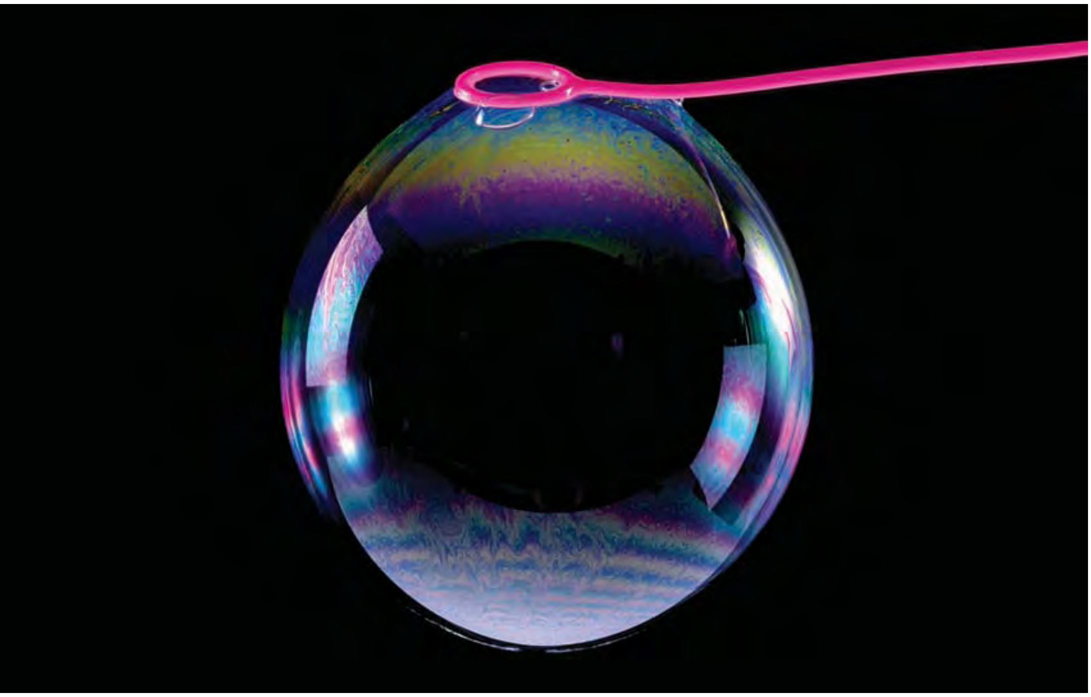
C H A P T E R

33



WHITE LIGHT IS REFLECTED OFF A SOAP BUBBLE. WHEN LIGHT OF ONE WAVELENGTH IS INCIDENT ON A THIN SOAP-AND-WATER FILM, LIGHT IS REFLECTED FROM BOTH THE FRONT AND THE BACK SURFACES OF THE FILM. IF THE ORDER OF MAGNITUDE OF THE THICKNESS OF THE FILM IS ONE WAVELENGTH OF THE LIGHT, THE TWO REFLECTED LIGHT WAVES INTERFERE. IF THE TWO REFLECTED WAVES ARE 180° OUT OF PHASE, THE REFLECTED WAVES INTERFERE DESTRUCTIVELY, SO THE NET RESULT IS THAT NO LIGHT IS REFLECTED. IF WHITE LIGHT, WHICH CONTAINS A CONTINUUM OF WAVELENGTHS, IS INCIDENT ON THE THIN FILM, THEN THE REFLECTED WAVES WILL INTERFERE DESTRUCTIVELY ONLY FOR CERTAIN WAVELENGTHS, AND FOR OTHER WAVELENGTHS THEY WILL INTERFERE CONSTRUCTIVELY. THIS PROCESS PRODUCES THE COLORED FRINGES THAT YOU SEE IN THE SOAP BUBBLE. *(Aaron Haupt/ Photo Researchers.)*

Interference and Diffraction

33-1 Phase Difference and Coherence
33-2 Interference in Thin Films
33-3 Two-Slit Interference Pattern
33-4 Diffraction Pattern of a Single Slit
*33-5 Using Phasors to Add Harmonic Waves
33-6 Fraunhofer and Fresnel Diffraction
33-7 Diffraction and Resolution
*33-8 Diffraction Gratings

? Have you ever wondered if the phenomenon that produces the bands that you see in the light reflected off a soap bubble has any practical applications? (See Example 33-2.)

Interference and diffraction are the important phenomena that distinguish waves from particles.* Interference is the formation of a lasting intensity pattern by two or more waves that superpose in space. Diffraction is the bending of waves around corners that occurs when a portion of a wavefront is cut off by a barrier or obstacle.

In this chapter, we will see how the pattern of the resulting wave can be calculated by treating each point on the original wavefront as a point source, according to Huygens's principle, and calculating the interference pattern resulting from these sources.

* Before you study this chapter, you may wish to review Chapter 15 and Chapter 16, where the general topics of interference and diffraction of waves are first discussed.

33-1 PHASE DIFFERENCE AND COHERENCE

When two harmonic sinusoidal waves of the same frequency and wavelength but of different phase combine, the resultant wave is a harmonic wave whose amplitude depends on the phase difference. If the phase difference is zero, or an integer multiplied by 360°, the waves are in phase and interfere constructively. The resultant amplitude equals the sum of the two individual amplitudes, and the intensity (which is proportional to the square of the amplitude) is maximum. (If the amplitudes are equal and the waves are in phase, the intensity is four times that of either individual wave.) If the phase difference is 180° or any odd integer multiplied by 180°, the waves are out of phase and interfere destructively. The resultant amplitude is then the difference between the two individual amplitudes, and the intensity is a minimum. (If the amplitudes are equal and the waves are 180° out of phase, the intensity is zero.)

A phase difference between two waves is often the result of a difference in path lengths. When a light wave reflects from a thin transparent film, such as a soap bubble, the reflected light is a superposition of the light reflected from the front surface of the film and the light reflected from the back surface of the film. The additional distance traveled by the light reflected from the back surface is called the path-length difference between the two reflected waves. A path-length difference of one wavelength produces a phase difference of 360°, which is equivalent to no phase difference at all. A path-length difference of one-half wavelength produces a 180° phase difference. In general, a path-length difference of Δr contributes a phase difference δ given by

$$\delta = \frac{\Delta r}{\lambda}2\pi = \frac{\Delta r}{\lambda}360^\circ \qquad 33\text{-}1$$

PHASE DIFFERENCE DUE TO A PATH-LENGTH DIFFERENCE

Example 33-1 Phase Difference

(*a*) What is the minimum path-length difference that will produce a phase difference of 180° for light of wavelength 800 nm? (*b*) What phase difference will that path-length difference produce in light of wavelength 700 nm?

PICTURE The phase difference is to 360° as the path-length difference is to the wavelength.

SOLVE

(*a*) The phase difference δ is to 360° as the path-length difference Δr is to the wavelength λ. We know that $\lambda = 800$ nm and $\delta = 180°$:

$$\frac{\delta}{360^\circ} = \frac{\Delta r}{\lambda}$$

$$\Delta r = \frac{\delta}{360^\circ}\lambda = \frac{180^\circ}{360^\circ}(800\text{ nm}) = \boxed{400\text{ nm}}$$

(*b*) Set $\lambda = 700$ nm, $\Delta r = 400$ nm, and solve for δ:

$$\delta = \frac{\Delta r}{\lambda}360^\circ = \frac{400\text{ nm}}{700\text{ nm}}360^\circ = \boxed{206^\circ = 3.59\text{ rad}}$$

CHECK The Part (*b*) result is somewhat larger than 180°. This result is expected because 400 nm is longer than half of the 700-nm wavelength.

Another cause of phase difference is the 180° phase change a wave sometimes undergoes upon reflection from a surface. This phase change is analogous to the inversion of a pulse on a string when it reflects from a point where the density suddenly increases, such as when a light string is attached to a heavier string or rope. The inversion of the reflected pulse is equivalent to a phase change of 180° for a sinusoidal wave (which can be thought of as a series of pulses). When light traveling in air strikes the surface of a medium in which light travels more slowly, such as glass or water, there is a 180° phase change in the reflected light.

When light is traveling in the liquid wall of a soap bubble, there is no phase change in the light reflected from the surface between the liquid and the air. This situation is analogous to the reflection without inversion of a pulse on a heavy string at a point where the heavy string is attached to a lighter string.

> If light traveling in one medium strikes the surface of a medium in which light travels more slowly, there is a 180° phase change in the reflected light.
>
> PHASE DIFFERENCE DUE TO REFLECTION

As we saw in Chapter 16, interference of waves is observed when two or more coherent waves overlap. Interference of overlapping waves from two sources is not observed unless the sources are coherent. Because the light from each source is usually the result of millions of atoms radiating independently, the phase difference between the waves from such sources fluctuates randomly many times per second, so two light sources are usually not coherent. Coherence in optics is often achieved by splitting the light beam from a single source into two or more beams that can then be combined to produce an interference pattern. The light beam can be split by reflecting the light from the two surfaces of a thin film (Section 33-2), by diffracting the beam through two small openings or slits in an opaque barrier (Section 33-3), or by using a single point source and its image in a plane mirror for the two sources (Section 33-3). Today, lasers are the most important sources of coherent light in the laboratory.

Light from an ideal monochromatic source is a sinusoidal wave of infinite duration, and light from certain lasers approaches this ideal. However, light from conventional *monochromatic* sources, such as gas discharge tubes designed for this purpose, consists of packets of sinusoidal light that are only a few million wavelengths long. The light from such a source consists of many such packets, each approximately the same length. The packets have essentially the same wavelength, but the packets differ in phase in a random manner. The length of the individual packets is called the **coherence length** of the light, and the time it takes one of the packets to pass a point in space is the **coherence time.** The light emitted by a gas discharge tube designed to produce monochromatic light has a coherence length of only a few millimeters. By comparison, some highly stable lasers produce light that has a coherence length many kilometers long.

33-2 INTERFERENCE IN THIN FILMS

You have probably noticed the colored bands in a soap bubble or in the film on the surface of oily water. These bands are due to the interference of light reflected from the top and bottom surfaces of the film. The different colors arise because of variations in the thickness of the film, causing interference for different wavelengths at different points.

When waves traveling in a medium cross a surface where the wave speed changes, part of the wave is reflected and part is transmitted. In addition, the reflected wave undergoes a 180° phase shift upon reflection if the transmitted wave travels at a slower speed than do the incident and reflected waves. (This 180° phase shift is established for waves on a string in Section 15-4 of Chapter 15.) The reflected wave does not undergo a phase shift upon reflection if the transmitted wave travels at a faster speed than do the incident and reflected waves.

Consider a thin film of water (such as a small section of a soap bubble) of uniform thickness viewed at small angles with the normal, as shown in Figure 33-1. Part of the light is reflected from the upper air–water interface where it undergoes a 180° phase change. Most of the light enters the film and part of it is reflected by the bottom water–air interface. There is no phase change in this reflected light. If the light is nearly perpendicular to the surfaces, both the light reflected from the top surface and the light reflected from the bottom surface can enter the eye. The path-length

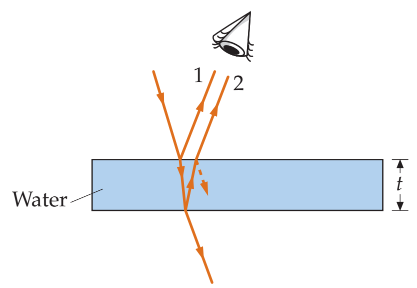


FIGURE 33-1 Light rays reflected from the top and bottom surfaces of a thin film are coherent because both rays come from the same source. If the light is incident almost normally, the two reflected rays will be very close to each other and will produce interference.

difference between these two rays is $2t$, where t is the thickness of the film. This path-length difference produces a phase difference of $(2t/\lambda')360°$, where $\lambda' = \lambda/n$ is the wavelength of the light in the film, n is the index of refraction of the film, and λ is the wavelength of the light in vacuum. The total phase difference between the two rays is thus 180° plus the phase difference due to the path-length difference. Destructive interference occurs when the path-length difference $2t$ is zero or a whole number of wavelengths λ' (in the film). Constructive interference occurs when the path-length difference is an odd number of half-wavelengths.

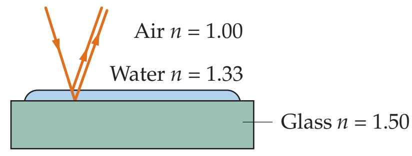


FIGURE 33-2 The interference of light reflected from a thin film of water resting on a glass surface. In this case, both rays undergo a change in phase of 180° upon reflection.

When a thin film of water lies on a glass surface, as in Figure 33-2, the ray that reflects from the lower water–glass interface also undergoes a 180° phase change, because the index of refraction of glass (approximately 1.50) is greater than that of water (approximately 1.33). Thus, both the rays shown in the figure have undergone a 180° phase change upon reflection. The phase difference δ between these rays is due solely to the path-length difference and is given by $\delta = (2t/\lambda')360°$.

When a thin film of varying thickness is viewed with monochromatic light, such as the yellow light from a sodium lamp, alternating bright and dark bands or lines, called **interference fringes,** are observed. The distance between a bright fringe and a dark fringe is that distance over which the film's thickness t changes so that the path-length difference $2t$ changes by $\lambda'/2$. Figure 33-3a shows the interference pattern observed when light is reflected from an air film between a spherical glass surface and a plane glass surface in contact. These circular interference fringes are known as **Newton's rings.** Typical rays reflected at the top and bottom of the air film are shown in Figure 33-3b. Near the point of contact of the surfaces, where the path-length difference between the ray reflected from the upper glass–air interface and the ray reflected from the lower air–glass interface is approximately zero (it is small compared with the wavelength of light) the interference is destructive because of the 180° phase shift of the ray reflected from the lower air–glass interface. This central region in Figure 33-3a is therefore dark. The first bright fringe occurs at the radius at which the path-length difference is $\lambda/2$, which contributes a phase difference of 180°. This adds to the phase shift due to reflection to produce a total phase difference of 360°, which is equivalent to a zero phase difference. The second dark region occurs at the radius at which the path-length difference is λ, and so on.

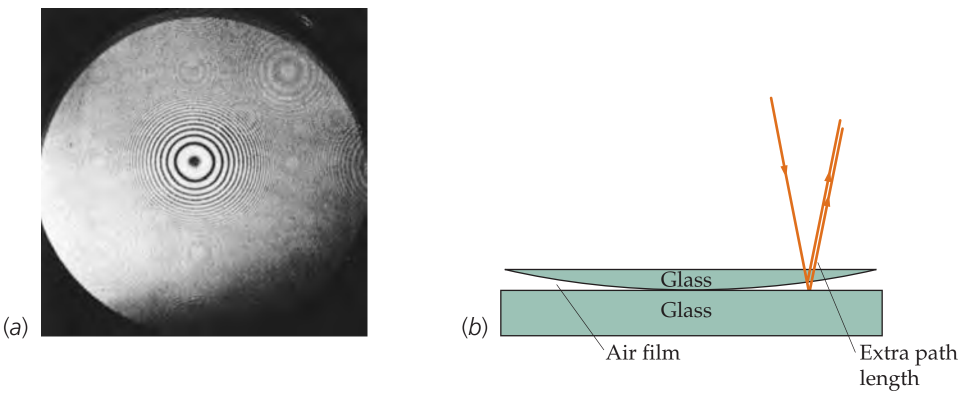


FIGURE 33-3 (a) Newton's rings observed when light is reflected from a thin film of air between a plane glass surface and a spherical glass surface. At the center, the thickness of the air film is negligible and the interference is destructive because of the 180° phase change of one of the rays upon reflection. (b) Glass surfaces for the observation of Newton's rings shown in Figure 33-3a. The thin film in this case is the film of air between the glass surfaces. *(Courtesy of Bausch & Lomb.)*

Example 33-2 A Wedge of Air

A wedge-shaped film of air is made by placing a small slip of paper between the edges of two flat pieces of glass, as shown in Figure 33-4. Light of wavelength 500 nm is incident normally on the glass, and interference fringes are observed by reflection. If the angle θ made by the plates is 3.0×10^{-4} rad (0.017°), how many dark interference fringes per centimeter are observed?

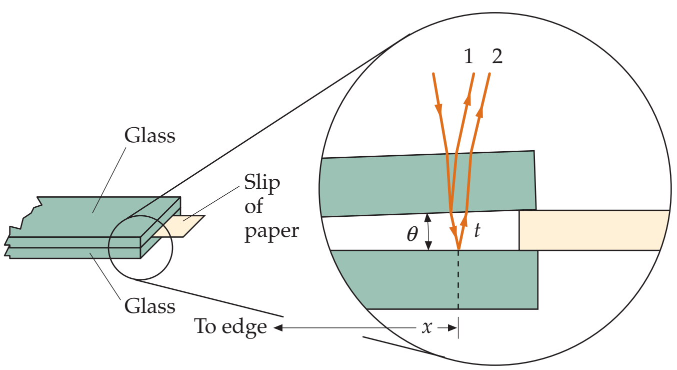


FIGURE 33-4 The angle θ, which is less than 0.02°, is exaggerated. The incoming and outgoing rays are virtually perpendicular to all air–glass interfaces.

PICTURE We find the number of fringes per centimeter by finding the horizontal distance x to the mth fringe and solving for m/x. Because the ray reflected from the bottom plate undergoes a 180° phase shift, the point of contact (where the path-length difference is zero) will be dark. The mth dark fringe after the contact point occurs when $2t = m\lambda'$, where $\lambda' = \lambda$ is the wavelength in the air film, and t is the plate separation at x, as shown in Figure 33-4. Because the angle θ is small, we can use the small-angle approximation $\theta \approx \tan\theta = t/x$.

SOLVE

1. The mth dark fringe from the contact point occurs when the path-length difference $2t$ equals m wavelengths:
$$2t = m\lambda' = m\lambda$$
$$m = \frac{2t}{\lambda}$$
2. The thickness t is related to the angle θ:
$$\theta = \frac{t}{x}$$
3. Substitute $t = x\theta$ into the equation for m:
$$m = \frac{2x\theta}{\lambda}$$
4. Calculate m/x:
$$\frac{m}{x} = \frac{2\theta}{\lambda} = \frac{2(3.0 \times 10^{-4})}{5.0 \times 10^{-7}\text{ m}} = 1200\text{ m}^{-1} = \boxed{12\text{ cm}^{-1}}$$

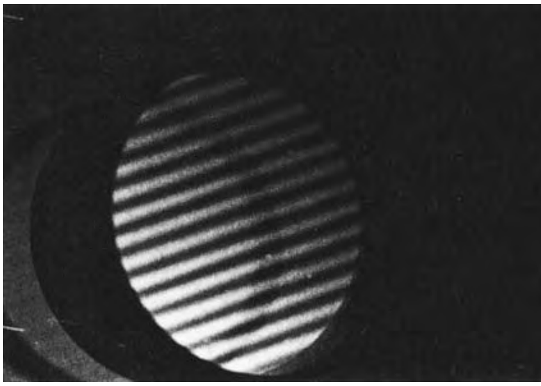
CHECK The expression for the number of dark fringes per unit length in step 4 shows that the number per centimeter would decrease if light of a longer wavelength is used. This result is as expected.

