

There are 10 multiple choice questions. Select the correct answer for each one and mark it on the bubble form. Each question has only one correct answer. (2 marks each)

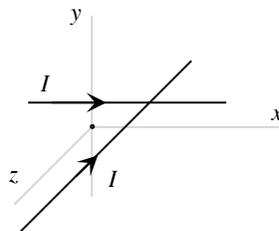
1. How can we explain that the north pole of a compass needle seems to be attracted to the north pole of the planet Earth?
- (a) The end labelled "north" of the compass needle is actually the south magnetic pole of the compass needle.
 - (b) Correct: The geographic north pole of the earth is actually the south magnetic pole of the Earth.
 - (c) Like poles of magnets attract each other.

That's the way it is.

2. A beam of electrons is sent horizontally down the axis of a tube to strike a fluorescent screen at the end of the tube. On the way, the electrons encounter a magnetic field directed vertically upward. The spot on the screen will therefore be deflected
- (a) upward.
 - (b) downward.
 - (c) to the right as seen from the electron source.
 - (d) Correct: to the left as seen from the electron source.
 - (e) not at all.

The trick here is to read carefully "as seen from the electron source". Electrons experience force in the direction of $-\vec{v} \times \vec{B}$ which is to the left when one is looking in the direction of \vec{v} .

3. Two long straight current-carrying parallel wires carry current I in each. One is parallel to the x axis and passes through the y axis at $y = 1$ m with I in the $+x$ direction. The other is parallel to the z axis, passes through the y axis at $y = -1$ m with its I in the $-z$ direction.



Which expression describes the magnetic field at the origin assuming B_0 is positive?

- (a) $B_0(\hat{i} + \hat{k})$
- (b) $B_0(-\hat{i} + \hat{k})$
- (c) $B_0(\hat{i} - \hat{j})$
- (d) $B_0(-\hat{i} - \hat{j})$
- (e) Correct, it's $-\hat{k} + \hat{i}$: none of the above or zero.

The top wire contributes a component along $-\hat{k}$ and the bottom one along $+\hat{i}$.

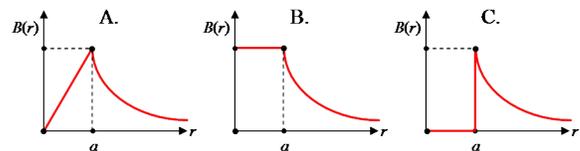
4. Consider a long wire running in the vertical direction with a rectangular loop of wire beside it as shown. Which of the following situations would result in a clockwise induced current in the loop?

- (a) A current in the long wire, directed upward, is increasing in magnitude.
- (b) With a constant current in the long wire directed upward the loop is moved toward the top of the page, parallel to the long wire.
- (c) Correct: The loop is held stationary and the long wire, while carrying a constant upward current, is moved away from the loop.



The magnetic field will be into the page and decreasing. A clockwise current would bolster the decreasing B field.

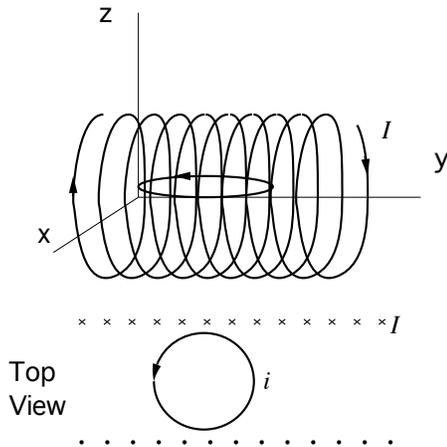
5. A long, straight wire of radius a carries a steady current I . The graph of magnetic field strength $B(r)$ as a function of perpendicular distance r from the centre of the wire is:



Answer is A. Use Ampère's law to show that the field inside the wire constantly increases. Outside the wire the magnetic field decreases as $1/r$.

The next two questions relate to the situation described below.

A tiny wire loop with n turns of radius a , each carrying a counterclockwise current i , is placed inside a solenoid as shown. The solenoid has N turns, radius b , length d , and carries a current I .



6. What is the direction of the torque on the loop?

- (a) Correct: $+\hat{i}$
- (b) $-\hat{i}$
- (c) $-\hat{j}$
- (d) $+\hat{k}$
- (e) $-\hat{k}$

the B-field inside the solenoid is to the left. The magnetic moment of the loop is upwards. The tendency is to turn to align the magnetic moment with the field. The turning axis is x and the rhr gives a positive sign.

