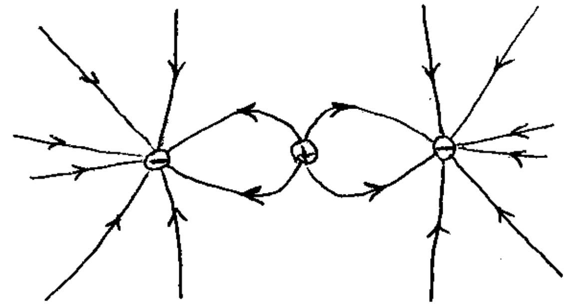
Unit 20 - Homework Solution

23-70



# of electric field lines is proportional to the electric charge.

central one: $4 = K\,(1.0\,\mu C)$ ⇒ $K = \dfrac{4}{1.0\,\mu C}$

constant

left one: # of lines $= K q$

$8 = \dfrac{4}{1.0\,\mu C}\, q \Rightarrow q = 2.0\,\mu C$

right one: # of lines $= 8 = \dfrac{4}{1.0\,\mu C}\, q \Rightarrow q = 2.0\,\mu C$

23-76

$A = B + C$

a) D is true, because the # of lines leaving area A is the same as the # of lines entering that.

b) B is true, because all of the lines leaving area B.

c) E is true, the charge contained in C is negative and equal to the charge in B.

SP 20 - 1

No, there might be some electric charges in that region but they neutralize each other.

SP 20 - 2

Net charge is negative.

SP 20 - 3

No, according to Gauss' Law, only extra charges reside on the surface. not the conduction electrons.

24 - 5

a) $\vec{E} = (3.00\ \text{N/cm})\, y\, \hat{j}$

Electric field is not constant and depends on y.

$\Phi^{net} = \Phi_1 + \Phi_2 + \cdots + \Phi_6$ where Φ_i is the flux passing through face i.

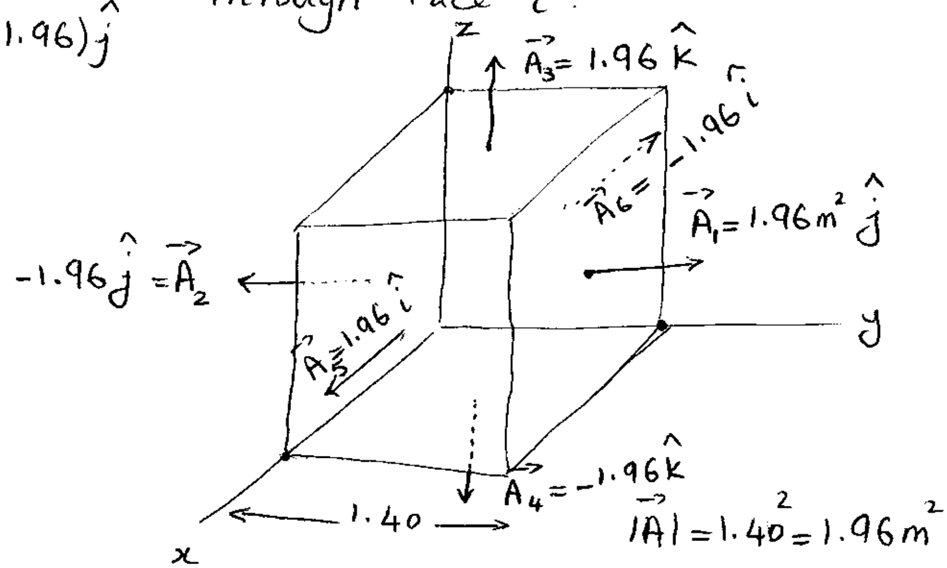
$\Phi_1 = \vec{E_1} \cdot \vec{A_1} = (3 \times 1.40)\hat{j} \cdot (1.96)\hat{j}$

$\boxed{\Phi_1 = 8.232}$

$\Phi_2 = \vec{E_2} \cdot \vec{A_2}$

$= (3 \times 0)\hat{j} \cdot (-1.96\hat{j}) = 0$

$\boxed{\Phi_2 = 0}$



$\Phi_3 = \vec{E}_3 \cdot \vec{A}_3 \qquad \vec{E}_3 \perp \vec{A}_3 \Rightarrow \boxed{\Phi_3 = 0}$

$\Phi_4 = \vec{E}_4 \cdot \vec{A}_4 \qquad \vec{E}_4 \perp \vec{A}_4 \Rightarrow \boxed{\Phi_4 = 0} \quad \ldots \Phi_5 = \Phi_6 = 0$

$$\Phi_a^{net} = 8.232 \ \frac{N m^2}{c}$$

b) $\vec{E} = -(4.00 \frac{N}{c})\hat{i} + (6.00 \frac{N}{c} + 3.00 \frac{N}{c} m y)\hat{j}$

Electric field in not constant and depends on y coordinate and it's always on x-y plane.