TAKING IT FURTHER We observe 12 dark fringes per centimeter. In practice, the number of fringes per centimeter, which is easy to count, can be used to determine the angle. Note that if the angle of the wedge is increased, the fringes become more closely spaced.

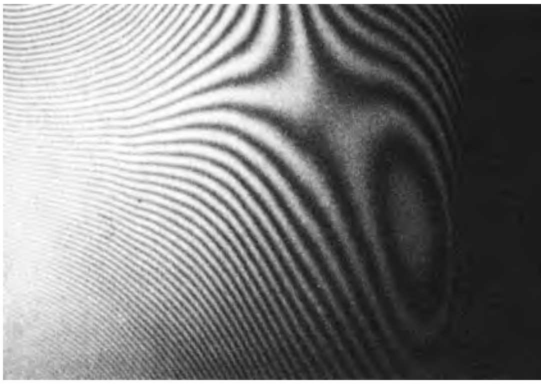
PRACTICE PROBLEM 33-1 How many dark fringes per centimeter are observed if light of wavelength 650 nm is used?

Figure 33-5*a* shows interference fringes produced by a wedge-shaped air film between two flat glass plates, as in Example 33-2. Plates that produce straight fringes, such as those in Figure 33-5*a*, are said to be **optically flat.** To be optically flat, a surface must be flat to within a small fraction of a wavelength. A similar wedge-shaped air film formed by two ordinary glass plates yields the irregular fringe pattern in Figure 33-5*b*, which indicates that these plates are not optically flat.

One application of interference effects in thin films is in nonreflecting lenses, which are made by coating the surface of a lens with a thin film of a material that has an index of refraction equal to approximately 1.38, which is between the index of refraction of glass and that of air. The intensities of the light reflected from the top and bottom surfaces of the film are approximately equal, and because the reflected rays undergo a 180° phase change at both surfaces there is no phase difference due to reflection between the two rays. The thickness of the film is chosen to be $\frac{1}{4}\lambda' = \frac{1}{4}\lambda n$, where λ is the wavelength, in vacuum, that is in the middle of the visible spectrum, so that there is a phase change of 180° due to the path-length difference of $\lambda'/2$ for light of normal incidence. Reflection from the coated surface is thus minimized, which means that transmission through the surface is maximized.



(*a*)



(*b*)

FIGURE 33-5 (*a*) Straight-line fringes from a wedge-shaped film of air, like that shown in Figure 33-4. The straightness of the fringes indicates that the glass plates are optically flat. (*b*) Fringes from a wedge-shaped film of air between glass plates that are not optically flat. *(Courtesy T. A. Wiggins.)*

33-3 TWO-SLIT INTERFERENCE PATTERN

Interference patterns of light from two or more sources can be observed only if the sources are coherent. The interference in thin films discussed previously can be observed because the two beams come from the same light source but are separated by reflection. In Thomas Young's famous 1801 experiment, in which he demonstrated the wave nature of light, two coherent light sources are produced by illuminating two very narrow parallel slits using a single light source. We saw in Chapter 15 that when a wave encounters a barrier that has a very small opening, the opening acts as a point source of waves (Figure 33-6).

During Young's experiment, diffraction causes each slit to act as a line source (which is equivalent to a point source in two dimensions). The interference pattern is observed on a screen far from the slits (Figure 33-7*a*). At very large distances from the slits, the lines from the two slits to some point P on the screen are approximately parallel, and the path-length difference is approximately $d \sin\theta$, where d is the separation of the slits, as shown in Figure 33-7*b*. When the path-length difference is equal to an integral number of wavelengths, the interference is constructive. We thus have interference maxima at an angle θ_m given by

$$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, \ldots \qquad 33\text{-}2$$

TWO-SLIT INTERFERENCE MAXIMA

where m is called the **order number.** The interference minima occur at

$$d \sin\theta_m = \left(m - \tfrac{1}{2}\right)\lambda \qquad m = 1, 2, 3, \ldots \qquad 33\text{-}3$$

TWO-SLIT INTERFERENCE MINIMA

The phase difference δ at a point P is related to the path-length difference $d \sin\theta$ by

$$\delta = \frac{\Delta r}{\lambda}2\pi = \frac{d \sin\theta}{\lambda}2\pi \qquad 33\text{-}4$$

We can relate the distance y_m measured along the screen from the central point to the mth bright fringe (see Figure 33-7*b*) to the distance L from the slits to the screen:

$$\tan\theta_m = \frac{y_m}{L}$$

For small angles, $\tan\theta \approx \sin\theta$. Substituting y_m/L for $\sin\theta_m$ in Equation 33-2 and solving for y_m gives

$$y_m = m\frac{\lambda L}{d} \qquad 33\text{-}5$$

From this result, we see that for small angles the fringes are equally spaced on the screen.

FIGURE 33-6 Plane water waves in a ripple tank encountering a barrier that has a small opening. The waves to the right of the barrier are circular waves that are concentric about the opening, just as if there were a point source at the opening. *(From PSSC Physics, 2nd Edition, 1965. D. C. Heath & Co. and Education Development Center, Newton MA.)*

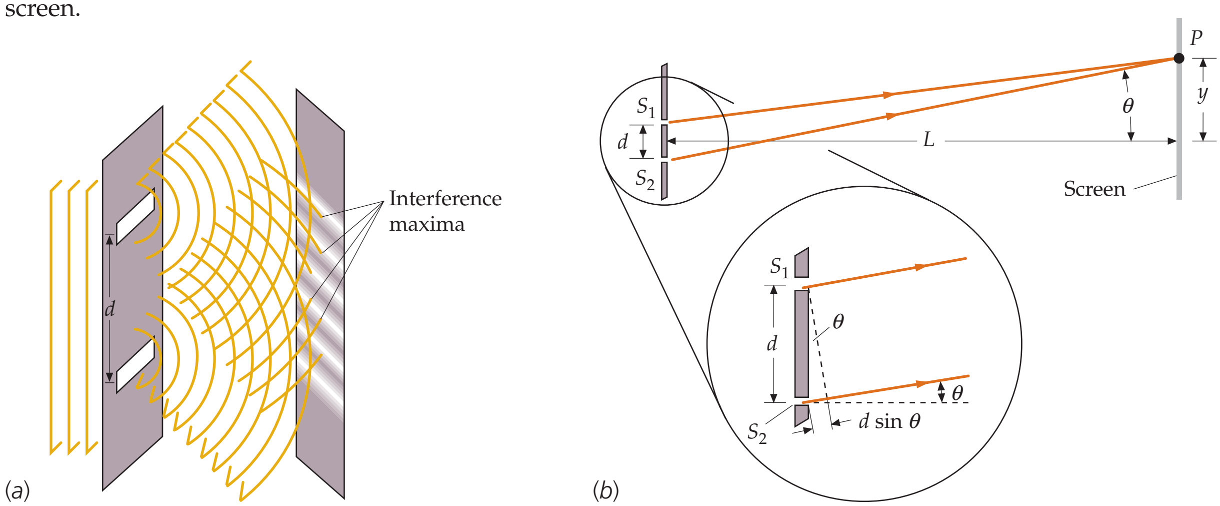


FIGURE 33-7 (*a*) Two slits act as coherent sources of light for the observation of interference in Young's experiment. Cylindrical waves from the slits overlap and produce an interference pattern on a screen. (*b*) Geometry for relating the distance y measured along the screen to L and θ. When the screen is very far away compared with the slit separation, the rays from the slits to a point on the screen are approximately parallel, and the path-length difference between the two rays is $d \sin\theta$.

Example 33-3 Fringe Spacing from Slit Spacing *Try It Yourself*

Two narrow slits separated by 1.50 mm are illuminated by yellow light from a sodium lamp that has a wavelength equal to 589 nm. Find the spacing of the bright fringes observed on a screen 3.00 m away.

PICTURE The distance y_m measured along the screen to the mth bright fringe is given by Equation 33-5, where $L = 3.00$ m, $d = 1.50$ mm, and $\lambda = 589$ nm.

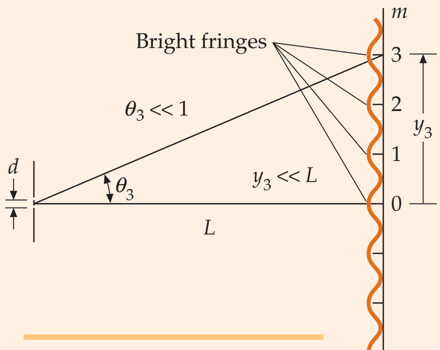


FIGURE 33-8 The vertical scale of the figure is expanded.

SOLVE

Cover the column to the right and try these on your own before looking at the answers.

Steps	Answers
1. Make a sketch of the situation (Figure 33-8).	
2. The fringe spacing is the distance between the mth bright fringe and the $(m+1)$th bright fringe. Using the sketch, obtain an expression for the spacing between fringes.	fringe spacing $= y_{m+1} - y_m$
3. Apply Equation 33-5 to the mth and $(m+1)$th fringe.	$y_m = m\dfrac{\lambda L}{d}$ and $y_{m+1} = (m+1)\dfrac{\lambda L}{d}$
4. Substitute into the step-2 result and simplify.	$y_{m+1} - y_m = \dfrac{\lambda L}{d}$
5. Substitute into the step-4 result and solve for the fringe spacing.	fringe spacing $= \boxed{1.18 \text{ mm}}$

TAKING IT FURTHER The fringes are uniformly spaced only to the degree that the small-angle approximation is valid, that is, to the degree that $\lambda/d \ll 1$. In this example, $\lambda/d = (589 \text{ nm})/(1.50 \text{ mm}) \approx 0.0004$.

See **Math Tutorial** *for more information on* **Trigonometry**

Example 33-4 How Many Fringes? *Conceptual Example*

Two narrow slits are illuminated by monochromatic light. If the distance between the slits is equal to 2.75 wavelengths, what is the maximum number of bright fringes that can be seen on a screen? (*a*) 1, (*b*) 2, (*c*) 3, (*d*) 4, (*e*) 5, (*f*) 6 or more

PICTURE A bright fringe (constructive interference) exists at points on the screen for which the distance to the two slits differs by an integer multiplied by the wavelength. However, the maximum difference in distance possible is equal to the distance between the two slits.

SOLVE

1. Find the maximum difference in distance from points on the screen to the two slits:	At all points on the screen, the difference in distance from the two slits is 2.75 wavelengths or less.
2. A bright fringe (constructive interference) exists at points on the screen for which the distance to the two slits differs by an integer multiplied by the wavelength:	Bright fringes exist on the screen at places where the difference in distance to the slits is 2 wavelengths, 1 wavelength, or zero wavelengths.
3. Count up the bright fringes. There is the central maximum and two on either side of the central maximum:	$\boxed{(e)\ 5}$

CONCEPT CHECK 33-1

What is the maximum number of dark fringes that can be seen on a screen?

CALCULATION OF INTENSITY

To calculate the intensity of the light on the screen at a general point P, we need to add two harmonic wave functions that differ in phase.* The wave functions for electromagnetic waves are the electric field vectors. Let E_1 be the electric field at some point P on the screen due to the waves from slit 1, and let E_2 be the electric field at that point due to waves from slit 2. Because the angles of interest are small, we can treat the fields as though they are parallel. Both electric fields oscillate with the same frequency (they result from a single source that illuminates both slits) and they have the same amplitude. (The path-length difference is only of the order of a few wavelengths of light at most.) They have a phase difference δ given by Equation 33-4. If we represent the wave functions by

$$E_1 = A_0 \sin \omega t$$

and

$$E_2 = A_0 \sin(\omega t + \delta)$$

the resultant wave function is

$$E = E_1 + E_2 = A_0 \sin \omega t + A_0 \sin(\omega t + \delta) \qquad 33\text{-}6$$

By making use of the identity

$$\sin \alpha + \sin \beta = 2 \cos \tfrac{1}{2}(\alpha - \beta) \sin \tfrac{1}{2}(\alpha + \beta)$$

the resultant wave function is given by

$$E = \left[2A_0 \cos \tfrac{1}{2}\delta\right] \sin\left(\omega t + \tfrac{1}{2}\delta\right) \qquad 33\text{-}7$$

The amplitude of the resultant wave is thus $2A_0 \cos \frac{1}{2}\delta$. It has its maximum value of $2A_0$ when the waves are in phase and is zero when they are 180° out of phase. Because the intensity is proportional to the square of the amplitude, the intensity at any point P is

$$I = 4I_0 \cos^2 \tfrac{1}{2}\delta \qquad 33\text{-}8$$

INTENSITY IN TERMS OF PHASE DIFFERENCE

where I_0 is the intensity of the light reaching the screen from either slit separately. The phase angle δ is related to the position on the screen by $\delta = (d \sin\theta/\lambda)2\pi$ (Equation 33-4).

Figure 33-9*a* shows the intensity pattern as seen on a screen. A graph of the intensity as a function of $\sin\theta$ is shown in Figure 33-9*b*. For small θ, this graph is equivalent to a plot of intensity versus y (because $y = L \tan\theta \approx L \sin\theta$). The intensity I_0 is the intensity from each slit separately. The dashed line in Figure 33-9*b* shows the average intensity $2I_0$, which is the result of averaging over a distance containing many interference maxima and minima. This is the intensity that would arise from the two sources if they acted independently without interference, that is, if they were not coherent. Then, the phase difference between the two sources would fluctuate randomly, so that only the average intensity would be observed.

Figure 33-10 shows another method of producing the two-slit interference pattern, an arrangement known as **Lloyd's mirror.** A monochromatic horizontal line source is placed at a distance $\frac{1}{2}d$ above the plane of a mirror. Light striking the screen directly from the source interferes with the light that is reflected from the mirror. The reflected light can be considered to come from the virtual image of the line source formed by the mirror. Because of the 180° change in phase upon reflection at the mirror, the interference pattern is that of two coherent line sources that differ in phase by 180°. The pattern is the same as that shown in Figure 33-9 for two slits, except that the maxima and minima are interchanged. Constructive interference occurs at points for which the path-length difference is a half-wavelength or any odd number of half-wavelengths. At those points, the 180° phase difference due to the path-length difference combines with the 180° phase difference of the sources to produce constructive interference.

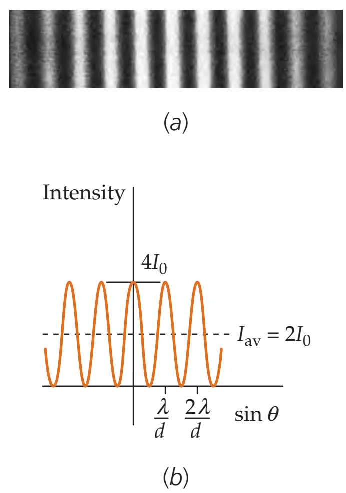


FIGURE 33-9 (*a*) The interference pattern observed on a screen far away from the two slits shown in Figure 33-7. (*b*) Plot of intensity versus $\sin\theta$. The maximum intensity is $4I_0$, where I_0 is the intensity due to each slit separately. The average intensity (dashed line) is $2I_0$. *(Courtesy of Michael Cagnet.)*

* We did this in Chapter 16 where we first discussed the superposition of two waves.

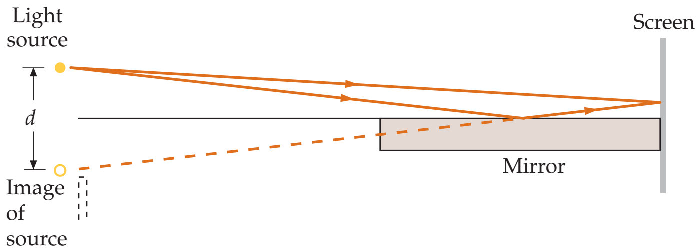


FIGURE 33-10 Lloyd's mirror for producing a two-slit interference pattern. The two sources (the source and its virtual image) are coherent and are 180° out of phase.

PRACTICE PROBLEM 33-2

A point source of light (λ = 589 nm) is placed 0.40 mm above the surface of a glass mirror. Interference fringes are observed on a screen 6.0 m away, and the interference is between the light reflected from the front surface of the glass and the light traveling from the source directly to the screen. Find the spacing of the fringes on the screen.

The physics of Lloyd's mirror was used in the early days of radio astronomy to determine the location of distant radio sources on the celestial sphere. A radio-wave receiver was placed on a cliff overlooking the sea, and the surface of the sea served as the mirror.

33-4 DIFFRACTION PATTERN OF A SINGLE SLIT

In our discussion of the interference patterns produced by two or more slits, we assumed that the slits were very narrow so that we could consider the slits to be line sources of cylindrical waves, which in our two-dimensional diagrams are point sources of circular waves. We could therefore assume that the value of the intensity due to one slit acting alone was the same (I_0) at any point P on the screen, independent of the angle θ made between the ray to point P and the normal line between the slit and the screen. When the slit is not narrow, the intensity on a screen far away is not independent of angle but decreases as the angle increases. Consider a slit of width a. Figure 33-11 shows the intensity pattern on a screen far away from the slit of width a as a function of $\sin\theta$. We can see that the intensity is maximum in the forward direction ($\sin\theta = 0$) and decreases to zero at an angle that depends on the slit width a and the wavelength λ.

Most of the light intensity is concentrated in the broad **central diffraction maximum,** although there are minor secondary maxima bands on either side of the central maximum. The first zeroes in the intensity occur at angles specified by

$$\sin\theta_1 = \lambda/a \qquad 33\text{-}9$$

Note that for a given wavelength λ, Equation 33-9 describes how variations in the slit width result in variations in the angular width of the central maximum. If we *increase* the slit width a, the angle θ_1 at which the intensity first becomes zero *decreases*, giving a more narrow central diffraction maximum. Conversely, if we *decrease* the slit width, the angle of the first zero *increases*, giving a wider central diffraction maximum. When a is smaller than λ, then $\sin\theta_1$ would have to exceed 1

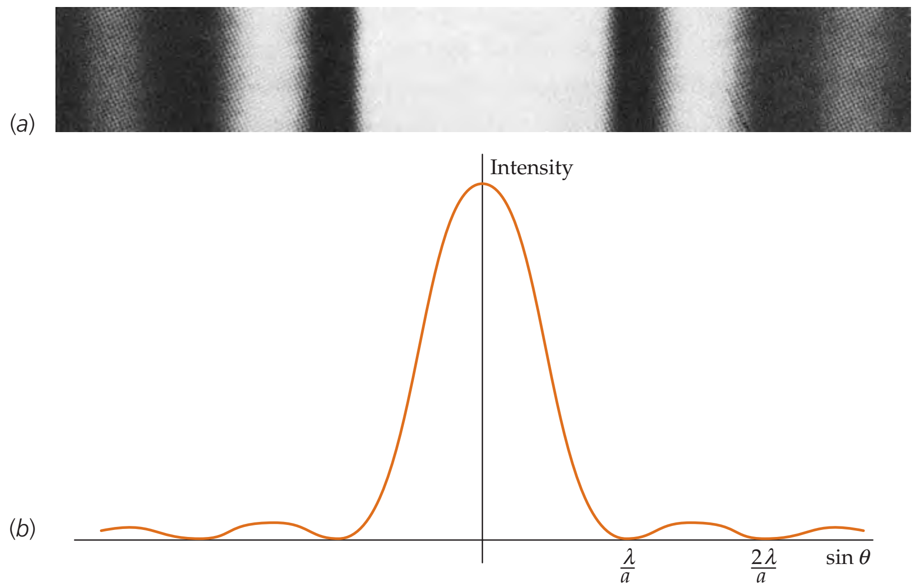


FIGURE 33-11 (*a*) Diffraction pattern of a single slit as observed on a screen far away. (*b*) Plot of intensity versus $\sin\theta$ for the pattern in Figure 33-11*a*. *(Courtesy of Michael Cagnet.)*

to satisfy Equation 33-9. Thus, for a less than λ, there are no points of zero intensity in the pattern, and the slit acts as a line source (a point source in two dimensions) radiating light energy essentially equal in all directions.

