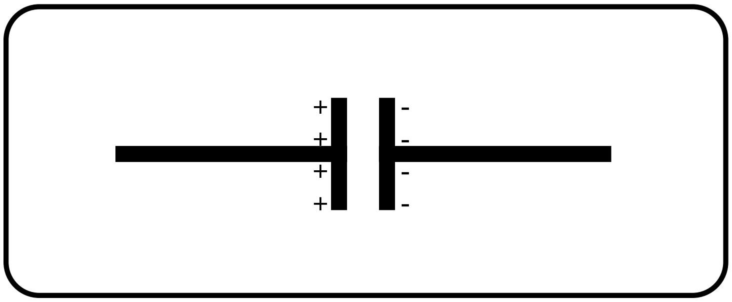
Name ______________________ Date(YY/MM/DD) ______/_________/_______
St.No. __ __ __ __ __-__ __ __ __ Section_________Group #________

UNIT 24: CAPACITORS AND RC CIRCUITS



The most universal and significant concept to come out of the work on the telegraph was that of capacitance.

H. I. Sharlin

I get a real charge out of capacitors.

P. W. Laws

1. To define capacitance and learn how to measure it with a digital multimeter.

2. To discover how the capacitance of parallel plates is related to the area of the plates and the separation between them.

3. To determine how capacitance changes when capacitors are wired in parallel and when they are wired in series by using physical reasoning, mathematical reasoning, and direct measurements.

4. To discover how the charge on a capacitor and the electric current change with time when a charged capacitor is placed in a circuit with a resistor.

SESSION ONE: CAPACITANCE, AREA, AND DISTANCE AND CHARGE BUILD UP AND DECAY: QUALITATIVE

Overview

Any two conductors separated by an insulator can be electrically charged so that one conductor has a positive charge and the other conductor has an equal amount of negative charge; such an arrangement is called a *capacitor*. A capacitor can be made up of two strange shaped blobs of metal or it can have any number of regular symmetric shapes such as that of one hollow sphere inside another, or one hollow rod inside another.

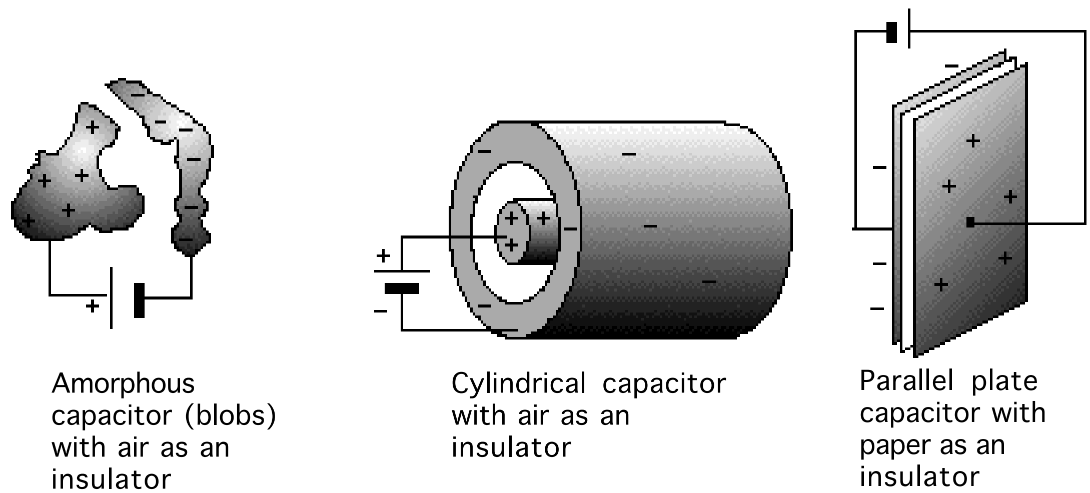


Figure 24-1: Some Different Capacitor Geometries

The type of capacitor that is of the most practical interest is the parallel plate capacitor. Thus, we will focus exclusively on the study of the properties of parallel plate capacitors. There are a couple of reasons why you will be studying parallel plate capacitors. First, the parallel plate capacitor is the easiest to use when making mathematical calculations or using physical reasoning. Second, it is relatively easy to construct. Third, parallel plate capacitors are used widely in electronic circuits to do such diverse things as defining the flashing rate of a neon tube, determining what radio station will be tuned in, and storing electrical energy to run an electronic flash unit. Materials other than conductors separated by an insulator can be used to make a system that behaves like a simple capacitor. Although many of the most interesting properties of capacitors come in the operation of alternating current circuits, we will limit our present study to the properties of the parallel plate capacitor and its behaviour in direct current circuits like those you have been constructing in the last couple of units. The circuit symbol for a capacitor is a pair of lines as shown in the figure below.