$E_3 \perp A_3$ and $E_4 \perp A_4 \Rightarrow \boxed{\Phi_3 = \Phi_4 = 0}$

$\Phi_1 = \cancel{4}\ \vec{E}_1 \cdot \vec{A}_1 = (-4\hat{i} + 6\hat{j} + 3 \times 1.40\hat{j}) \cdot 1.96\hat{j}$
$= 19.992 \ \frac{Nm^2}{c}$

$\Phi_2 = \vec{E}_2 \cdot \vec{A}_2 = (-4\hat{i} + 6\hat{j}) \cdot (-1.96\hat{j}) = -11.76 \ \frac{Nm^2}{c}$

$\Phi_5 = \vec{E}_5 \cdot \vec{A}_5 = (-4\hat{i} + (6+3y)\hat{j}) \cdot (1.96\hat{i}) = -7.84 \ \frac{Nm^2}{c}$

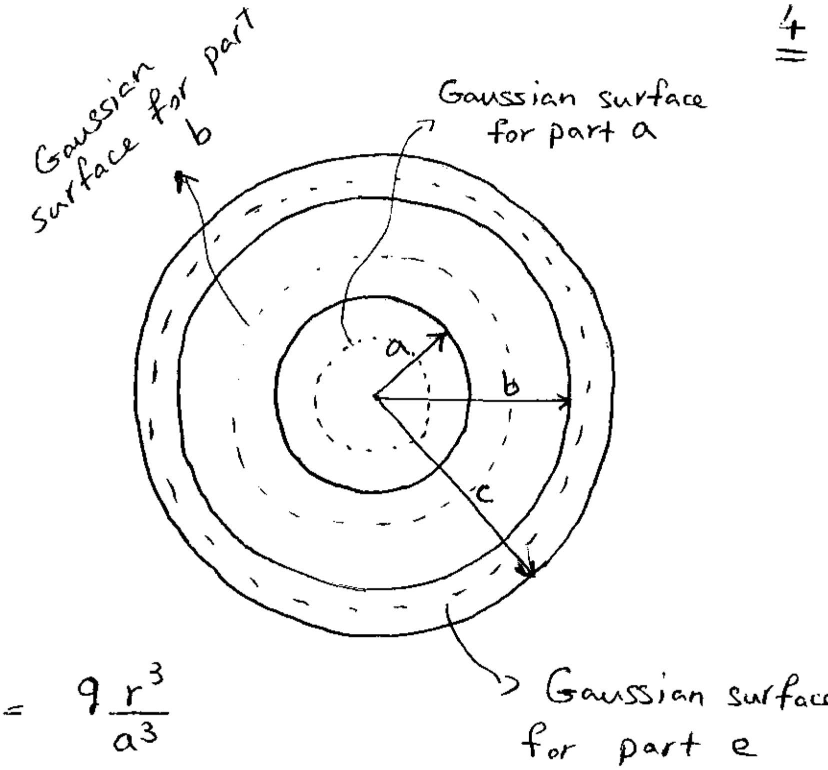
$\Phi_6 = \vec{E}_6 \cdot \vec{A}_6 = (-4\hat{i} + (6+3y)\hat{j}) \cdot (-1.96\hat{i}) = 7.84 \ \frac{Nm^2}{c}$

$$\boxed{\Phi_b^{net} = 8.23 \ \frac{Nm^2}{c}}$$

c) $\Phi_a^{net} = \frac{q_a}{\epsilon_0} \Rightarrow q_a = 8.85 \times 10^{-12} \times 8.23 = 7.28 \times 10^{-11}\ C$

$q_b = 8.85 \times 10^{-12} \times 8.23 = 7.28 \times 10^{-11}\ C$

24-19



a) $r < a$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q^{encl.}}{\epsilon_0}$$

$$\rho = \frac{+q}{V_a} = \frac{+q}{\frac{4}{3}\pi a^3}$$

$$q^{encl} = \rho V = \frac{+q}{\frac{4}{3}\pi a^3}\left(\frac{4}{3}\pi r^3\right) = q\frac{r^3}{a^3}$$

$$\vec{E} \parallel d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = E\,dA$$

$$\Rightarrow E\oint dA = \frac{q r^3}{\epsilon_0 a^3} \Rightarrow E(4\pi r^2) = \frac{q r^3}{\epsilon_0 a^3} \Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q r}{a^3}\hat{r}}$$

b) $a < r < b$ $\quad \oint \vec{E} \cdot d\vec{A} = \frac{q^{encl.}}{\epsilon_0}$ $\quad q^{encl.} = +q$ (the whole charge on the insulating sphere)

$$\Rightarrow E\oint dA = \frac{+q}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$\Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$$

c) $b < r < c$ Electric field inside a conducting material is zero. $\vec{E} = 0$

d) $r > c$ $\quad q^{encl} = +q + (-q) = 0 \Rightarrow \vec{E} = 0$

e) We can use Gauss' Law to find the charge on the inner surface of the shell.