Multiplying both sides of Equation 33-9 by $a/2$ gives

$$\tfrac{1}{2}a\sin\theta_1 = \tfrac{1}{2}\lambda \qquad 33\text{-}10$$

The quantity $\frac{1}{2}a\sin\theta_1$ is the path-length difference between a light ray leaving the middle of the upper half of the slit and one leaving the middle of the lower half of the slit. We see that the first diffraction *minimum* occurs when these two rays are 180° out of phase, that is, when their path-length difference equals a half-wavelength. We can understand this result by considering each point on a wavefront to be a point source of light in accordance with Huygens's principle. In Figure 33-12, we have placed a line of dots on the wavefront at the slit to represent these point sources schematically. Suppose, for example, that we have 100 such dots and that we look at an angle θ_1 for which $a\sin\theta_1 = \lambda$. Let us consider the slit to be divided into two halves, with sources 1 through 50 in the upper half and sources 51 through 100 in the lower half. When the path-length difference between the middle of the upper half and the middle of the lower half of the slit equals a half-wavelength, the path-length difference between source 1 (the first source in the upper half) and source 51 (the first source in the lower half) is also $\frac{1}{2}\lambda$. The waves from those two sources will be out of phase by 180° and will thus cancel. Similarly, waves from the second source in each region (source 2 and source 52) will cancel. Continuing this argument, we can see that the waves from each pair of sources separated by $a/2$ will cancel. Thus, there will be no light energy at that angle. We can extend this argument to the second and third minima in the diffraction pattern of Figure 33-11. At an angle θ_2 where $a\sin\theta_2 = 2\lambda$, we can divide the slit into four regions, two regions for the top half and two regions for the bottom half. Using this same argument, the light intensity from the top half is zero because of the cancellation of pairs of sources; similarly, the light intensity from the bottom half is zero. The general expression for the points of zero intensity in the diffraction pattern of a single slit is thus

$$a\sin\theta_m = m\lambda \qquad m = 1, 2, 3, \ldots \qquad 33\text{-}11$$

POINTS OF ZERO INTENSITY FOR A SINGLE-SLIT DIFFRACTION PATTERN

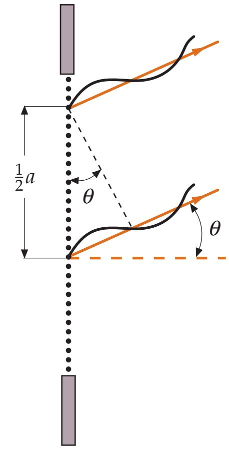


FIGURE 33-12 A single slit is represented by a large number of point sources of equal amplitude. At the first diffraction minimum of a single slit, the waves from each point source in the upper half of the slit are 180° out of phase with the wave from the point source a distance $a/2$ lower in the slit. Thus, the interference from each such pair of point sources is destructive.

Usually, we are just interested in the first occurrence of a minimum in the light intensity because nearly all of the light energy is contained in the central diffraction maximum.

In Figure 33-13, the distance y_1 from the central maximum to the first diffraction minimum is related to the angle θ_1 and the distance L from the slit to the screen by

$$\tan\theta_1 = \frac{y_1}{L}$$

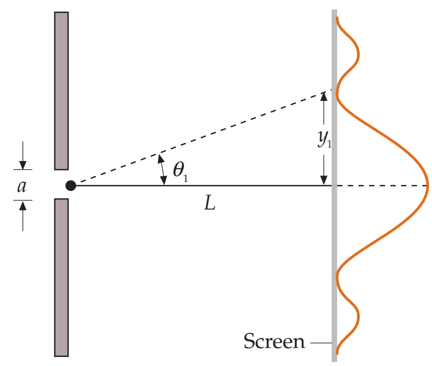


FIGURE 33-13 The distance y_1 measured along the screen from the central maximum to the first diffraction minimum is related to the angle θ_1 by $\tan\theta_1 = y_1/L$, where L is the distance to the screen.

Example 33-5 Width of the Central Diffraction Maximum

During a lecture demonstration of single-slit diffraction, a laser beam that has a wavelength equal to 700 nm passes through a vertical slit 0.20 mm wide and hits a screen 6.0 m away. Find the width of the central diffraction maximum on the screen; that is, find the distance between the first minimum on the left and the first minimum on the right of the central maximum.

PICTURE Referring to Figure 33-13, the width of the central diffraction maximum is $2y_1$.

SOLVE

1. The half-width of the central maximum y_1 is related to the angle θ_1 by: $\qquad \tan\theta_1 = \dfrac{y_1}{L}$

2. The angle θ_1 is related to the slit width a by Equation 33-11: $\sin\theta_1 = \lambda/a$

3. Solve the step-2 result for θ_1, substitute into the step-1 result, and solve for $2y_1$:

$$2y_1 = 2L\tan\theta_1 = 2L\tan\left(\sin^{-1}\frac{\lambda}{a}\right)$$
$$= 2(6.0\text{ m})\tan\left(\sin^{-1}\frac{700\times10^{-9}\text{ m}}{0.00020\text{ m}}\right)$$
$$= 4.2\times10^{-2}\text{ m} = \boxed{4.2\text{ cm}}$$

CHECK Because $\sin\theta_1 = \lambda/a = (700\text{ nm})/(0.20\text{ mm}) = 0.0035$, we can use the small-angle approximation to evaluate $2y_1$. In this approximation, $\sin\theta_1 = \tan\theta_1$, so $\lambda/a = y_1/L$ and $2y_1 = 2L\lambda/a = 2(6.0\text{ m})(700\text{ nm})/(0.20\text{ mm}) = 4.2$ cm. (This approximate value is in agreement with the exact value to within 0.0006 percent.)

INTERFERENCE–DIFFRACTION PATTERN OF TWO SLITS

When there are two or more slits, the intensity pattern on a screen far away is a combination of the single-slit diffraction pattern of the individual slits and the multiple-slit interference pattern we have studied. Figure 33-14 shows the intensity pattern on a screen far from two slits whose separation d is $10a$, where a is the width of each slit. The pattern is the same as the two-slit pattern that has very narrow slits (Figure 33-11) except that it is modulated by the single-slit diffraction pattern; that is, the intensity due to each slit separately is now not constant but decreases with angle, as shown in Figure 33-14*b*.

Note that the central diffraction maximum in Figure 33-14 has 19 interference maxima—the central interference maximum and 9 maxima on either side. The tenth interference maximum on either side of the central one is at the angle θ_{10}, given by $\sin\theta_{10} = 10\lambda/d = \lambda/a$, because $d = 10a$. This coincides with the position of the first diffraction minimum, so this interference maximum is not seen. At these points, the light from the two slits would be in phase and would interfere constructively, but there is no light coming from either slit because the points are at diffraction minima of each slit. In general, we can see that if $m = d/a$, the mth interference maximum will fall at the first diffraction minimum. Because the mth fringe is not seen, there will be $m - 1$ fringes on each side of the central fringe for a total of N fringes in the central maximum, where N is given by

$$N = 2(m-1) + 1 = 2m - 1 \qquad 33\text{-}12$$

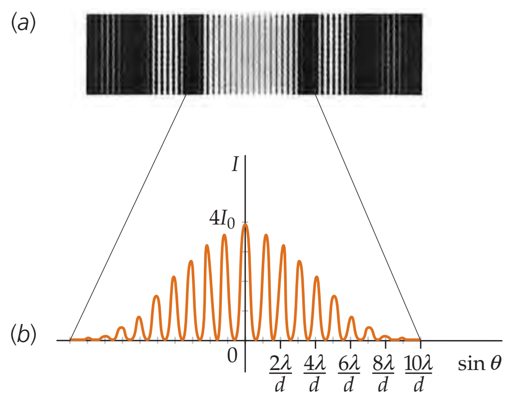


FIGURE 33-14 (*a*) Interference–diffraction pattern for two slits whose separation d is equal to 10 times their width a. The tenth interference maximum on either side of the central interference maximum is missing because it falls at the first diffraction minimum. (*b*) Plot of intensity versus $\sin\theta$ for the central band of the pattern in Figure 33-14*a*. *(Courtesy of Michael Cagnet.)*

Example 33-6 Interference and Diffraction

Two slits that each have a width $a = 0.015$ mm are separated by a distance $d = 0.060$ mm and are illuminated by light of wavelength $\lambda = 650$ nm. How many bright fringes are seen in the central diffraction maximum?

PICTURE We need to find the value of m for which the mth interference maximum coincides with the first diffraction minimum. Then there will be $N = 2m - 1$ fringes in the central maximum.

SOLVE

1. Relate the angle θ_1 of the first diffraction minimum to the slit width a: $\sin\theta_1 = \dfrac{\lambda}{a}$ (first diffraction minimum)

2. Relate the angle θ_m of the mth interference maxima to the slit separation d: $\sin\theta_m = \dfrac{m\lambda}{d}$ (mth interference maxima)

3. Set the angles equal and solve for m:

$$\frac{m\lambda}{d} = \frac{\lambda}{a}$$

$$m = \frac{d}{a} = \frac{0.060\text{ mm}}{0.015\text{ mm}} = 4.0$$

4. The first diffraction minimum coincides with the fourth bright fringe. Therefore, there are 3 bright fringes visible on either side of the central diffraction maximum. These 6 maxima, plus the central interference maximum, combine for a total of 7 bright fringes in the central diffraction maximum:

$$N = \boxed{7\text{ bright fringes}}$$

* 33-5 USING PHASORS TO ADD HARMONIC WAVES

To calculate the interference pattern produced by three, four, or more coherent light sources and to calculate the diffraction pattern of a single slit, we need to combine several harmonic waves of the same frequency that differ in phase. A simple geometric interpretation of harmonic wave functions leads to a method of adding harmonic waves of the same frequency by geometric construction.

Let the wave functions for two waves at some point be $E_1 = A_1 \sin\alpha$ and $E_2 = A_2 \sin(\alpha + \delta)$, where $\alpha = \omega t$. Our problem is then to find the sum:

$$E_1 + E_2 = A_1 \sin\alpha + A_2 \sin(\alpha + \delta)$$

We can represent each wave function by the y component of a two-dimensional vector, as shown in Figure 33-15. The geometric method of addition is based on the fact that the y component of the sum of two or more vectors equals the sum of the y components of the vectors, as illustrated in the figure. The wave function E_1 is represented by the y component of the vector $\vec{A}_1$. As the time continues on, this vector rotates in the xy plane with angular frequency ω. The vector $\vec{A}_1$ is called a **phasor.** (We encountered phasors in our study of ac circuits in Section 29-5.) The wave function E_2 is the y component of a phasor of magnitude A_2 that makes an angle $\alpha + \delta$ with the x axis. By the laws of vector addition, the sum of the y components of the individual phasors equals the y component of the resultant phasor $\vec{A}$, as shown in Figure 33-15. The y component of the resultant phasor, $A \sin(\alpha + \delta')$, is a harmonic wave function that is the sum of the two original wave functions. That is,

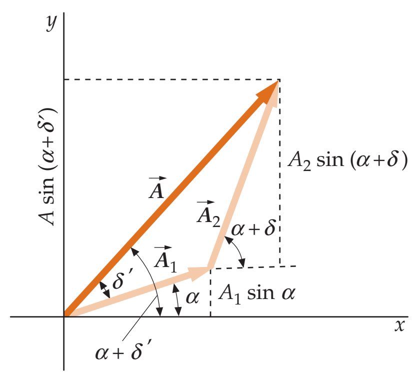


FIGURE 33-15 Phasor representation of wave functions.

$$A_1 \sin\alpha + A_2 \sin(\alpha + \delta) = A \sin(\alpha + \delta') \qquad 33\text{-}13$$

where A (the amplitude of the resultant wave) and δ' (the phase of the resultant wave relative to the phase of the first wave) are found by adding the phasors representing the waves. As time varies, α varies. The phasors representing the two wave functions and the resultant phasor representing the resultant wave function rotate in space, but their relative positions do not change because they all rotate with the same angular velocity ω.

Example 33-7 Wave Superposition Using Phasors *Try It Yourself*

Use the phasor method of addition to derive $E = \left[2A_0 \cos\frac{1}{2}\delta\right] \sin\left(\omega t + \frac{1}{2}\delta\right)$ (Equation 33-7) for the superposition of two waves of the same amplitude.

PICTURE Represent the waves $y_1 = A_0 \sin\alpha$ and $y_2 = A_0 \sin(\alpha + \delta)$ by vectors (phasors) of length A_0 making an angle δ with one another. The resultant wave $y_r = A \sin(\alpha + \delta')$ is represented by the sum of these vectors, which form an isosceles triangle, as shown in Figure 33-16.

SOLVE

Cover the column to the right and try these on your own before looking at the answers.

Steps	Answers
1. Relate δ and δ' using the theorem: "An external angle to a triangle is equal to the sum of the two non-adjacent internal angles."	$\delta' + \delta' = \delta$
2. Solve for δ'.	$\delta' = \frac{1}{2}\delta$
3. Write $\cos\delta'$ in terms of A and A_0.	$\cos\delta' = \dfrac{\frac{1}{2}A}{A_0}$
4. Solve for A in terms of δ.	$A = 2A_0\cos\delta' = 2A_0\cos\frac{1}{2}\delta$
5. Use your results for A and δ' to write the resultant wave function.	$y_r = A\sin(\alpha + \delta')$ $= \boxed{\left[2A_0\cos\frac{1}{2}\delta\right]\sin\left(\alpha + \frac{1}{2}\delta\right)}$

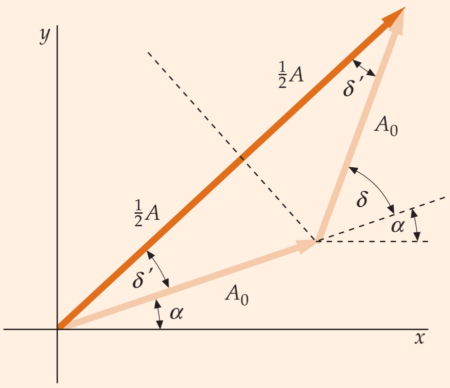


FIGURE 33-16

CHECK The step-5 result is identical to Equation 33-7 (see Problem statement).

PRACTICE PROBLEM 33-3 Find the amplitude and phase constant of the resultant wave function produced by the superposition of the two waves $E_1 = (4.0\text{ V/m})\sin(\omega t)$ and $E_2 = (3.0\text{ V/m})\sin(\omega t + 90°)$.

*THE INTERFERENCE PATTERN OF THREE OR MORE EQUALLY SPACED SOURCES

We can apply the phasor method of addition to calculate the interference pattern of three or more coherent sources that are equally spaced and in phase. We are most interested in the location of the interference maxima and minima. Figure 33-17 illustrates the case of three such sources. The geometry is the same as for two sources. At a great distance from the sources, the rays from the sources to a point P on the screen are approximately parallel. The path-length difference between the first and second source is then $d\sin\theta$, as before, and the path-length difference between the first and third source is $2d\sin\theta$. The wave at point P is the sum of the three waves. Let $\alpha = \omega t$ be the phase of the first wave at point P. We thus have the problem of adding three waves of the form

$$E_1 = A_0\sin\alpha$$

$$E_2 = A_0\sin(\alpha + \delta) \qquad 33\text{-}14$$

$$E_3 = A_0\sin(\alpha + 2\delta)$$

where

$$\delta = \frac{2\pi}{\lambda}d\sin\theta \approx \frac{2\pi}{\lambda}\frac{yd}{L} \qquad 33\text{-}15$$

as in the two-slit problem.

At $\theta = 0$, $\delta = 0$, so all the waves are in phase. The amplitude of the resultant wave is 3 times that of each individual wave and the intensity is 9 times that due to each source acting separately. As the angle θ increases from $\theta = 0$, the phase angle δ increases and the intensity decreases. The position $\theta = 0$ is thus a position of maximum intensity.

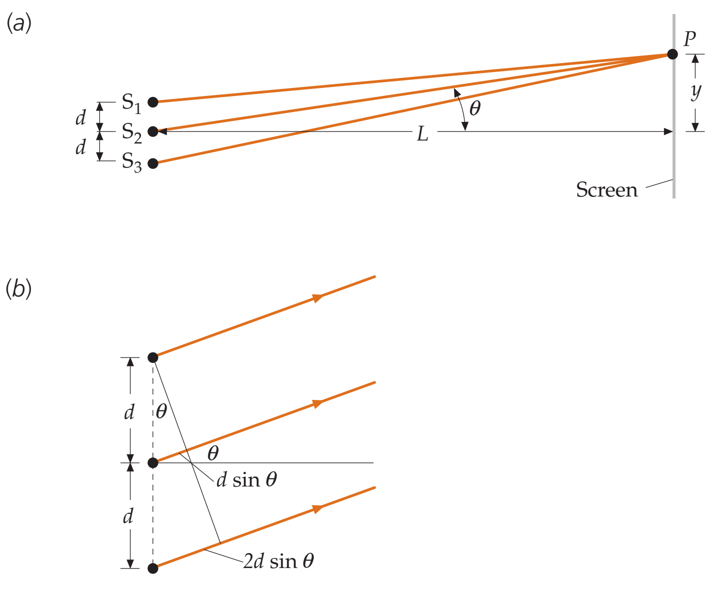


FIGURE 33-17 Geometry for calculating the intensity pattern far away from three equally spaced, coherent sources that are in phase.

