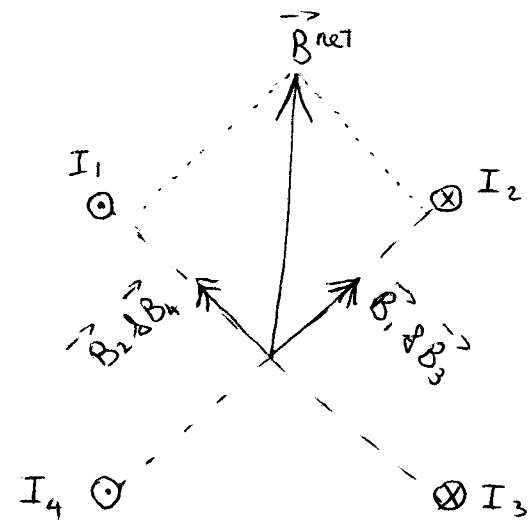
Unit 26 – Homework Solution

30-27

$$|\vec{B}_1| = |\vec{B}_2| = |\vec{B}_3| = |\vec{B}_4| = \frac{\mu_0 I}{2\pi (a/\sqrt{2})}$$

$$= \frac{4\pi \times 10^{-7} \times 20}{2\pi \times 0.2/\sqrt{2}} = 2.8 \times 10^{-5}\ \text{T}$$

$$|\vec{B}_{net}| = \sqrt{2(2\times 2.8\times 10^{-5})^2} = 7.9\times 10^{-5}\ \text{T}$$



30-29

First we should find the net magnetic field due to I_1, I_2, and I_3 at the location of I_4.

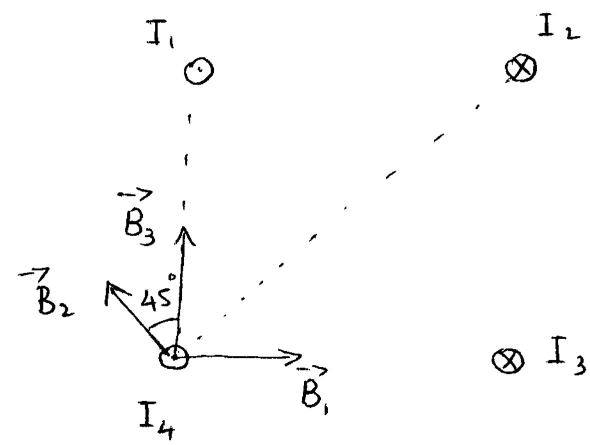
$$I_1 = I_2 = I_3 = I_4 = i$$

$$|\vec{B}_1| = \frac{\mu_0 I_1}{2\pi a} = \frac{\mu_0 i}{2\pi a}$$

$$|\vec{B}_2| = \frac{\mu_0 I_2}{2\pi a\sqrt{2}} = \frac{\mu_0 i}{2\sqrt{2}\,\pi a}$$

$$|\vec{B}_3| = |\vec{B}_1| = \frac{\mu_0 i}{2\pi a}$$

$\cos 45° = \frac{\sqrt{2}}{2}$



$$B_x^{net} = B_1 - B_2\cos 45° = \frac{\mu_0 i}{2\pi a} - \frac{\mu_0 i}{2\sqrt{2}\pi a}\frac{\sqrt{2}}{2}$$

$$= \frac{\mu_0 i}{4\pi a}$$

$$B_y^{net} = B_3 + B_2 \sin 45° = \frac{\mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\sqrt{2}\pi a}\frac{\sqrt{2}}{2} = \frac{3\mu_0 i}{4\pi a}$$

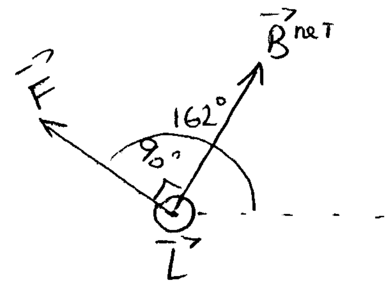
$$|\vec{B}^{net}| = \sqrt{\left(\frac{\mu_0 i}{4\pi a}\right)^2 + \left(\frac{3\mu_0 i}{4\pi a}\right)^2} = 0.79\,\frac{\mu_0 i}{\pi a}\ (\text{T})$$

$$\theta = \tan^{-1}\frac{B_y^{net}}{B_x^{net}} = 72°$$ relative to $+x$ direction

