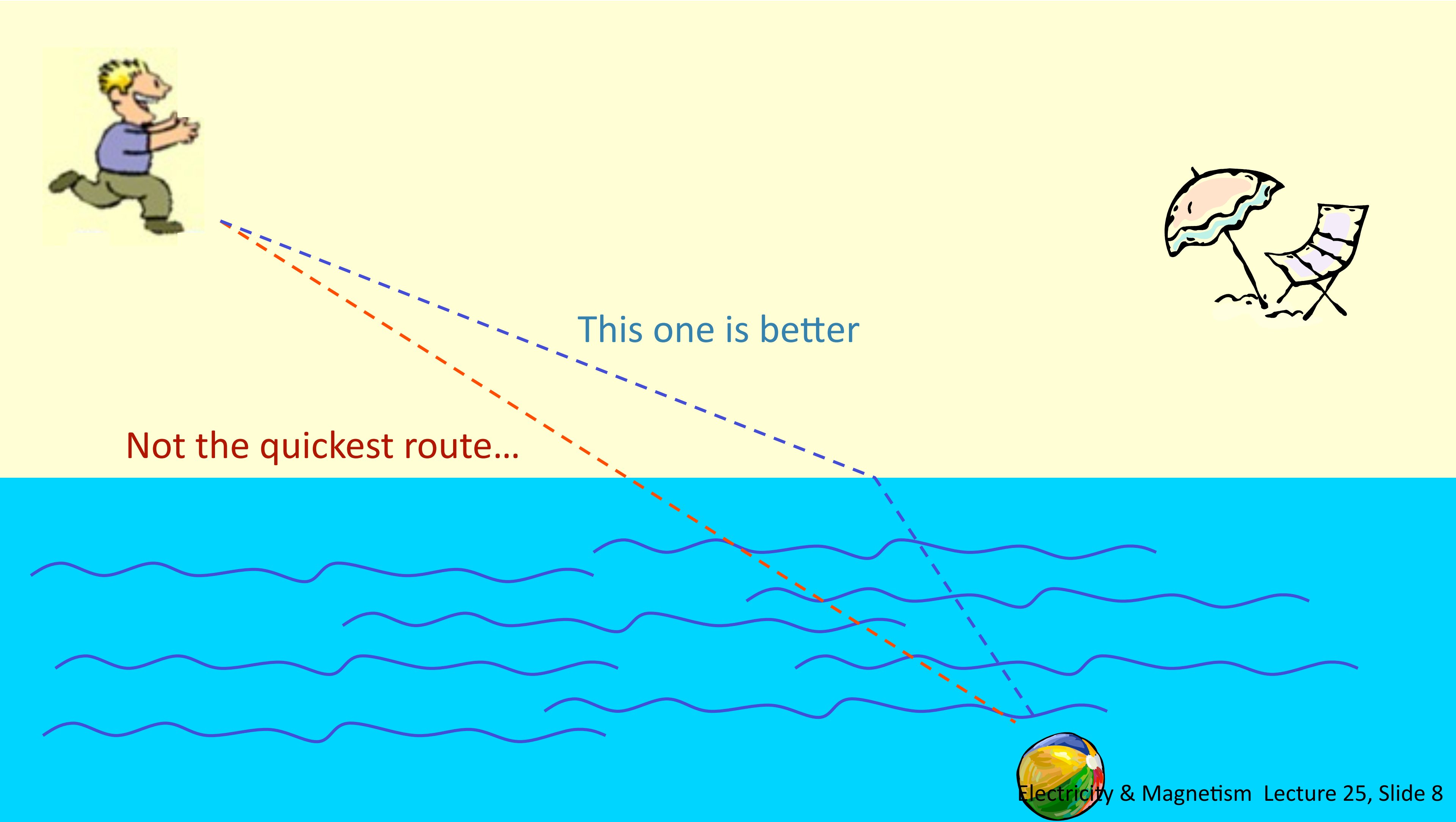
Refraction: Snell's Law



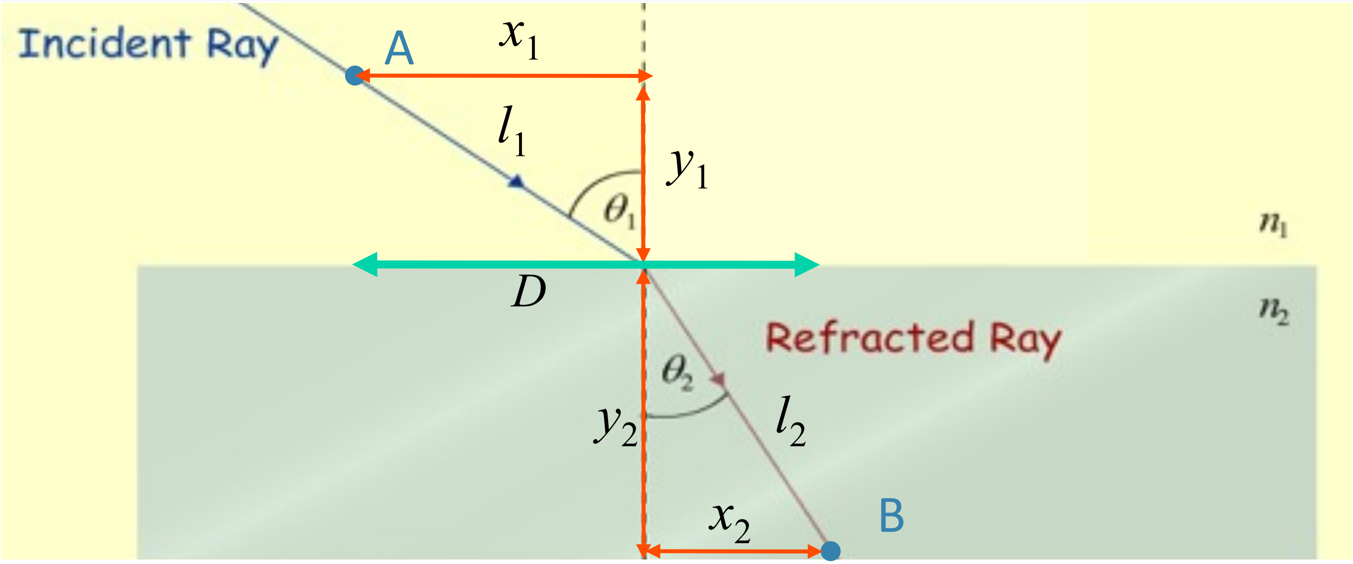
$$\frac{D\sin\theta_2}{c/n_2} = \frac{D\sin\theta_1}{c/n_1} \rightarrow n_2 \sin\theta_2 = n_1 \sin\theta_1$$

Think of a Day at the Beach

What's the fastest path to the ball knowing
you can run faster than you can swim?



Same Principle works for Light!