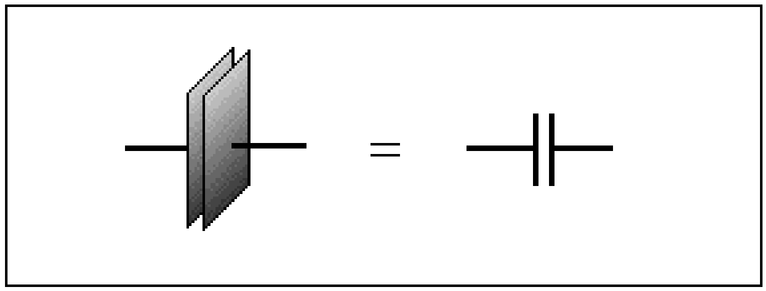


Figure 24-2: The Circuit Diagram Symbol for a Capacitor

The Parallel Plate Capacitor

The typical method for transferring equal and opposite charges to a capacitor is to use a voltage source such as a battery or power supply to impress a potential difference between the two conductors. Electrons will then flow off of one conductor (leaving positive charges) and on to the other until the potential difference between the two conductors is the same as that of the voltage source. In general, the amount of charge needed to reach the impressed potential difference will depend on the size, shape, and location of the conductors relative to each other. The capacitance of a given capacitor is *defined mathematically as the ratio of the magnitude of the excess charge, $|q|$, on either one of the conductors to the size of the potential difference, $|V|$, across the two conductors* so that

$$C = \frac{|q|}{|V|} \qquad \text{[Eq. 24-1]}$$

Thus, capacitance is *defined* as a measure of the amount of net or excess charge on either one of the conductors per unit potential difference (coulombs per volt in SI units).

You can draw on some of your experiences with electrostatics to think about what might happen to a parallel plate capacitor when it is hooked to a battery as shown in Figure 24-3. This thinking can give you an intuitive feeling for the meaning of capacitance. For a fixed voltage from a battery, the net charge found on either plate is proportional to the capacitance of the pair of conductors.

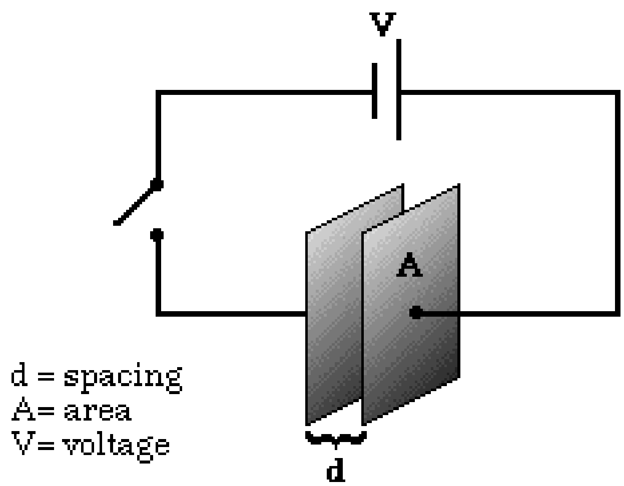


Figure 24-3: A parallel plate capacitor with a voltage V across it.

✍ Activity 24-1: Predicting How Capacitance Depends on Area and Separation.

(a) Consider two identical metal plates of area *A*, separated by a non-conducting material which has a thickness *d*. They are connected in a circuit with a battery and a switch, as shown above. When the switch is open, there is no excess charge on either plate. The switch is then closed. What will happen to the amount of net excess charge on the metal plate that is attached to the negative terminal of the battery? What will happen to the amount of net excess charge on the plate that is connected to the positive terminal of the battery? Explain.

(b) Can excess charges on one plate of a charged parallel plate capacitor interact with excess charges on the other plate? If so how? **Note:** To say that two charges interact is to say that they exert forces on each other from a distance.

