Unit 21 - Homework Solution

14-12

A : $m_A = 400$ kg , $(0, 50)$ cm

B : $m_B = 350$ kg , $(0, 0)$ cm

C : $m_C = 2000$ kg , $(-80, 0)$ cm

D : $m_D = 500$ kg , $(40, 0)$ cm

$$\vec{F}_B = \vec{F}_{AB} + \vec{F}_{CB} + \vec{F}_{DB} = \frac{+Gm_Am_B}{r_{AB}^2}\hat{r}_{AB} + \frac{+Gm_Cm_B}{r_{CB}^2}\hat{r}_{CB} + \frac{+Gm_Dm_B}{r_{DB}^2}$$

$$r_{AB}^2 = (x_B - x_A)^2 + (y_B - y_A)^2 = 0 + 2500 = 2500\,\text{cm}^2 \qquad \hat{r}_{AB} = \hat{j}$$

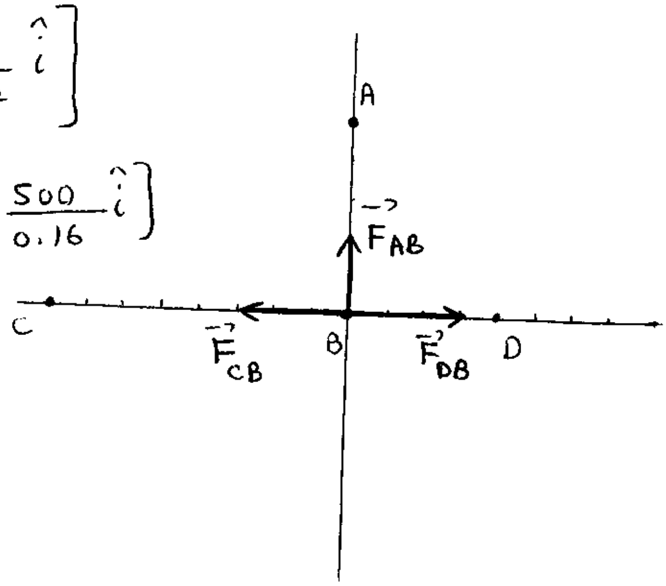
$$r_{CB}^2 = (x_B - x_C)^2 + (y_B - y_C)^2 = 6400 + 0 = 6400\,\text{cm}^2 \qquad \hat{r}_{CB} = -\hat{i}$$

$$r_{DB}^2 = (x_B - x_D)^2 + (y_B - y_D)^2 = 1600 + 0 = 1600\,\text{cm}^2 \qquad \hat{r}_{DB} = \hat{i}$$

$$\vec{F}_B = +Gm_B\left[\frac{m_A}{r_{AB}^2}\hat{j} + \frac{m_C}{r_{CB}^2}(-\hat{i}) + \frac{m_D}{r_{DB}^2}\hat{i}\right]$$

$$= (+6.67\times10^{-11}\times350)\left[\frac{400}{0.25}\hat{j} - \frac{2000}{0.64}\hat{i} + \frac{500}{0.16}\hat{i}\right]$$

$$= 3.73\times10^{-5}\,\text{N}\,\hat{j}$$



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a) Using Gauss' Law for gravitational force:

$$\gamma = \frac{F_g}{m} \qquad \oint \vec{\gamma}_A \cdot \vec{dA} = 4\pi G\, m^{encl.}$$

Gaussian surface: A sphere Passing through A

$$\gamma_A \oint dA = 4\pi G\,(M_1 + M_2)$$

$$\gamma_A (4\pi r^2) = 4\pi G (M_1 + M_2) \Rightarrow \gamma_A = \frac{G(M_1+M_2)}{r^2}$$

$$\Rightarrow \boxed{|\vec{F}_g| = \frac{G(M_1+M_2)m}{a^2}} \qquad r = a$$

b) $r = b$

$$\oint \vec{\gamma}_B \cdot \vec{dA} = 4\pi G M_1 \Rightarrow \gamma_B (4\pi b^2) = 4\pi G M_1$$

$$\Rightarrow \gamma_B = \frac{GM_1}{b^2} \Rightarrow \boxed{F_g = \frac{GM_1 m}{b^2}}$$

