Waveplate design

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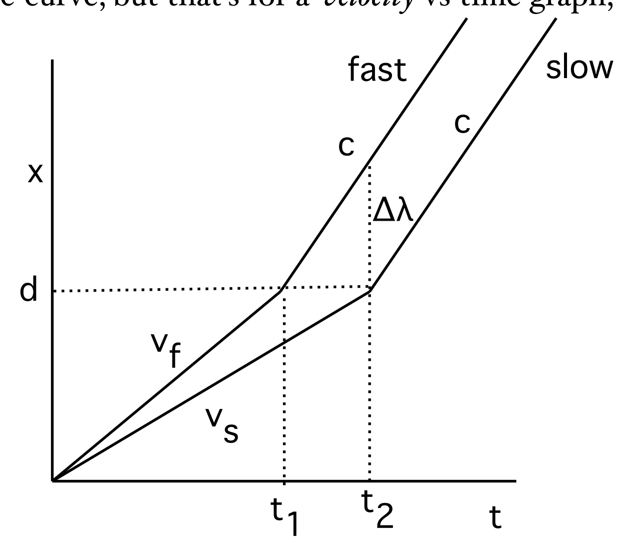
After our discussion on the wave plates today I was a little dissatisfied because I think things were a little unclear. Kirill was right. You can think of this as just two cars going over a bad section of road where both cars have the same normal speed, c, but have two speeds on the rough section, v_f and v_s.

The math becomes clear if you just draw a position vs time graph of both cars. The length of the bad section of road is d. What we want is both cars enter the bad section together but have a definite separation distance between them after they both reach the good section of the road again. That separation distance is $\Delta\lambda$.

(I made the mistake of thinking that the distance is represented by the area under the curve, but that's for a *velocity* vs time graph, not *distance* vs time graph.)



You can see from the graph that the separation of the cars after they get back on the good road is

$$\Delta\lambda = c(t_2 - t_1)$$

$$t_1 = \frac{d}{v_f} = \frac{n_f d}{c}$$

$$t_2 = \frac{d}{v_s} = \frac{n_s d}{c}$$

Therefore,

$$\Delta\lambda = c\left(\frac{n_s d}{c} - \frac{n_f d}{c}\right) = d(n_s - n_f)$$

Solving for the thickness

$$d = \frac{\Delta\lambda}{n_s - n_f}$$

If we want a quarter wave plate, then $\Delta\lambda = \lambda/4$ where λ is the wavelength in vacuum.

$$d = \frac{\lambda}{4(n_s - n_f)}$$

In the case that $\lambda = 633$ nm, $n_f = 1.48$ and $n_s = 1.50$ one gets $31\,650$ nm or 31.65 µm.

The way I posed the question there was no minimum thickness, but you can add any integer N to the $1/4$ like this

$$d = \frac{\lambda(\frac{1}{4} + N)}{(n_s - n_f)}$$