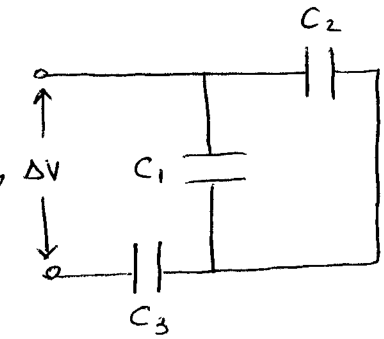
Unit 24 - Homework Solution

28-13

C_1 and C_2 are connected in parallel, because both of their paltes are directly connected to each other.



C_{12} is in Series with C_3.

$$C_{12} = C_1 + C_2 = 10 + 5 = 15\ \mu F$$

$$C_{eq} = \left(\frac{1}{C_{12}} + \frac{1}{C_3}\right)^{-1} = \left(\frac{1}{15} + \frac{1}{4}\right)^{-1} = 3.16\ \mu F$$

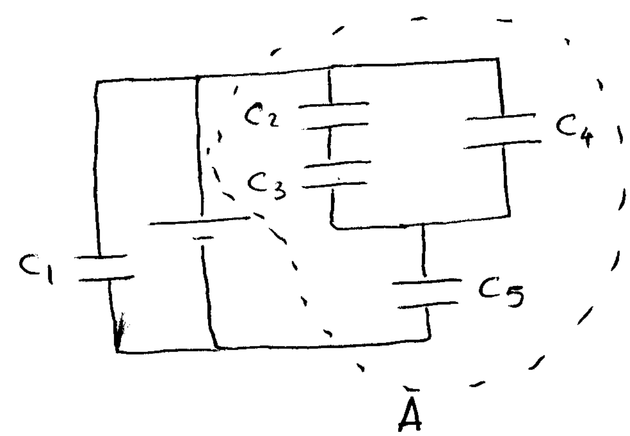
28-16

a) ΔV_1 (potential difference across C_1) is equal to 10 V, because C_1 is in parallel with the battery.

$$q_1 = C_1 \Delta V_1 = (10\times10^{-6}\ F)(10\ V) = 10^{-4}\ C = 100\ \mu C$$

b)

$$C_A = \left(\frac{1}{C_5} + \frac{1}{C_{234}}\right)^{-1}$$



$$C_{234} = C_4 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

$$= 10 + 5 = 15\ \mu F$$

$$C_A = \left(\frac{1}{10} + \frac{1}{15}\right)^{-1} = 6.0\ \mu F$$

$$q_A = C_A \Delta V_A = 6\times10 = 60\ \mu C = q_5 = q_{234}$$

$$\Rightarrow q_4 + q_{23} = 60 \Rightarrow q_4 + q_2 = 60$$

$$\Delta V_4 = \Delta V_{23} = 2\Delta V_2 \Rightarrow \frac{q_4}{C_4} = \frac{2q_2}{C_2}$$

$$\Rightarrow \boxed{q_4 = 43\ \mu C}$$

28-20

a) C_1 and C_3 : Series

C_2 and C_4 : Series

C_{13} and C_{24} : parallel

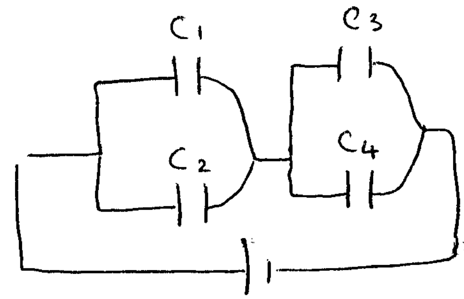
$$q_1 = q_3 = q_{13} = C_{13}\,\Delta V_B = \left(\frac{C_1 C_3}{C_1 + C_3}\right)\Delta V_B = \frac{(1.0)(3.0)}{1.0+3.0} \times 12 = 9.0\,\mu C$$

$$q_2 = q_4 = q_{24} = C_{24}\,\Delta V_B = \left(\frac{C_2 C_4}{C_2 + C_4}\right)\Delta V_B = \frac{2\times 4}{2+4} \times 12 = 16\,\mu C$$

b) C_1 and C_2 : parallel

C_3 and C_4 : parallel

C_{12} and C_{24} : Series



$$q_{12} = q_{34} = q_{tot}$$

$$q_{tot} = C_{tot}\,\Delta V_B = \left(\frac{1}{C_1+C_2} + \frac{1}{C_3+C_4}\right)^{-1}\Delta V_B = \left(\frac{1}{3} + \frac{1}{7}\right)^{-1} \times 12$$

$$= 25\cancel{8}\,\mu C$$

$$\left.\begin{array}{l} q_{12} = q_1 + q_2 = 25\cancel{8}\,\mu C \\ \Delta V_1 = \Delta V_2 \Rightarrow \dfrac{q_1}{C_1} = \dfrac{q_2}{C_2} \Rightarrow \dfrac{q_1}{1} = \dfrac{q_2}{2} \end{array}\right\} \Rightarrow \boxed{q_1 = 8.4\,\mu C,\ q_2 = 17\,\mu C}$$

$$\left.\begin{array}{l} q_{34} = q_3 + q_4 = 25\cancel{8}\,\mu C \\ \Delta V_3 = \Delta V_4 \Rightarrow \dfrac{q_3}{C_3} = \dfrac{q_4}{C_4} \Rightarrow \dfrac{q_3}{3} = \dfrac{q_4}{4} \end{array}\right\} \Rightarrow \boxed{q_4 = 14\,\mu C,\ q_3 = 11\,\mu C}$$