c) $r = c$

$$m^{encl} = 0 \Rightarrow \gamma_c = 0 \Rightarrow \boxed{F_g = 0}$$

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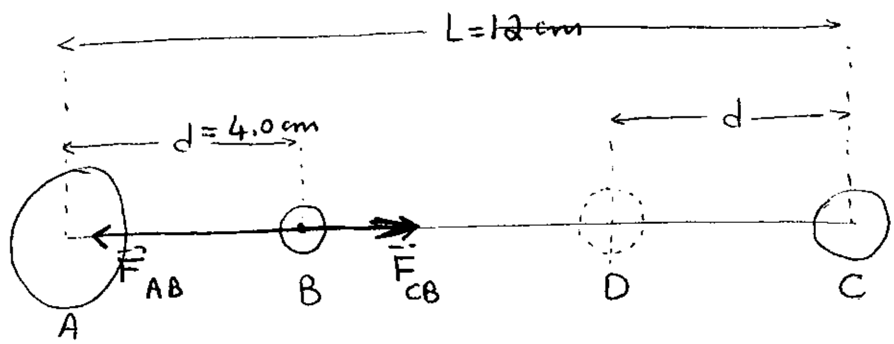


a) $W_{you} = \Delta U \qquad U = -\frac{GMM}{R}$

$$= U_D - U_B$$

$$= \left(-\frac{Gm_Bm_A}{r_{AD}} - \frac{Gm_Bm_c}{r_{DC}}\right) - \left(-\frac{Gm_Bm_A}{r_{AB}} - \frac{Gm_Bm_c}{r_{BC}}\right)$$

$$= Gm_B\left[-\frac{m_A}{L-d} - \frac{m_c}{d} + \frac{m_A}{d} + \frac{m_c}{L-d}\right]$$

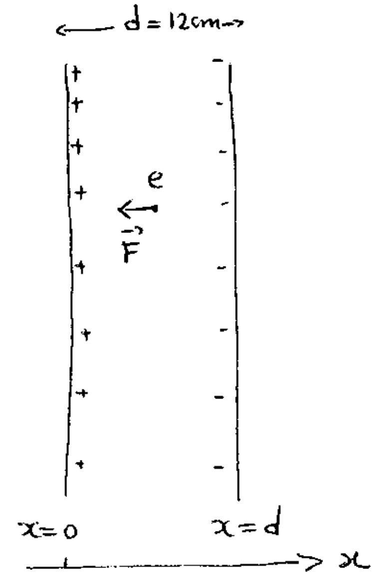
$$= 5.00 \times 10^{-11}\ J$$

$$\begin{cases} |\vec{F}_{CB}| = \dfrac{Gm_Bm_c}{(L-d)^2} = 2.08 \times 10^{-10}\ N \\ |\vec{F}_{AB}| = 3 \times 10^{-9}\ N \end{cases}$$

Don't need to calculate the forces!

b) $W_G = -\Delta U = -5.00 \times 10^{-11}\ J$

25-6



a) $|\vec{F}| = 3.9 \times 10^{-15}$ N

$$\vec{E} = \frac{\vec{F} \text{ (acting on a positive test charge)}}{q}$$

$$= \frac{3.9 \times 10^{-15}}{1.6 \times 10^{-19}} \hat{i} = 2.4 \times 10^{4} \, \text{N/C} \, \hat{i}$$

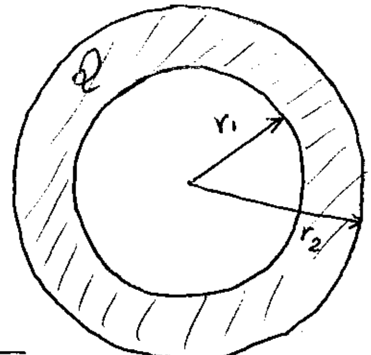
b) $$\Delta V = \int_0^d \vec{E} \cdot d\vec{x} = \int_0^d E \, dx$$

$$= E x \Big|_0^d = Ed = 2.4 \times 10^4 \times 0.12$$

$$= 2.9 \times 10^3 \text{ V}$$

25-11

We can calculate the electric field at each point using Gauss' Law and then calculate the potential using $\Delta V = -\int_\infty^r \vec{E} \cdot d\vec{r}$



a) $r > r_2$ $\quad \oint E \cdot dA = \frac{q^{encl}}{\epsilon_0}$

$$\Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

$$\Delta V = -\int_\infty^r \vec{E} \cdot d\vec{r} = \frac{-Q}{4\pi\epsilon_0} \int_\infty^r \frac{1}{r^2} \underbrace{\hat{r} \cdot d\vec{r}}_{dr}$$

$$= \frac{+Q}{4\pi\epsilon_0} \frac{1}{r} \Rightarrow \boxed{V(r) = +\frac{1}{4\pi\epsilon_0} \frac{Q}{r}}$$

b) $r_1 < r < r_2$ $\quad \oint E \cdot dA = \frac{q^{encl}}{\epsilon_0}$

Gaussian surface is a sphere passing through r between r_1 and r_2.

