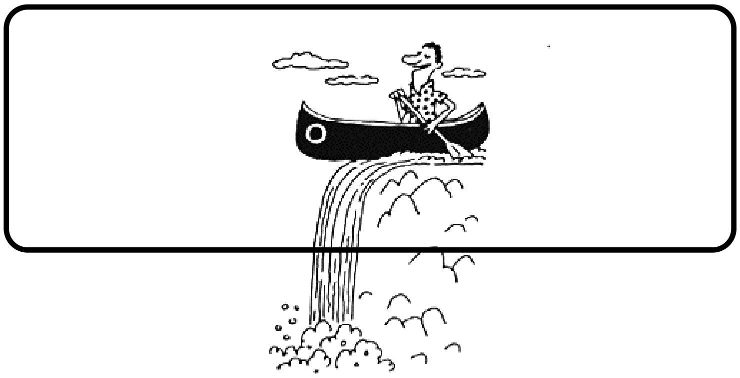
Name ______________________ Date(YY/MM/DD) ______/_________/_______
St.No. __ __ __ __ __-__ __ __ __ Section__________

UNIT 21: ELECTRICAL AND GRAVITATIONAL POTENTIAL



I began to think of gravity extending to the orb of the moon, and . . . I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centres about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them to answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since.

– Isaac Newton

OBJECTIVES

1. To understand the similarities in the mathematics used to describe gravitational and electrical forces.

2. To review the mathematical definition of work and potential in a conservative force field.

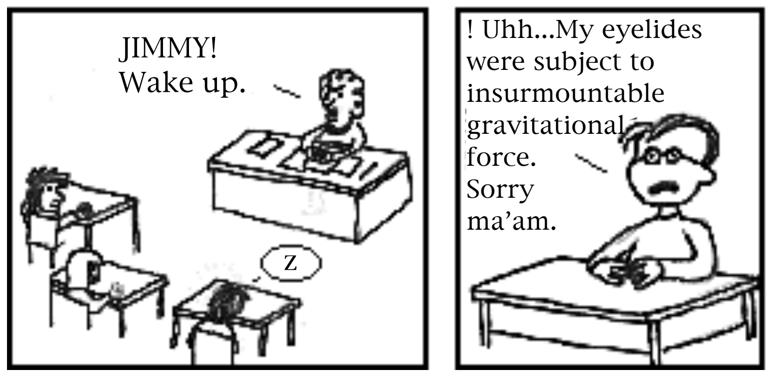
3. To understand the definition of electrical potential, or *voltage*, and its similarity to the concept of gravitational potential.

4. To learn how to determine electric field lines from equipotential surfaces and vice versa.

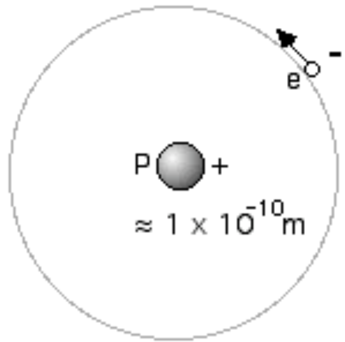
5. To learn how to map equipotentials in a plane resulting from two point charges and two line electrodes.

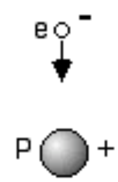
OVERVIEW

The enterprise of physics is concerned ultimately with mathematically describing the fundamental forces of nature. Nature offers us several fundamental forces, which include a strong force that holds the nuclei of atoms together, a weak force that helps us describe certain kinds of radioactive decay in the nucleus, the force of gravity, and the electromagnetic force.



Two kinds of force dominate our everyday reality – the gravitational force acting between masses and the Coulomb force acting between electrical charges. The gravitational force allows us to describe mathematically how objects near the surface of the earth are attracted toward the earth and how the moon revolves around the earth and planets revolve around the sun. The genius of Newton was to realize that objects as diverse as falling apples and revolving planets are both moving under the action of the same gravitational force. Similarly, the Coulomb force allows us to describe how one charge "falls" toward another or how an electron orbits a proton in a hydrogen atom.





The fact that both the Coulomb and the gravitational forces lead to objects falling and to objects orbiting around each other suggests that these forces might have the same mathematical form.

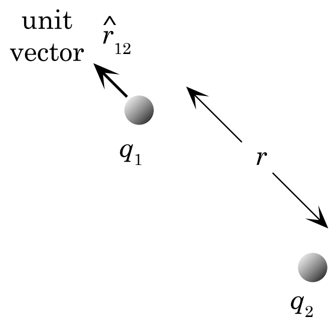
In this unit we will explore the mathematical symmetry between electrical and gravitational forces for two reasons. First, it is beautiful to behold the unity that nature offers us as we use the same type of mathematics to predict the motion of planets and galaxies, the falling of objects, the flow of electrons in circuits, and the nature of the hydrogen atom and of other chemical elements. Second, what you have already learned about the influence of the gravitational force on a mass and the concept of potential energy in a gravitational field can be applied to aid your understanding of the forces on charged particles. Similarly, a "gravitational" Gauss' law can be used to find the forces on a small mass in the presence of a large spherically symmetric mass.

