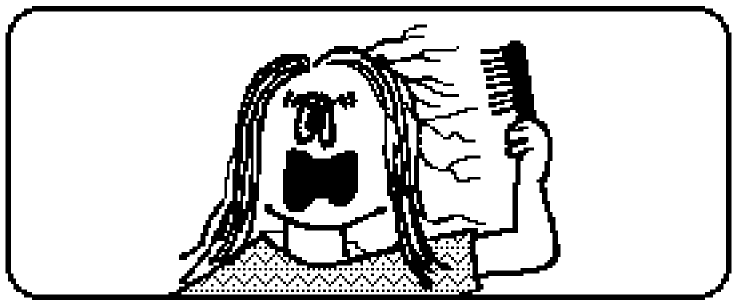
Name ______________________ Date(YY/MM/DD) ______/_________/_______
St.No. __ __ __ __ __ __ __ __ __ Section__________Group#_______

UNIT 19: ELECTRIC FIELDS

Approximate time three 100-minute sessions



Electricity is a quality universally expanded in all the matter we know, and which influences the mechanism of the universe far more than we think.

Charles Dufay (1698-1739)

OBJECTIVES

1. To discover some of the basic properties of particles which carry electric charges.

2. To understand how Coulomb's law describes the forces between charged objects.

3. To understand the concept of electric fields.

4. To learn how to calculate the electric field associated with charges that are distributed throughout an object.

Credits: Some of the activities in this unit have been adapted from those designed by the Physics Education Group at the University of Washington.

OVERVIEW

10 min

On cold, clear days, rubbing almost any object seems to cause it to be attracted to or repelled from other objects. After being used, a plastic comb will pick up bits of paper, hair, and cork, and people wearing polyester clothing in the winter walk around cursing the phenomenon dubbed in TV advertisements as "static cling". We are going to begin a study of electrical phenomena by exploring the nature of the forces between objects that have been rubbed or that have come into contact with objects that have been rubbed. These forces are attributed to a fundamental property of the constituents of atoms known as *charge*. The forces between particles that are not moving or that are moving relatively slowly are known as *electrostatic forces*.

We start our study in the first session by exploring the circumstances under which electrostatic forces are attractive and under which they are repulsive. Then we can proceed to a qualitative study of how the forces between charged objects depend on the amount of charge the objects carry and on the distance between them. This will lead to a formulation of *Coulomb's law* which expresses the mathematical relationship of the vector force between two small charged objects in terms of both distance and quantity of charge. In the second session you will be asked to verify Coulomb's law quantitatively by performing a video analysis of the repulsion between two charged objects as they get closer and closer together.

Finally, in the third session we will define a quantity called *electric field* which can be used to determine the net force on a small test charge due to the presence of other charges. You will then use Coulomb's law to calculate the electric field, at various points of interest, arising from some simply shaped charged objects.

SESSION ONE: ELECTROSTATIC FORCES

Exploring the Nature of Electrical Interactions

You can investigate some properties of electrical interactions with the following equipment. Each student should have:

- Scotch tape and a hard non-conducting surface
- 1 stand for suspending charged balls
- 2 small Styrofoam balls attached to threads
- 2 small Styrofoam balls covered with metallic paint or aluminum foil attached to threads
- A hard plastic rod and fur
- A glass rod and a plastic bag
- A metal rod

The nature of electrical interactions is not obvious without careful experimentation and reasoning. We will first state two hypotheses about electrical interactions. We will then observe some electrical interactions and determine whether our observations are consistent with these hypotheses.

Hypothesis One: The interaction between objects that have been rubbed is due to a *property* of matter which we will call *charge*.* There are *two* types of electrical charge which we will call, for the sake of convenience, positive charge and negative charge.

*Note: A property of matter is not the same thing as the matter itself. For instance, a full balloon has several properties at once – it can be made of rubber or plastic, have the colour yellow or blue, have a certain surface area, and so on. Thus, *we don't think of charge as a substance but rather as a property that certain substances can have at times.* It is easy when speaking and writing casually to refer to charge as if it were a substance. Don't be misled by this practice which we will all indulge in at times during the next few units.

Hypothesis Two: Charge moves readily on certain materials, known as conductors, and not on others, known as insulators. In general, metals are good conductors while glass, rubber, Styrofoam and plastic tend to be insulators.

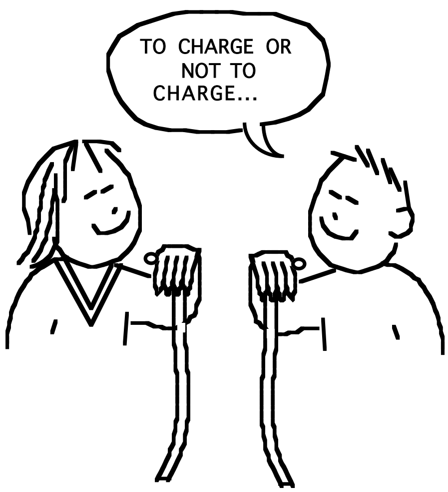
Note: In completing the activities that follow, *you are not allowed to state results that you have memorized previously. You must devise a sound and logical set of reasons to support the hypotheses.*

Hypothesis One: Testing for Different Types of Charge

Try the activities suggested below. Mess around and see if you can design careful, logical procedures to demonstrate that there are at least two types of charge. Carefully explain your observations and reasons for any conclusions you draw.

Hint: What procedures should you use to generate two objects that carry the same type of charge?

✍ Activity 19-1: Interactions of Scotch Tape Strips



(a) You and your partner should each stick a 10 cm or so strip of scotch tape on the lab table with the end curled over to make a non-stick handle. Peel the tape off the table and bring the non-sticky side of the tape toward your partner's strip. What happens? How does the distance between the strips affect the interaction between them?