Time from A to B :

$$t = \frac{l_1}{v_1} + \frac{l_2}{v_2} = \frac{\sqrt{x_1^2 + y_1^2}}{v_1} + \frac{\sqrt{x_2^2 + y_2^2}}{v_2}$$

To find minimum time, differentiate t wrt x_1 and set $= 0$

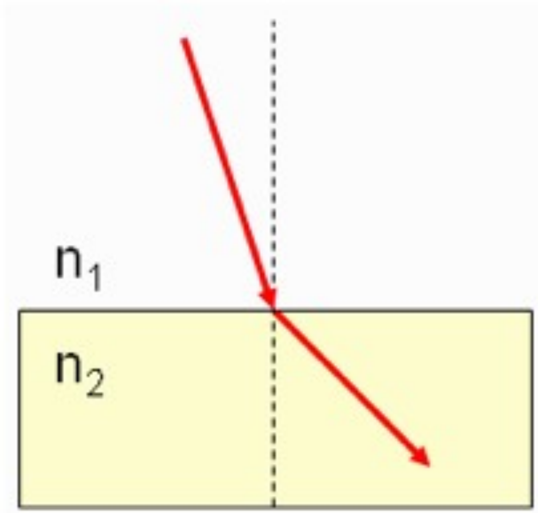
$$\frac{dt}{dx_1} = \frac{x_1}{v_1\sqrt{x_1^2 + y_1^2}} + \frac{x_2}{v_2\sqrt{x_2^2 + y_2^2}}\frac{dx_2}{dx_1}$$

How is x_2 related to x_1? $x_2 = D - x_1$ → $\frac{dx_2}{dx_1} = -1$

Setting $dt/dx_1 = 0$ → $\frac{x_1}{v_1 l_1} - \frac{x_2}{v_2 l_2} = 0$ → $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$ $\xrightarrow{v = c/n}$ $n_1 \sin\theta_1 = n_2 \sin\theta_2$

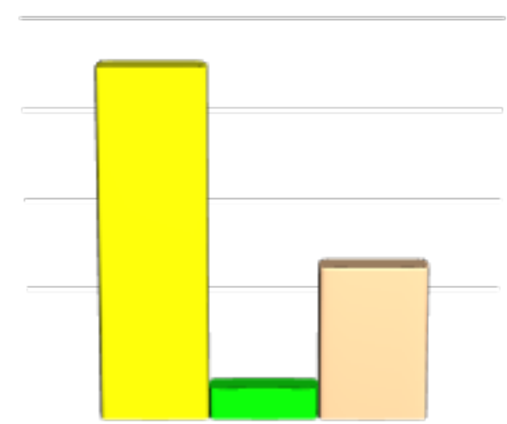
CheckPoint 6

The path of light is bent as passes from medium 1 to medium 2.



Compare the indexes of refraction in the two mediums.

- $n_1 > n_2$
- $n_1 = n_2$
- $n_1 < n_2$

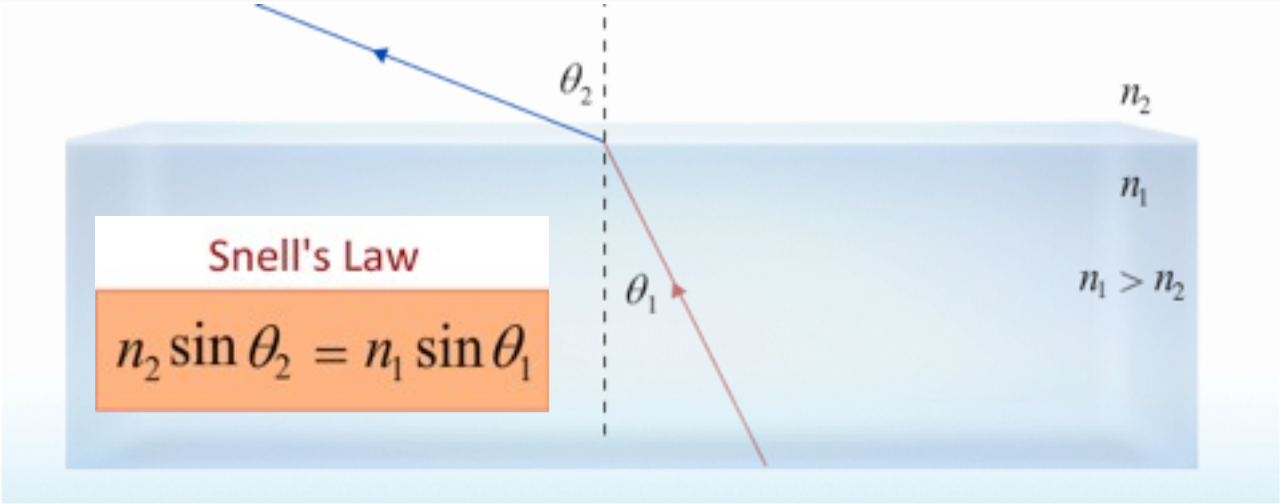


Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

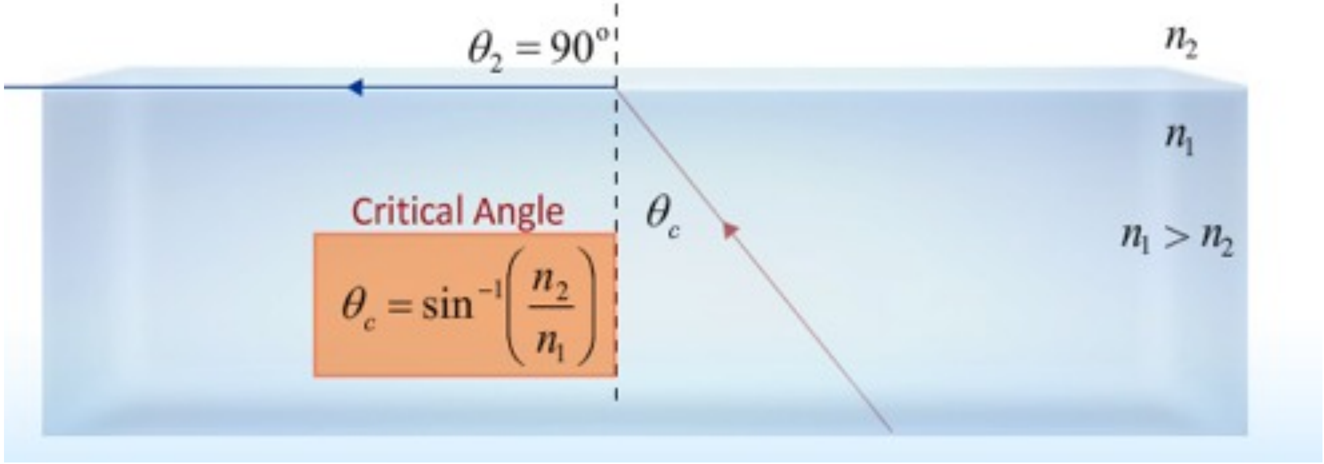
n decreases Þ θ increases

Total Internal Reflection



NOTE: $n_1 > n_2$ implies $\theta_2 > \theta_1$

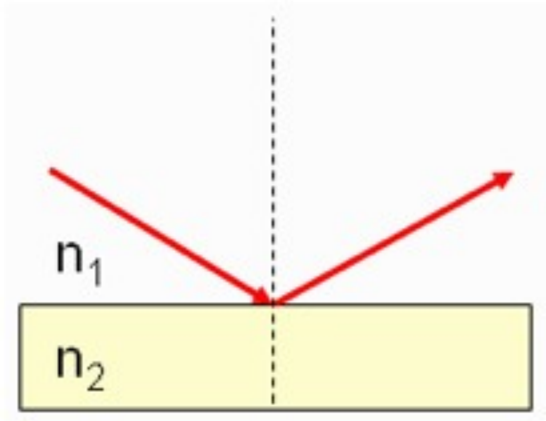
BUT: θ_2 has max value = 90^o!