28 - 29

a) Initial charge : $q_i = C_i \Delta V_i = \frac{K\epsilon_0 A}{d} \Delta V_i$

As capacitor is disconnected, charge remains the same : and d is changed

$$q_f = q_i \Rightarrow C_f \Delta V_f = q_i$$

$$\Rightarrow \frac{K\epsilon_0 A}{2d} \Delta V_f = \frac{K\epsilon_0 A}{d} \Delta V_i$$

$$\Rightarrow \boxed{\Delta V_f = 2\,\Delta V_i}$$

b) $U_i = \frac{q_i^2}{2C_i}$ $\quad U_f = \frac{q_f^2}{2C_f}$ $\quad q_i = q_f$ and $C_i = 2C_f$

$$\Rightarrow \boxed{U_f = 2U_i}$$

c) $W = \Delta U = U_f - U_i = U_i = \frac{1}{2} C_i V_i^2 = \frac{1}{2} \frac{K\epsilon_0 A}{d} \Delta V_i^2$

28 - 49

a) Capacitor is charging up, therefore the potential difference across its plates is increasing:

$$\Delta V_c = \Delta V_0 \left(1 - e^{-\frac{t}{RC}}\right) \qquad RC = \tau$$

$t = 1.3\ \mu s$: $\Delta V_c = 5$ V , $\Delta V_0 = 12.0$ V

$$5 = 12\left(1 - e^{-\frac{1.3}{\tau}}\right) \Rightarrow \boxed{\tau = 2.41\ \mu s}$$

b) $\tau = RC \Rightarrow 2.41\ \mu s = (15 \times 10^3\ \Omega)\ C$

$\Rightarrow C = 1.6 \times 10^{-4}\ \mu F = 160\ pF$

28-51

a) $\Delta V_c = \Delta V_0 \, e^{-\frac{t}{\tau}}$ $\quad \tau = RC$

$1.00 = 100 \, e^{-10/\tau} \Rightarrow \boxed{\tau = 2.17 \text{ s}}$

$\Delta V_c = 100 \, e^{-\frac{17}{2.17}} = 0.04 \text{ V}$

28-46

$q_c = q_0 \, e^{-t/\tau}$

a) $q_c = q_0 - \frac{1}{3} q_0 = \frac{2}{3} q_0$

$\frac{2}{3} \cancel{q_0} = \cancel{q_0} \, e^{-t/\tau} \Rightarrow \boxed{t = 0.41 \tau}$

b) $q_c = \frac{1}{3} \cancel{q_0} = \cancel{q_0} \, e^{-t/\tau} \Rightarrow \boxed{t = 1.10 \tau}$

28-53

a) $U_i = \frac{q_i^2}{2C} \Rightarrow 0.50 = \frac{q_i^2}{2 \times 10^{-6}} \Rightarrow q_i^2 = 1 \times 10^{-6}$

$\Rightarrow \boxed{q_i = 10^{-3} \text{ C}}$

$\tau = RC$
$= 10^{6} \times 10^{-6}$
$= 1 \text{ s}$

b) $I_R = -\frac{dq_c}{dt} = -\frac{d}{dt}\left(q_i \, e^{-t/\tau}\right)$

$= \frac{q_i}{\tau} e^{-t/\tau} = \frac{10^{-3}}{1} e^{-t/1} = 10^{-3} e^{-t}$

discharge starts at $t = 0$

$\Rightarrow \boxed{I_R = 10^{-3} \text{ A} = 1 \text{ mA}}$

c) $\Delta V_c = \Delta V_i \, e^{-\frac{t}{\tau}} = \frac{q_i}{C} e^{-\frac{t}{1}} = \frac{10^{-3}}{10^{-6}} e^{-t} = 10^{3} e^{-t}$

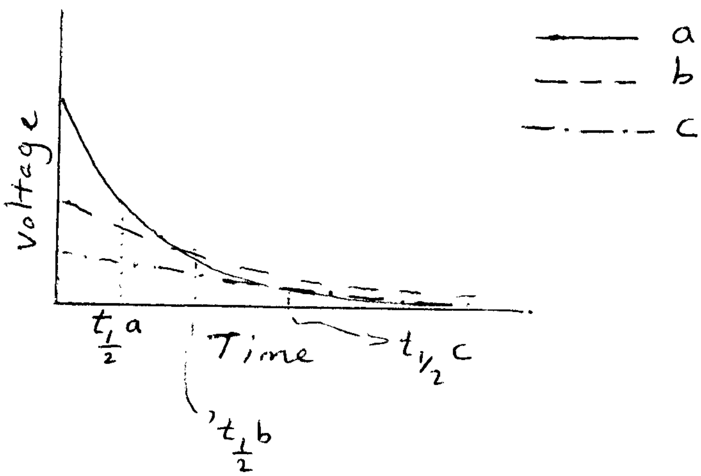
$\Delta V_R = \Delta V_c = 10^{3} e^{-t}$

d) $P = RI^2 = (10^6 \, \Omega) \times (10^{-3} e^{-t})^2 = e^{-2t} \text{ W}$

28-62

These are similar to the graphs that you drew in the class.

By comparing the half lives, you can find out which one is a bigger capacitor.



$$t_{1/2}c > t_{1/2}b > t_{1/2}a \Rightarrow 0.69\tau_c > 0.69\tau_b > 0.69\tau_a$$

$$\Rightarrow RC_c > RC_b > RC_a$$

$$\Rightarrow C_c > C_b > C_a$$