We will introduce the concept of electrical potential differences, which are analogous to gravitational potential energy differences. An understanding of *electrical potential difference*, commonly called *voltage*, is essential to understanding the electrical circuits which are used in physics research and which dominate this age of electronic technology. The unit will culminate in the actual measurement of electrical potential differences due to an electric field from two "line" conductors that lie in a plane and the mathematical prediction of the potential differences using a derivation based on Gauss' law.

SESSION ONE: ELECTRICAL AND GRAVITATIONAL FORCES

Comparison of Electrical and Gravitational Forces

Let's start our discussion of this comparison with the familiar expression of the Coulomb force exerted on charge 1 by charge 2.

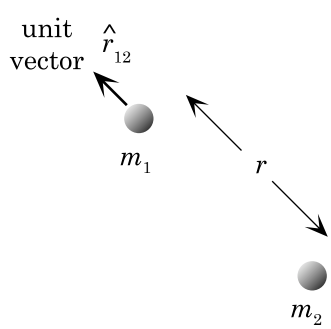


$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$k_e = 8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

Charles Coulomb did his experimental investigations of this force in the 18th century by exploring the forces between two small charged spheres. Much later, in the 20th century, Coulomb's law enabled scientists to design cyclotrons and other types of accelerators for moving charged particles in circular orbits at high speeds.

Newton's discovery of the universal law of gravitation came the other way around. He thought about orbits first. This was back in the 17th century, long before Coulomb began his studies. A statement of Newton's universal law of gravitation describing the force experienced by mass 1 due to the presence of mass 2 is shown below in modern mathematical notation:



$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12}$$

$$G = 6.67 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$$

About the time that Coulomb did his experiments with electrical charges in the 18th century, one of his contemporaries, Henry Cavendish, did a direct experiment to determine the nature of the gravitational force between two spherical masses in a laboratory. This confirmed Newton's gravitational force law and allowed him to determine the gravitational constant, G. A fact emerges that is quite amazing. *Both types of forces, electrical and gravitational, are very similar. Essentially the same mathematics can be used to describe orbital and linear motions due to either electrical or gravitational interactions of the tiniest fundamental particles or the largest galaxies.* [This statement needs to be qualified a bit when electrons, protons and other fundamental particles are considered. A new field of study called wave mechanics was developed in the early part of this century to take into account the wave nature of matter, which we don't actually study in introductory physics. However, even in wave mechanical calculations electrical forces like those shown above are used.]

✍ Activity 21-1: The Electrical vs. the Gravitational Force

Examine the mathematical expression for the two force laws.

(a) What is the same about the two force laws?

(b) What is different? For example, is the force between two like masses attractive or repulsive? How about two like charges? What part of each equation determines whether the like charges or masses are attractive or repulsive?

(c) Do you think negative mass could exist? If there is negative mass, would two negative masses attract or repel?

Which Force is Stronger-- Electrical or Gravitational?
Gravitational forces hold the planets in our solar system in orbit and account for the motions of matter in galaxies. Electrical forces serve to hold atoms and molecules together. If we consider two of the most common fundamental particles, the electron and the proton, how do their electrical and gravitational forces compare with each other?

Let's peek into the hydrogen atom and compare the gravitational force on the electron due to interaction of its mass with that of the proton to the electrical force between the two particles as a result of their charge. In order to do the calculation you'll need to use some well known constants.

Electron: $m_e = 9.1 \times 10^{-31}$ kg $q_e = -1.6 \times 10^{-19}$ C
Proton: $m_p = 1.7 \times 10^{-27}$ kg $q_p = +1.6 \times 10^{-19}$ C
Distance between the electron and proton: $r = 1.0 \times 10^{-10}$ m

---

✍ Activity 21-2: The Electrical vs. the Gravitational Force in the Hydrogen Atom

(a) Calculate the magnitude of the electrical force on the electron. Is it attractive or repulsive?

(b) Calculate the magnitude of the gravitational force on the electron. I s it attractive or repulsive?

(c) Which is larger? By what factor (i.e. what is the ratio)?

(d) Which force are you more aware of on a daily basis? If your answer does not agree with that in part (c), explain why.