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi(r_2^3 - r_1^3)}$$

$$q^{encl} = \rho \underset{\text{of Gaussian sphere}}{v} = \rho \frac{4}{3}\pi(r^3 - r_1^3) = \frac{Q}{\cancel{\frac{4}{3}\pi}(r_2^3 - r_1^3)} \cancel{\frac{4}{3}\pi}(r^3 - r_1^3) = \frac{Q(r^3 - r_1^3)}{(r_2^3 - r_1^3)}$$

$$\Rightarrow E(4\pi r^2) = \frac{Q(r^3 - r_1^3)}{\epsilon_0 (r_2^3 - r_1^3)} \Rightarrow E = \frac{Q}{4\pi\epsilon_0 (r_2^3 - r_1^3)} \left(r - \frac{r_1^3}{r^2}\right)$$

$$\Delta V = V(r) - V(r_2) = -\int_{r_2}^{r} \vec{E} \cdot d\vec{r} = \frac{-Q}{4\pi\epsilon_0 (r_2^3 - r_1^3)} \int_{r_2}^{r} \left(r - \frac{r_1^3}{r^2}\right) dr$$

$$= \frac{-Q}{4\pi\epsilon_0 (r_2^3 - r_1^3)} \left(\frac{1}{2} r^2 + \frac{r_1^3}{r}\right)\Bigg|_{r_2}^{r}$$

$$= \underbrace{\frac{-Q}{4\pi\epsilon_0 (r_2^3 - r_1^3)} \left(\frac{1}{2} r^2 + \frac{r_1^3}{r} - \frac{1}{2} r_2^2 - \frac{r_1^3}{r_2}\right)}$$

$$\Rightarrow V(r) = V(r_2) -$$

$$V(r_2) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_2} \quad \text{from part (a)}$$

$$\Rightarrow V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_2} + \frac{Q}{4\pi\epsilon_0 (r_2^3 - r_1^3)} \left(\frac{1}{2} r^2 + \frac{r_1^3}{r} - \frac{1}{2} r_2^2 - \frac{r_1^3}{r_2}\right)$$

$$= \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_2} - \frac{\frac{1}{2} r^2 + \frac{r_1^3}{r} - \frac{1}{2} r_2^2 - \frac{r_1^3}{r_2}}{r_2^3 - r_1^3}\right]$$

$$= \frac{Q}{4\pi\epsilon_0} \frac{(r_2^3 - \cancel{r_1^3}) - \frac{1}{2} r_2 r^2 - \frac{r_1^3 r_2}{r} + \frac{1}{2} r_2^3 + \cancel{r_1^3}}{(r_2^3 - r_1^3) r_2}$$

$$= \frac{\rho}{3\epsilon_0} \left(\frac{3}{2} r_2^3 - \frac{1}{2} r^2 - \frac{r_1^3}{r}\right)$$

c) $r < r_1$ $\quad q^{encl} = 0 \Rightarrow E = 0 \Rightarrow V(r) = \text{constant} = V(r_1)$

$$\Rightarrow V(r) = \frac{\rho}{3\epsilon_0} \left(\frac{3}{2} r_2^3 - \frac{1}{2} r_1^2 - \frac{r_1^3}{r_1}\right) = \frac{\rho}{2\epsilon_0} (r_2^2 - r_1^2)$$

d) Parts b and c should give us the same results for $r = r_1$
and parts b and a 〃 〃 〃 〃 〃 〃 〃 $r = r_2$

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$$\Delta V = \frac{\Delta U}{q} = + \frac{W_{required}}{q}$$

Note : $W_{required} = -W_{electric\ force}$

In this case charge $q = +5q$ is brough from ∞ to that point.

$$\Rightarrow W = q\ \Delta V$$

$$\Delta V = \frac{k(+4q)}{2d} + \frac{k(-2q)}{d} = 0 \implies \boxed{W = 0J}$$

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$$V = \left(2.0\ \frac{V}{m^2}\right) x^2 - \left(3.0\ \frac{V}{m^2}\right) y^2$$

$$E_x = -\frac{dV}{dx} = -2\left(2.0\ \frac{V}{m^2}\right) x = \left(-4.0\ \frac{V}{m^2}\right) x$$

$$E_y = -\frac{dV}{dy} = +2\left(3.0\ \frac{V}{m^2}\right) y = \left(6.0\ \frac{V}{m^2}\right) y$$

$$\boxed{\vec{E} = \left(-12\ \frac{V}{m}\right)\hat{i} + \left(12\ \frac{V}{m}\right)\hat{j}}$$