7. What is the magnitude of the torque on the loop?

- (a) $\mu_0abiInN/d$
- (b) Correct: $\mu_0\pi a^2iInN/d$
- (c) $\mu_0\pi^2a^2b^2iInN/d$
- (d) $\mu_0\pi a(i/I)nN/d$
- (e) $\mu_0\pi(a/b)^2iI(n/N)$

Maybe you have the formula for the torque on a magnetic dipole in a B field, or maybe not. But remember that the field inside the solenoid doesn't depend on the solenoids radius, so any formula with b in it is wrong. Then reason that the torque increases if either or both of i or I

increases and you're left with only one choice, b.

8. An inductor, capacitor and a resistor are in series with an ac sine-wave voltage source with frequency ω . The reactance of the inductor with inductance L is $X_L = \omega L$. The reactance of the capacitor with capacitance C is $X_C = 1/\omega C$. Under which condition is the current from the generator in phase with the emf (voltage)?

- (a) $X_C = R$
- (b) $X_C + X_L + R = 0$
- (c) $X_C + X_L = 0$
- (d) $X_L = R$
- (e) Correct: $X_C = X_L$

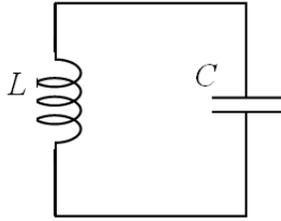
Resonance occurs when X_L and X_C are equal, which means that the total impedance is totally resistive and current and voltage are in phase.

9. In the Faraday's Law experiment you did in class, which of the following describes your qualitative results when the oscilloscope channel 1 displays the voltage across the resistor attached to the *slinky* and channel 2 displays the emf from the *pick-up coil*.

- (a) When channel 1 displays a square wave, channel 2 displays a square wave
- (b) When channel 1 displays a square wave, channel 2 displays a triangle wave
- (c) Correct: When channel 1 displays a triangle wave, channel 2 displays a square wave
- (d) When channel 1 displays a square wave, channel 2 displays a sine wave
- (e) When channel 1 displays a sine wave, channel 2 displays a triangle wave

The induced emf (CH2) must be the negative of the derivative of the B field created by the current (CH1) in the solenoid.

10. At time $t = 0$ the capacitor in the circuit below is fully charged with Q_{max} , and the current through the circuit is 0.



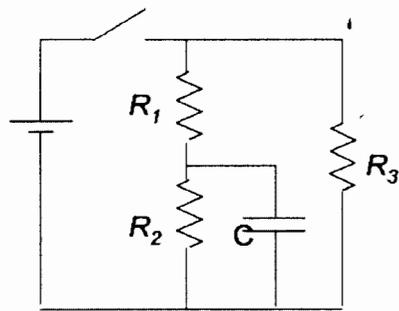
How much energy is stored in the capacitor when the current is maximum?

- (a) $U = Q_{max}^2/2C$
- (b) $U = Q_{max}^2/4C$
- (c) Correct: 0

When current is maximum, all the energy is in the inductor.

There are four written problems. Show all your work to get full credit.

11. The values of all circuit elements are given in the figure. The capacitor is initially uncharged. Then, at time $t = 0$, the switch is closed.



$$E = 6 \text{ V}$$

$$R_1 = R_2 = R_3 = 10 \ \Omega$$

$$C_1 = 1 \ \mu\text{F}$$

- (a) Calculate the current through the battery immediately after the switch is closed.

$$I(t=0) = \frac{V_0}{\left(\frac{R_1 R_3}{R_1 + R_3} \right)} = \frac{6 \text{ V}}{5 \ \Omega} = 1.2 \text{ A}$$

- (b) Calculate the charge on the capacitor a long time after the switch is closed.

$$V_c = V_b \frac{R_2}{R_1 + R_2} = 6 \text{ V} / 2 = 3 \text{ V}$$

$$Q = C V_c = (1 \ \mu\text{F})(3 \text{ V}) = 3 \ \mu\text{C}$$

- (c) A long time after the switch has been closed, it is re-opened. What is the time constant for discharging the capacitor?