Gauss' Law for Electrical and Gravitational Forces
Gauss' law states that the net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0 . Mathematically this is represented by an integral of the dot product of the electric field and the normal to each element of area over the closed surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q^{enc}}{\varepsilon_0}$$

The fact that electric field lines spread out so that their density (and hence the strength of the electric field) decreases at the same rate that the area of an enclosing surface increases can ultimately be derived from the $1/r^2$ dependence of electrical force on distance. Thus, *Gauss' law should also apply to gravitational forces*.

✍ **Activity 21-3: The Gravitational Gauss' Law**
State a gravitational version of Gauss' law in words and then represent it with an equation. Hint: If the electric field vector, $\vec{E}$, is defined as $\vec{F}_e/q$, let's define the gravitational field vector $\vec{Y}$ as $\vec{F}_g/m$ by analogy.

We can use the new version of Gauss' law to calculate the gravitational field at some distance from the surface of the earth just as we can use the electrical Gauss' law to determine the electric field at some distance from a uniformly charged sphere. This is useful in figuring out the familiar force "due to gravity" near the surface of the earth and at other locations.

✍ Activity 21-4: The Gravitational Force of the Earth

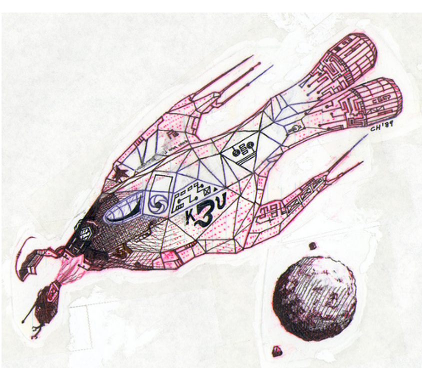
(a) Use Gauss' law to show that the magnitude of the gravitational field, $\vec{Y}$, at a height h above the surface of the earth is given by $\dfrac{GM_e}{\left(R_e + h\right)^2}$.

Hints: How much mass is enclosed by a spherical shell of radius $R_e + h$? Does $\vec{Y}$ have a constant magnitude everywhere on the surface of the spherical shell? Why? If so, can you pull it out of the Gauss' law integral?

(b) Calculate the gravitational field, $\vec{Y}$, at the surface of the earth ($h = 0$). The radius of the earth is $R_e \approx 6.38 \times 10^3$ km and its mass $M_e \approx 5.98 \times 10^{24}$ kg. Does the result look familiar? How is $\vec{Y}$ related to the gravitational acceleration g?

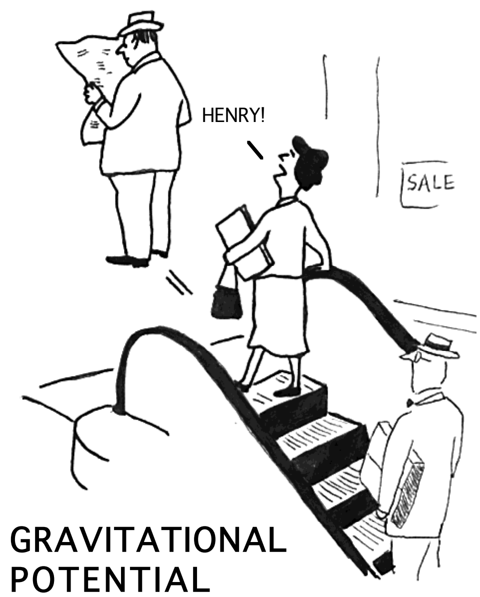
(c) Use the equation you derived in part (a) to calculate the value of $\vec{Y}$ at the ceiling of the room you are now in. How does it differ from the value of $\vec{Y}$ at the floor? Can you measure the difference in the lab using the devices available?

(d) Suppose you travel halfway to the moon. What is the new value of $\vec{Y}$? Can you measure the difference? (Recall that the earth-moon distance is about 384,000 km.)



(e) Is the gravitational acceleration "constant", g, really a constant? Explain.

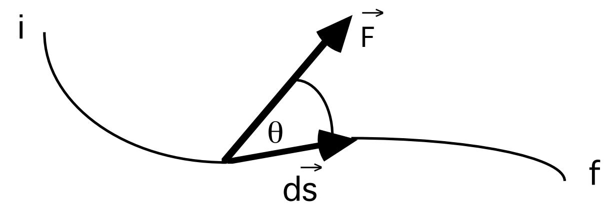
(f) In part (d) you showed that there is a significant gravitational attraction halfway between the earth and the moon. Why, then, do astronauts experience weightlessness when they are orbiting a mere 120 km above the earth?