Force acting on I_4 : $\vec{F} = I_4 \vec{L} \times \vec{B}$

$$|\vec{F}| = i L B = i (1m) \left(0.79 \frac{\mu_0 i}{\pi a}\right)$$

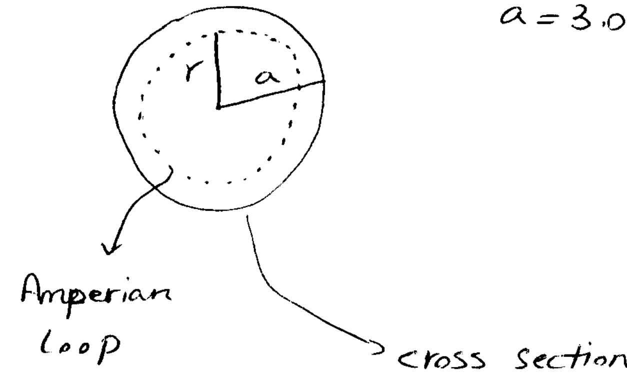
$$= 0.79 \frac{\mu_0 i^2}{\pi a} \ (N)$$



30 - 38

a) We can use Ampere's law to calculate the magnetic field.

a = 3.0mm



$$\oint_{loop} \vec{B} \cdot \vec{ds} = \mu_0 I^{encl} \qquad \vec{B} \parallel \vec{ds}$$

$$B \oint ds = \mu_0 I^{encl} \Rightarrow B(2\pi r) = \mu_0 I^{encl}$$

☆ Not the total curren is enclosed by Amperian loop!

$$\text{current density} = \frac{I}{A} \Rightarrow 100 \ A/m^2 = \frac{I^{encl}}{A_{\text{Ampere loop}}} = \frac{I^{encl}}{\pi r^2}$$

area

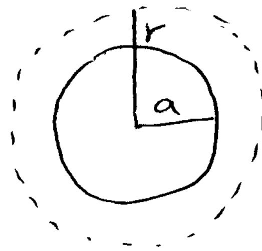
$$\Rightarrow I^{encl} = 100 \pi r^2$$

$$\Rightarrow B = \frac{\mu_0 I^{encl}}{2\pi r} = \frac{\mu_0 100 \cancel{\pi} r^{\cancel{2}}}{2 \cancel{\pi} \cancel{r}} = 50 \mu_0 r = 50 \times 4\pi \times 10^{-7} \times 2 \times 10^{-3}$$

$$= 1.25 \times 10^{-7} \ T$$

b) $r = 4.0$ mm

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 I^{encl}$$



$$B(2\pi r) = \mu_0 \left(100 \times \pi a^2\right)$$

$$B = \frac{100\, \cancel{\pi} a^2 \mu_0}{2 \cancel{\pi} r} = \frac{100 \times (3 \times 10^{-3})^2 \times 4\pi \times 10^{-7}}{2 \times 4 \times 10^{-3}}$$

$$= 1.41 \times 10^{-7}\ T$$

31 – 5

a) $R = \rho \frac{L}{A} = \frac{(1.69 \times 10^{-8}\ \Omega \cdot m)\ 2\pi(0.05)}{\pi\left(\frac{2.5 \times 10^{-3}}{2}\right)^2}$

$= 1.08 \times 10^{-3}\ \Omega$

b) $I = \frac{\mathcal{E}mf}{R} \Rightarrow emf = (1.08 \times 10^{-3}\Omega)(10\ A) = 1.08 \times 10^{-2}\ V$

$|emf| = N \frac{d\Phi}{dt} = NA \frac{dB}{dt}$

$\Rightarrow 1.08 \times 10^{-2} = \pi(0.05)^2 \frac{dB}{dt} \Rightarrow \frac{dB}{dt} = 1.37\ T/s$

31 – 20

a) $\theta = \omega t = (2\pi f)t$

angle between $\vec{B}$ and $\vec{A}$

$\Phi_m = \vec{B} \cdot \vec{A} = BA\cos\theta = BA\cos(2\pi f t)$

$A = \frac{1}{2}\pi a^2 \Rightarrow \Phi_m = \frac{1}{2} B\pi a^2 \cos(2\pi f t)$