(b) Place two strips of tape on the table sticky side down and label them "B" for bottom. Press another strip of tape on top of each of the B pieces; label these strips "T" for top. Pull each pair of strips off of the table. Then pull the top and bottom strips apart.

1. Describe the interaction between two top strips when they are brought toward one another.

2. Describe the interaction between two bottom strips

3. Describe the interaction between a top and bottom strip.

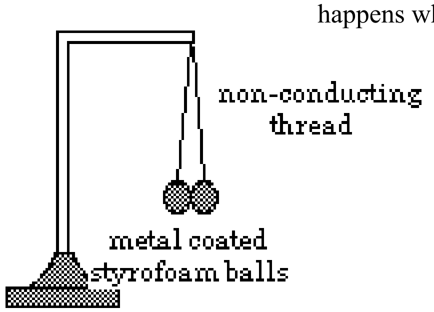
(c) Are your observations of the tape strip interactions consistent with the hypothesis that there are two types of charge? Please explain your answer carefully, in complete sentences, and cite the outcomes of *all* of your observations.

Hypothesis Two: Testing for Conductors and Non-conductors

Try the activities suggested below using rods charged by rubbing to charge small balls. Mess around and see if you can design careful, logical procedures to demonstrate that there are at least two types of objects with regard to how easily charges move on them.

Note: *In the activities that follow your observations will not be valid if you touch the balls with your hands after charging them.*

✍ **Activity 19-2: Charging Styrofoam Balls with Rods**
(a) Try rubbing a black plastic rod with fur and then use the rod to touch a pair of small metal-coated Styrofoam balls hanging from nonconducting threads. What happens to the hanging balls? What happens when you bring the plastic rod near the balls?

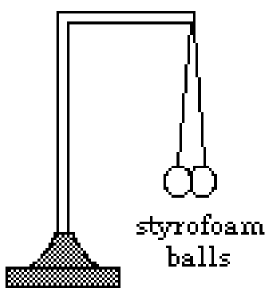


(b) What happens if you rub a glass rod with a plastic bag and then bring it into the vicinity of the balls that were charged with the plastic rod?

(c) Recalling the interactions between like and unlike charged objects that you observed before, can you explain your observations?

(d) Touch each of the two charged metal-coated balls with a large metal rod. Now what happens when you let them hang again? Is there an interaction between them? If so, describe it.

(e) Repeat observations (a) through (d) using a similar pair of hanging Styrofoam balls that have no *metal coating*. Describe the outcome of your observations in the space below. In particular, what happens after you touch each of the charged Styrofoam balls with the metal rod?



(f) What happens when you bring a charged rubber or glass rod near a metal-covered ball that has not been charged? What happens when you bring the same rod near an uncoated Styrofoam ball that has not been charged?

(g) Using Hypothesis Two, which claims that metals are electrical conductors and Styrofoam is not, explain why the metal covered ball is always attracted to a charged object *even when it is not charged itself.*

The process by which charges rearrange themselves in a conductor so that an uncharged conductor is always attracted to a charged object is known as *induction*.

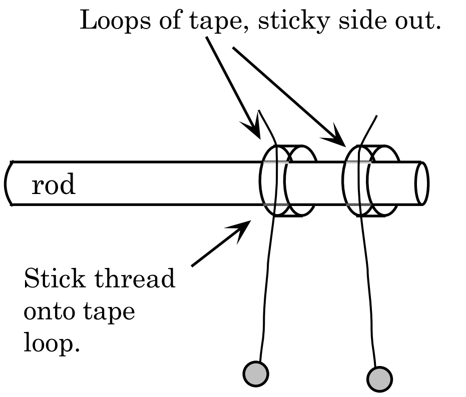
Benjamin Franklin *arbitrarily* assigned the term "negative" to the nature of the charge that results when a hard plastic rod (or, in his day, a rubber rod) is rubbed with fur. Conversely, the nature of the charge found on the glass rod after it is rubbed with silk is defined as "positive". (The term "negative" could just as well have been assigned to the charge on the glass rod; the choice was purely arbitrary.)

Forces between Charged Particles – Coulomb's Law
Coulomb's law is a mathematical description of the fundamental nature of the electrical forces between charged objects that are either spherical in shape or small compared to the distance between them)so that they act more or less like point particles). This law relates the force between small charged objects to the charges on the objects and the distance between them. Coulomb's law is usually stated without experimental proof in most introductory physics textbooks. Instead of just accepting the textbook statement of Coulomb's law, you are going to determine qualitatively how the charge on two objects and their separation affect the mutual force between them. These objects could be, for instance, two metal-covered Styrofoam balls, or perhaps a small metal ball affixed to the tip of an insulated rod and one of the metal-covered balls. For this set of observations you will need:

- 1 or 2 stands for suspending charged balls
- 2 small Styrofoam balls covered with metallic paint
  or 2 tiny balls of aluminum foil attached to threads
- A hard plastic rod and fur, or a vinyl or acetate strip and paper.
- A metal rod

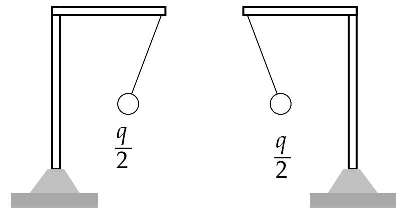
Note: Coulomb devised a clever trick for determining how much force charged objects exert on each other without knowing the actual amount of charge on the objects. Coulomb transferred an unknown amount of charge, q, to a conductor. He then touched the newly charged conductor to an identical uncharged one. The conducting objects would quickly exchange charge until both had $q/2$ on them. After observing the effects with $q/2$, Coulomb would discharge one of the conductors by touching a large piece of metal to it and then repeat the procedure to get $q/4$ on each conductor, and so on.