$\theta_1 > \theta_c$ → Total Internal Reflection

CheckPoint 8

A light ray travels in a medium with n_1 and completely reflects from the surface of a medium with n_2.



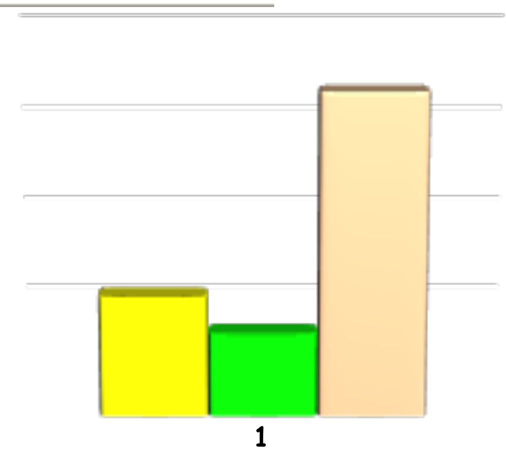
The critical angle depends on:

- ○ n_1 only
- ○ n_2 only
- ○ both n_1 and n_2

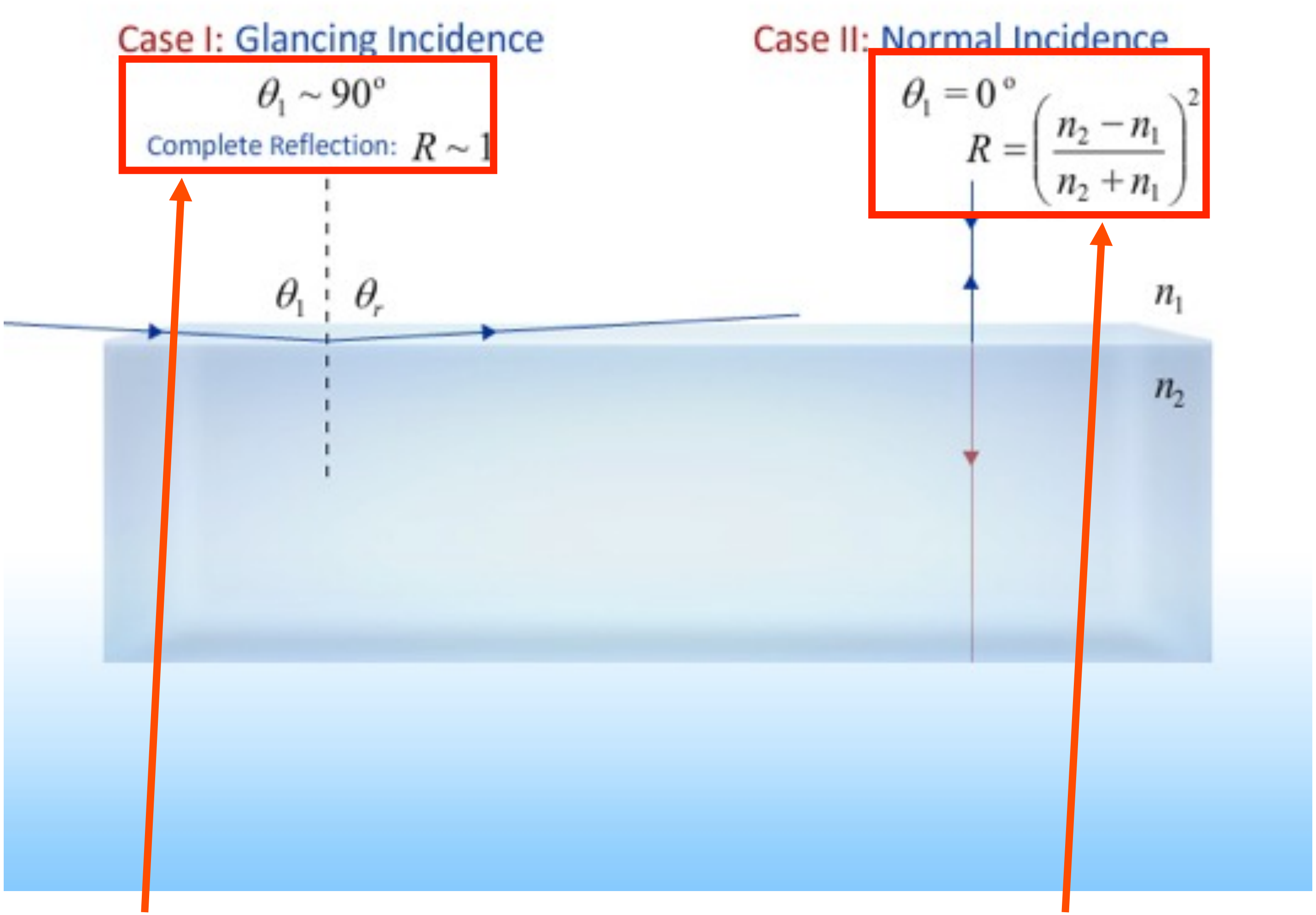
Critical Angle

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

θ_c clearly depends on both n_2 and n_1



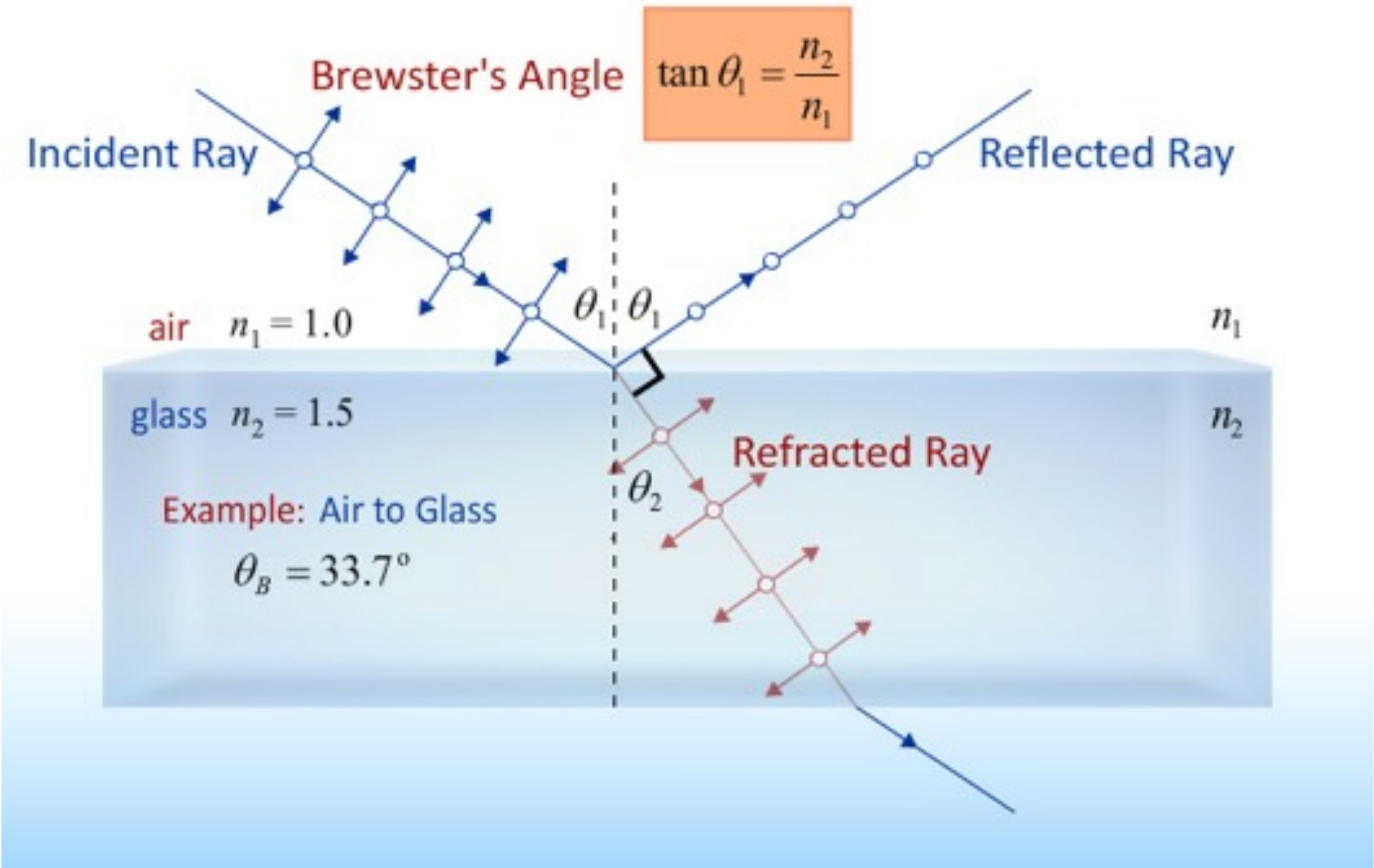
Intensity



Anything looks like a mirror if light is just glancing off it.