Figure 33-18 shows the phasor addition of three waves for a phase angle $\delta = 30° = \pi/6$ rad. This corresponds to a point P on the screen for which θ is given by $\sin\theta = \lambda\delta/(2\pi d) = \lambda/(12d)$. The resultant amplitude A is considerably less than 3 times the amplitude A_0 of each source. As δ increases, the resultant amplitude decreases until the amplitude is zero at $\delta = 120°$. For this value of δ, the three phasors form an equilateral triangle (Figure 33-19). This first interference minimum for three sources occurs at a smaller value of δ (and therefore at a smaller angle θ) than it does for only two sources (for which the first interference minimum occurs at $\delta = 180°$). As δ increases from 120°, the resultant amplitude increases, reaching a secondary maximum at $\delta = 180°$. At the phase angle $\delta = 180°$, the amplitude is the same as that from a single source, because the waves from the first two sources cancel each other, leaving only the third. The intensity of the secondary maximum is one-ninth that of the maximum at $\theta = 0$. As δ increases beyond 180°, the amplitude again decreases and is zero at $\delta = 180° + 60° = 240°$. For δ greater than 240°, the amplitude increases and is again 3 times that of each source when $\delta = 360°$. This phase angle corresponds to a path-length difference of 1 wavelength for the waves from the first two sources and 2 wavelengths for the waves from the first and third sources. Hence, the three waves are in phase at this point. The largest maxima, called the principal maxima, are at the same positions as for just two sources, which are those points corresponding to the angles θ given by

$$d\sin\theta_m = m\lambda \quad m = 0, 1, 2, \ldots \tag{33-16}$$

These maxima are stronger and narrower than those for two sources. They occur at points for which the path-length difference between adjacent sources is zero or an integral number of wavelengths.

These results can be generalized to more than three sources. For four coherent sources that are equally spaced and in phase, the principal interference maxima are again given by Equation 33-16, but the maxima are even more intense, they are narrower, and there are two small secondary maxima between each pair of principal maxima. At $\theta = 0$, the intensity is 16 times that due to a single source. The first interference minimum occurs when δ is 90°, as can be seen from the phasor diagram of Figure 33-20. The first secondary maximum is near $\delta = 132°$. The intensity of the secondary maximum is about one-fourteenth that of the central maximum. There is another minimum at $\delta = 180°$, another secondary maximum near $\delta = 228°$, and another minimum at $\delta = 270°$ before the next principal maximum at $\delta = 360°$.

Figure 33-21 shows the intensity patterns for two, three, and four equally spaced coherent sources. Figure 33-22 shows a graph of I/I_0, where I_0 is the intensity due to each source acting separately. For three sources, there is a very small secondary maximum between each pair of principal maxima, and the principal maxima are sharper and more intense than those due to just two sources. For four sources, there are two small secondary maxima between each pair of principal maxima, and the principal maxima are even more narrow and intense.

From this discussion, we can see that as we increase the number of sources, the intensity becomes more and more concentrated in the principal maxima given by Equation 33-16, and these maxima become narrower. For N sources, the intensity of the principal maxima is N^2 times that due to a single source. The first minimum occurs at a phase angle of $\delta = 360°/N$, for which the N phasors form a closed polygon of N sides. There are $N - 2$ secondary maxima between each pair of principal maxima. These secondary maxima are very weak compared with the principal maxima.

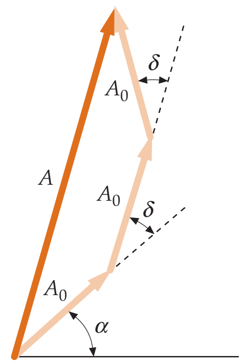


FIGURE 33-18 Phasor diagram for determining the resultant amplitude A due to three waves, each of amplitude A_0, that have phase differences of δ and 2δ due to path-length differences of $d\sin\theta$ and $2d\sin\theta$. The angle $\alpha = \omega t$ varies with time, but this does not affect the calculation of A.

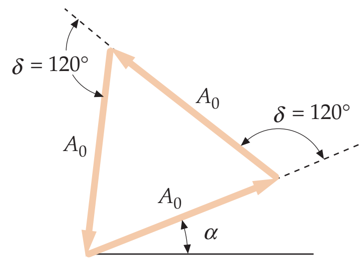


FIGURE 33-19 The resultant amplitude for the waves from three sources is zero when δ is 120°. This interference minimum occurs at a smaller angle θ than does the first minimum for two sources, which occurs when δ is 180°.

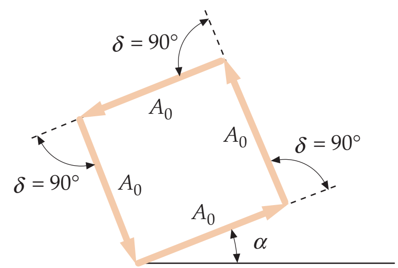


FIGURE 33-20 Phasor diagram for the first minimum for four coherent sources that are equally spaced and in phase. The amplitude is zero when the phase difference of the waves from adjacent sources is 90°.

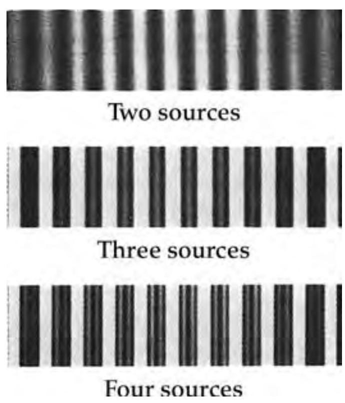


FIGURE 33-21 Intensity patterns for two, three, and four coherent sources that are equally spaced and in phase. There is a secondary maximum between each pair of principal maxima for three sources, and two secondary maxima between each pair of principal maxima for four sources. *(Courtesy of Michael Cagnet.)*

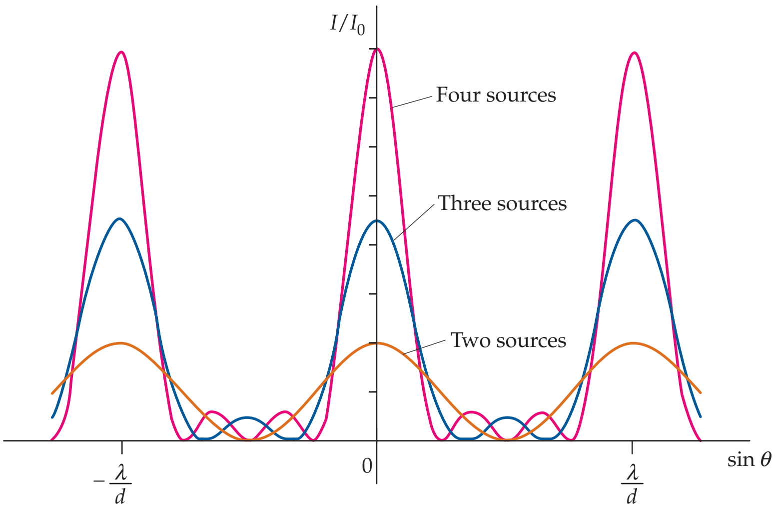


FIGURE 33-22 Plot of relative intensity versus $\sin\theta$ for two, three, and four coherent sources that are equally spaced and in phase.

As the number of sources is increased, the principal maxima become sharper and more intense, and the intensities of the secondary maxima become negligible compared to those of the principal maxima.

*CALCULATING THE SINGLE-SLIT DIFFRACTION PATTERN

We now use the phasor method for the addition of harmonic waves to calculate the intensity pattern shown in Figure 33-11. We assume that the slit of width a is divided into N equal intervals and that there is a point source of waves at the midpoint of each interval (Figure 33-23). If d is the distance between two adjacent sources and a is the width of the opening, we have $d = a/N$. Because the screen on which we are calculating the intensity is far from the sources, the rays from the sources to a point P on the screen are approximately parallel. The path-length difference between any two adjacent sources is $\delta \sin\theta$, and the phase difference δ is related to the path-length difference by

$$\delta = \frac{d\sin\theta}{\lambda}2\pi$$

If A_0 is the amplitude due to a single source, the amplitude at the central maximum, where $\theta = 0$ and all the waves are in phase, is $A_{\max} = NA_0$ (Figure 33-24).

We can find the amplitude at some other point at an angle θ by using the phasor method for the addition of harmonic waves. As in the addition of two, three, or four waves, the intensity is zero at any point where the phasors representing the waves form a closed polygon. In this case, the polygon has N sides (Figure 33-25). At the first minimum, the wave from the first source just below the top of the opening and the wave from the source just below the middle of the opening are 180° out of phase. In this case, the waves from the source near the top of the opening differ

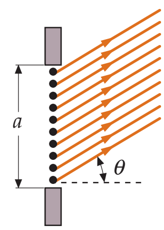


FIGURE 33-23 Diagram for calculating the diffraction pattern far away from a narrow slit. The slit width a is assumed to contain a large number of in-phase, equally spaced point sources separated by a distance d. The rays from the sources to a point far away are approximately parallel. The path-length difference for the waves from adjacent sources is $d\sin\theta$.

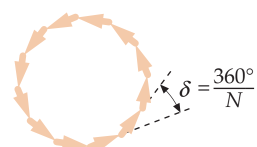


FIGURE 33-25 Phasor diagram for calculating the first minimum in the single-slit diffraction pattern. When the waves from the N sources completely cancel, the N phasors form a closed polygon. The phase difference between the waves from adjacent sources is then $\delta = 360°/N$. When N is very large, the waves from the first and last sources are approximately in phase.

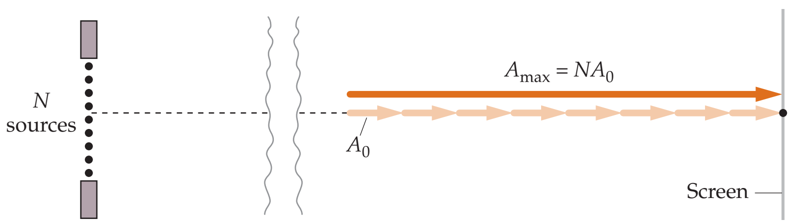


FIGURE 33-24 A single slit is represented by N sources, each of amplitude A_0. At the central maximum point, where $\theta = 0$, the waves from the sources add in phase, giving a resultant amplitude $A_{\max} = NA_0$.

from those from the bottom of the opening by nearly 360°. [The phase difference is, in fact, $360° - (360°/N)$.] Thus, if the number of sources is very large, $360°/N$ is negligible and we get complete cancellation if the waves from the first and last sources are out of phase by 360°, corresponding to a path-length difference of one wavelength, in agreement with Equation 33-11.

We will now calculate the amplitude at a general point at which the waves from two adjacent sources differ in phase by δ. Figure 33-26 shows the phasor diagram for the addition of N waves, where the subsequent waves differ in phase from the first wave by $\delta, 2\delta, \ldots, (N-1)\delta$. When N is very large and δ is very small, the phasor diagram approximates the arc of a circle. The resultant amplitude A is the length of the chord of this arc. We will calculate this resultant amplitude in terms of the phase difference ϕ between the first wave and the last wave. From Figure 33-26, we have

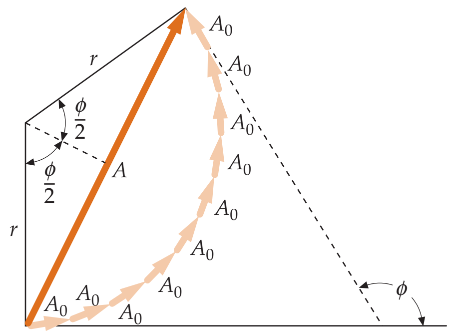


FIGURE 33-26 Phasor diagram for calculating the resultant amplitude due to the waves from N sources in terms of the phase difference ϕ between the wave from the first source just below the top of the slit and the wave from the last source just above the bottom of the slit. When N is very large, the resultant amplitude A is the chord of a circular arc of length $NA_0 = A_{max}$.

$$\sin\tfrac{1}{2}\phi = \frac{A/2}{r}$$

or

$$A = 2r\sin\tfrac{1}{2}\phi \qquad 33\text{-}17$$

where r is the radius of the arc. Because the length of the arc is $A_{max} = NA_0$ and the angle subtended is ϕ, we have

$$\phi = \frac{A_{max}}{r} \qquad 33\text{-}18$$

or

$$r = \frac{A_{max}}{\phi}$$

Substituting this into Equation 33-17 gives

$$A = \frac{2A_{max}}{\phi}\sin\tfrac{1}{2}\phi = A_{max}\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}$$

Because the amplitude at the center of the central maximum ($\theta = 0$) is A_{max}, the ratio of the intensity at any other point to that at the center of the central maximum is

$$\frac{I}{I_0} = \frac{A^2}{A_{max}^2} = \left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2$$

or

$$I = I_0\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2 \qquad 33\text{-}19$$

INTENSITY FOR A SINGLE-SLIT DIFFRACTION PATTERN

The phase difference ϕ between the first and last waves is related to the path-length difference $a\sin\theta$ between the top and bottom of the opening by

$$\phi = \frac{a\sin\theta}{\lambda}2\pi \qquad 33\text{-}20$$

Equation 33-19 and Equation 33-20 describe the intensity pattern shown in Figure 33-11. The first minimum occurs at $a\sin\theta = \lambda$, which is the point where the waves from the middle of the upper half and the middle of the lower half of the slit have a path-length difference of $\lambda/2$ and are 180° out of phase. The second minimum occurs at $a\sin\theta = 2\lambda$, where the waves from the upper half of the upper half of the slit and those from the lower half of the upper half of the slit have a path-length difference of $\lambda/2$ and are 180° out of phase.

There is a secondary maximum approximately midway between the first and second minima at $a \sin\theta \approx \frac{3}{2}\lambda$. Figure 33-27 shows the phasor diagram for determining the approximate intensity of this secondary maximum. The phase difference ϕ between the first and last waves is approximately $2\pi + \pi$. The phasors thus complete $1\frac{1}{2}$ circles. The resultant amplitude is the diameter of a circle that has a circumference which is two-thirds the total length A_{max}. If $C = \frac{2}{3}A_{max}$ is the circumference, the diameter A is

$$A = \frac{C}{\pi} = \frac{\frac{2}{3}A_{max}}{\pi} = \frac{2}{3\pi}A_{max}$$

and

$$A^2 = \frac{4}{9\pi^2}A_{max}^2$$

The intensity at this point is

$$I = \frac{4}{9\pi^2}I_0 = \frac{1}{22.2}I_0 \qquad 33\text{-}21$$

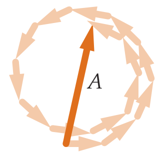


FIGURE 33-27 Phasor diagram for calculating the approximate amplitude of the first secondary maximum of the single-slit diffraction pattern. The secondary maximum occurs near the midpoint between the first and second minima when the N phasors complete $1\frac{1}{2}$ circles.

*CALCULATING THE INTERFERENCE–DIFFRACTION PATTERN OF MULTIPLE SLITS

The intensity of the two-slit interference–diffraction pattern can be calculated from the two-slit pattern (Equation 33-8) where the intensity of each slit (I_0 in that equation) is replaced by the diffraction pattern intensity due to each slit, I, given by Equation 33-19. The intensity for the two-slit interference–diffraction pattern is thus

$$I = 4I_0\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2\cos^2\tfrac{1}{2}\delta \qquad 33\text{-}22$$

INTERFERENCE–DIFFRACTION INTENSITY FOR TWO SLITS

where ϕ is the difference in phase between rays from the top and bottom of each slit, which is related to the width of each slit by

$$\phi = \frac{a\sin\theta}{\lambda}2\pi$$

and δ is the difference in phase between rays from the centers of two adjacent slits, which is related to the slit separation by

$$\delta = \frac{d\sin\theta}{\lambda}2\pi$$

In Equation 33-22, the intensity I_0 is the intensity at $\theta = 0$ due to one slit alone.

Example 33-8 Five-Slit Interference–Diffraction Pattern

Find the interference–diffraction intensity pattern for five equally spaced slits, where a is the width of each slit and d is the distance between adjacent slits.

PICTURE First, find the interference intensity pattern for the five slits, assuming no angular variations in the intensity due to diffraction. To do this, first construct a phasor diagram to find the amplitude of the resultant wave in an arbitrary direction θ. Intensity is proportional to the square of the amplitude. Next, correct for the variation of intensity with θ by using the single-slit diffraction pattern intensity relation (Equation 33-20 and Equation 33-20).

SOLVE

1. The diffraction pattern intensity I' due to a slit of width a is given by Equation 33-19 and Equation 33-20:

$$I' = I_0\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2$$

where

$$\phi = \frac{2\pi}{\lambda}a\sin\theta$$

2. The interference pattern intensity I is proportional to the square of the amplitude A of the superposition of the wave functions for the light from the five slits:

$$I \propto A^2$$

where

$$A\sin(\alpha + \delta') = A_0\sin\alpha + A_0\sin(\alpha + \delta) + A_0\sin(\alpha + 2\delta) + A_0\sin(\alpha + 3\delta) + A_0\sin(\alpha + 4\delta)$$

and where $\alpha = \omega t$ and $\delta = \dfrac{d\sin\theta}{\lambda}2\pi$

3. To solve for A, we construct a phasor diagram (Figure 33-28). The amplitude A equals the sum of the projections of the individual phasors onto the resultant phasor:

$$\delta' = \beta + \delta$$

so

$$\beta = \delta' - \delta = 2\delta - \delta = \delta$$

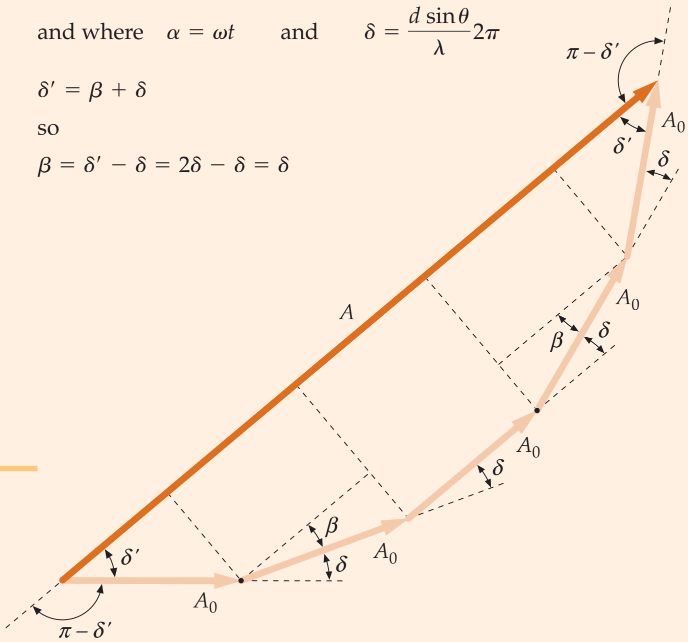


FIGURE 33-28

4. To find δ', we add the exterior angles. The sum of the exterior angles equals 2π (if you walk the perimeter of any polygon you rotate through the sum of the exterior angles, and you rotate through 2π radians):

$$2(\pi - \delta') + 4\delta = 2\pi \quad \Rightarrow \quad \delta' = 2\delta$$

5. Solve for A from the figure:

$$A = 2A_0\cos\delta' + 2A_0\cos\beta + A_0$$

6. Substitute for δ' using the step-4 result, and substitute for β using the relation $\beta = \delta$. (That β and δ are equal follows from the theorem "If two parallel lines are cut by a transversal, the interior and exterior angles on the same side of the transversal are equal."):

$$A = A_0(2\cos 2\delta + 2\cos\delta + 1)$$

7. Square both sides to relate the intensities. Recall, I' is the intensity from a single slit, and A_0 is the amplitude from a single slit:

$$A^2 = A_0^2(2\cos 2\delta + 2\cos\delta + 1)^2$$

so

$$I = I'(2\cos 2\delta + 2\cos\delta + 1)^2$$

8. Substitute for I' using the step-1 result:

$$\boxed{I = I_0\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2(2\cos 2\delta + 2\cos\delta + 1)^2}$$

where $\phi = \dfrac{a\sin\theta}{\lambda}2\pi$ and $\delta = \dfrac{d\sin\theta}{\lambda}2\pi$

CHECK If $\theta = 0$, both $\phi = 0$ and $\delta = 0$. So, for $\theta = 0$, step 5 becomes $A = 5A_0$ and step 8 becomes $I = 5^2 I_0 = 25I_0$ as expected.