✍ Activity 19-3: Dependence of Force on Charge, Distance, and Direction – Qualitative Observations



You can suspend the balls from a rod by wrapping two loops of tape around the rod with the sticky sides of the tape facing out. Then just push the thread onto the tape's adhesive. The distance between balls can be changed by sliding a loop left or right. The height of the ball can be adjusted by carefully rotating the tape loop

Consider a pair of conductors, each initially having charge $q_1 = q_2 = q/2$. These conductors are hanging from strings in the configuration shown in the diagram below.

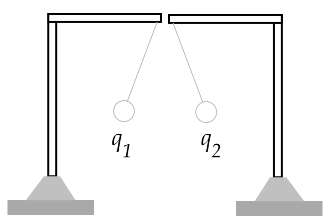


Use the diagrams below to sketch what you predict will happen to the positions of charged objects 1 and 2 as compared to their initial positions when $q_1=q_2 = q/2$. In each case give the reasons for your prediction. Then make the observation and sketch what you observed.

(a) What if the charged conductors still each have a charge of $q_1=q_2 = q/2$ but the suspension points are moved closer together as shown in the diagram below?

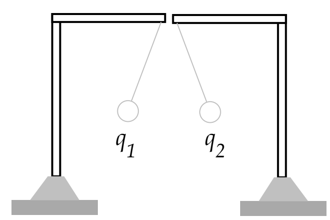
In these diagrams the original positions are indicated with light lines, Draw your prediction and observation in dark pen or pencil.

Prediction



Reasons:

Observation

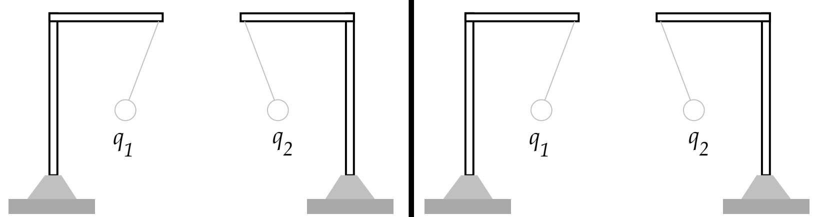


New Explanation:
if observation and prediction disagree.

(b) What seems to happen to the force of interaction between the charged conductors as the distance between them *decreases*?

(c) What if the suspension points are moved back to their original position but the amount of charge on each conductor is decreased so that $q_1 = q_2 = q/4$?

Prediction | Observation

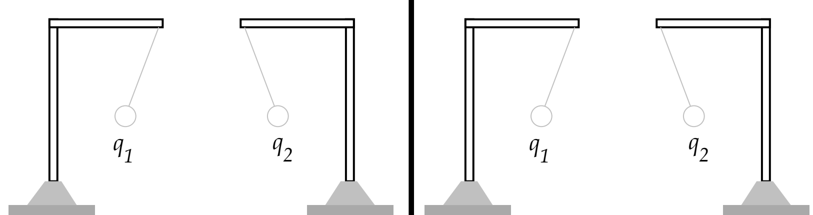


Reasons: | New Explanation: if observation and prediction disagree.

(d) Does the force of interaction between charged objects seem to increase or decrease as the charge decreases?

(e) What if *one* of the conductors q_1 still has a charge of $q/4$ while the other one is discharged completely so $q_2 = 0$. The observation may surprise you. Can you explain it?

Prediction | Observation

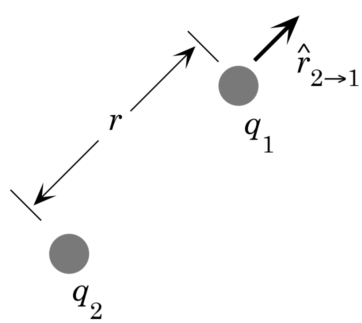


Reasons: | New Explanation: if observation and prediction disagree.

Hint: Did Newton's Third Law or the idea of induction come into play?

(f) Explain on the basis of the observations you have already made why the force between the two charged objects seems to lie along a line between them. **Hint**: What would happen to the mutual repulsion or attraction if the force did not lie on a line between the two charged objects?

The Mathematical Formulation of Coulomb's Law
Coulomb's law asserts that the magnitude of the force between two electrically charged spherical objects is *directly proportional to the product of the amount of charge on each object and inversely proportional to the square of the distance* between the centres of the spherical objects. The direction of the force is along a line between the two objects and is attractive if the particles have opposite signs and repulsive if the particles have like signs. All of this can be expressed by the equation below in which $\vec{F}_{e,2\to1}$ represents the electrostatic force exerted on q_1 due to q_2.

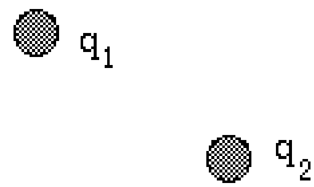


$$\vec{F}_{e,2\to1} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{2\to1}$$

The $\hat{r}_{2\to1}$ with a "hat" over it is a *unit* vector directed from q_2 to q_1, r^2 is the square of the distance between the two charged objects in metres, k_e is a constant that equals 9.0×10^9 N m^2/C^2, and q is the charge in Coulombs.

✍ Activity 19-4: "Reading" the Coulomb Equation

(a) Draw the direction of the unit vector $\hat{r}_{2\to1}$ in the diagram below. **Note:** The direction of this vector does *not* depend on the signs or the magnitudes of the charges.

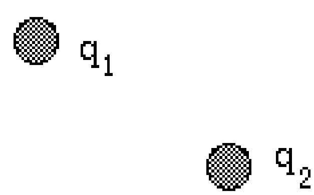


(b) If the force vector $\vec{F}_{e,2\to1}$ is in the opposite direction from the unit vector $\hat{r}_{2\to1}$, the unit vector must be multiplied by a negative number. Where does this negative number come from in the Coulomb equation? (Think about the signs of the two charges.) Does this negative number indicate a repulsive force or an attractive force?

(c) In the Coulomb equation, what happens to the magnitude of the force as either q_1 or q_2 decreases? Why?