Work in a Gravitational Field – A Review

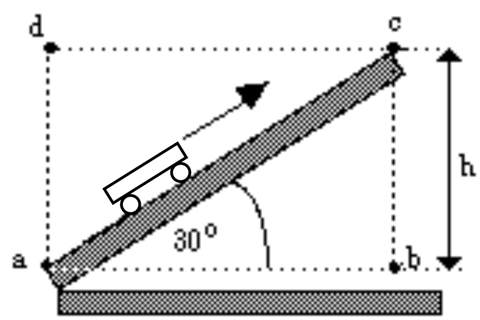
Let's review some old definitions in preparation for tackling the idea of work and energy expended in moving through an electric field. Work, like flux, is a scalar rather than a vector quantity. It is defined mathematically as a line integral over a path between locations i and f. Each element of $\vec{F} \bullet d\vec{s}$ represents the dot product or projection of $\vec{F}$ along $d\vec{s}$ at each place along a path.

$$W = \int_i^f \vec{F} \bullet d\vec{s}$$



$$\vec{F} \bullet d\vec{s} = F\, ds \cos\theta$$

Let's review the procedures for calculating work and for determining that the work done in a conservative force field is independent of path by taking some work measurements along two different paths, ac and adc, as shown in the diagram below.



In order to take measurements you will need the following equipment:

- an inclined plane
- a cart
- a 5-N spring scale
- a metre stick
- a protractor

Set the inclined plane to an angle of about 30° to the horizontal and take any necessary length measurements. Don't forget to list your units!

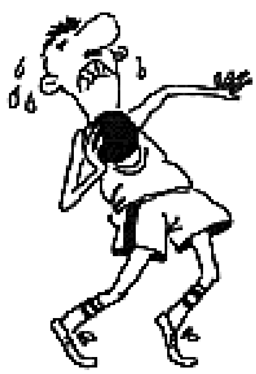
✍ Activity 21-5: Earthly Work

(a) Use path ac to raise the cart up an inclined plane that makes an angle of about 30 degrees with the earth's surface (see diagram above). Use a spring balance to measure the force you are exerting, and calculate the work you performed *using the definition of work.*

(b) Raise the cart directly to the same height as the top of the inclined plane along path ad and then move the cart horizontally from point d to point c. Again, use the *definition of work* to calculate the work done in raising the cart along path adc.

(c) How does the work measured in (a) compare to that measured in (b)? Is this what you expected? Why or why not? Hint: Is the force field conservative?

Work and the Electric Field

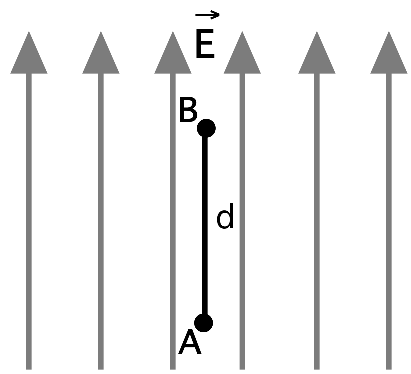


It takes work to lift an object in the earth's gravitational field. Lowering the object releases the energy that was stored as potential energy when it was lifted. Last semester, we applied the term *conservative* to the gravitational force because it "releases" <u>all</u> of the stored energy. We found experimentally that the work required to move a mass in the gravitational field was path independent. This is an important property of any conservative force. Given the mathematical similarity between the Coulomb force and the gravitational force, it should come as no surprise that experiments confirm that an electric field is also conservative. This means that the work needed to move a charge from point A to point B is independent of the path taken between points. A charge could be moved directly between two points or looped around and the work expended to take either path would be the same. Work done by an electric field on a test charge q_o traveling between points A and B is given by

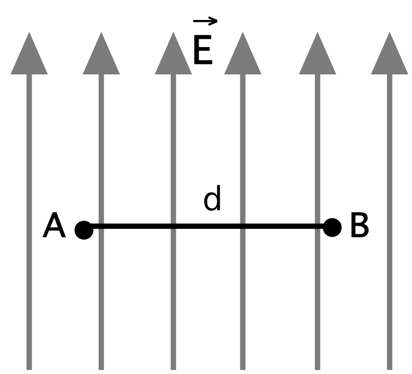
$$W = \int_A^B \vec{F} \bullet d\vec{s} = \int_A^B q_0 \vec{E} \bullet d\vec{s}$$

✍ Activity 21-6: Work Done on a Charge Traveling in a Uniform Electric Field

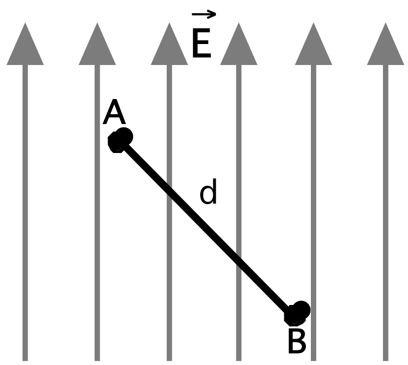
(a) A charge *q* travels a distance *d* from point A to point B; the path is parallel to a uniform electric field of magnitude *E*. What is the work done by the field on the charge? How does the form of this equation compare to the work done on a mass *m* travelling a distance *d* in the almost uniform gravitational field near the surface of the earth?