33-6 FRAUNHOFER AND FRESNEL DIFFRACTION

Diffraction patterns, like the single-slit pattern shown in Figure 33-11, that are observed at points for which the rays from an aperture or an obstacle are nearly parallel are called **Fraunhofer diffraction patterns.** Fraunhofer patterns can be observed at great distances from the obstacle or the aperture so that the rays reaching any point are approximately parallel, or they can be observed using a lens to focus parallel rays on a viewing screen placed in the focal plane of the lens.

The diffraction pattern observed near an aperture or an obstacle is called a **Fresnel diffraction pattern.** Because the rays from an aperture or an obstacle close to a screen cannot be considered parallel, Fresnel diffraction is much more difficult to analyze. Figure 33-29 illustrates the difference between the Fresnel and the Fraunhofer patterns for a single slit.*

Figure 33-30*a* shows the Fresnel diffraction pattern of an opaque disk. Note the bright spot at the center of the pattern caused by the constructive interference of the light waves diffracted from the edge of the disk. This pattern is of some historical interest. In an attempt to discredit Augustin Fresnel's wave theory of light, Siméon Poisson pointed out that it predicted a bright spot at the center of the shadow, which he assumed was a ridiculous contradiction of fact. However, Fresnel immediately demonstrated experimentally that such a spot does, in fact, exist. This demonstration convinced many doubters of the validity of the wave theory of light. The Fresnel diffraction pattern of a circular aperture is shown in Figure 33-30*b*. Comparing this with the pattern of the opaque disk in Figure 33-30*a*, we can see that the two patterns are complements of each other.

Figure 33-31*a* shows the Fresnel diffraction pattern of a straightedge illuminated by light from a point source. A graph of the intensity versus distance (measured along a line perpendicular to the edge) is shown in Figure 33-31*b*. The light intensity does not fall abruptly to zero in the geometric shadow, but it decreases rapidly and is negligible within a few wavelengths of the edge. The Fresnel diffraction pattern

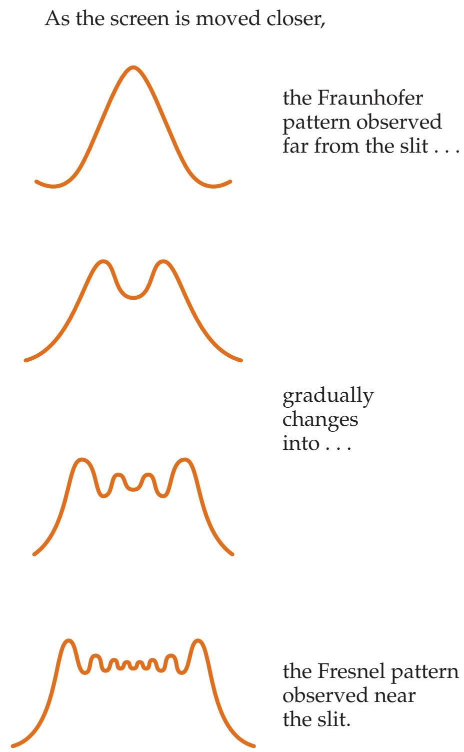


FIGURE 33-29 Diffraction patterns for a single slit at various screen distances.

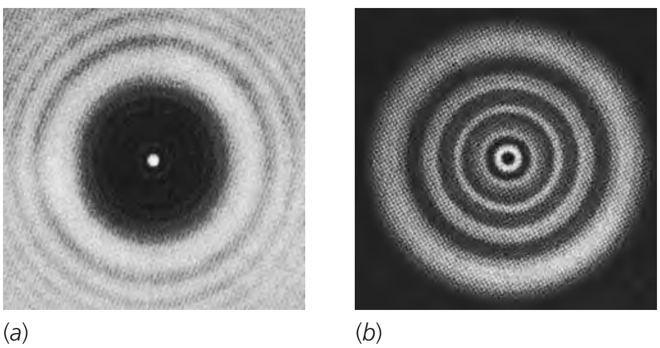


FIGURE 33-30 (*a*) The Fresnel diffraction pattern of an opaque disk. At the center of the shadow, the light waves diffracted from the edge of the disk are in phase and produce a bright spot called the *Poisson spot.* (*b*) The Fresnel diffraction pattern of a circular aperture. Compare this with Figure 33-30*a*. *((a) and (b) M. Cagnet, M. Françon, J. C. Thrierr, Atlas of Optical Phenomena.)*

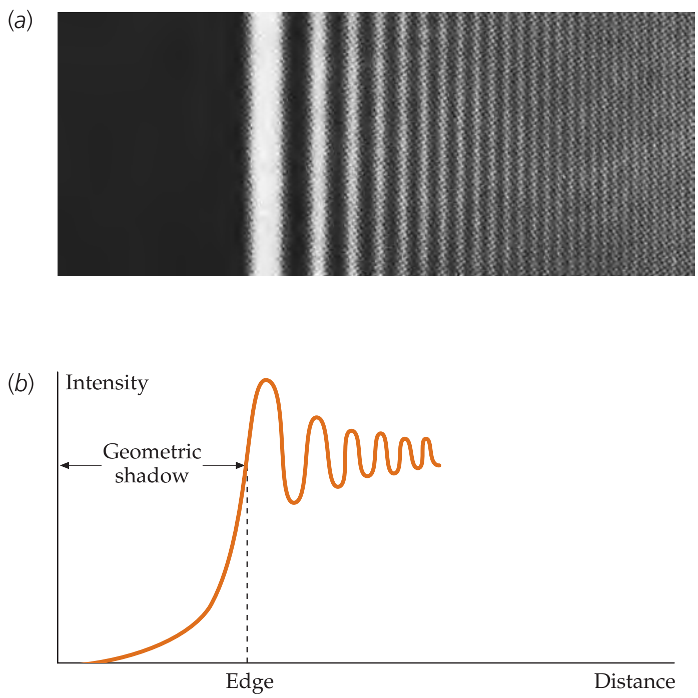


FIGURE 33-31 (*a*) The Fresnel diffraction of a straightedge. (*b*) A graph of intensity versus distance along a line perpendicular to the edge. *(Courtesy Battelle-Northwest Laboratories.)*

* See Richard E. Haskel, "A Simple Experiment on Fresnel Diffraction," *American Journal of Physics* 38 (1970): 1039.

of a rectangular aperture is shown in Figure 33-32. These patterns cannot be seen using extended light sources like an ordinary lightbulb, because the dark fringes of the pattern produced by light from one point on the source overlap the bright fringes of the pattern produced by light from another point.

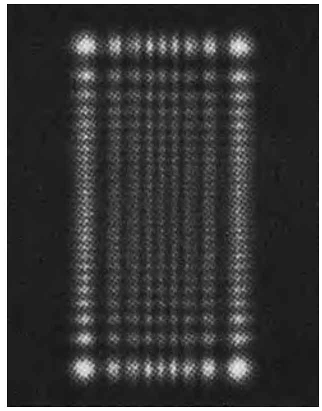


FIGURE 33-32 The Fresnel diffraction pattern of a rectangular aperture. *(Courtesy of Michael Cagnet.)*

33-7 DIFFRACTION AND RESOLUTION

Diffraction due to a circular aperture has important implications for the resolution of many optical instruments. Figure 33-33 shows the Fraunhofer diffraction pattern of a circular aperture. The angle θ subtended by the first diffraction minimum is related to the wavelength and the diameter of the opening D by

$$\sin\theta = 1.22\frac{\lambda}{D} \qquad 33\text{-}23$$

Equation 33-23 is similar to Equation 33-9 except for the factor 1.22, which arises from the mathematical analysis, and is similar to the equation for a single slit but more complicated because of the circular geometry. In many applications, the angle θ is small, so $\sin\theta$ can be replaced by θ. The first diffraction minimum is then at an angle θ given by

$$\theta \approx 1.22\frac{\lambda}{D} \qquad 33\text{-}24$$

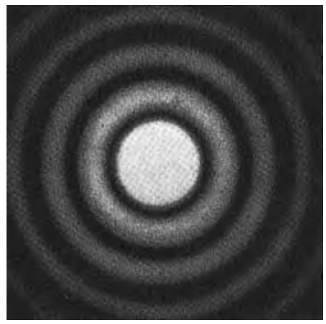


FIGURE 33-33 The Fraunhofer diffraction pattern of a circular aperture. *(Courtesy of Michael Cagnet.)*

Figure 33-34 shows two point sources that subtend an angle α at a circular aperture far from the sources. The intensities of the Fraunhofer diffraction pattern are also indicated in this figure. If α is much greater than $1.22\lambda/D$, the sources will be seen as two sources. However, as α is decreased, the overlap of the diffraction patterns increases, and it becomes difficult to distinguish the two sources from one source. At the critical angular separation, α_c, given by

$$\alpha_c = 1.22\frac{\lambda}{D} \qquad 33\text{-}25$$

the first minimum of the diffraction pattern of one source falls on the central maximum of the other source. These objects are said to be just resolved by **Rayleigh's criterion for resolution.** Figure 33-35 shows the diffraction patterns for two sources when α is greater than the critical angle for resolution and when α is just equal to the critical angle for resolution.

Equation 33-25 has many applications. The *resolving power* of an optical instrument, such as a microscope or telescope, is the ability of the instrument to resolve two objects that are close together. The images of the objects tend to overlap because of diffraction at the entrance aperture of the instrument. We can see from Equation 33-25 that the resolving power can be increased either by increasing the diameter D of the lens (or mirror) or by decreasing the wavelength λ. Astronomical telescopes use large objective lenses or mirrors to increase their resolution as well as to increase their light-gathering power. An array of 27 radio antennas (Figure 33-36) mounted on rails can be configured to form a single telescope that has a resolution distance D of 36 km (22 mi). In a microscope, a film of transparent oil that has an index of refraction

(a)

(b)

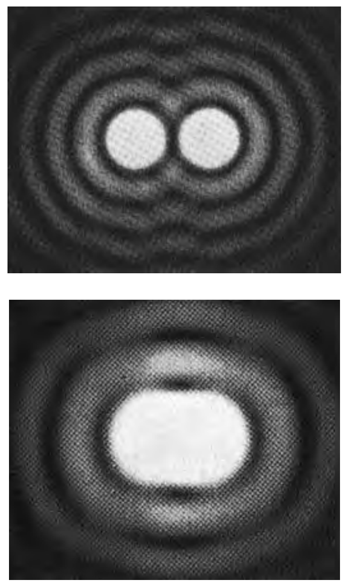


FIGURE 33-35 The diffraction patterns for a circular aperture and two incoherent point sources when (*a*) α is a factor of 2 or so greater than $\alpha_c = 1.22\lambda/D$ and (*b*) when α is equal to the limit of resolution, $\alpha_c = 1.22\lambda/D$. *((a) and (b) Courtesy of Michael Cagnet.)*

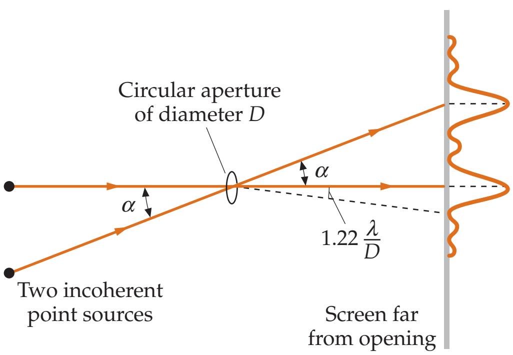


FIGURE 33-34 Two distant sources that subtend an angle α. If α is much greater than $1.22\lambda/D$, where λ is the wavelength of light and D is the diameter of the aperture, the diffraction patterns have little overlap and the sources are easily seen as two distinct sources. If α is not much greater than $1.22\lambda/D$, the overlap of the diffraction patterns makes it difficult to distinguish two sources from one.

of approximately 1.55 is sometimes used under the objective to decrease the wavelength of the light ($\lambda' = \lambda/n$). The wavelength can be reduced further by using ultraviolet light and photographic film; however, ordinary glass is opaque to ultraviolet light, so the lenses in an ultraviolet microscope must be made from quartz or fluorite. To obtain very high resolutions, electron microscopes are used—microscopes that use electrons rather than light. The wavelengths of electrons vary inversely with the square root of their kinetic energy and can be made as small as desired.*

FIGURE 33-36 The very large array (VLA) of radio antennas is located near Socorro, New Mexico. The 25-m-diameter antennas are mounted on rails, which can be arranged in several configurations, and can be extended over a diameter of 36 km. The data from the antennas are combined electronically, so the array is really a single high-resolution telescope. *(Courtesy of National Radio Astronomy Observatory/Associated Universities, Inc./National Science Foundation. Photographer: Kelly Gatlin. Digital composite: Patricia Smiley.)*

Example 33-9 Physics in the Library

Context-Rich

While studying in the library, you lean back in your chair and ponder the small holes you notice in the ceiling tiles. You notice that the holes are approximately 5.0 mm apart. You can clearly see the holes directly above you, about 2 m up, but the tiles far away do not appear to have the holes. You wonder if the reason you cannot see the distant holes is because they are not within the criteria for resolution established by Rayleigh. Is this a feasible explanation for the disappearance of the holes? You notice the holes disappear about 20 m from you.

PICTURE We will need to make assumptions about the situation. If we use Equation 33-25, we will need to know the wavelength of light and the aperture diameter. Assuming our pupil is the aperture, we can assume approximately 5.0 mm for the diameter. (This is the value used in this physics textbook.) The wavelength of the light is probably about 500 nm or so.

SOLVE

1. The angular limit for resolution by the eye depends on the ratio of the wavelength and the diameter of the pupil:

$$\theta_c \approx 1.22\frac{\lambda}{D}$$

2. The angle subtended by two holes depends on their separation distance d and their distance L from your eye:

$$\theta \approx \frac{d}{L}$$

3. Equating the two angles and putting in the numbers give:

$$\frac{d}{L} \approx 1.22\frac{\lambda}{D}$$

$$\frac{5.0\text{ mm}}{L} \approx 1.22\frac{500\text{ nm}}{5.0\text{ mm}}$$

4. Solving for L gives:

$$L = 41\text{ m}$$

5. By a factor of 2, 41 m is too large. However, you are suspect of the value given for the pupil diameter in your physics textbook. You know the pupil is smaller when the light is bright, and the library ceiling is very bright and colored white. An online search for eye pupil diameter soon yields the information you need. The pupil diameter ranges from 2 to 3 mm up to 7 mm:

> Sucess. If the pupil diameter is 2.5 mm, the value of L is 20 m.

It is instructive to compare the limitation on resolution of the eye due to diffraction, as seen in Example 33-9, with the limitation on resolution due to the separation of the receptors (cones) on the retina. To be seen as two distinct objects, the images of the objects must fall on the retina on two nonadjacent cones. (See Problem 65 in Chapter 32.) Because the retina is about 2.5 cm from the cornea, the distance y on the retina corresponding to an angular separation of 1.5×10^{-4} rad is found from

$$\alpha_c = 1.5 \times 10^{-4}\text{ rad} = \frac{y}{2.5\text{ cm}}$$

or

$$y = 3.8 \times 10^{-4}\text{ cm} = 3.8 \times 10^{-6}\text{ m} = 3.8\ \mu\text{m}$$

The actual separation of the cones in the fovea centralis, where the cones are the most tightly packed, is about 1 μm. Outside this region, they are about 3 μm to 5 μm apart.

CONCEPT CHECK 33-2

True or False:
Fraunhofer diffraction is a limiting case of Fresnel diffraction.

* The wave properties of electrons are discussed in Chapter 34.

* 33-8 DIFFRACTION GRATINGS

A widely used tool for measuring the wavelength of light is the **diffraction grating,** which consists of a large number of equally spaced lines or slits on a flat surface. Such a grating can be made by cutting parallel, equally spaced grooves on a glass or metal plate with a precision ruling machine. With a reflection grating, light is reflected from the ridges between the lines or grooves. Phonograph records and compact disks exhibit some of the properties of reflection gratings. In a transmission grating, the light passes through the clear gaps between the rulings. Inexpensive, optically produced plastic gratings that have 10 000 or more slits per centimeter are common items in teaching laboratories. The spacing of the slits in a grating that has 10 000 slits per centimeter is $d = (1\text{ cm})/10\,000\text{ slits} = 10^{-4}\text{ cm slit}$.

Consider a plane wave of monochromatic light that is incident normally on a transmission grating (Figure 33-37). Assume that the width of each slit is very small so that it produces a widely diffracted beam. The interference pattern produced on a screen a large distance from the grating is due to a large number of coherent, equally spaced light sources. Suppose we have N slits that have separation d between adjacent slits. At $\theta = 0$, the light from each slit is in phase with that from all the other slits, so the amplitude of the wave is NA_0, where A_0 is the amplitude from each slit, and the intensity is N^2I_0, where I_0 is the intensity due to a single slit alone. At an angle θ_1, where $d\sin\theta_1 = \lambda_1$, the path-length difference between any two successive slits is λ_1, so again the light from each slit is in phase with that from all the other slits and the intensity is N^2I_0. The interference maxima are thus at angles θ given by

$$d\sin\theta_m = m\lambda \qquad m = 0, 1, 2, \ldots \qquad \text{33-26}$$

The positions of the interference maxima do not depend on the number of sources, but the more sources there are, the sharper (narrower) and more intense the maxima will be.

To see that the interference maxima will be sharper when there are many slits, consider the case of N illuminated slits, where N is large ($N \gg 1$). The distance from the first slit to the Nth slit is $(N-1)d \approx Nd$. When the path-length difference for the light from the first slit and that from the Nth slit is λ, the resulting intensity will be zero because the light from any two slits separated by $\frac{1}{2}Nd$ interferes destructively. (We saw this in our discussion of single-slit diffraction in Section 33-4.) Because the first and Nth slits are separated by approximately Nd, the intensity will be zero at angle $\theta_{\min}$ given by

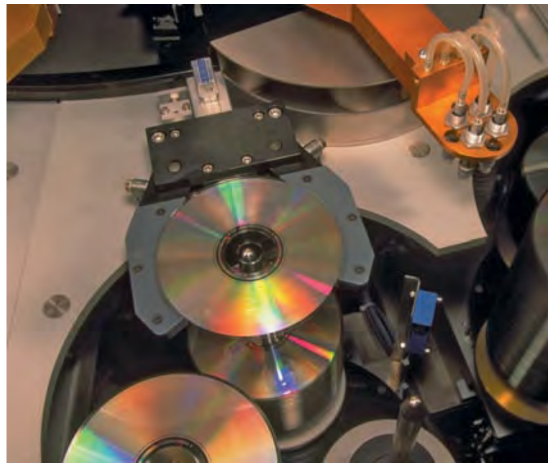
$$Nd\sin\theta_{\min} = \lambda$$

so

$$\theta_{\min} \approx \sin\theta_{\min} = \frac{\lambda}{Nd}$$

The angular width of the interference maximum, which is equal to $2\theta_{\min}$, is thus inversely proportional to N. Therefore, the greater the number of illuminated slits N, the sharper the maximum. Because the intensity in the maximum is proportional to N^2I_0, the intensity in the maximum multiplied by the width of the maximum is proportional to NI_0. The intensity multiplied by the width is a measure of power per unit length in the maximum.