(d) In the equation, what happens to the magnitude of the force as the distance between the charged objects decreases? Why?

(e) In the diagram below, show the direction of the unit vector $\hat{r}_{1\to2}$.



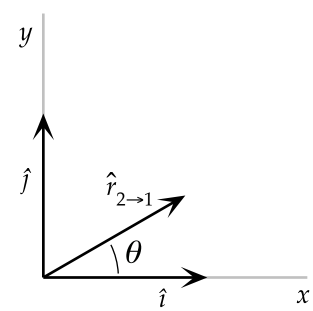
(f) Is Coulomb's law consistent with Newton's Third Law? In particular, how do $\vec{F}_{e,2\to1}$ and $\vec{F}_{e,1\to2}$ compare in magnitude? In direction?

In order to get some more practice with reading and using the Coulomb's law equation you should do the following vector calculations. You may need to brush up on vectors!

✍ Activity 19-5: Using Coulomb's Law for Calculations

(a) Consider two charged objects lying along the x-axis. A 2.0 μC point charge is located at $x = 3.0$ cm and a –3.0 μC point charge is located at $x = 5.0$ cm. What is the magnitude of the force exerted by the positively charged object on the negatively charged object? What is its direction? Express the force as a vector quantity using unit vector notation.

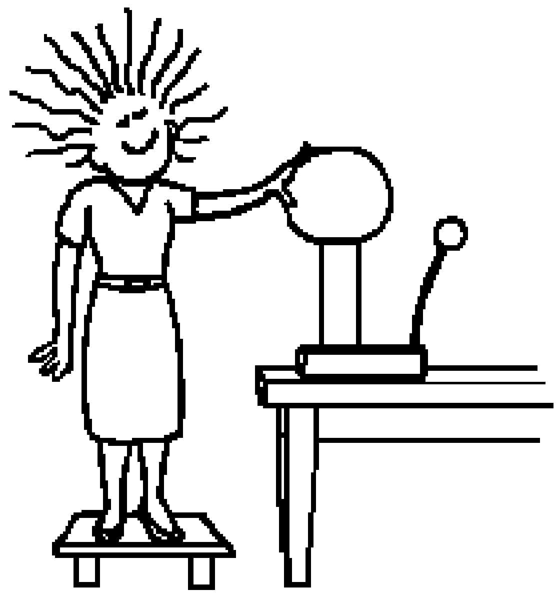
(b) Suppose the unit vector $\hat{r}_{2\to1}$ makes an angle θ with the x axis as shown in the diagram below. Use unit vector notation to express $\hat{r}_{2\to1}$ in Cartesian coordinates in terms of sinθ and cosθ. **Hint**: $\hat{r}_{2\to1}$ is a unit vector and hence has a magnitude of 1.



(c) Suppose the –3.0 μC point charge is moved to $x = 5.0$ cm and $y = 6.0$ cm. What is the magnitude of the force exerted by the negative point charge on the positive point charge? What is its direction? Express the force as a vector quantity using unit vector notation. Then draw a diagram of this situation, indicating the positions of the charges and the force vector. **Hint:** (1) Calculate the magnitude of the force. (2) Figure out what angle the force vector makes with respect to the *x*-axis. (3) Resolve the force vector into *x* and *y* components.

SESSION TWO: QUANTITATIVE ASPECTS OF COULOMB'S LAW

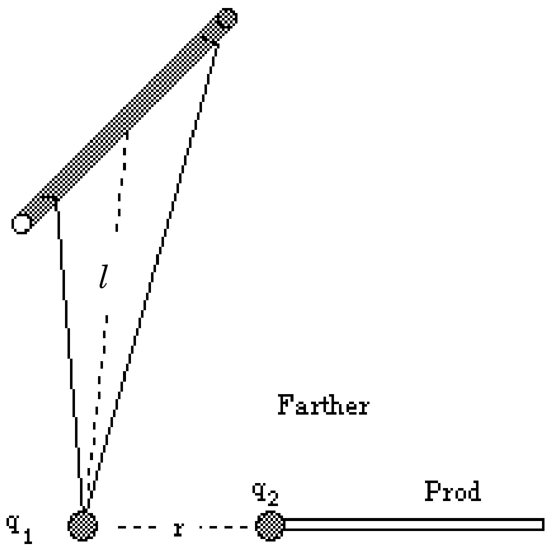
Demonstration of Electrostatic Discharges
In addition to exploring the nature of the relatively small collections of electrical charge that result from rubbing objects together, you can examine two demonstrations involving relatively high levels of electrical charge being "discharged."

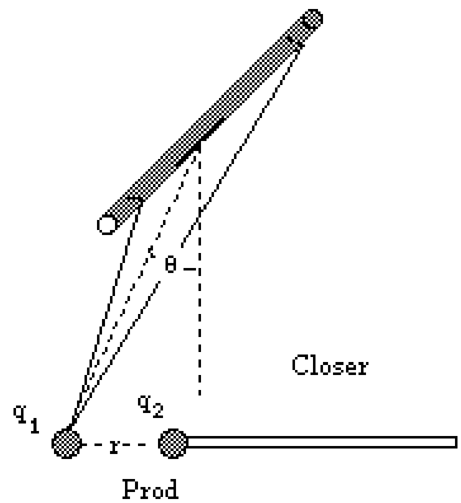


The Van de Graaff Generator:
Ben Franklin and others recognized that electrical charge can be "produced" by doing mechanical work. The Van de Graaff Generator will be demonstrated briefly. This device produces a relatively high density of electrical charge.

Quantitative Verification of Coulomb's Law
In the late eighteenth century Charles Coulomb used a torsion balance and a great deal of patience to verify that the force of interaction between small spherical charged objects varied as the inverse square of the distance between them. Verification of the inverse square law can also be attempted using modern apparatus.