(b) The charge *q* travels a distance *d* from point A to point B in a uniform electric field of magnitude *E*, but this time the path is perpendicular to the field lines. What is the work done by the field on the charge?



(c) The charge q travels a distance d from point A to point B in a uniform electric field of magnitude E. The path lies at a 45° angle to the field lines. What is the work done by the field on the charge?



SESSION TWO: ELECTRIC POTENTIAL DIFFERENCE

Potential Energy and Potential Difference

Recall that by definition the work done by a conservative force equals the negative of the change in potential energy, so that the change in potential energy of a charge moving from point A to point B under the influence of an electrical force can be found as follows:

$$-\Delta E_{pot} = W = \int_A^B \vec{F} \bullet d\vec{s} = \int_A^B q_0 \vec{E} \bullet d\vec{s}$$

By analogy to the definition of the electric field, we are interested in defining the *electric potential difference* $\Delta V = V_B - V_A$ as the change in electrical potential energy ΔE_{pot} per unit charge. Formally, *the potential difference is defined as the work per unit charge that an external agent must perform to move a test charge from A to B without changing its kinetic energy* The potential difference has units of joules per coulomb. Since 1 J/C is defined as one *volt*, the potential difference is often referred to as *voltage*.

✍ Activity 21-7: The Equation for Potential Difference

Write the equation for the potential difference as a function of $\vec{E}, d\vec{s}$, A, and B.

The Potential Difference for a Point Charge

The simplest charge configuration that can be used to consider how voltage changes between two points in space is a single point charge. We will start by considering a single point charge and then move on to more complicated configurations of charge.

A point charge q produces an electric field that moves out radially in all directions. The line integral equation for the potential difference can be evaluated to find the potential difference between any two points in space A and B.

It is common to choose the reference point for the determination of voltage to be at ∞ so that we are determining the work per unit charge that is required to bring a test charge from infinity to a certain point in space. Let's choose a coordinate system so that the point charge is conveniently located at the origin. In this case we will be interested in the potential difference between infinity and some point which is a distance r from the point charge. Thus, we can write the equation for the potential difference, or voltage, as

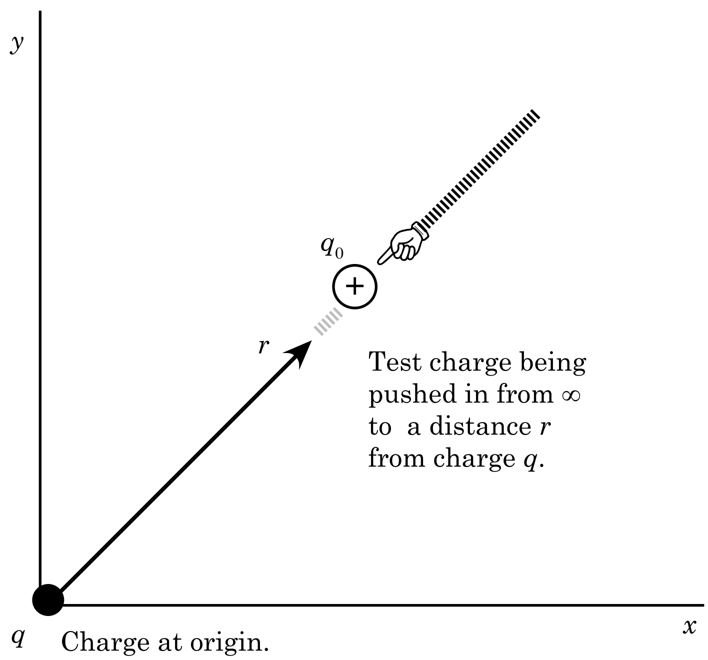
$$\Delta V = V_B - V_A = V_r - V_\infty = -\int_\infty^r \vec{E} \bullet d\vec{s}$$

Often, when the reference point for the potential difference is at infinity, this difference is simply referred to as "the potential" and the symbol ΔV is just replaced with the symbol V.

✍ Activity 21-8: Potential at a Distance *r* from a Charge

Show that, if A is at infinity and B is a distance r from a point charge q, then the potential V is given by the expression

$$V = \frac{kq}{r}$$

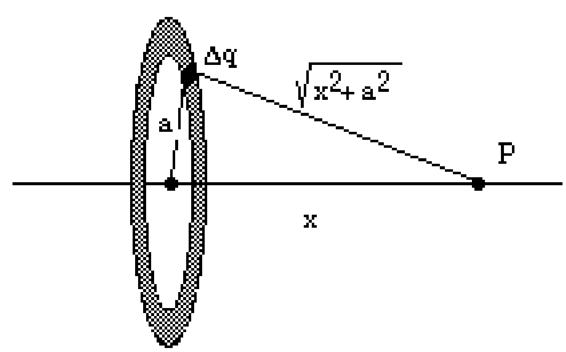


Hint: What is the mathematical expression for an E-field from a point charge?