Figure 33-38*a* shows a student spectroscope that uses a diffraction grating to analyze light. In student laboratories, the light source is typically a glass tube containing atoms of a gas (for example, helium or sodium vapor) that are excited by a bombardment of electrons accelerated by high voltage across the tube. The light emitted by such a source contains only certain wavelengths that are characteristic of the atoms in the source. Light from the source passes through a narrow collimating slit and is made parallel by a lens. Parallel light from the lens is incident on the grating. Instead of falling on a screen a large distance away, the parallel light from the grating is focused by a telescope and viewed by the eye. The telescope is



Compact disks act as reflection gratings. *(Kevin R. Morris/Corbis.)*

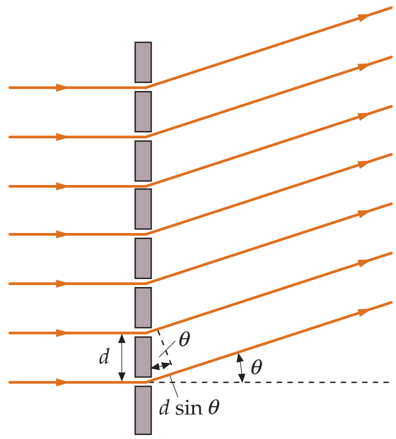


FIGURE 33-37 Light incident normally on a diffraction grating. At an angle θ, the path-length difference between rays from adjacent slits is $d\sin\theta$.


(a)

(b)

mounted on a rotating platform that has been calibrated so that the angle θ can be measured. In the forward direction ($\theta = 0$), the central maximum for all wavelengths is seen. If light of a particular wavelength λ is emitted by the source, the first interference maximum is seen at the angle θ given by $d \sin\theta_m = m\lambda$ (Equation 33-26) with $m = 1$. Each wavelength emitted by the source produces a separate image of the collimating slit in the spectroscope called a **spectral line.** The set of lines corresponding to $m = 1$ is called the **first-order spectrum.** The **second-order spectrum** corresponds to $m = 2$ for each wavelength. Higher orders may be seen, providing the angle θ given by $d \sin\theta_m = m\lambda$ is less than 90°. Depending on the wavelengths, the orders may be mixed; that is, the third-order line for one wavelength may occur at a smaller value of θ than does the second-order line for another wavelength. If the spacing of the slits in the grating is known, the wavelengths emitted by the source can be determined by measuring the angles.

FIGURE 33-38 (*a*) A typical student spectroscope. Light from a collimating slit near the source is made parallel by a lens and falls on a grating. The diffracted light is viewed with a telescope at an angle that can be accurately measured. (*b*) Aerial view of the very large array (VLA) radio telescope in New Mexico. Radio signals from distant galaxies add constructively when Equation 33-26 is satisfied, where *d* is the distance between two adjacent telescopes. *((a) Clarence Bennett/ Oakland University, Rochester, Michigan. (b) NRAO/AUI/Science Photo Library/ Photo Researchers.)*

Example 33-10 Sodium D Lines

Sodium light is incident on a diffraction grating with 12 000 lines per centimeter. At what angles will the two yellow lines (called the sodium D lines) of wavelengths 589.00 nm and 589.59 nm be seen in the first order?

PICTURE Apply $d \sin\theta_m = m\lambda$ to each wavelength, with $m = 1$ and $d = (1/12\,000)$ cm.

SOLVE

1. The angle θ_m is given by $d \sin\theta_m = m\lambda$ with $m = 1$:
$$\sin\theta_1 = \frac{\lambda}{d}$$

2. Calculate θ_1 for $\lambda = 589.00$ nm:
$$\theta_1 = \sin^{-1}\left[\frac{589.00 \times 10^{-9}\text{ m}}{(1/12\,000)\text{ cm}} \times \left(\frac{100\text{ cm}}{1\text{ m}}\right)\right] = \boxed{44.98°}$$

3. Repeat the calculation for $\lambda = 589.59$ nm:
$$\theta_1 = \sin^{-1}\left[\frac{589.59 \times 10^{-9}\text{ m}}{(1/12\,000)\text{ cm}} \times \left(\frac{100\text{ cm}}{1\text{ m}}\right)\right] = \boxed{45.03°}$$

CHECK The first-order intensity maximum for the longer wavelength appears at the larger angle, as expected.

PRACTICE PROBLEM 33-4 Find the angles for the first-order intensity maxima of the two yellow lines if the grating has 15 000 lines per centimeter.

An important feature of a spectroscope is its ability to resolve spectral lines of two nearly equal wavelengths λ_1 and λ_2. For example, the two prominent yellow lines in the spectrum of sodium have wavelengths 589.00 and 589.59 nm. These can be seen as two separate wavelengths if their interference maxima do not overlap. According to Rayleigh's criterion for resolution, these wavelengths are resolved if the angular separation of their interference maxima is greater than the angular separation between an interference maximum and the first interference minimum on either side of it. The **resolving power** of a diffraction grating is defined to be $\lambda/|\Delta\lambda|$, where $|\Delta\lambda|$ is the smallest observable difference between two nearby wavelengths, each approximately equal to λ, that may be resolved. The resolving power is proportional to the number of slits illuminated because the more slits illuminated, the sharper the interference maxima. The resolving power R can be shown to be

$$R = \frac{\lambda}{|\Delta\lambda|} = mN \qquad 33\text{-}27$$

where N is the number of illuminated slits and m is the order number (see Problem 78). We can see from Equation 33-27 that to resolve the two yellow lines in the first order ($m = 1$) of the sodium spectrum the resolving power must be

$$R = 1 \times \frac{589.00\text{ nm}}{589.59\text{ nm} - 589.00\text{ nm}} = 998$$

Thus, to resolve the two yellow sodium lines in the first order, we need a grating containing 998 or more slits in the area illuminated by the light.

*HOLOGRAMS

An interesting application of diffraction gratings is the production of a three-dimensional photograph called a **hologram** (Figure 33-39). In an ordinary photograph, the intensity of reflected light from an object is focused on a light-sensitive surface. As a result, a two-dimensional image is recorded. In a hologram, a beam from a laser is split into two beams, a reference beam and an object beam. The object beam reflects from the object to be photographed, and the interference pattern between it and the reference beam is recorded on a transparent film coated with a photosensitive emulsion. This can be done because the laser beam is coherent so that the relative phase difference between the reference beam and the object beam can be kept constant during the exposure. The film can be used to produce a holographic image after the emulsion is developed (chemically processed). The interference fringes on the film act as a diffraction grating. When the developed film is illuminated with a laser, a three-dimensional holographic image of the object is produced.

Holograms that you see on credit cards or postage stamps, called rainbow holograms, are more complicated. A horizontal strip of the original hologram is used to make a second hologram. The three-dimensional image can be seen as the viewer moves from side to side, but if viewed using monochromatic light, the image disappears when the viewer's eyes move above or below the slit image. When viewed using white light, the image is seen in different colors as the viewer moves in the vertical direction.

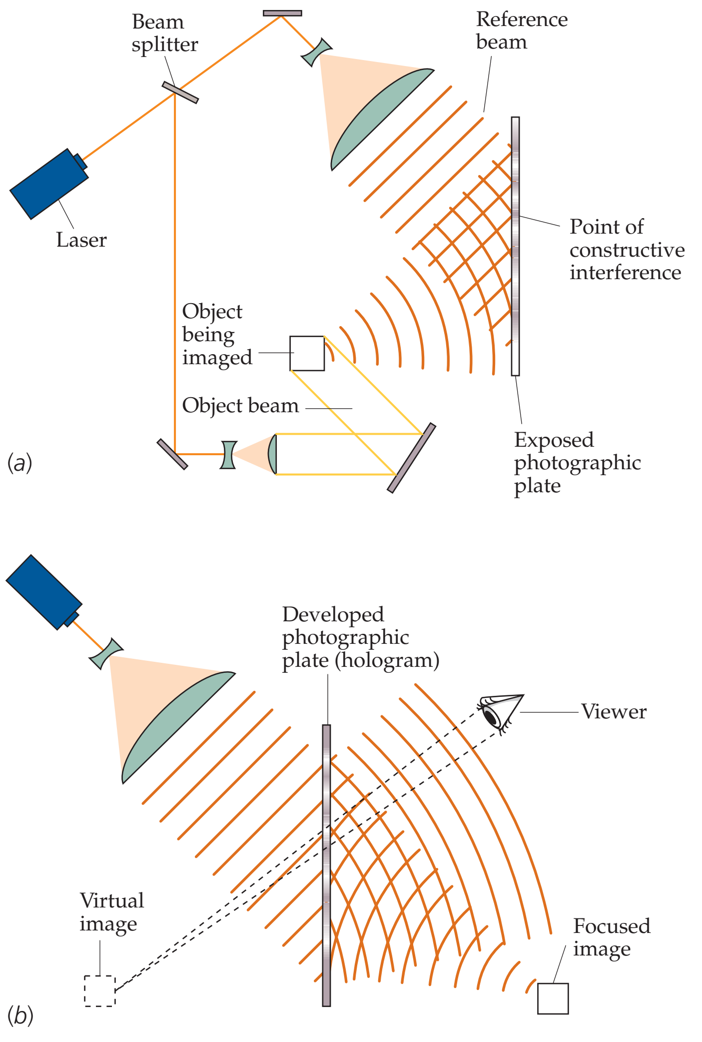
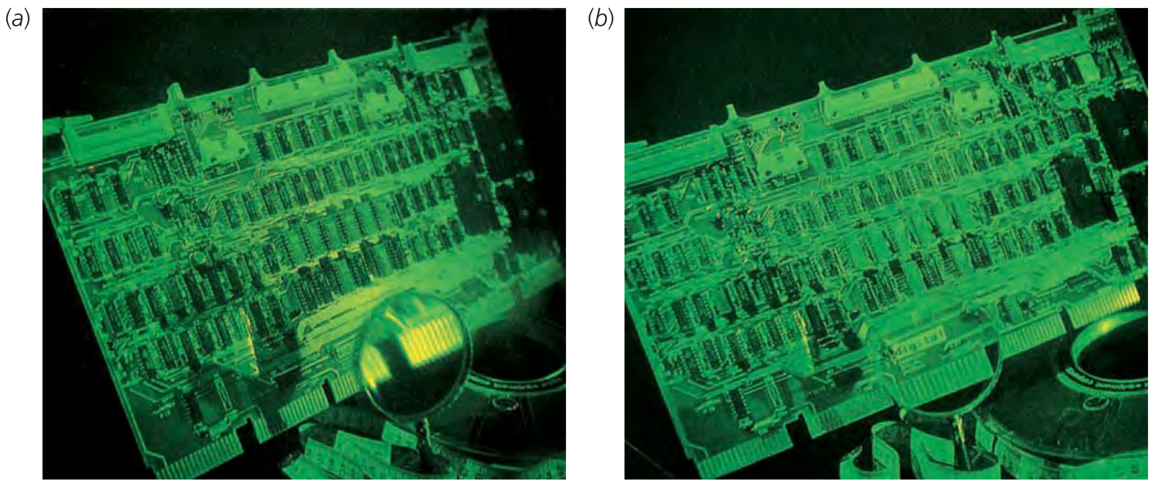


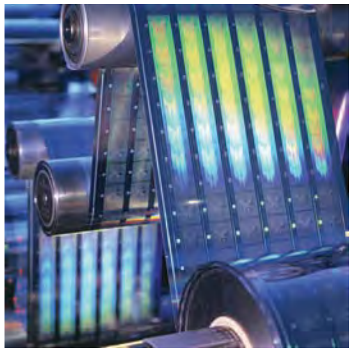
FIGURE 33-39 (*a*) The production of a hologram. The interference pattern produced by the reference beam and object beam is recorded on a photographic film. (*b*) When the film is developed and illuminated by coherent laser light, a three-dimensional image is seen.



A hologram viewed from two different angles. Note that different parts of the circuit board appear behind the front magnifying lens. *(© 1981 by Ronald R. Erickson, Hologram by Nicklaus Phillips, 1978, for Digital Equipment Corporation.)*

Physics Spotlight

Holograms: Guided Interference



The machine is using heat to emboss holograms on plastic cards (perhaps credit cards). Such holograms serve as both a security feature and aesthetic enrichment. *(Pascal Goetgheluck/Photo Researchers.)*

Holography was invented by Dennis Gabor in 1948 when he tried to improve the resolution of electron microscopy.[*] He reconstructed wavefronts using interference on the photographic plate to make a picture that contained phase information as well as intensity information. He named this type of imagery holography, after the Greek words for "whole" and "writing," for he felt that including the phase information gave a complete picture.[†]

It was extremely difficult to create those first few holograms, and they did not achieve the desired resolution. He used mercury vapor lamps as a light source. The light was highly monochromatic, but incoherent. (The phase of the light fluctuated randomly.) A decade or so later, after the laser was invented, the use of coherent laser light made holography practical for many purposes.

Embossed holograms are frequently used because they are inexpensive. Embossed holograms are made by hot-stamping a metallized plastic film[‡] with a die that is a negative copy of the extremely shallow (around 0.3–0.5 micron deep) interference lines present in a hologram.[#] The plastic film is then a duplicate of the very tiny interference lines in the original hologram. When light shines through the film and reflects from the metallic backing, the holographic image is reconstructed. Almost all embossed holograms are rainbow holograms—able to be viewed without a laser. Creating the master of a rainbow hologram is a complex process involving multiple exposures at precise angles.[°]

Embossed holograms are highly visible, easy to recognize, and difficult to forge.[§] Because they can take the place of paper labels or be added to paper or plastic, they are used on credit cards, pharmaceutical packaging, currency, and traveler's checks as a quick method for authentication.[¶**]

In January 1999, the Ford Motor Company used a series of digital holograms to create a 10 foot by 4 foot hologram of a concept car. The holograms were printed directly from computer design data.[††] Digital holography is now used to help physicians visualize the results of either computed tomography scans or magnetic resonance scans.[‡‡] The output from a series of MR or CT slices is collected, digitally processed, and then printed onto a single hologram, which can be viewed on a portable viewer. The resulting hologram allows surgeons to prepare for difficult surgeries[##] and may also have biomedical and industrial engineering applications.[°°] Digital holography is beginning to be used in holographic video applications.[§§]

Holograms have also been used as substitutes for traditional lenses. Holographic optical elements allow smaller and more compact displays to be built. Heads-up displays for airplane pilots are created using holographic optical elements.[¶¶] An extremely compact system that uses digitally calculated holograms as the optical element has been tested for use as a cell-phone based projector.[***] The use of holograms as optical elements and in optical data storage depends on advances in materials that are lightweight, tough, and have the desired optical properties.[†††]

Recently, holograms have been used to measure the electrostatic potential[‡‡‡] and magnetic fields[###] of very small objects. They have also been used to create higher resolution optics for X-ray lenses.[°°°] More than fifty years after holograms were invented, they are used to improve the resolution of microscopic images.

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† Scanlon, L., "The Whole Picture." *Technology Review*, Dec. 2002/Jan. 2003, Vol. 105, No. 10, p. 88.
‡ Ruschmann, H. W., "Apparatus for Embossing Holograms on Metallized Thermoplastic Films." *United States Patent 4,547,141*, Oct. 15, 1985.
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° Benton, S., Houde-Walter, W., and Mingace, Jr., H., "Methods of Making Holographic Images." *United States Patent 4,415,225.*, Nov. 15, 1983.
§ Cross, L., "Brand Security." *Graphic Arts Monthly*, Jan. 2006, Vol. 78, No. 1, pp. 32–33.
¶ "MasterCard Renews Hologram Contract." *American Banker*, Mar. 3, 2003, Vol. 168, No. 44, p. 18.
** Miller, H. I., "Fear and Pharmaceutical Failure." *The Washington Times*, Oct. 5, 2006, p. A16.
†† Mahoney, D. P., "Ford Drives Holography Development." *Computer Graphics World*, Feb. 1999, Vol. 22, No. 2, pp. 12–13.
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§§ Freedman, D. H., "Holograms in Motion." *Technology Review*, Nov. 2002, Vol. 105, No. 9, pp. 48–55.
¶¶ Stevens, T., "Holograms: More than Pretty Pictures." *Industry Week*, Oct. 4, 1993, Vol. 242, No. 19, pp. 34–46.
*** Buckley, E., "Miniature Projectors Based on LBO Technology." *SID Mobile Displays Conference*, San Diego: Oct. 3–5. 2006. At http://www.lightblueoptics.com/images/news/SID_Mobile_Displays_2006.pdf As of Nov. 2006.
††† Huang, G. T., "Holographic Memory." *Technology Review*, Sept. 2005, Vol. 108, No. 9, pp. 64–67.
‡‡‡ Chou, L.-J., Chang, M.-T., and Chueh, Y.-L., "Electron Holography for Improved Measurement of Microfields in Nanoelectrode Assemblies." *Applied Physics Letters*, Jul. 10, 2006, Vol. 89, No. 2, Letter 023112, 3 pp.
\### Nepijko, S., and Wiesendanger, R., "Studies of Magnetic Properties of Small Particles by Electron Holography." *Applied Physics A, Materials Science and Processing*, 1997, Vol. 65, No. 4/5, pp. 361–366.
°°° "Solak, H. H., David, C., and Gobrecht, J., "Fabrication of High-Resolution Zone Plates with Wideband Extreme-Ultraviolet Holography." *Applied Physics Letters*, Oct. 4, 2004, Vol. 85, No. 14, pp. 2700–2702.