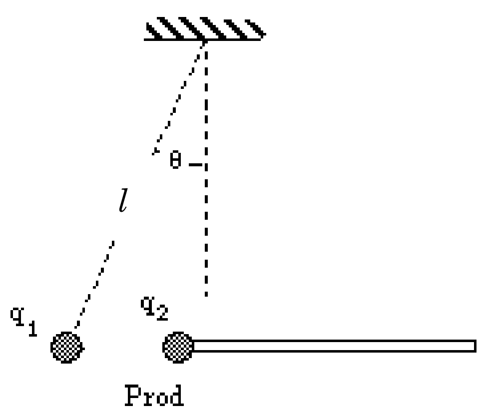
A small, conducting sphere can be placed on the end of an insulating rod and can then be charged negatively using a rubber rod that has been rubbed with fur. This charged sphere can be used as a prod to cause another charged sphere, suspended from two threads, to rise to a higher and higher angle as the prod comes closer, as shown in the diagram below. A video camera can be used to record the angle of rise, θ, of the suspended object as well as the distance between the prod and the suspended object.





Using the laws of mechanics, it is possible to determine the relationship between the Coulomb force on the small sphere and the angle through which it rises above a vertical line. Thus, you should be able to measure the Coulomb force on q_1 as a function of the distance between q_1 and q_2.

Before proceeding with the video analysis, lets take time out to determine the angle of rise, θ, of a charged sphere of mass m due to a Coulomb force on it. This force is the result of the presence of another charged object that lies in the same horizontal plane as the suspended mass. This situation is shown in the diagram below.



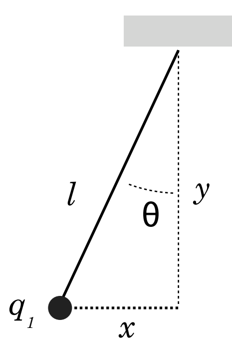
✍ Activity 19-6: Forces on a Suspended Charged Object–Theory

(a) Draw a vector diagram with arrows showing the direction of each of the forces on the mass *m*, including the gravitational force, $\vec{F}_g$, the tension in the string, $\vec{F}_T$, and a horizontal electrostatic force due to the charge on the prod, $\vec{F}_e$.

(b) Show that, when there is no motion along the vertical direction, $|\vec{F}_T|\cos\theta - mg = 0$ so that $|\vec{F}_T| = mg/\cos\theta$

(c) Show that, if there is no motion along the horizontal direction, $|\vec{F}_e| - |\vec{F}_T| \sin\theta = 0$.

(d) Show that $|\vec{F}_e| = mg \tan\theta$



(e) Find $\tan\theta$ as a function of *x* and *l*.
Hint: Now find *y* as a function of *x* and *l*.

(f) Now you can use the equations in (d) and (e) to find the magnitude of the electrostatic force, $|\vec{F}_e|$ as a function of *l, x, m,* and *g*.

Finally, we can turn to the task of obtaining a movie of how the mass rises as a function of its horizontal distance from the prod. Then, using the equation you obtained in Activity 19-6, you can analyse the video data to determine F_e as a function of r. (For simplicity we can write $|\vec{F}_e|$ as F_e.) To do this we will need the following apparatus:

- A small metal-coated Styrofoam ball
- Two long polyester threads
- A set-up for suspending the ball
- A prod (conducting ball with an insulating handle)
- Video analysis software (Logger Pro)

Your instructor will do this activity as a demonstration and then each group will individually analyse a pre-made movie.

Making a Movie of the Force Law Experiment

The purpose of this experiment is to verify that the forces of interaction between two small foil covered Styrofoam spheres varies as $1/r^2$, where r is the distance between the spheres. To obtain data in movie form that can be analysed one has to:

1. Suspend a small foil-covered Styrofoam sphere from two long polyester threads. Record the vertical distance from the point of suspension to the centre of the hanging sphere. Place a ruler horizontally under the hanging sphere.

2. Use fur to charge a rubber rod and transfer charge from the rod to both the prod and the hanging ball.

3. *Carefully touch the sphere with the prod so that they contain the same amount of charge.*

4. Practice bringing the charged prod closer and closer to the hanging sphere, slowly and steadily. The trick is to *keep the line between the sphere and the prod horizontal at all times*.

5. Once you get good at step 4, repeat it two or more times while the video camera is running. Start each movie with the prod far enough away from the sphere that there is no noticeable interaction between the two.

✍ Activity 19-7: Verifying Coulomb's Law Experimentally

1. Pick the best movie segment for analysis.

2. Analyse the movie frames by recording the position of each mass so that you can determine:

   (i) *r* the distance between the charged objects
   (ii) *x* the distance from the suspended mass to a vertical line

You will need at least six data points for each mass. Make sure the positions of each mass are recorded at the same times.

3. Carefully record your length and mass data in the space below.

4. Analyse your data with a spreadsheet program. Use the effective length of the strings suspending the mass *l* , the mass of the hanging sphere *m* and *x* to calculate the horizontal force on the suspended ball due to the prod in each of the frames you are analysing. Next determine *r;* the horizontal distance between the prod and the suspended mass, for each frame.

5. Plot F_e vs. $1/r^2$. Is it a straight line? If so, perform a linear fit and determine the slope and y-intercept. Submit a copy of your spreadsheet online.

6. Draw conclusions. Does the $1/r^2$ relationship hold?

7. Describe the most plausible sources of uncertainty in your data.

✍ Activity 19-8: How Much Charge is on the Hanging Sphere?