If two materials have the same n then its hard to tell them apart.

Polarization

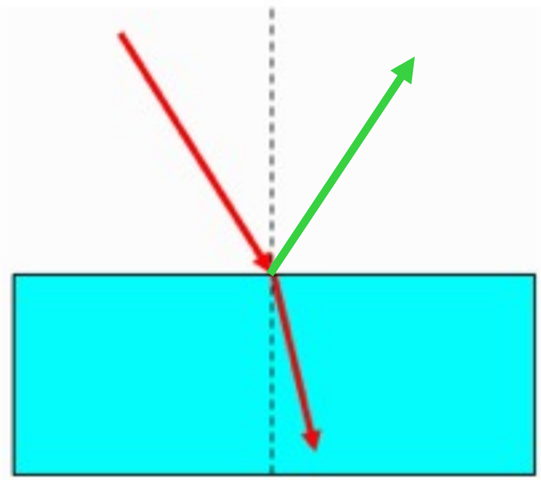


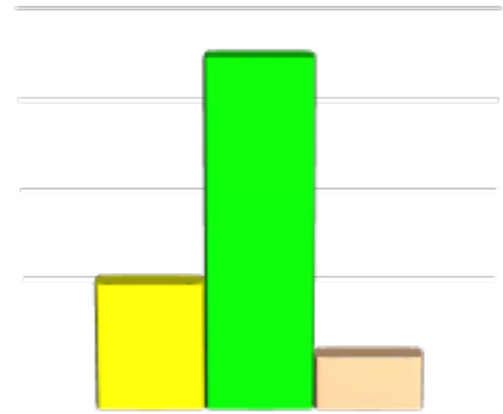
$$\theta_1 + \theta_2 = 90^\circ \quad \longrightarrow \quad \sin\theta_2 = \sin(90^\circ - \theta_1) = \cos\theta_1$$

Snell's Law: $n_2 \sin\theta_2 = n_2 \cos\theta_1 = n_1 \sin\theta_1 \quad \longrightarrow \quad \tan\theta_1 = \frac{n_2}{n_1}$

CheckPoint 4

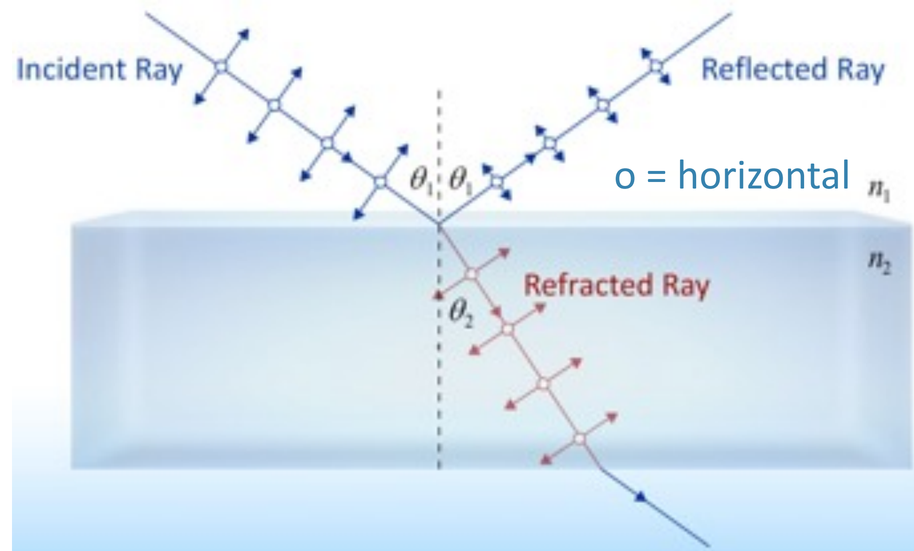
A ray of light passes from air into water with an angle of incidence of 30 degrees.