Summary

TOPIC	RELEVANT EQUATIONS AND REMARKS	
1. Interference	Two superposing light waves interfere if their phase difference remains constant for a time long enough to observe. They interfere constructively if their phase difference is zero or an integer multiplied by 360°. They interfere destructively if their phase difference is 180° or an odd integer multiplied by 180°.	
Phase difference due to a path-length difference	$\delta = \frac{\Delta r}{\lambda} 2\pi$	33-1
Phase difference due to reflection	A phase difference of 180° is introduced when a light wave is reflected from a boundary between two media for which the wave speed is greater on the incident-wave side of the boundary.	
Thin films	The interference of light waves reflected from the front and back surfaces of a thin film produces interference fringes, commonly observed in soap films or oil films. The difference in phase between the two reflected waves results from the path-length difference of twice the thickness of the film plus any phase change due to reflection of one or both of the rays.	
Two slits	The path-length difference at an angle θ on a screen far away from two narrow slits separated by a distance d is $d \sin\theta$. If the intensity due to each slit separately is I_0, the intensity at points of constructive interference is $4I_0$, and the intensity at points of destructive interference is zero.	
Two slit interference maxima (sources in phase)	$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, \ldots$	33-2
Two slit interference maxima (sources 180° out of phase)	$d \sin\theta_m = (m - \frac{1}{2})\lambda \qquad m = 1, 2, 3, \ldots$	33-3
2. Diffraction	Diffraction occurs whenever a portion of a wavefront is limited by an obstacle or an aperture. The intensity of light at any point in space can be computed using Huygens's construction by taking each point on the wavefront to be a point source and computing the resulting interference pattern.	
Fraunhofer patterns	Fraunhofer patterns are observed at great distances from the obstacle or aperture so that the rays reaching any point are approximately parallel, or they can be observed using a lens to focus parallel rays on a viewing screen placed in the focal plane of the lens.	
Fresnel patterns	Fresnel patterns are observed at points not necessarily far from the source.	
Single slit	When light is incident on a single slit of width a, the intensity pattern on a screen far away shows a broad central diffraction maximum that decreases to zero at an angle θ_1 given by $\sin\theta_1 = \frac{\lambda}{a}$ (33-9). The width of the central maximum is inversely proportional to the width of the slit. The zeros in the single-slit diffraction pattern occur at angles given by $a \sin\theta_m = m\lambda \qquad m = 1, 2, 3, \ldots$ (33-11). The maxima on either side of the central maximum have intensities that are much smaller than the intensity of the central maximum.	33-9, 33-11
Two slits	The interference-diffraction pattern of two slits is the two-slit interference pattern modulated by the single-slit diffraction pattern.	
Resolution of two sources	When light from two point sources that are close together passes through an aperture, the diffraction patterns of the sources may overlap. If the overlap is too great, the two sources cannot be resolved as two separate sources. When the central diffraction maximum of one source falls at the diffraction minimum of the other source, the two sources are said to be just resolved by Rayleigh's criterion for resolution. For a circular aperture of diameter D, the critical angular separation of two sources for resolution by Rayleigh's criterion is given by	
Rayleigh's criterion	$\alpha_c = 1.22\frac{\lambda}{D}$	33-25

TOPIC	RELEVANT EQUATIONS AND REMARKS
*Gratings	A diffraction grating consisting of a large number of equally spaced lines or slits is used to measure the wavelength of light emitted by a source. The positions of the mth-order interference maxima from a grating are at angles given by $$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, \ldots \qquad \text{33-26}$$ The resolving power of a grating is $$R = \frac{\lambda}{\lvert\Delta\lambda\rvert} = mN \qquad \text{33-27}$$ where N is the number of slits of the grating that are illuminated and m is the order number.
3. *Phasors	Two or more harmonic waves can be added by representing each wave as the y component of a two-dimensional vector called a phasor. The phase difference between two harmonic waves is represented as the angle between the phasors.

Answers to Concept Checks

33-1 6

33-2 True. Fresnel diffraction is the name that describes the observations when the observing screen is any distance from the source of the diffraction. Fraunhofer diffraction is the name that describes the observations in the limit that the observing screen is far from the source of the diffraction.

Answers to Practice Problems

33-1 9.2 cm^{-1}

33-2 4.4 mm

33-3 $A = 5.0$ V/m, $\delta = 37°$

33-4 62.07° and 62.18°

Problems

In a few problems, you are given more data than you actually need; in a few other problems, you are required to supply data from your general knowledge, outside sources, or informed estimate.

Interpret as significant all digits in numerical values that have trailing zeros and no decimal points.

- • Single-concept, single-step, relatively easy
- •• Intermediate-level, may require synthesis of concepts
- ••• Challenging
- SSM Solution is in the *Student Solutions Manual*
- Consecutive problems that are shaded are paired problems.

CONCEPTUAL PROBLEMS

1 • A phase difference due to a path-length difference is observed for monochromatic visible light. Which phase difference requires the least (minimum) path-length difference? (*a*) 90°,(*b*) 180°, (*c*) 270°,(*d*) The answer depends on the wavelength of the light.

2 • Which of the following pairs of light sources are coherent: (*a*) two candles, (*b*) one point source and its image in a plane mirror, (*c*) two pinholes uniformly illuminated by the same point source, (*d*) two headlights of a car, (*e*) two images of a point source due to reflection from the front and back surfaces of a soap film?

3 • The spacing between Newton's rings decreases rapidly as the diameter of the rings increases. Explain why this result occurs. SSM

4 • If the angle of a wedge-shaped air film such as the angle in Example 33-2 is too large, fringes are not observed. Why?

5 • Why must a film that is used to observe interference colors be thin?

6 • A loop of wire is dipped in soapy water and held up so that the soap film is vertical. (*a*) Viewed by reflection using white light, the top of the film appears black. Explain why. (*b*) Below the black region are colored bands. Is the first band red or violet?

7 • A two-slit interference pattern is formed using monochromatic laser light that has a wavelength of 640 nm. At the second maximum from the central maximum, what is the path-length difference between the light coming from each of the slits? (*a*) 640 nm, (*b*) 320 nm, (*c*) 960 nm, (*d*) 1280 nm SSM

8 • A two-slit interference pattern is formed using monochromatic laser light that has a wavelength of 640 nm. At the first minimum from the central maximum, what is the path-length difference between the light coming from each of the slits? (*a*) 640 nm, (*b*) 320 nm, (*c*) 960 nm, (*d*) 1280 nm

9 • A two-slit interference pattern is formed using monochromatic laser light that has a wavelength of 450 nm. What happens to the distance between the first maximum and the central maximum as the two slits are moved closer together? (*a*) The distance increases. (*b*) The distance decreases. (*c*) The distance remains the same.

10 • A two-slit interference pattern is formed using two different monochromatic lasers, one green and one red. Which color light has its first maximum closer to the central maximum? (*a*) green, (*b*) red, (*c*) Both maxima are at the same location.

11 • A single-slit diffraction pattern is formed using monochromatic laser light that has a wavelength of 450 nm. What happens to the distance between the first maximum and the central maximum as the slit is made narrower? (*a*) The distance increases. (*b*) The distance decreases. (*c*) The distance remains the same.

12 • Equation 33-2, which is $d \sin\theta_m = m\lambda$, and Equation 33-11, which is $a \sin\theta_m = m\lambda$, are sometimes confused. For each equation, define the symbols and explain the equation's application.

13 • When a diffraction grating is illuminated by white light, the first-order maximum of green light (*a*) is closer to the central maximum than the first-order maximum of red light, (*b*) is closer to the central maximum than the first-order maximum of blue light, (*c*) overlaps the second-order maximum of red light, (*d*) overlaps the second-order maximum of blue light.

14 • A double-slit interference experiment is set up in a chamber that can be evacuated. Using light from a helium–neon laser, an interference pattern is observed when the chamber is open to air. As the chamber is evacuated, one will note that (*a*) the interference fringes remain fixed, (*b*) the interference fringes move closer together, (*c*) the interference fringes move farther apart, (*d*) the interference fringes disappear completely.

15 • True or false:

(*a*) When waves interfere destructively, the energy is converted into heat.
(*b*) Interference patterns are observed only if the relative phases of the waves that superpose remain constant.
(*c*) In the Fraunhofer diffraction pattern for a single slit, the narrower the slit, the wider the central maximum of the diffraction pattern.
(*d*) A circular aperture can produce both a Fraunhofer diffraction pattern and a Fresnel diffraction pattern.
(*e*) The ability to resolve two point sources depends on the wavelength of the light. SSM

16 • You observe two very closely spaced sources of white light through a circular opening using various filters. Which color filter is most likely to prevent your resolving the images on your retinas as coming from two distinct sources? (*a*) red, (*b*) yellow, (*c*) green, (*d*) blue, (*e*) The filter choice is irrelevant.

17 •• Explain why the ability to distinguish the two headlights of an oncoming car at a given distance is easier for a human eye at night than during the daytime. Assume the headlights of the oncoming car are on both during both daytime and nighttime hours.

ESTIMATION AND APPROXIMATION

18 • It is claimed that the Great Wall of China is the only human-made object that can be seen from space using no equipment. Check to see if this claim is true, based on the resolving power of the human eye. Assume the observers are in a low-Earth orbit that has an altitude of about 250 km.

19 •• (*a*) Estimate how close an approaching car at night on a flat, straight part of a highway must be before its headlights can be distinguished from the single headlight of a motorcycle. (*b*) Estimate how far ahead of you a car is if its two red taillights merge to look as if they were one. SSM

20 •• A small loudspeaker is located at a large distance to the east from you. The loudspeaker is driven by a sinusoidal current whose frequency can be varied. Estimate the lowest frequency for which one of your ears would receive the sound waves exactly out of phase with that received by your other ear when you are facing north.

21 •• Estimate the maximum distance at which a binary star system could be resolvable by the human eye. Assume the two stars are about fifty times farther apart than Earth and the Sun are. Neglect any atmospheric effects. (A test similar to this "eye test" was used in ancient Rome to test for eyesight acuity before entering the army. A person who had normal eyesight could just barely resolve two well-known stars that appear close in the sky. Anyone who could not tell there were two stars failed the test.) SSM

PHASE DIFFERENCE AND COHERENCE

22 • Light that has a wavelength of 500 nm is incident normally on a film of water 1.00 μm thick. (*a*) What is the wavelength of the light in the water? (*b*) How many wavelengths are contained in the distance $2t$, where t is the thickness of the film? (*c*) The film has air on both sides. What is the phase difference between the wave reflected from the front surface and the wave reflected from the back surface in the region where the two reflected waves superpose?

23 •• Two coherent microwave sources both produce waves that each have a wavelength equal to 1.50 cm. The sources are located in the $z = 0$ plane, one at $x = 0$, $y = 15.0$ cm and the other at $x = 3.00$ cm, $y = 14.0$ cm. If the sources are in phase, find the difference in phase between the two waves for a receiver located at the origin. SSM

INTERFERENCE IN THIN FILMS

24 • A wedge-shaped film of air is made by placing a small slip of paper between the edges of two flat plates of glass. Light that has a wavelength of 700 nm is incident normally on the glass plates, and interference fringes are observed by reflection. (*a*) Is the first fringe near the point of contact of the plates dark or bright? Why? (*b*) If there are five dark fringes per centimeter, what is the angle of the wedge?

25 •• The diameters of fine fibers can be accurately measured using interference patterns. Two optically flat pieces of glass that each have a length L are arranged with the fiber between them, as shown in Figure 33-40. The setup is illuminated by monochromatic light, and the resulting interference fringes are observed. Suppose that L is 20.0 cm and that yellow sodium light (590 nm) is used for illumination. If 19 bright fringes are seen along this 20.0-cm distance, what are the limits on the diameter of the fiber? *Hint: The nineteenth fringe might not be right at the end, but you do not see a twentieth fringe at all.* SSM

26 •• Light that has a wavelength equal to 600 nm is used to illuminate two glass plates at normal incidence. The plates are 22 cm in length, touch at one end, and are separated at the other end by a wire that has a radius equal to 0.025 mm. How many bright fringes appear along the total length of the plates?

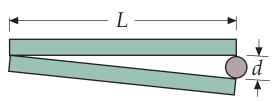

FIGURE 33-40 Problem 25

27 •• A thin film having an index of refraction of 1.50 is surrounded by air. It is illuminated normally by white light. Analysis of the reflected light shows that the wavelengths 360, 450, and 602 nm are the only missing wavelengths in or near the visible portion of the spectrum. That is, for those wavelengths, there is destructive interference. (*a*) What is the thickness of the film? (*b*) What visible wavelengths are brightest in the reflected interference pattern? (*c*) If this film were resting on glass that has an index of refraction of 1.60, what wavelengths in the visible spectrum would be missing from the reflected light?

28 •• A drop of oil (refractive index of 1.22) floats on water (refractive index of 1.33). When reflected light is observed from above, as shown in Figure 33-41, what is the thickness of the drop at

the point where the second red fringe, counting from the edge of the drop, is observed? Assume red light has a wavelength of 650 nm.

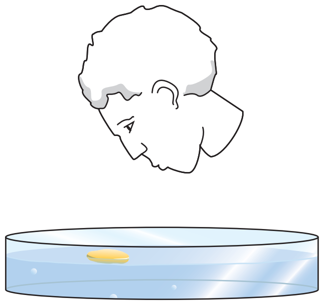

FIGURE 33-41 Problem 28

29 •• A film of oil that has an index of refraction of 1.45 rests on an optically flat piece of glass that has an index of refraction of 1.60. When illuminated by white light at normal incidence, light of wavelengths 690 nm and 460 nm is predominant in the reflected light. Determine the thickness of the oil film. SSM

30 •• A film of oil that has an index of refraction equal to 1.45 floats on water. When illuminated by white light at normal incidence, light of wavelengths 700 nm and 500 nm is predominant in the reflected light. Determine the thickness of the oil film.

NEWTON'S RINGS

31 •• A Newton's ring apparatus consists of a plano-convex glass lens that has a radius of curvature R and rests on a flat glass plate, as shown in Figure 33-42. The thin film is air of variable thickness. The apparatus is illuminated from above by light from a sodium lamp that has a wavelength equal to 590 nm. The pattern is viewed by reflected light. (*a*) Show that for a thickness t the condition for a bright (constructive) interference ring is $2t = \left(m + \frac{1}{2}\right)\lambda$, where $m = 0, 1, 2, \ldots$. (*b*) Show that for $t \ll R$, the radius r of a fringe is related to t by $r = \sqrt{2tR}$. (*c*) For a radius of curvature of 10.0 m and a lens diameter of 4.00 cm, how many bright fringes would you see in the reflected light? (*d*) What would be the diameter of the sixth bright fringe? (*e*) If the glass used in the apparatus has an index of refraction $n = 1.50$ and water replaces the air between the two pieces of glass, explain qualitatively the changes that will take place in the bright-fringe pattern. SSM

32 •• A plano-convex glass lens of radius of curvature 2.00 m rests on an optically flat glass plate. The arrangement is illuminated from above using monochromatic light that has a 520-nm wavelength. The indices of refraction of the lens and plate are 1.60. Determine the radii of the first and second bright fringe from the center in the reflected light.

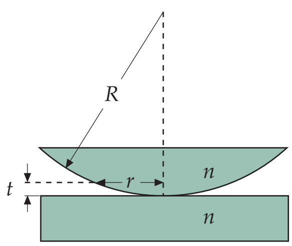

FIGURE 33-42 Problem 31

33 ••• Suppose that before the lens of Problem 32 is placed on the plate, a film of oil that has a refractive index equal to 1.82 is deposited on the plate. What will then be the radii of the first and second bright fringes?

TWO-SLIT INTERFERENCE PATTERNS

34 • Two narrow slits separated by 1.00 mm are illuminated by light that has a 600-nm wavelength, and the interference pattern is viewed on a screen 2.00 m away. Calculate the number of bright fringes per centimeter on the screen in the region near the center fringe.

35 • Using a conventional two-slit apparatus and light that has a 589-nm wavelength, 28 bright fringes per centimeter are observed near the center of a screen 3.00 m away. What is the slit separation? SSM

36 • Light that has a 633-nm wavelength and is from a helium–neon laser is shone normal to a plane having two slits. The first interference maximum is 82 cm from the central maximum on a screen 12 m away. (*a*) Find the separation of the slits. (*b*) How many interference maxima is it, in principle, possible to observe?

37 •• Two narrow slits are separated by a distance d. Their interference pattern is to be observed on a screen a large distance L away. (*a*) Calculate the spacing between successive maxima near the center fringe for light that has a 500-nm wavelength, when L is 1.00 m and d is 1.00 cm. (*b*) Would you expect to be able to observe the interference of light on the screen for this situation? (*c*) How close together should the slits be placed for the maxima to be separated by 1.00 mm for this wavelength and screen distance?

38 •• Light is incident at an angle ϕ with the normal to a vertical plane that has two slits of separation d (Figure 33-43). Show that the interference maxima are located at angles θ_m given by $\sin\theta_m + \sin\phi = m\lambda/d$.

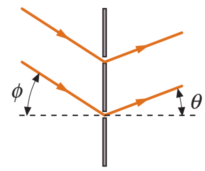

FIGURE 33-43 Problems 38 and 39

39 •• White light falls at an angle of 30° to the normal of a plane that has a pair of slits separated by 2.50 μm. What visible wavelengths give a bright interference maximum in the transmitted light in the direction normal to the plane? (See Problem 38.) SSM

40 •• Two small loudspeakers are separated by a distance of 5.0 cm, as shown in Figure 33-44. The speakers are driven in phase with a sine wave signal of frequency 10 kHz. A small microphone is placed a distance 1.00 m away from the speakers on the axis running through the middle of the two speakers, and the microphone is then moved perpendicular to the axis. Where does the microphone record the first minimum and the first maximum of the interference pattern from the speakers? The speed of sound in air is 343 m/s.

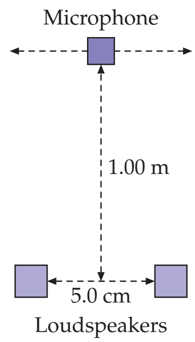

FIGURE 33-44 Problem 40

DIFFRACTION PATTERN OF A SINGLE SLIT

41 • Light that has a 600-nm wavelength is incident on a long narrow slit. Find the angle of the first diffraction minimum if the width of the slit is (*a*) 1.0 mm, (*b*) 0.10 mm, and (*c*) 0.010 mm.

42 • Plane microwaves are incident on a thin metal sheet that has a long, narrow slit of width 5.0 cm in it. The microwave radiation strikes the sheet at normal incidence. The first diffraction minimum is observed at $\theta = 37°$. What is the wavelength of the microwaves?

43 ••• Measuring the distance to the moon (lunar ranging) is routinely done by firing short-pulse lasers and measuring the time it takes for the pulses to reflect back from the moon. A pulse is fired from Earth. To send the pulse out, the pulse is expanded so that it fills the aperture of a 6.00-in-diameter telescope. Assuming the only thing spreading the beam out is diffraction and that the light wavelength is 500 nm, how large will the beam be when it reaches the moon, 3.82×10^5 km away? SSM

INTERFERENCE–DIFFRACTION PATTERN OF TWO SLITS

44 • How many interference maxima will be contained in the central diffraction maximum in the interference–diffraction pattern of two slits if the separation of the slits is exactly 5 times their width? How many will there be if the slit separation is an integral multiple of the slit width (that is, $d = na$ for any value of n)?

45 •• A two-slit Fraunhofer interference–diffraction pattern is observed using light that has a wavelength equal to 500 nm. The slits have a separation of 0.100 mm and an unknown width. (*a*) Find the width if the fifth interference maximum is at the same angle as the first diffraction minimum. (*b*) For that case, how many bright interference fringes will be seen in the central diffraction maximum? SSM

46 •• A two-slit Fraunhofer interference–diffraction pattern is observed using light that has a wavelength equal to 700 nm. The slits have widths of 0.010 mm and are separated by 0.20 mm. How many bright fringes will be seen in the central diffraction maximum?