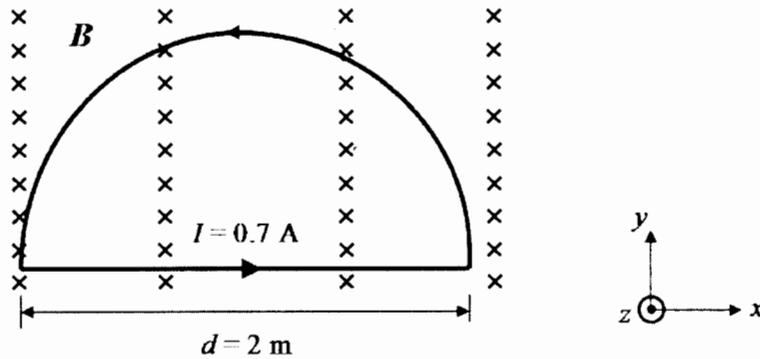
$$\tau = 20$$

$$R_{eq} = \frac{20}{3}$$

$$(1 \ \mu\text{F}) \left(\frac{20}{3} \ \Omega \right)$$

$$= 6.6 \ \mu\text{sec}$$

12. A semi-circular loop consisting of one turn of wire is placed in the xy plane. A constant magnetic field of magnitude $B = 1.7$ T points along the negative z -axis (into the page), and a current $I = 0.7$ A flows counter-clockwise as viewed from the positive z -axis.



(a) Find the magnitude and direction of the net magnetic force on the circular section of the loop.

$$F_{\text{net}} = 0 \rightarrow I \ell B = F$$

$$(0.7)(1.7)(2) = \boxed{2.38 \text{ N}}$$

(b) Calculate the magnetic dipole moment of this loop.

$$\mu = NIA$$

$$= (0.7)(1.7)(1 \text{ m})^2$$

$$= \boxed{1.1 \frac{\text{Nm}}{\text{T}}}$$

Out of paper, in the z direction.

(c) What is the torque on the loop?

$$\vec{\mu} \times \vec{B} = \vec{\tau}$$

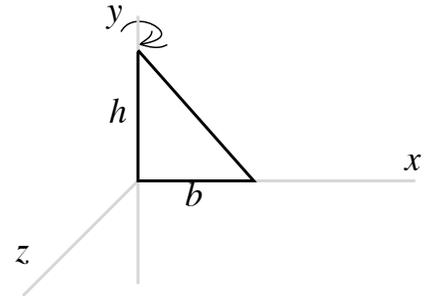
$$\left(1.1 \frac{\text{Nm}}{\text{T}}\right)(1.7) \sin(180^\circ)$$

$$= 0 \text{ n}\cdot\text{m}$$

13. A conducting wire formed in the shape of a right triangle with base $b = 39$ cm and height $h = 51$ cm and having resistance $R = 1.6 \Omega$, rotates uniformly around the y -axis in the direction indicated by the arrow (clockwise as viewed from above (looking down in the negative y -direction)). The triangle makes one complete rotation in time $t = T = 1.5$ seconds. A constant magnetic field $B = 1.5$ T pointing in the positive z -direction (out of the page) exists in the region where the wire is rotating.

- (a) What is the angular frequency of rotation? [2 marks]

Angular frequency is ω in radians/second. $\omega = 2\pi/T = 6.28/1.5 = 4.19$ radians/second.



- (b) What is the magnitude of the maximum current induced in the loop? [2 marks]

Max current occurs when the area vector of the triangular loop is perpendicular to the field. If it starts at $t = 0$ in the orientation shown in the figure then

$$\Phi_B = \vec{A} \cdot \vec{B} = 0.5abB \cos(\omega t)$$

$$\frac{d\Phi_B}{dt} = -0.5abB\omega \sin(\omega t) = \mathcal{E}_{\max} \sin(\omega t)$$

$$I_{\max} = R/\mathcal{E}_{\max} = R/(0.5abB\omega)$$

$$I_{\max} = 0.62484 \text{ V}/1.6 \Omega = 0.39 \text{ A}$$

- (c) The loop is oriented as shown in the figure at $t = 0$ s. What is the magnitude of the magnetic flux at $t = t_1 = 0.5625$ s? [2 marks]