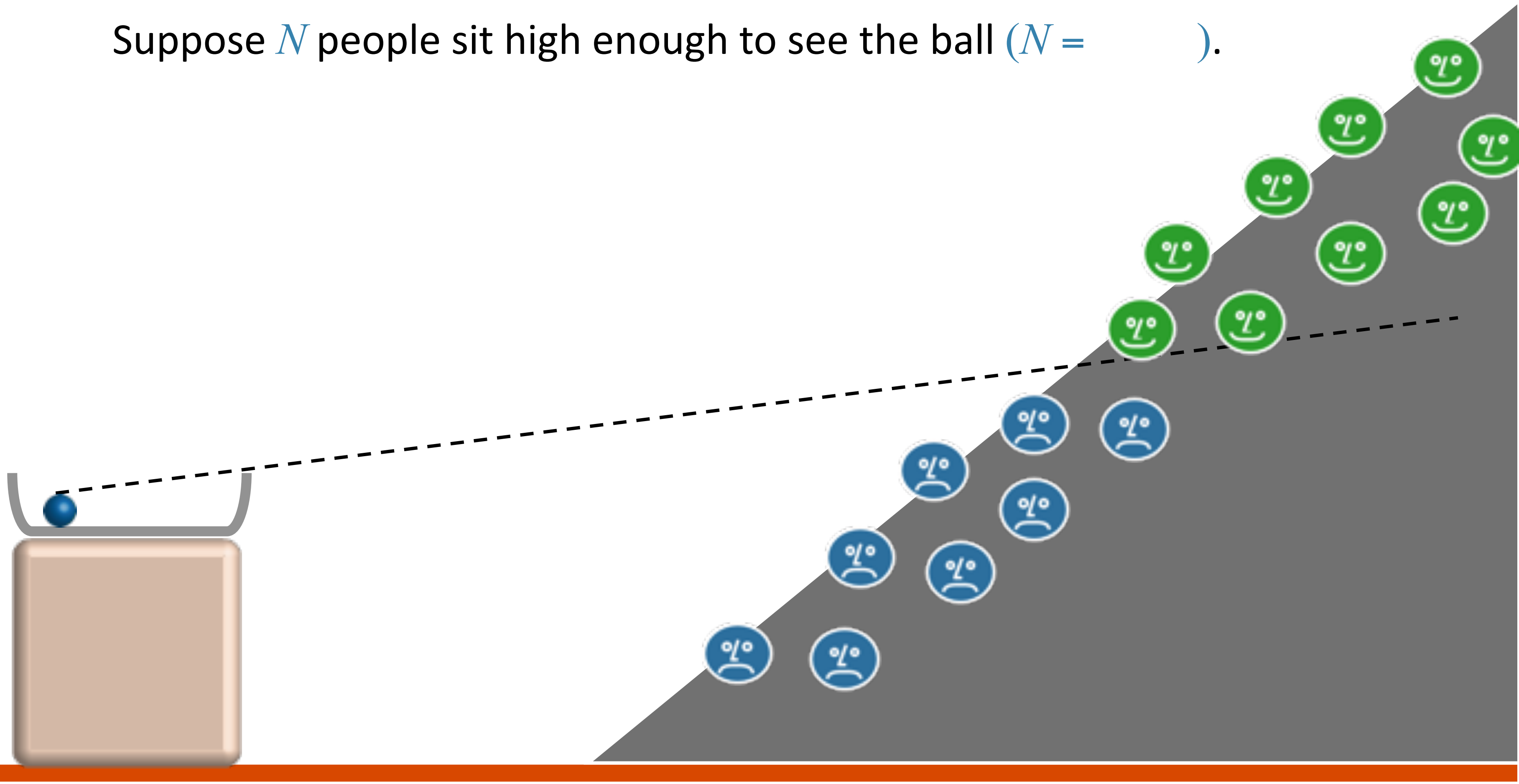
Some of the light also reflects off the surface of the water. If the incident light is initially unpolarized, the reflected light will be

- ○ unpolarized
- ○ somewhat horizontally polarized
- ○ somewhat vertically polarized



A ball sits in the bottom of an otherwise empty tub at the front of the room.

Suppose N people sit high enough to see the ball ($N =$).



A ball sits in the bottom of an otherwise empty tub at the front of the room.

Suppose N people sit high enough to see the ball ($N =$).

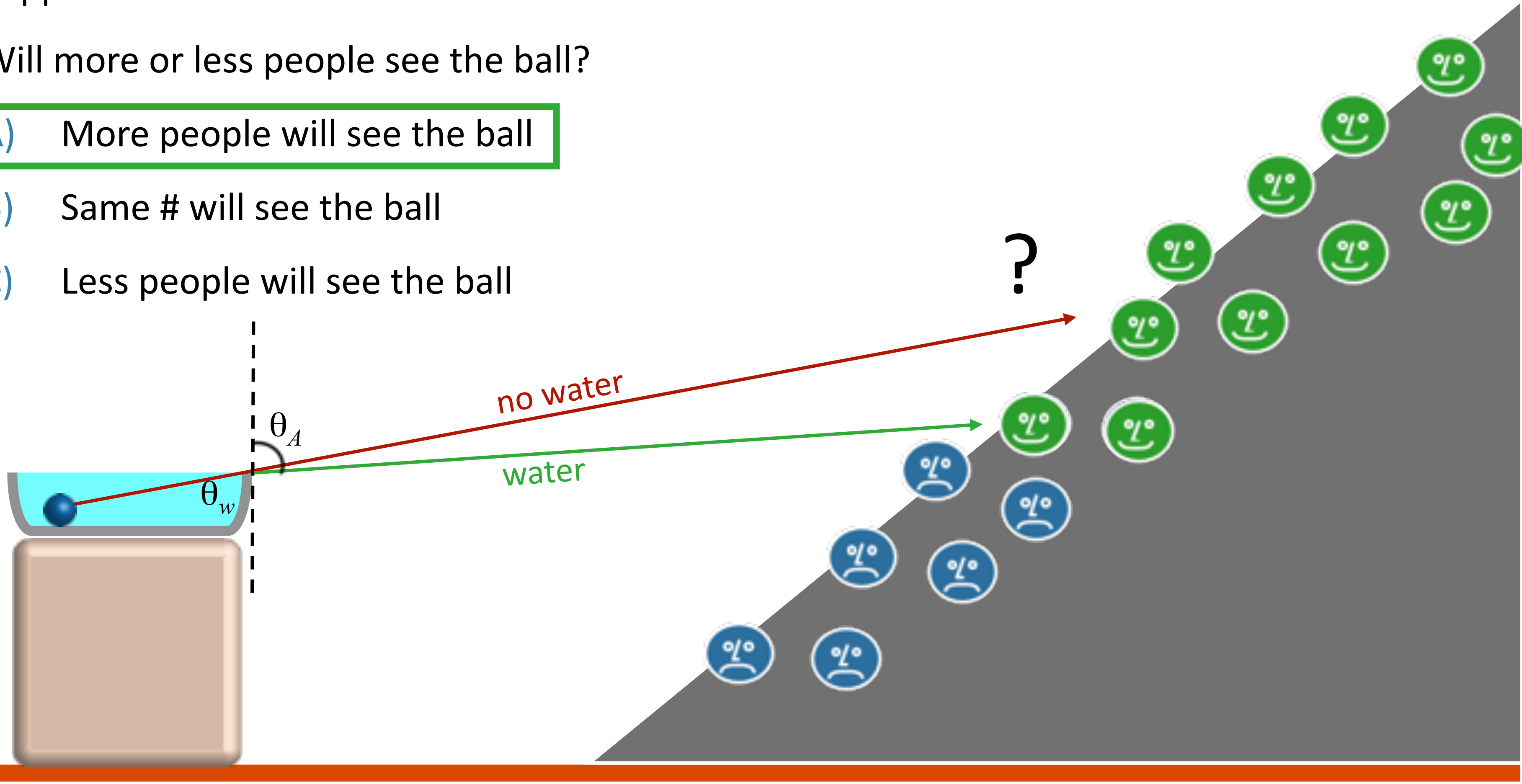
Suppose I fill the tub with water but the ball doesn't move.

Will more or less people see the ball?

A) More people will see the ball

B) Same # will see the ball

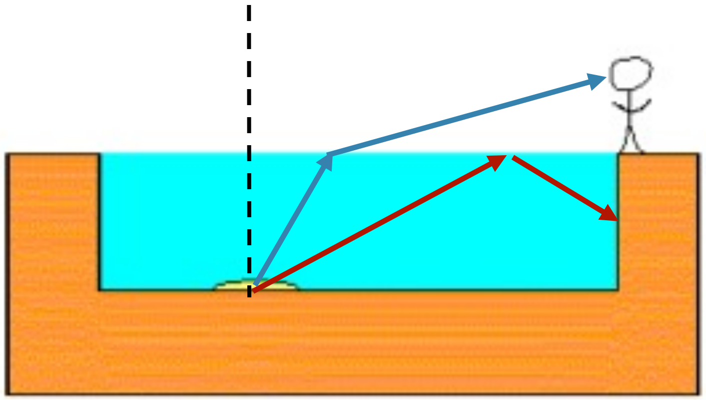
C) Less people will see the ball



Snell's Law: ray bent away from normal going from water to air

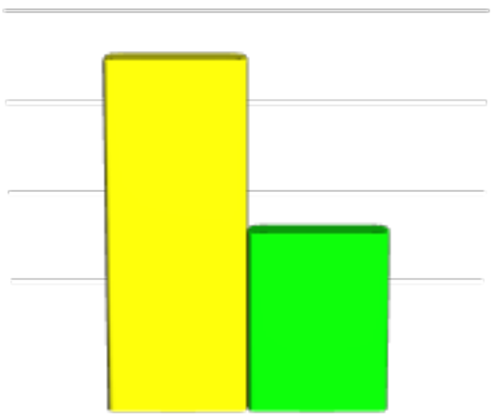
CheckPoint 10

A light is shining at the bottom of a swimming pool (shown in yellow in the figure). A person is standing at the edge of the pool.