47 •• Suppose that the *central* diffraction maximum for two slits has 17 interference fringes for some wavelength of light. How many interference fringes would you expect in the diffraction maximum adjacent to one side of the central diffraction maximum?

48 •• Light that has a wavelength equal to 550 nm illuminates two slits that both have widths equal to 0.030 mm and separations equal to 0.15 mm. (*a*) How many interference maxima fall within the full width of the central diffraction maximum? (*b*) What is the ratio of the intensity of the third interference maximum to one side of the center interference maximum to the intensity of the center interference maximum?

*USING PHASORS TO ADD HARMONIC WAVES

49 • Find the resultant of the two waves whose electric fields at a given location vary with time as follows: $\vec{E}_1 = 2.0A_0 \sin\omega t\,\hat{i}$ and $\vec{E}_2 = 3.0A_0 \sin\left(\omega t + \frac{3}{2}\pi\right)\hat{i}$. SSM

50 • Find the resultant of the two waves whose electric fields at a given location vary with time as follows: $\vec{E}_1 = 4.0A_0 \sin\omega t\,\hat{i}$ and $\vec{E}_2 = 3.0A_0 \sin\left(\omega t + \frac{1}{6}\pi\right)\hat{i}$.

51 •• Monochromatic light is incident on a sheet with a long narrow slit (Figure 33-45). Let I_0 be the intensity at the central maximum of the diffraction pattern on a distant screen, and let I be the intensity at the second intensity maximum from the central intensity maximum. The distance from this second intensity maximum to the far edge of the slit is longer than the distance from the second intensity maximum to the near edge of the slit by approximately 2.5 wavelengths. What is the ratio of I to I_0?

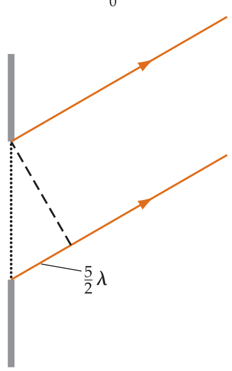

FIGURE 33-45 Problem 51

52 •• Monochromatic light is incident on a sheet that has three long narrow parallel equally spaced slits a distance d apart. (*a*) Show that the positions of the interference minima on a screen a large distance L away from the sheet that has the three equally spaced slits (spacing d, with $d \gg \lambda$) are given approximately by $y_m = m\lambda L/3d$, where $m = 1, 2, 4, 5, 7, 8, 10, \ldots,$ that is, m is *not* a multiple of 3. (*b*) For a screen distance of 1.00 m, a light wavelength of 500 nm, and a source spacing of 0.100 mm, calculate the width of the principal interference maxima (the distance between successive minima) for three sources.

53 •• Monochromatic light is incident on a sheet that has four long narrow parallel equally spaced slits a distance d apart. (*a*) Show that the positions of the interference minima on a screen a large distance L away from the sheet that has the four equally spaced slits (spacing d, with $d \gg \lambda$) are given approximately by $y_m = m\lambda L/4d$, where $m = 1, 2, 3, 5, 6, 7, 9, 10, \ldots,$ that is, m is *not* a multiple of 4. (*b*) For a screen distance of 2.00 m, a light wavelength of 600 nm, and a source spacing of 0.100 mm, calculate the width of the principal interference maxima (the distance between successive minima) for four sources. Compare the width with that for two sources with the same spacing. SSM

54 •• Light that has a wavelength equal to 480 nm falls normally on four slits. Each slit is 2.00 μm wide and the center-to-center separation between it and the next slit is 6.00 μm. (*a*) Find the angular width of the central intensity maximum of the single-slit diffraction pattern on a distant screen. (*b*) Find the angular position of all interference intensity maxima that lie inside the central diffraction maximum. (*c*) Find the angular width of the central interference maximum. That is, find the angle between the first interference intensity minima on either side of the central interference maximum. (*d*) Sketch the relative intensity as a function of the sine of the angle.

55 ••• Three slits, each separated from its neighbor by 60.0 μm, are illuminated at the central intensity maximum by a coherent light source that has a wavelength equal to 550 nm. The slits are extremely narrow. A screen is located 2.50 m from the slits. The intensity is 50.0 mW/m^2. Consider a location 1.72 cm from the central maximum. (*a*) Draw a phasor diagram suitable for the addition of the three harmonic waves at that location. (*b*) From the phasor diagram, calculate the intensity of light at that location. SSM

56 ••• In single-slit Fraunhofer diffraction, the intensity pattern (Figure 33-11) consists of a broad central maximum with a sequence of secondary maxima to either side of the central maximum. The intensity is given by $I = I_0\left(\dfrac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2$, where ϕ is the phase difference between the wavelets arriving from the opposite edges of the slits. Calculate the values of ϕ for the first three seconday maxima to one side of the central maximum by finding the values of ϕ for which $dI/d\phi$ is equal to zero. Check your results by comparing your answers with approximate values for ϕ of 3π, 5π and 7π. (That these values for ϕ are approximatly correct at the secondary intensity maxima is discussed in the discussion surrounding Figure 33-27.)

DIFFRACTION AND RESOLUTION

57 • Light that has a wavelength equal to 700 nm is incident on a pinhole of diameter 0.100 mm. (*a*) What is the angle between the central maximum and the first diffraction minimum for a Fraunhofer diffraction pattern? (*b*) What is the distance between the central maximum and the first diffraction minimum on a screen 8.00 m away? SSM

58 • Two sources of light that both have wavelengths equal to 700 nm are 10.0 m away from the pinhole of Problem 57. How far apart must the sources be for their diffraction patterns to be resolved by Rayleigh's criterion?

59 • Two sources of light that both have wavelengths equal to 700 nm are separated by a horizontal distance x. They are 5.00 m from a vertical slit of width 0.500 mm. What is the smallest value of x for which the diffraction pattern of the sources can be resolved by Rayleigh's criterion?

60 •• The ceiling of your lecture hall is probably covered with acoustic tile, which has small holes separated by about 6.0 mm. (*a*) Using light that has a wavelength of 500 nm, how far could you be from this tile and still resolve the holes? Assume the diameter of the pupil of each of your eyes is about 5.0 mm. (*b*) Could you resolve the holes better using red light or violet light? Explain your answer.

61 •• The telescope on Mount Palomar has a diameter of 200 in. Suppose a double star were 4.00 light-years away. Under ideal conditions, what must be the minimum separation of the two stars for their images to be resolved using light that has a wavelength equal to 550 nm? SSM

62 •• The star Mizar in Ursa Major is a binary system of stars that have nearly equal magnitudes. The angular separation between the two stars is 14 seconds of arc. What is the minimum diameter of the pupil that allows resolution of the two stars using light that has a wavelength equal to 550 nm?

*DIFFRACTION GRATINGS

63 • A diffraction grating that has 2000 slits per centimeter is used to measure the wavelengths emitted by hydrogen gas. (*a*) At what angles in the first-order spectrum would you expect to find the two violet lines that have wavelengths of 434 nm and 410 nm? (*b*) What are the angles if the grating has 15 000 slits per centimeter? SSM

64 • Using a diffraction grating that has 2000 slits per centimeter, two lines in the first-order hydrogen spectrum are found at angles of 9.72×10^{-2} rad and 1.32×10^{-1} rad. What are the wavelengths of the lines?

65 • The colors of many butterfly wings and beetle carapaces are due to the effects of diffraction. The *Morpho* butterfly has structural elements on its wings that effectively act as a diffraction grating that has an 880-nm spacing. At what angle will the first diffraction maximum occur for normally incident light diffracted by the butterfly's wings? Assume the light is blue and has a wavelength of 440 nm.

66 •• A diffraction grating that has 2000 slits per centimeter is used to analyze the spectrum of mercury. (*a*) Find the angular separation in the first-order spectrum of the two lines that have wavelengths equal to 579 nm and 577 nm. (*b*) How wide must the beam on the grating be for the lines to be resolved?

67 •• A diffraction grating that has 4800 lines per centimeter is illuminated at normal incidence using white light (wavelength range of 400 nm to 700 nm). How many orders of complete spectra can one observe in the transmitted light? Do any of these orders overlap? If so, describe the overlapping regions. SSM

68 •• A square diffraction grating that has an area of 25.0 cm^2 has a resolution of 22 000 in the fourth order. At what angle should you look to see a wavelength of 510 nm in the fourth order?

69 •• Sodium light that has a wavelength equal to 589 nm falls normally on a 2.00-cm-square diffraction grating ruled with 4000 lines per centimeter. The Fraunhofer diffraction pattern is projected onto a screen a distance of 1.50 m from the grating by a 1.50-m-focal-length lens that is placed immediately in front of the grating. Find (*a*) the distances of the first and second order intensity maxima from the central intensity maximum, (*b*) the width of the central maximum, and (*c*) the resolution in the first order. (Assume the entire grating is illuminated.)

70 •• The spectrum of neon is exceptionally rich in the visible region. Among the many lines are two lines at wavelengths of 519.313 nm and 519.322 nm. If light from a neon discharge tube is normally incident on a transmission grating that has 8400 lines per centimeter and the spectrum is observed in second order, what must be the width of the grating that is illuminated, so that the two lines can be resolved?

71 •• Mercury has several stable isotopes, among them ^{198}Hg and ^{202}Hg. The strong spectral line of mercury, at about 546.07 nm, is a composite of spectral lines from the various mercury isotopes. The wavelengths of the line for ^{198}Hg and ^{202}Hg are 546.07532 nm and 546.07355 nm, respectively. What must be the resolving power of a grating capable of resolving the two isotopic lines in the third-order spectrum? If the grating is illuminated over a 2.00-cm-wide region, what must be the number of lines per centimeter of the grating? SSM

72 ••• A diffraction grating has n lines per unit length. Show that the angular separation ($\Delta\theta$) of two lines of wavelengths λ and $\lambda + \Delta\lambda$ is approximately $\Delta\theta = \Delta\lambda/\sqrt{\dfrac{1}{n^2m^2} - \lambda^2}$, where m is the order number.

73 ••• For a diffraction grating in which all the surfaces are normal to the incident radiation, most of the energy goes into the zeroth order, which is useless from a spectroscopic point of view, because in zeroth order all the wavelengths are at 0°. Therefore, modern reflection gratings have shaped, or *blazed*, grooves, as shown in Figure 33-46. This shifts the specular reflection, which contains most of the energy, from the zeroth order to some higher order. (*a*) Calculate the *blaze angle* ϕ_m in terms of the groove separation d, the wavelength λ, and the order number m in which specular reflection is to occur for $m = 1, 2, \ldots$. (*b*) Calculate the proper blaze angle for the specular reflection to occur in the second order for light of wavelength 450 nm incident on a grating with 10 000 lines per centimeter. SSM

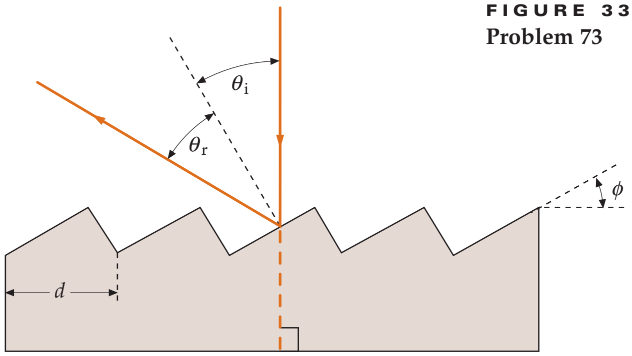

FIGURE 33-46 Problem 73

74 ••• In this problem, you will derive the relation $R = \lambda/|\Delta\lambda| = mN$ (Equation 33-27) for the resolving power of a diffraction grating having N slits separated by a distance d. To do this, you will calculate the angular separation between the intensity maximum and intensity minimum for some wavelength λ and set it equal to the angular separation of the mth-order maximum for two nearby wavelengths. (*a*) First show that the phase difference ϕ waves between the waves from two adjacent slits is given by $\phi = \dfrac{2\pi d}{\lambda}\sin\theta$. (*b*) Next, differentiate that expression to show that a small change in angle $d\theta$ results in a change in phase of $d\phi$ given by $d\phi = \dfrac{2\pi d}{\lambda}\cos\theta\, d\theta$. (*c*) Then for N slits, the angular separation between an interference maximum and an interference minimum corresponds to a phase change of $d\phi = 2\pi/N$. Use that to show that the angular separation $d\theta$ between the intensity maximum and intensity minimum for some wavelength λ is given by $d\theta = \dfrac{\lambda}{Nd\cos\theta}$. (*d*) Next, use the fact that the angle of the mth-order interference maximum for wavelength λ is specified by $d\sin\theta = m\lambda$ (Equation 33-26). Compute the differential of each side of the equation to show that angular separation of the mth-order

maximum for two nearly equal wavelengths differing by $d\lambda$ is given by $d\theta = \dfrac{m\,d\lambda}{d\cos\theta}$. (*e*) According to Rayleigh's criterion, two wavelengths will be resolved in the *m*th order if the angular separation of the wavelengths, given by the Part (*d*) result, equals the angular separation of the interference maximum and the interference minimum given by the Part (*c*) result. Use this to arrive at $R = \lambda/|\Delta\lambda| = mN$ (Equation 33-27) for the resolving power of a grating.

GENERAL PROBLEMS

75 • Naturally occuring coronas (brightly colored rings) are sometimes seen around the moon or the Sun when viewed through a thin cloud. (Warning: When viewing a Sun corona, be sure that the entire Sun is blocked by the edge of a building, a tree, or a traffic pole to safeguard your eyes.) These coronas are due to diffraction of light by small water droplets in the cloud. A typical angular diameter for a coronal ring is about 10°. From this, estimate the size of the water droplets in the cloud. Assume that the water droplets can be modeled as opaque disks that have the same radius as the droplet, and that the Fraunhofer diffraction pattern from an opaque disk is the same as the pattern from an aperture of the same diameter. (This last statement is known as *Babinet's principle*.) SSM

76 • An artificial corona (see Problem 75) can be made by placing a suspension of polystyrene microspheres in water. Polystyrene microspheres are small, uniform spheres that are made of plastic and have indices of refraction equal to 1.59. Assuming that the water has an index of refraction equal to 1.33, what is the angular diameter of such an artificial corona if 5.00-μm-diameter particles are illuminated by light from a helium–neon laser that has a wavelength in air of 632.8 nm?

77 • Coronas (see Problem 75) can be caused by pollen grains, typically of birch or pine. Such grains are irregular in shape, but they can be treated as if they had an average diameter of about 25 μm. What is the angular diameter (in degrees) of the corona for blue light? What is the angular diameter (in degrees) of the corona for red light?

78 • Light from a He–Ne laser (632.8 nm) is directed upon a human hair, in an attempt to measure its diameter by examining the diffraction pattern. The hair is mounted in a frame 7.5 m from a wall, and the central diffraction maximum is measured to be 14.6 cm wide. What is the diameter of the hair? (The diffraction pattern of a hair that has a diameter *d* is the same as the diffraction pattern of a single slit that has a width $a = d$. See Babinet's principle discussed in Problem 75.)

79 • A long, narrow horizontal slit lies 1.00 μm above a plane mirror, which is in the horizontal plane. The interference pattern produced by the slit and its image is viewed on a screen 1.00 m from the slit. The wavelength of the light is 600 nm. (*a*) Find the distance from the mirror to the first maximum. (*b*) How many dark bands per centimeter are seen on the screen? SSM

80 • A radio telescope is situated at the edge of a lake. The telescope is looking at light from a radio galaxy that is just rising over the horizon. If the height of the antenna is 20 m above the surface of the lake, at what angle above the horizon will the radio galaxy be when the telescope is centered in the first intensity interference maximum of the radio waves? Assume the wavelength of the radio waves is 20 cm. *Hint: The interference is caused by the light reflecting off the lake and remember that this reflection will result in a 180° phase shift.*

81 • The diameter of the radio telescope at Arecibo, Puerto Rico, is 300 m. What is the smallest angular separation of two objects that this telescope can detect when it is tuned to detect microwaves of 3.2-cm wavelength?

82 •• A thin layer of a transparent material that has an index of refraction of 1.30 is used as a nonreflective coating on the surface of glass that has an index of refraction of 1.50. What should the minimum thickness of the material be for the material to be nonreflecting for light that has a wavelength of 600 nm?

83 •• A *Fabry–Perot interferometer* (Figure 33-47) consists of two parallel, half-silvered mirrors that face each other and are separated by a small distance *a*. A half-silvered mirror is one that transmits 50% of the incident intensity and reflects 50% of the incident intensity. Show that when light is incident on the interferometer at an angle of incidence θ, the transmitted light will have maximum intensity when $2a = m\lambda\cos\theta$. SSM

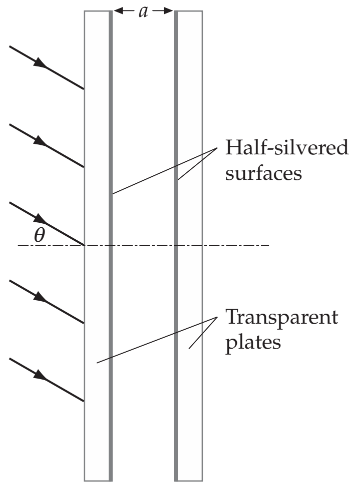

FIGURE 33-47 Problem 83

84 •• A mica sheet that is 1.20 μm thick is suspended in air. In reflected light, there are gaps in the visible spectrum at 421, 474, 542, and 633 nm. Find the index of refraction of the mica sheet.

85 •• A camera lens is made of glass that has an index of refraction of 1.60. This lens is coated with a magnesium fluoride film (index of refraction equal to 1.38) to enhance its light transmission. The purpose of this film is to produce zero reflection for light that has a wavelength of 540 nm. Treat the lens surface as a flat plane and the film as a uniformly thick flat film. (*a*) What minimum thickness of this film will accomplish its objective? (*b*) Would there be destructive interference for any other visible wavelengths? (*c*) By what factor would the reflection for light of 400 nm wavelength be reduced by the presence of this film? Neglect the variation in the reflected light amplitudes from the two surfaces. SSM

86 •• In a pinhole camera, the image is fuzzy because of geometry (rays arrive at the film after passing through different parts of the pinhole) and because of diffraction. As the pinhole is made smaller, the fuzziness due to geometry is reduced, but the fuzziness due to diffraction is increased. The optimum size of the pinhole for the sharpest possible image occurs when the spread due to diffraction equals the spread due to the geometric effects of the pinhole. Estimate the optimum size of the pinhole if the distance from the pinhole to the film is 10.0 cm and the wavelength of the light is 550 nm.

87 •• The Impressionist painter Georges Seurat used a technique called *pointillism*, in which his paintings are composed of small, closely spaced dots of pure color, each about 2.0 mm in diameter. The illusion of the colors blending together smoothly is produced in the eyes of the viewer by diffraction effects. Calculate the minimum viewing distance for this effect to work properly. Use the wavelength of visible light that requires the *greatest* distance between dots, so that you are sure the effect will work for *all* visible wavelengths. Assume the pupil of the eye has a diameter of 3.0 mm. SSM