The Potential Difference Due to Continuous Charge Distributions

The potential from a continuous charge distribution can be calculated several ways. Each method should yield approximately the same result. First, we can use an integral method in which the potential dV from each element of charge dq is integrated mathematically to give a total potential at the location of interest. Second, we can approximate the value of the potential V by summing up several finite elements of charge Δq by using a computer spreadsheet or hand calculations. Finally, we can use Gauss' law to find the electric field along with the defining equation for potential difference to set up the appropriate line integral shown in Activity 21-7.

Again, let's consider a relatively simple charge distribution. In this case we will look at a ring with charge uniformly distributed on it. We will calculate the potential on the axis passing through the center of the ring as shown in the diagram below. (Later on you could find the potential difference from a disk or a sheet of charge by considering a collection of nested rings).



A ring of charge has a total charge of $Q = 20.0\ \mu C$. (i.e. 20.0 x 10 $^{-6}$ C). The radius of the ring, a, is 30.0 cm. What is the electric field, $\vec{E}$, at a distance of x cm from the ring along an axis that is perpendicular to the ring and passes through its centre? What is the potential, V? Let's begin by calculating the potential.

Hints: Since the potential is a scalar and not a vector we can calculate the potential at point P (relative to ∞) for each of the charge elements Δq and add them to each other. This looks like a big deal but it is actually a trivial problem because all the charge elements are the same distance from point P.

✍ Activity 21-9: Numerical Estimate of the Potential from a Charged Ring

(a) Divide the ring into 20 elements of charge Δq and calculate the total V at a distance of $x = 20$ cm from the centre of the ring. Show all of your work below.

✍ Activity 21-10: Calculation of the Potential from a Charged Ring

By following the steps below, you can use an integral to find a more exact value of the potential.

(a) Show that $V = k\int \frac{dq}{r} = k\int \frac{dq}{\sqrt{x^2 + a^2}}$

(b) Show that $k\int \frac{dq}{\sqrt{x^2 + a^2}} = \frac{k}{\sqrt{x^2 + a^2}}\int dq$ (i.e. show that $\sqrt{x^2 + a^2}$ is a constant and can thus be pulled out of the integral).

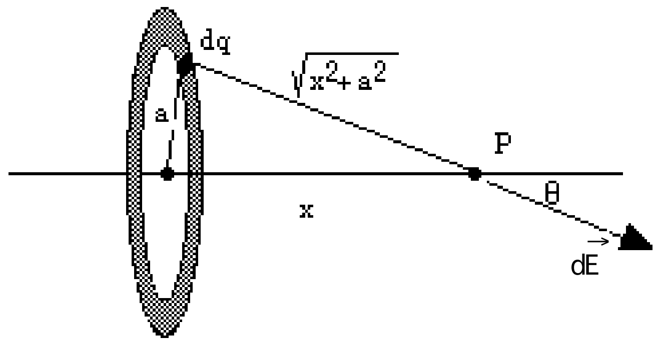
(c) Perform the integration in part (b) above. Then substitute values for a, x, and Q into the resulting expression in order to obtain a more "exact" value for the potential.

(d) How does the "numerical" value that you obtained in Activity 21-9 compare with the "exact" value you obtained in (c)?

Now let's take a completely different approach to this problem. If we can find the vector equation for the electric field at point P due to the ring of charge, then we can use the expression

$$\Delta V = V(x) - V_{\infty} = -\int_{\infty}^{x} \vec{E} \bullet d\vec{s}$$

as an alternative way to find a general equation for the potential at point P.



✍ Activity 21-11: ΔV from a Ring using the E-field Method

(a) Show that the electric field at point P from the charged ring is given by

$$\vec{E} = \frac{kQx}{\left(x^2+a^2\right)^{\frac{3}{2}}}\hat{\imath} = \frac{kQx}{\sqrt{x^2+a^2}^{\,3}}\hat{\imath}$$

Hints: (1) There is no y component of the E-field on the x-axis. Why?

(2) $\cos\theta = \dfrac{x}{\sqrt{x^2+a^2}}$

(b) Use the equation $\Delta V = V(x) - V_{\infty} = -\int_{\infty}^{x} \vec{E} \bullet d\vec{s}$ to find ΔV.

(c) How does the result compare to that obtained in Activity 21-10 (c) ?