For a spherical Gaussian surface inside the conducting shell we have :

$$\oint \vec{E} \cdot d\vec{A} = \frac{q^{encl}}{\epsilon_0}$$

We know the electric field on this surface is zero : $\vec{E} = 0 \implies q^{encl} = 0$

$$\left.\begin{array}{l} \vec{E} = 0 \implies q^{encl} = 0 \\ q^{encl} = \underbrace{+q}_{\text{charge on the insulating sphere}} + \underbrace{q_{inner}}_{\text{charge on the inner surface of the shell}} \end{array}\right\} \implies +q + q_{inner} = 0 \implies \boxed{q_{inner} = -q}$$

Therefore, the charge on the inner surface is $-q$ and the charge on the outer surface must be zero.

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24 - 27

a) For a cylindrical Gaussian surface outside the shell :

$$\oint \vec{E} \cdot d\vec{A} = \frac{q^{encl}}{\epsilon_0}$$

There is no flux passing through the bases of this cylinder because $\vec{E} \perp \vec{A}$

$$\implies E(2\pi r \underbrace{\cancel{\ell}}_{\text{length of the Gaussian cylinder}}) = \frac{(+q - 2q)\cancel{\ell}}{\epsilon_0 \underbrace{L}_{\text{length of the shell}}} \implies \boxed{\vec{E} = \frac{-q}{2\pi\epsilon_0} \frac{1}{rL} \hat{r}}$$

b) Using the same argument as the one used for part (e) of problem 24-19, we can say that the charge on the inner surface is $-q$. Therefore, the charge on the outer surface must be $-q$.

c) $\oint \vec{E} \cdot d\vec{A} = \dfrac{q^{encl}}{\epsilon_0}$ $\qquad q^{encl} = \Big(\underbrace{+\dfrac{q}{L}}_{\lambda}\Big)\, l$

$$\Rightarrow E\,(2\pi r \cancel{l}) = \frac{+q\cancel{l}}{\epsilon_0 L} \Rightarrow \boxed{\vec{E} = \frac{1}{2\pi\epsilon_0}\,\frac{+q}{rL}\,\hat{r}}$$

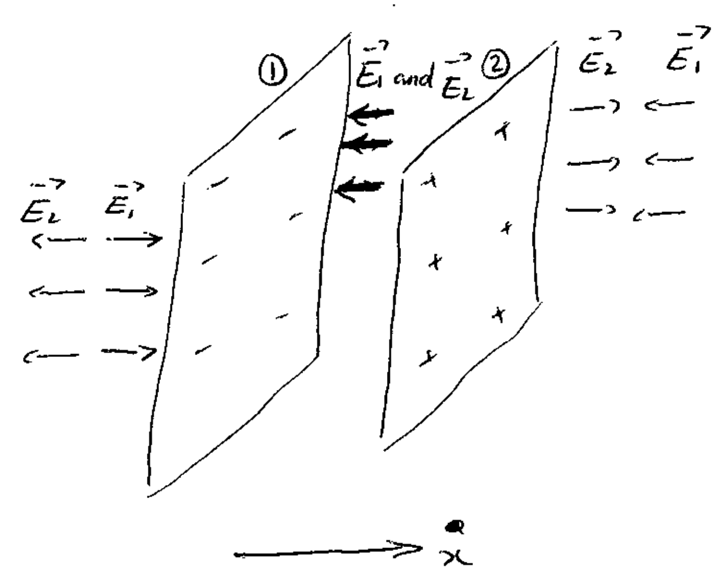
Note: The direction of the electric field the shell and rod is radially outward, but the direction of $\vec{E}$ field outside the shell is inward.

24-34

a) Magnitude of $\vec{E}$ due to each plate is $\dfrac{\sigma}{2\epsilon_0}$.

At the left:

$\vec{E}_1 = -\vec{E}_2 \Rightarrow E^{net} = 0$



b) At the right : $\vec{E}_1 = -\vec{E}_2 \Rightarrow \vec{E}^{net} = 0$

c) In between : $\vec{E}_1 = \vec{E}_2 \Rightarrow \vec{E}_{net} = 2\vec{E}_1 = 2\vec{E}_2$

$$|\vec{E}_{net}| = \frac{\sigma}{\epsilon_0}$$

$$\vec{E}_{net} = -\frac{\sigma}{\epsilon_0}\,\hat{i}$$