$$\Phi_B(t_1) = 0.5abB \cos(\omega t_1) = 0.105 \text{ T} \cdot \text{m}^2$$

- (d) What is the current induced at time t_1 , denoted I_1 ? [2 marks]

$$I_1 = I_{\max} \sin(\omega t_1) = 0.276 \text{ A}$$

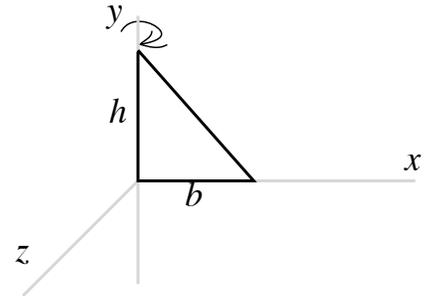
- (e) Calculate the *first* time after $t = 0$ s at which the magnitude of the current is maximum. [2 marks]

Max current occurs when the area vector is perpendicular to the B field because that's when the maximum rate of cutting through the field lines occurs, and when $|\sin(\omega t)|$ is largest. $T/4 = 0.374$ s.

13. A conducting wire formed in the shape of a right triangle with base $b = 39$ cm and height $h = 51$ cm and having resistance $R = 1.6 \Omega$, rotates uniformly around the y -axis in the direction indicated by the arrow (clockwise as viewed from above (looking down in the negative y -direction)). The triangle makes one complete rotation in time $t = T = 1.5$ seconds. A constant magnetic field $B = 1.5$ T pointing in the positive z -direction (out of the page) exists in the region where the wire is rotating.

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$$\Phi_B = \vec{A} \cdot \vec{B} = 0.5abB \cos(\omega t)$$

$$\frac{d\Phi_B}{dt} = -0.5abB\omega \sin(\omega t) = \mathcal{E}_{\max} \sin(\omega t)$$

$$I_{\max} = \mathcal{E}_{\max}/R = (0.5abB\omega)/R$$

$$I_{\max} = 0.62484 \text{ V}/1.6 \Omega = 0.39 \text{ A}$$

- (c) The loop is oriented as shown in the figure at $t = 0$ s. What is the magnitude of the magnetic flux at $t = t_1 = 0.5625$ s? [2 marks]

$$\Phi_B(t_1) = 0.5abB \cos(\omega t_1) = 0.105 \text{ T} \cdot \text{m}^2$$

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14. Consider a driven RLC circuit consisting of a resistor $R = 250 \Omega$, a capacitor $C = 0.40 \mu\text{F}$, and an inductor $L = 40 \text{ mH}$ connected in series. The AC generator provides a voltage $\mathcal{E}_{\text{rms}} = 120 \text{ V}$ at $\omega = 1.0 \times 10^4 \text{ rad/s}$.

- (a) Calculate the maximum EMF across by the generator, the inductor, and the capacitor. [2 marks]

$$\mathcal{E}_{\text{max}} = \sqrt{2}\mathcal{E}_{\text{rms}} = 170 \text{ V}$$

- (b) What is the magnitude and sign of the phase angle between the current and the generator voltage? Does the current lag the generator voltage? Explain. [2 marks].

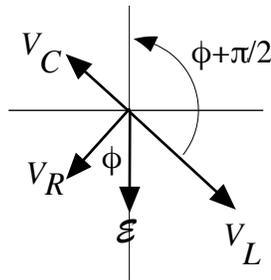
$$X_L = \omega L = 400\Omega,$$

$$X_C = 1/\omega C = 250\Omega,$$

$$\phi = \arctan \frac{X_L - X_C}{R} = +0.54 \text{ rad} = +31^\circ$$

The current lags because the voltage across R , which is proportional to I , reaches maximum after \mathcal{E} does.

- (c) Sketch the **voltage** phasor diagram at $t = 0$ given that $\mathcal{E}(t) = \mathcal{E}_{\text{max}} \sin(\omega t)$. [2 marks]



- (d) Calculate the resonant frequency. [2 marks].

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.6 \times 10^{-8}}} = 7.9 \times 10^3 \text{ rad/s}$$

- (e) Calculate the first time when the potential across the inductor is zero. [2 marks]

See the phasor diagram. The voltage across the inductor becomes zero when the phasor for V_L reaches the positive y axis.

$$t_1 = (\phi + \pi/2)/\omega = 2.1 \times 10^{-4} \text{ s}$$