Since you touched the sphere and probe together before starting, they should have the same amount of charge on them. In your experiment, a fit of the plot of F_e vs. $1/r^2$ should yield a value for the slope. Now,

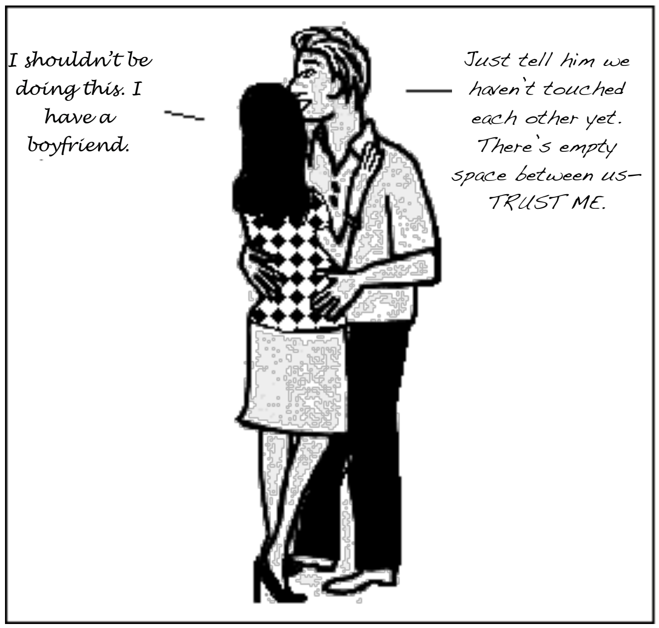
$$F_e = k\frac{q_1 q_2}{r^2} = kq^2\frac{1}{r^2} = (\text{slope})\frac{1}{r^2} \text{ where } q = q_1 = q_2$$

since $q_1 = q_2$. Thus, you should be able to calculate the amount of charge on the sphere (and probe). Go ahead and do it and show your calculations below.

SESSION THREE: THE ELECTRIC FIELD

The Electric Field

Until this week, most of the forces you studied resulted from the direct action or contact of one piece of matter with another. From your direct observations of charged, foil-covered Styrofoam balls, it should be obvious that charged objects can exert electrical forces on each other at a distance. How can this be? The action at a distance that characterizes electrical forces, or for that matter gravitational forces, is in some ways inconceivable to us. How can one charged object feel the presence of another and detect its motion with only empty space in between? Since all atoms and molecules are thought to contain electrical charges, physicists currently believe that all "contact" forces are actually electrical forces involving small separations. So, even though forces acting at a distance seem inconceivable to most people, physicists believe that *all* forces act at a distance.



Physicists now explain all forces between charged particles, including contact forces, in terms of the transmission of travelling electromagnetic waves. We will engage in a preliminary consideration of the electromagnetic wave theory toward the end of the semester. For the present, let's consider the attempts of Michael Faraday and others to explain action-at-a-distance forces back in the 19th century. Understanding more about these attempts should help you develop some useful models to describe the forces between charged objects in some situations.

To describe action at a distance, Michael Faraday introduced the notion of an *electric field* emanating from a collection of charged objects and extending out into space. More formally, the electric field due to a known collection of charged objects is represented by an electric field vector at every point in space. Thus, the electric field vector, $\vec{E}$, is defined as the force, $\vec{F}_e$, experienced by a very small positive test charge at a point in space divided by the magnitude of the test charge q_t. The electric field is in the direction of the force $\vec{F}_e$ on a small positive "test" charge and has the magnitude of

$$\vec{E} = \vec{F}_e/q_t$$

where q_t is the charge on a small test particle.

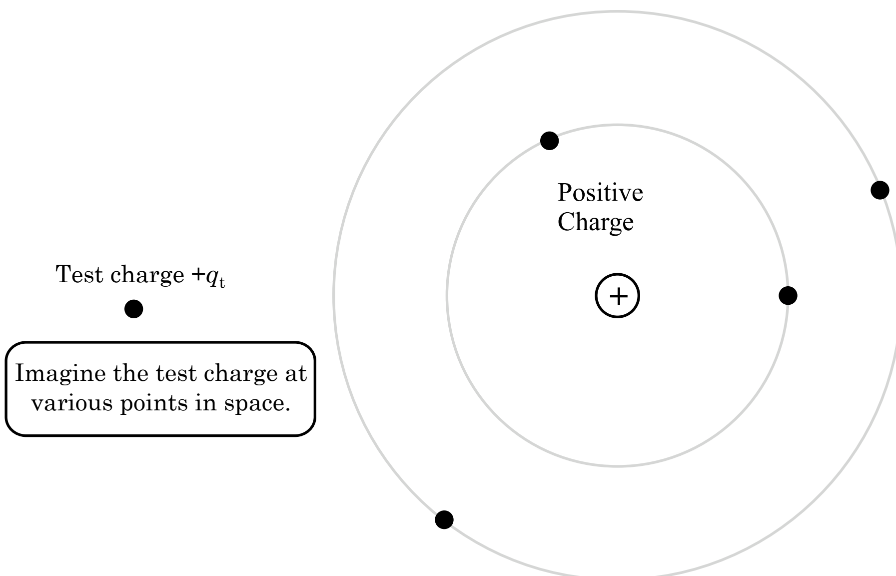
To investigate the vector nature of an electric field, we will do an experiment using a PhET simulation called Charges and Fields. It can be found at:

http://phet.colorado.edu/simulations/sims.php?sim=Charges_and_Fields

Note: By convention physicists always place the tail of the E-field vector at the point in space of interest rather than at the charged object that causes the field.