(c) Is there any limit to the amount of charge that can be put on a plate? Explain.

(d) Use qualitative reasoning to anticipate how the amount of charge a pair of parallel plate conductors can hold will change as the area of the plates increases. Explain your reasoning.

(e) Do you think that the amount of net excess charge a given battery can store on the plates will increase or decrease as the spacing, d, between the plates of the capacitor increases? Explain.

Capacitance Measurements for Parallel Plates
The unit of capacitance is the farad, F, named after Michael Faraday. One farad is equal to one coulomb/volt. As you will demonstrate shortly, one farad is a very large capacitance. Thus, actual capacitances are often expressed in smaller units with alternate notation as shown below:

Units of Capacitance
microfarad: 10^{-6} F = 1 μF
picofarad: 10^{-12} F = 1 pF = 1 μμF
nanofarad: 10^{-9} F = 1 nF = 1000μμF

Figure 24-4: Units of Capacitance

Note: Sometimes the symbol *m* is used instead of μ or U on capacitors to represent 10^{-6}, despite the fact that in other situations m always represents 10^{-3}!

Typically, there are several types of capacitors used in electronic circuits, including disk capacitors, foil capacitors, electrolytic capacitors and so on. You might want to examine some typical capacitors. To do this you'll need:

- A collection of four assorted capacitors

To complete the next few activities you will need to construct a parallel plate capacitor and use a multimeter to measure capacitance. Thus, you'll need the following items:

- 2 sheets of aluminum foil (12 cm x 12 cm)
- 1 "fat" textbook
- 1 digital multimeter (w/ a capacitance mode)
- 2 insulated wires, stripped at the ends, approx. 12" long
- 1 ruler
- 1 Vernier caliper

You can make a parallel plate capacitor out of two rectangular sheets of aluminum foil separated by pieces of paper. A textbook works well as the separator for the foil as you can slip the two foil sheets between any number of sheets of paper and weight the book down with something heavy and non-conducting like another massive textbook. You can then use your digital multimeter in its capacitance mode for the measurements.

When you measure the capacitance of your "parallel plates", be sure the aluminum foil pieces are arranged carefully so they don't touch each other and "short out".

✍ Activity 24-2: Measuring How Capacitance Depends on Area or on Separation

(a) Devise a way to measure how the capacitance depends on either the foil area or on the separation between foil sheets. If you hold the area constant and vary separation, record the dimensions of the foil so you can calculate the area. Alternatively, if you hold the distance constant, record its value. Take at least five data points in either case. Use the space below to create a data table with proper units of the results.

(b)Make a graph of capacitance vs. the variable you varied. Is your graph a straight line? If not, you should make a guess at the functional relationship it represents and create a model that fits the data using the Modeling Worksheet. Submit your spreadsheet showing both the data and the model you fitted to the data online.

(c) What is the function that best describes the relationship between spacing and capacitance or between area and capacitance? How do the results compare with your prediction based on physical reasoning?

Deriving a Mathematical Expression for Capacitance

We can use Gauss' law and the relationship between potential difference, *V*, and electric field to derive an expression for the capacitance, *C*, of a parallel plate capacitor in terms of the area, *A*, and separation, *d,* of the aluminum plates. The diagram in Figure 24-5 below is useful in this regard.

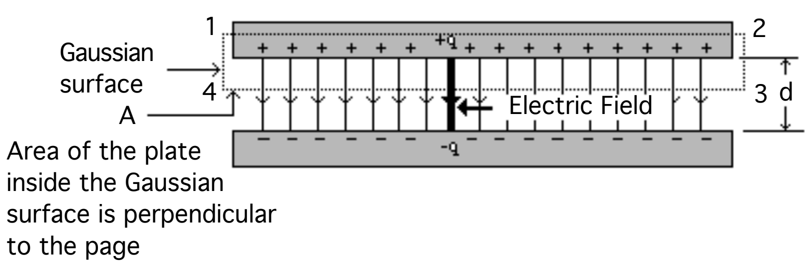


Figure 24-5: A charged capacitor. The light grey line represents a Gaussian surface enclosing positive charge.

✍ Activity 24-3: Derivation of Capacitance vs. *A* and *d*

(a) Write down the integral form of Gauss' law. However, use a new form of Gauss' law where the electric constant ε_0 is replaced with a new constant $\varepsilon = \kappa\varepsilon_0$. In this case, κ (kappa[1]) represents the dielectric constant of the pages in the textbook you are using as a spacer.