Equipotential Surfaces

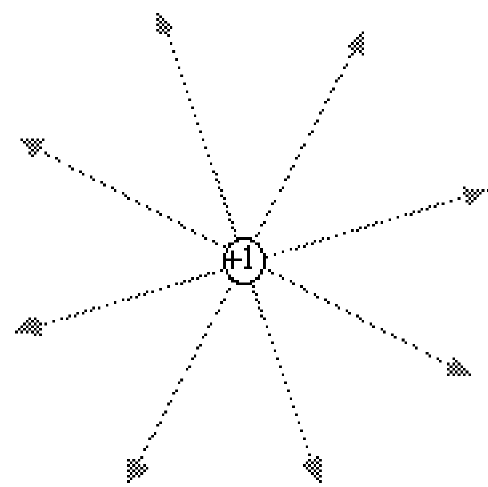
Sometimes it is possible to move along a surface without doing any work. Thus, it is possible to remain at the same potential energy anywhere along such a surface. If an electric charge can travel along a surface without doing any work, the surface is called an *equipotential surface.*

Consider the three different charge configurations shown below. Where are the equipotential surfaces? What shapes to they have?

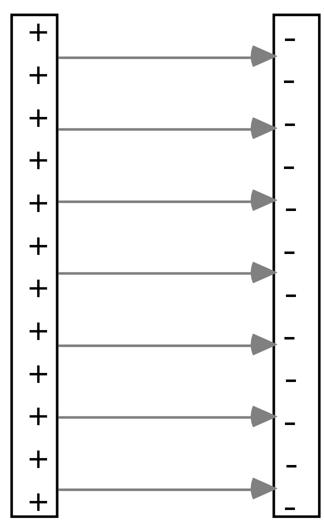
Hint: If you have any computer simulations available to you for drawing equipotential lines associated with electrical charges, you may want to check your guesses against the patterns drawn in one or more of the simulations.

✍ Activity 21-12: Sketches of Electric Field Lines and Equipotentials

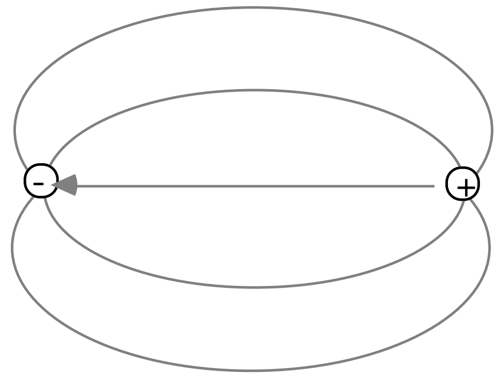
(a) Suppose that you are a test charge and you start moving at some distance from the charge below (such as 4 cm). What path could you move along without doing any work, i.e. $\vec{E} \cdot d\vec{s}$ is always zero? What is the shape of the equipotential surface? Remember that in general you can move in *three* dimensions.



(b) Find some equipotential surfaces for the charge configuration shown below, which consists of two charged metal plates placed parallel to each other. What is the shape of the equipotential surfaces?



(c) Find some equipotential surfaces for the electric dipole charge configuration shown below.