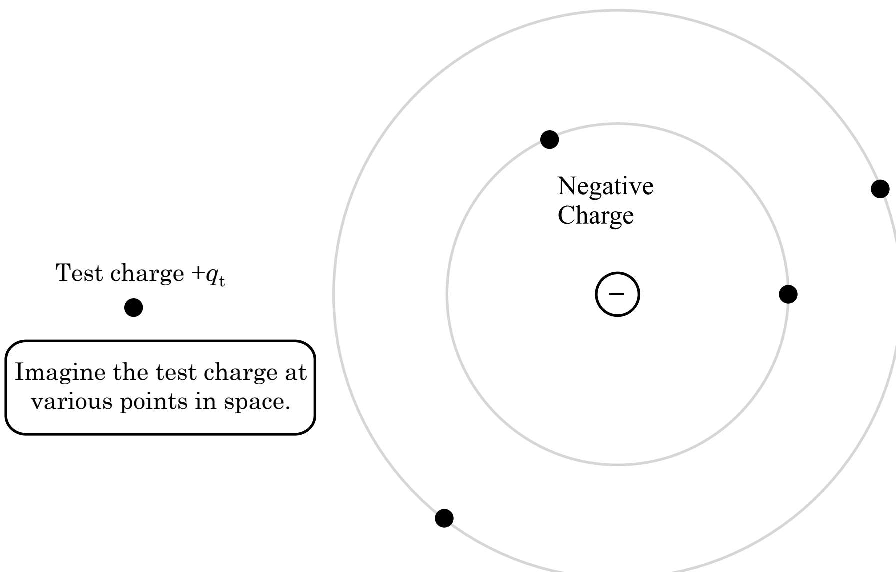
✍ Activity 19-9: Electric Field Vectors from a Positive Charge

Make a qualitative sketch of some electric field vectors around a positive charge. To do so, run the PhET Charges and Fields simulation and place E-Field sensors around a positive point charge at locations corresponding to the black dots shown in the diagram below. The length of each vector roughly indicates the *relative* magnitude of the field (i.e. if the E-field is stronger at one point than another, its vector will be longer). The direction of the vector indicates the direction of the field at that point in space.



✍ Activity 19-10: Electric Field from a Negatively Charged Rod

Make a qualitative sketch of some electric field vectors around a negative charge. To do so, run the PhET Charges and Fields simulation and place E-Field sensors around a negative point charge at locations corresponding to the black dots shown in the diagram below. The length of each vector roughly indicates the *relative* magnitude of the field (i.e. if the E-field is stronger at one point than another, its vector will be longer). The direction of the vector indicates the direction of the field at that point in space.



Superposition of Electric Field Vectors

The fact that electric fields from charged objects that are distributed at different locations act along a line between the charged objects and the point in space of interest is known as *linearity*. The fact that the vector fields due to charged objects at different points in space can be added together is known as *superposition*. These two properties of the Coulomb force and the electric field that derives from it are very useful in our endeavour to calculate the value of the electric fields due to a collection of point charges at different locations. This can be done by finding the value of the E-field vector from each point charge and then using the *principle of superposition* to determine the vector sum of these individual electric field vectors.

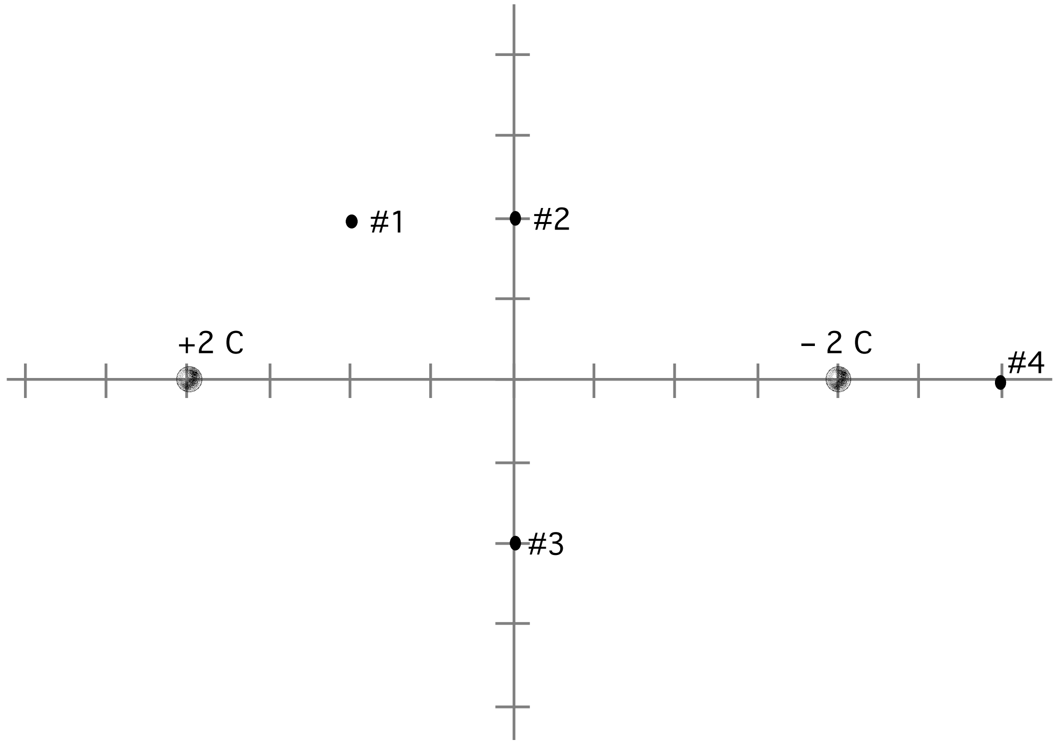
✍ Activity 19-11: Electric Field Vectors from Two Point Charges

(a) Look up the equations for Coulomb's law and the electric field from a point charge in your textbook. Also check out the value of any constants you would need to calculate the actual value of the electric field from a point charge. List the equations and any needed constants in the space below.

(b) Use a spreadsheet to calculate the *magnitude* of the electric field (in N/C) at distances of 0.5, 1.0, 1.5, ... , 10.0 cm. from a point charge of 2.0 C. Be careful to use the correct units (i.e., convert the distance to meters before doing the calculation). Save your file for later reference.