Can the person standing on the edge of the pool be prevented from seeing the light by total internal reflection at the water-air surface?

○ yes ○ no

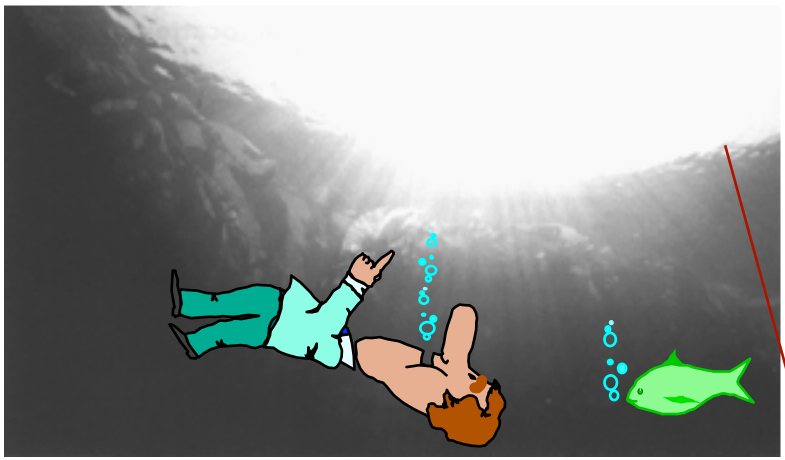


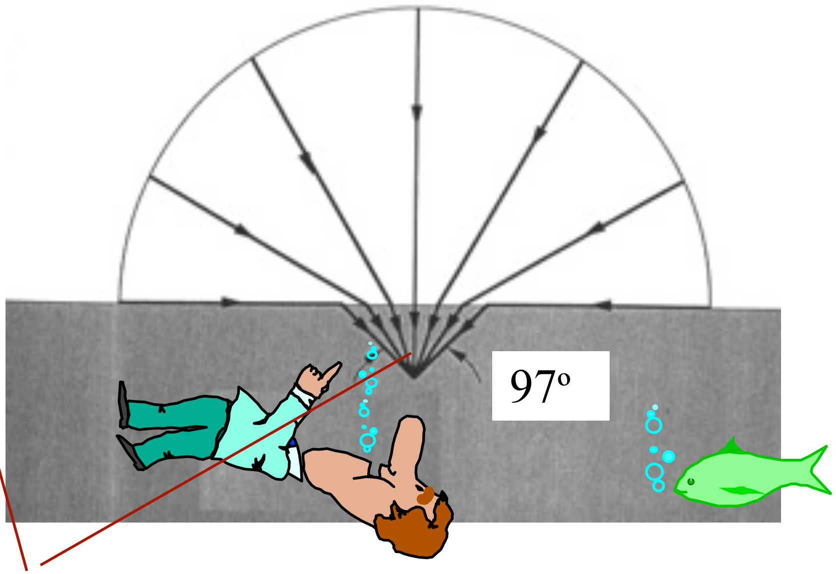
The light would go out in all directions, so only some of it would be internally reflected. The person would see the light that escaped after being refracted.

Draw some rays

Example: Refraction at Water/Air Interface

Diver's illusion





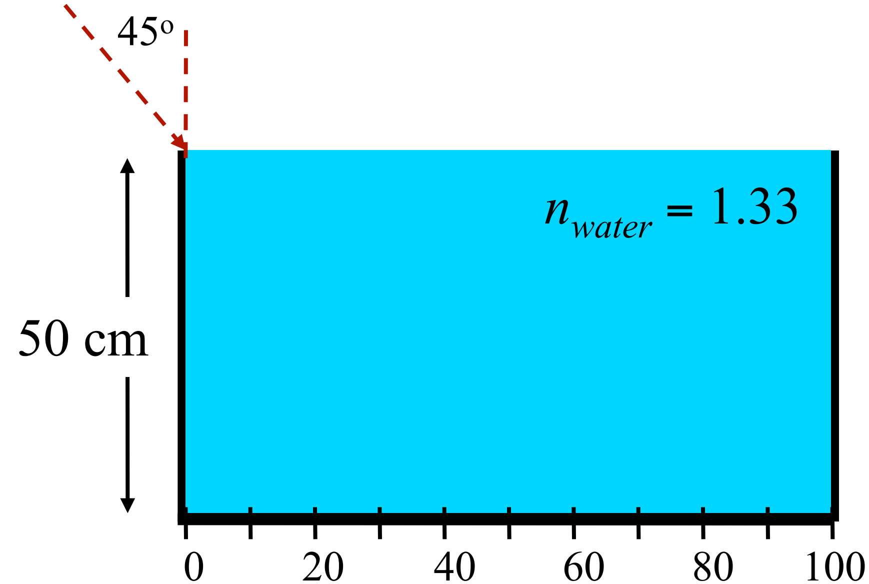
Diver sees all of horizon
refracted into a 97° cone

$$\theta_1 = 90^\circ \rightarrow \sin\theta_2 = \frac{n_1}{n_2}\sin 90^\circ = \frac{n_1}{n_2} = \frac{1}{1.33} \rightarrow \theta_2 = 48.5^\circ$$

Exercise

A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of 45° relative to vertical along a line that skims the top edge of the tank.

What is the smallest number on the ruler that you can see?



Conceptual Analysis:

- Light is refracted at the surface of the water

Strategy:

- Determine the angle of refraction in the water and extrapolate this to the bottom of the tank.

Exercise

A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of 45° relative to vertical along a line that skims the top edge of the tank.

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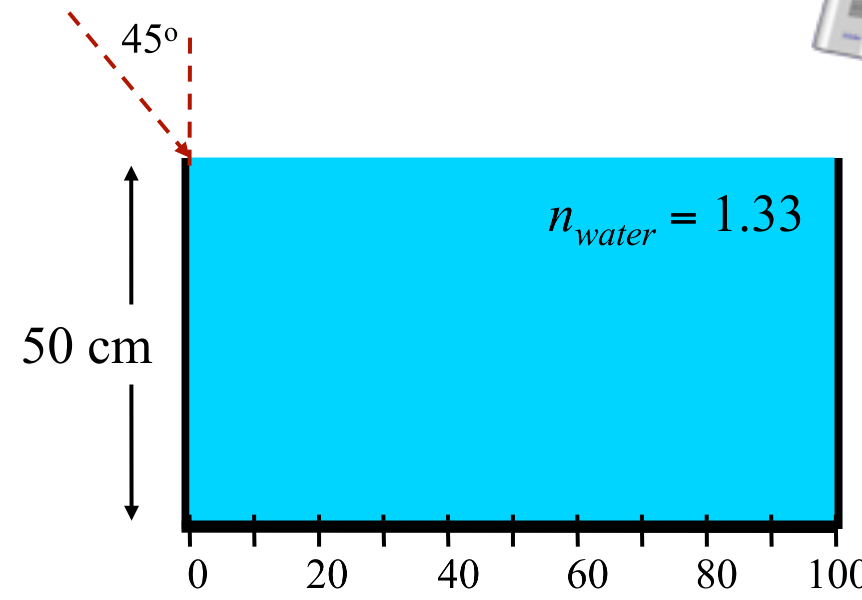


If you shine a laser into the tank at an angle of 45°, what is the refracted angle θ_R in the water ?