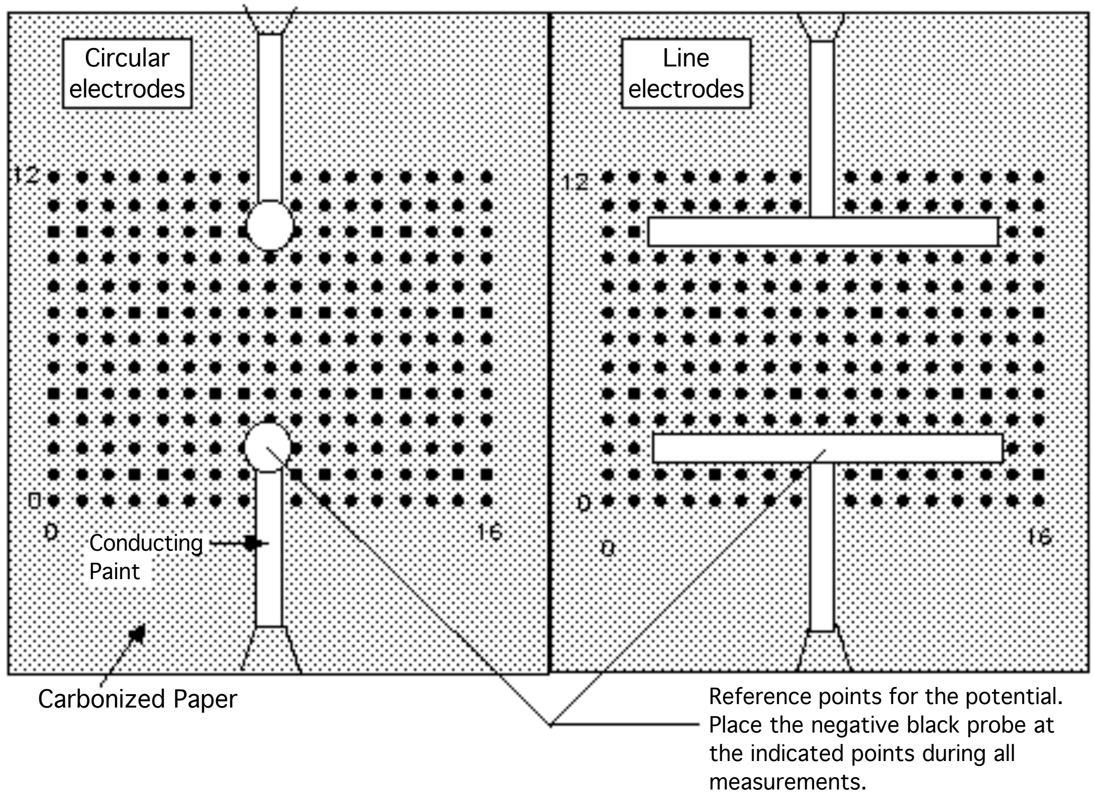


(d) In general, what is the relationship between the direction of the equipotential lines you have drawn (representing that part of the equipotential surface that lies in the plane of the paper) and the direction of the electric field lines?

Experiment on Equipotential Plotting

The purpose of this observation is to explore the pattern of potential differences in the space between conducting equipotential surfaces. This pattern of potential differences can be related to the electric field caused by the charges that lie on the conducting surfaces.

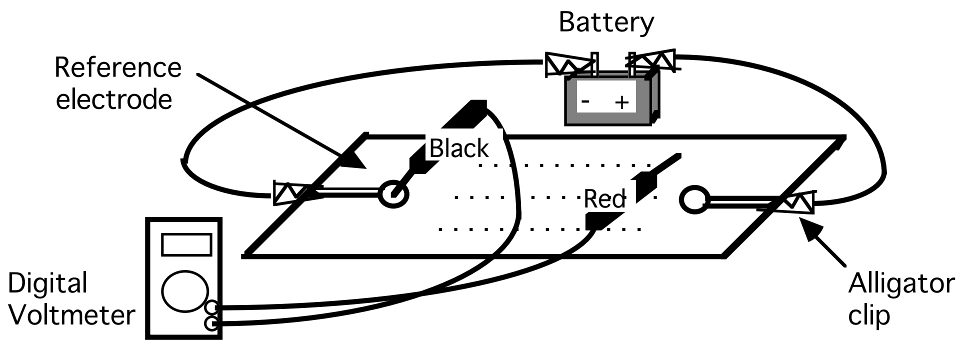
You will be using pieces of carbonized paper with conductors painted on them in different shapes to *simulate* the pattern of equipotential lines associated with metal electrodes in air. (Warning: this is only a simulation of "reality"!) One of the papers has two small circular conductors painted on it and the other has two linear conductors.



You can use a battery to set up a voltage (i.e. a potential difference) across a paper with two electrodes painted on it. You can then use a digital voltmeter to trace several equipotential lines on it. To do this observation you will need the following equipment:

- equipotential plotting paper with line and circular electrodes
- foam board
- metal push pins
- a digital voltmeter
- 4 D-cell batteries
- 2 alligator clip leads

Pick a piece of carbonized paper with either the two circular electrodes or the line electrodes. Use the alligator clips to connect the terminals of the battery to each of the electrodes on the paper as shown below.



Turn on the digital voltmeter. Set the tip of the black voltmeter probe (plugged into the COM input) on the centre of the negative circular electrode. Place the red probe (plugged into the V-Ω-S input) on any location on the paper. The reading on the voltmeter will determine the potential difference between the two points. What happens when you reverse the probes? Why?

After placing the black probe on the centre of an electrode, you will use the red probe to find the equipotential lines for potential differences of +1v, +2v, +3v, and +4v.

✍ Activity 21-13: Mapping the Equipotentials

(a) On the two sheets of grid paper provided draw diagrams of the two electrode sheets you are going to examine.

(b) For both electrode sheets, sketch the lines you mapped to scale on the diagrams. Attach these sheets to this Activity Guide when you hand it in.

(c) Compare the results to those you predicted in Activity 21-12. Is it what you expected? If not, explain how and why it differs. Make sketches in the space below, if that is helpful.

NOTE: BE SURE TO TURN THE VOLTMETER ***OFF*** WHEN FINISHED WITH OBSERVATIONS.