$\mathcal{E}mf = -N\frac{d\Phi_m}{dt} = -\frac{1}{2} B\pi a^2 \frac{d}{dt}(\cos 2\pi f t)$

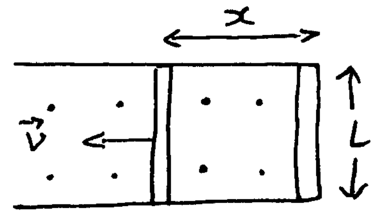
$= -\frac{1}{2}B\pi a^2 (-2\pi f \sin(2\pi f t))$

$= B\pi^2 a^2 f \sin(2\pi f t)$

$\Rightarrow$ frequency of emf $= f$

b) Amplitude $= B\pi^2 a^2 f$

31-29



a) $B = \text{constant}$

$A = Lx = Lvt$ $\qquad x = vt$

$\Rightarrow \Phi_m = \vec{B}\cdot\vec{A} = BLvt$

$\Rightarrow |\mathcal{E}mf| = \left|-\frac{d\Phi_m}{dt}\right| = +BLv = (+0.350)(0.25)(0.55)$
$= 0.048\ V$

b) $I = \frac{\mathcal{E}mf}{R} = \frac{0.048}{18} = 0.00267\ A = 2.67\ mA$

c) $P = RI^2 = 18 \times 0.00267^2 = 1.28 \times 10^{-4}\ W = 0.128\ mW$

32-8

$L = 4.6\ H$

$R = 12\ \Omega$

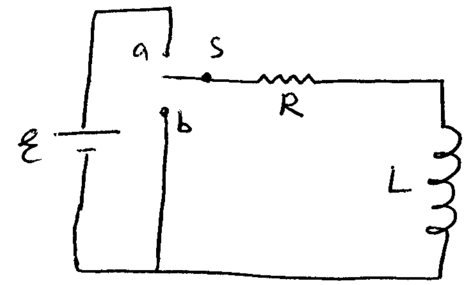
a) $|\mathcal{E}_L| = \left|L\frac{di}{dt}\right|$ for $0 < t < 2\,ms$: $\frac{di}{dt} = \frac{7}{0.002}$

$|\mathcal{E}_L| = (4.6)\left(\frac{7}{0.002}\right) = 16100\ V$

b) $\left|\frac{di}{dt}\right| = \frac{2}{0.003} \Rightarrow |\mathcal{E}_L| = (4.6)\left(\frac{2}{0.003}\right) = 3067\ V$

c) $\left|\frac{di}{dt}\right| = \frac{5}{0.001} \Rightarrow |\mathcal{E}_L| = (4.6)\left(\frac{5}{0.001}\right) = 23000\ V$

32 - 22



$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{\tau_L}}\right)$$

$$\mathcal{E}_L = -L\frac{di}{dt} = -L\left(-\frac{\mathcal{E}}{R}\left(-\frac{1}{\tau_L}\right)e^{-t/\tau_L}\right)$$

$$= -L\frac{\mathcal{E}}{R}\frac{R}{L}e^{-t/\tau_L} = -\mathcal{E}e^{-t/\tau_L}$$

a) at $t = 0 \Rightarrow |\mathcal{E}_L| = +\mathcal{E}$

b) $t = 2\tau_L \qquad \Delta V_2 = |\mathcal{E}_L| = \mathcal{E}e^{-2}$

c) $\Delta V_2 = \frac{1}{2}\mathcal{E} \Rightarrow \frac{1}{2}\cancel{\mathcal{E}} = \cancel{\mathcal{E}}e^{-t/\tau_L} \Rightarrow t = \tau_L \ln 2 = 0.693\,\tau_L$

32 - 35

$$\Delta V_s N_p = \Delta V_p N_s$$

a) $\Delta V_s = (120\text{ V})\frac{10}{500} = \frac{12}{5}\text{ V} = 2.4\text{ V}$

b) $I_s = \frac{\Delta V_s}{R} = \frac{2.4}{15} = 0.16\text{ A}$

$$I_p = \frac{i_s N_s}{N_p} = \frac{(0.16)(10)}{500} = 0.0032\text{ A}$$