A) $\theta_R = 28.3°$ B) $\boxed{\theta_R = 32.1°}$ C) $\theta_R = 38.7°$

Snell's Law: $n_{air} \sin(45) = n_{water} \sin(\theta_R)$

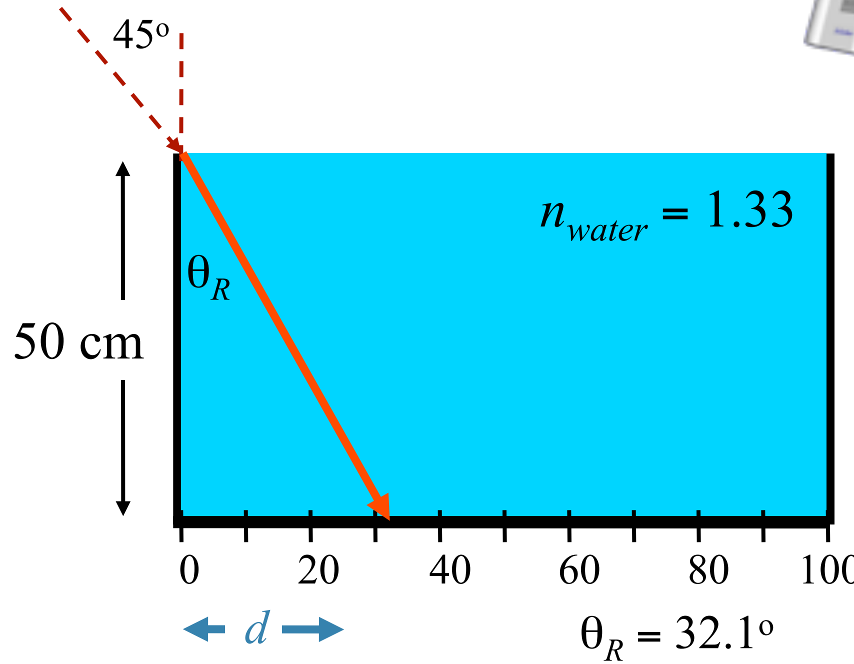
$$\rightarrow \sin(\theta_R) = n_{air} \sin(45°)/n_{water} = 0.532$$

$$\rightarrow \theta_R = \sin^{-1}(0.532) = 32.1°$$

Exercise

A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of 45° relative to vertical along a line that skims the top edge of the tank.

What is the smallest number on the ruler that you can see?



What number on the ruler does the laser beam hit?

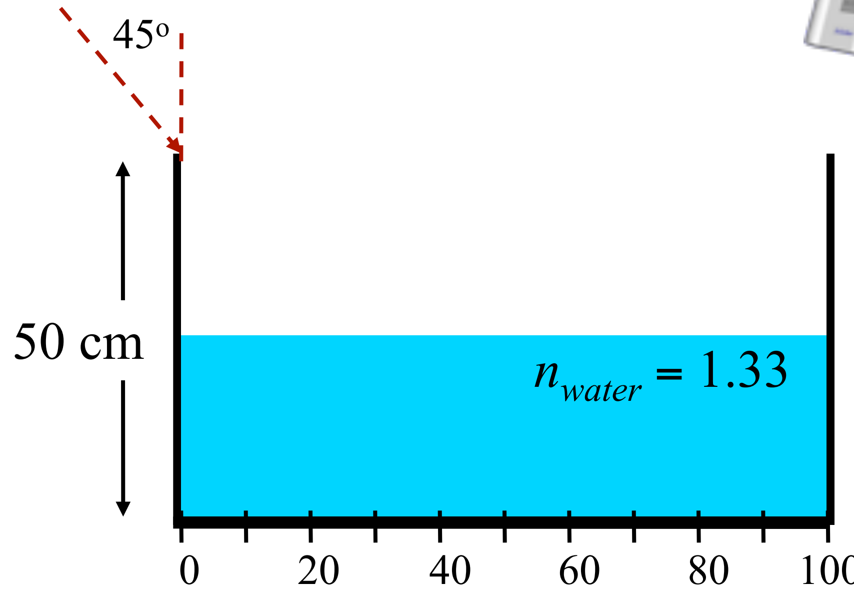
A) 31.4 cm B) 37.6 cm C) 44.1 cm

$\tan(\theta_R) = d/50$

$d = \tan(32.1°) \times 50\text{cm} = 31.4\text{cm}$

Follow-Up

A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of 45º relative to vertical along a line that skims the top edge of the tank.



If the tank were half full of water, what number would the laser hit?
(When full, it hit at 31.4 cm)