(c) The graph below shows two point particles with charges of +2C and –2C that are separated by a distance of 8.0 cm. Use the principle of *linearity* to draw the vector contribution of each of the point charges to the electric field at each of the four points in space shown below. Use your spreadsheet results and a scale in which the vector is 1 cm long for each electric field magnitude of 1.0×10^{13} N/C. Then use the principle of *superposition* and the rules of vector addition to draw the resultant $\vec{E}$ vector at each point. You do not need to find the vector components. A drawing of the vector addition at each location is sufficient.

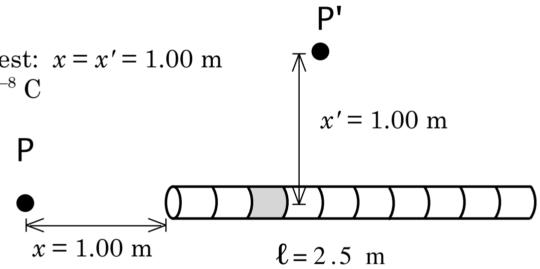
Hint: One of the point charges will attract a positive test charge and the other will repel it.



The Electric Field from an Extended Charge Distribution
If electrical charge is distributed uniformly throughout a continuous extended object, it can be divided into small segments each of which contains a charge Δq. Then, by assuming that each segment behaves like a tiny point charge, the electric field at a point **P** in space due to each segment can be calculated. The total electric field at **P** is simply the vector sum of the contributions of each of the charge segments. This process yields an *approximate* value of the electric field at point **P**. Such approximate values can be calculated quite readily using a computer spreadsheet. To get a more exact value we must sum up infinitely many infinitesimally small elements of charge dq. This is what mathematical integration is all about.

The goal of this section of the activity guide is to calculate the electric field $\vec{E}$ corresponding to a continuous charge distribution on a rod at two points in space, **P** and **P'**, as shown below.

Rod Length: $\ell = 2.5$ m
Closest distance to points of interest: $x = x' = 1.00$ m
Total charge on rod: $q = 1.00 \times 10^{-8}$ C

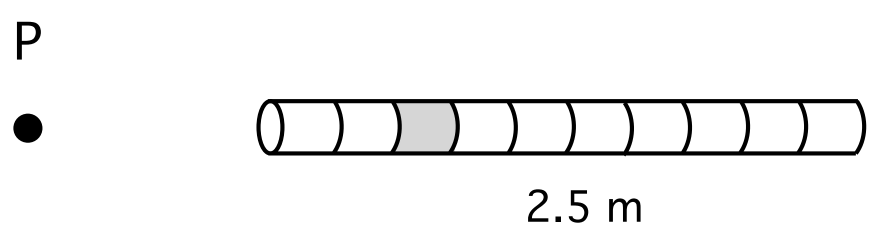


Each of these calculations will be done two ways: (1) doing an approximate numerical calculation with the spreadsheet, and (2) doing an "exact" integration. These two methods of calculation will be compared with each other. You could extend the calculation to other points in space and graph the change in field as a function of the distance from the rod along a line through the axis of the rod and along a line perpendicular to the rod.

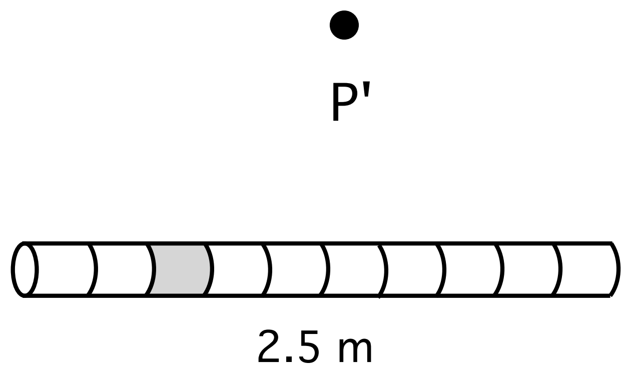
✍ Activity 19-12: E-Field Vectors from a Charge Distribution on a Rod

Draw the magnitude and direction of the ten vectors $\Delta\vec{E}_i$ to approximate relative scale (i.e. draw longer arrows for the vectors corresponding to charge elements closer to P or P') for each of the two points using the diagrams below. Draw a resultant vector in each case.

(a) *Parallel to the Axis of the Rod*



(b) *Perpendicular to the Axis of the Rod*



✍ Activity 19-13: Electric Field Calculations along the Axis of a Rod

(a) Consider a rod of length ℓ that is divided into n segments. If the total charge on the rod is given by q, show what equation you would use to calculate the charge Δq in each segment $\Delta\ell$ of the rod.
(Note that in general we can define a charge per unit length or linear charge density for the rod, λ, as q/ℓ)

(b) Use a spreadsheet to find the electric fields at the point **P** due to the ten elements numerically. You should probably define three columns: n (Charge Element #), x, and $\Delta\vec{E}_n$. Once the calculations are done you can sum up the $\Delta\vec{E}_n$'s to get the value of $\vec{E}$. Submit a copy of your spreadsheet online.

(c) Set up the integral for $\vec{E}$ and solve it to obtain the equation relating $\vec{E}$ to ℓ, x, and λ. Then substitute the values for ℓ, x, and λ to find a numerical value of $\vec{E}$.

Hints:

1) $\vec{E} = \int d\vec{E}$

2) What are the limits of integration. i.e. what is the range of x in which the charged rod exists?

3) How does dx relate to dq? You will need to integrate over dx.

4) Think about the best place to locate $x = 0$.

(d) How do the numerical and "exact" values compare? Compute the % discrepancy. How could you make the numerical method more "exact"?

The first set of calculations for the E-field along the axis of the rod was relatively easy because all of the electric field vectors lie along a single line. Now you are to consider a calculation of the E-field perpendicular to the axis of the rod. In this case we have to consider both the x and y components of the electric field resulting from the charge on each element.

✍ Activity 19-14 : Electric Field Calculations ⊥ to the Axis

Explain why the x-component of the total E-vector at point **P'** should be zero. **Hint**: the argument used to show this is known as a symmetry argument.