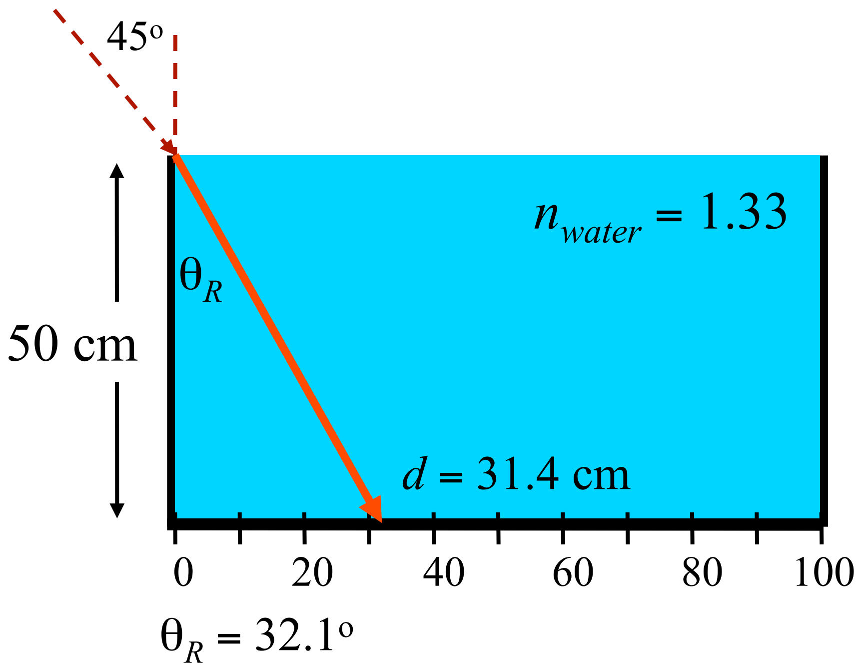
A) 25 cm

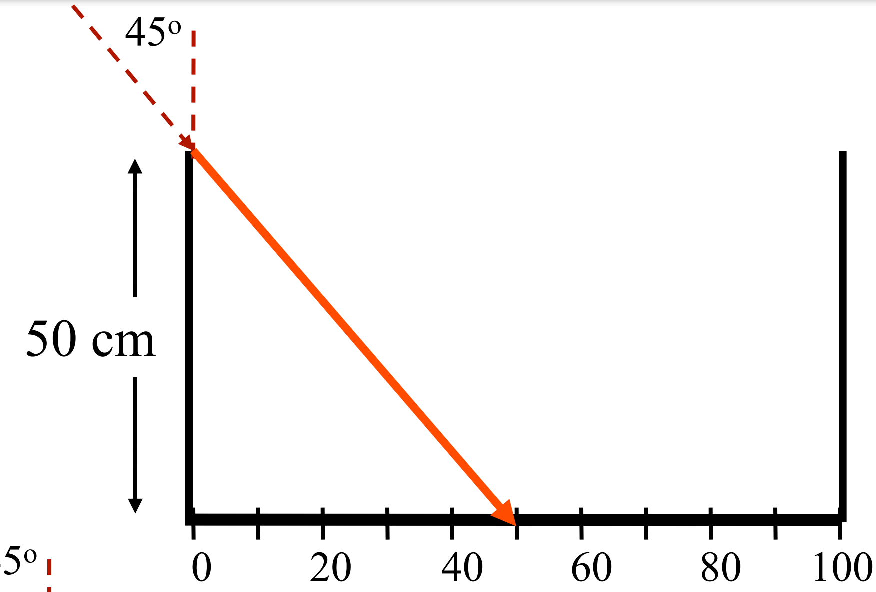
B) 31.4 cm

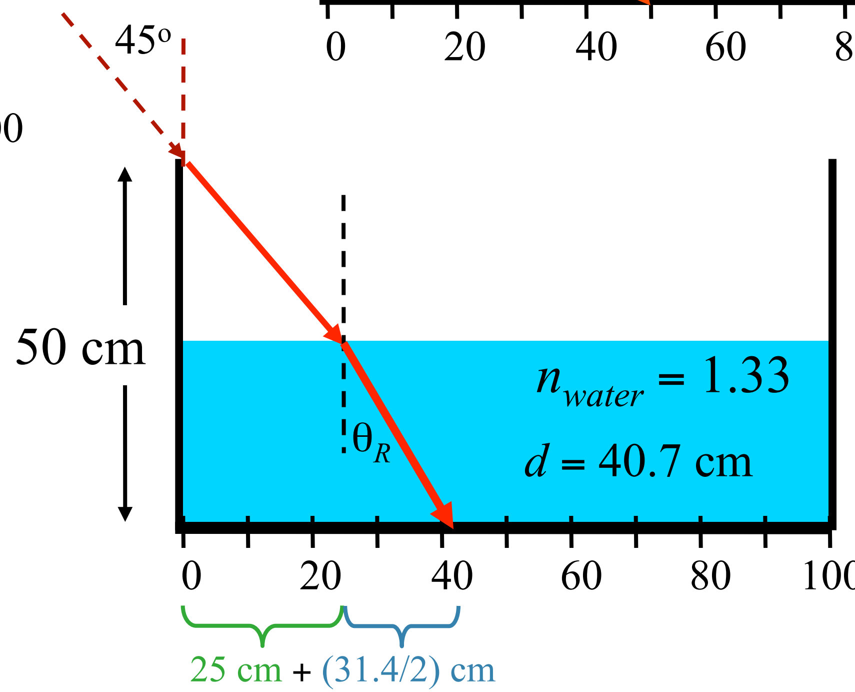
C) 32.0 cm

D) 40.7 cm

E) 44.2 cm

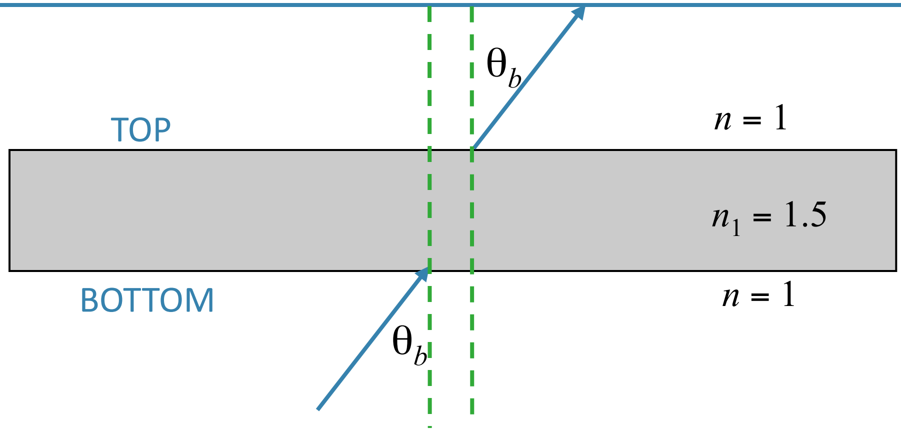

45°
n_{water} = 1.33
θ_R
50 cm
d = 31.4 cm
0 20 40 60 80 100
θ_R = 32.1°


45°
50 cm
0 20 40 60 80 100


45°
50 cm
n_{water} = 1.33
θ_R
d = 40.7 cm
0 20 40 60 80 100
25 cm + (31.4/2) cm

More Practice

A monochromatic ray enters a slab with $n_1 = 1.5$ at an angle θ_b as shown.



A) Total internal reflection at the top occurs for all angles θ_b, such that $\sin\theta_b < 2/3$

B) Total internal reflection at the top occurs for all angles θ_b, such that $\sin\theta_b > 2/3$

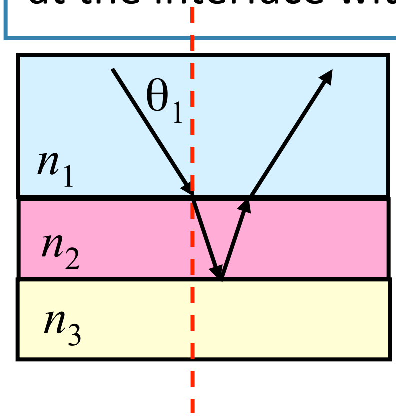
C) There is no angle θ_b $(0 < \theta_b < 90^o)$ such that total internal reflection occurs at top.

Snell's law:

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ → $n \sin\theta$ is "conserved" → Ray exits to air with same angle as it entered!

Follow-Up

A ray of light moves through a medium with index of refraction n_1 and is incident upon a second material (n_2) at angle θ_1 as shown. This ray is then totally reflected at the interface with a third material (n_3). Which statement must be true?

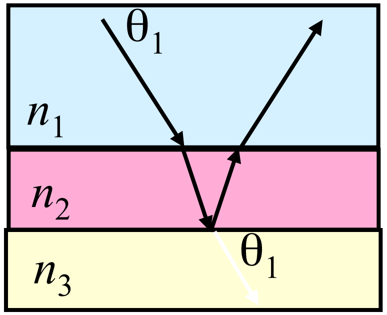


A) $n_3 < n_1$

B) $n_1 < n_3 \leq n_2$

C) $n_3 \geq n_2$

If $n_1 = n_3$



Want larger angle of refraction in n_3 → $n_3 < n_1$