(b) Examine the Gaussian surface shown in the diagram above. What is the value of the electric field *inside* the top plate? *What is the electric flux through the top surface of the Gaussian surface?* **Hint:** There is never an electric field inside a conductor when no electric current is present!

(c) Using the results above and the notation E to represent the magnitude of the uniform electric field *between* the two plates, *find the net flux through the six surfaces of the closed Gaussian surface and set the total flux equal to the charge enclosed.* Next show that if the net positive charge on the top plate is denoted by q, then $q = \varepsilon_0 EA$. **Note:** We are assuming that the excess charges reside on the inside surface of each plate.

[1] The Greek letter kappa can also be written as $\varkappa$.

(d) Remember that in a uniform electric field the size of the potential difference (or voltage drop) across a distance d is given by $|V| = |E|d$. Use this fact and the definition of capacitance to show that

$$C = \kappa\varepsilon_o A/d$$

where A is the area of the plates and d is their separation.

(e) Use one of your actual areas and spacings from the measurements you made in Activity 24-2 (a) above to calculate a value of C. Assume that the dielectric constant for paper is about 3.5. How does the calculated value of C compare with the directly measured value?

(f) Now for an unusual question. If you have two square foil sheets, separated by paper which is 1 mm thick, how long (in km) would each side of the sheets have to be in order to have C = 1 F?

L = km

(g) Your capacitor would make a mighty large circuit element! How could it be made smaller physically and yet still have the same value of capacitance? You may want to examine the collection of sample capacitors for some ideas.

Capacitors in Series and Parallel

You can observe and measure the equivalent capacitance for series and parallel combinations. For this study you can use two identical capacitors. You'll need:

- 2 capacitors (about 0.1 μF)
- Capacitance meter (Circuit-Test multimeter)

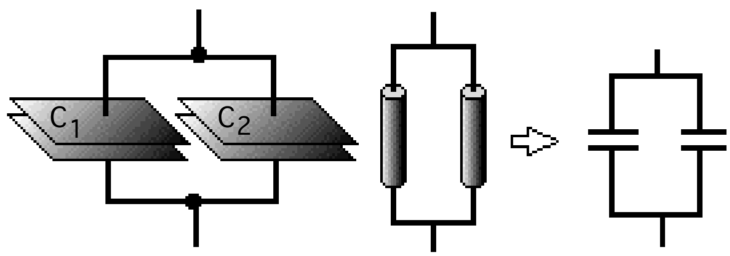


Figure 24-6: Capacitors wired in parallel.

✍ Activity 24-4: Capacitance for a Parallel Arrangement

(a) Use direct physical reasoning and the result: $C = \kappa\varepsilon_o A/d$ to predict the equivalent capacitance of a pair of identical capacitors wired in parallel. Explain your reasoning below. **Hint**: What is the effective area A of two parallel plate capacitors wired in parallel? Does the effective spacing between plates change?

(b) What is the equivalent capacitance when your two capacitors are wired in parallel? Summarize your *actual* data! Is the result compatible with the predicted value?

(c) Guess a general equation for the equivalent capacitance of a parallel network as a function of the two capacitances C_1 and C_2.

$C_{eq} =$

Next, consider how capacitors that are wired in series, as shown in the diagram below, behave.

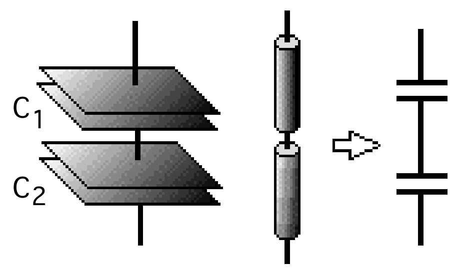


Figure 24-7: Capacitors wired in series.

✍ Activity 24-5: Capacitance for a Series Arrangement

(a) Use direct physical reasoning and the result: $C = \kappa\varepsilon_o A/d$ to predict the equivalent capacitance of a pair of capacitors wired in series. Explain your reasoning below. **Hint:** If you connect two capacitors in series what will happen to the charges along the conductor between them? What will the effective separation *d* of the "plates" be? Will the effective area *A* change?

(b) Measure the equivalent capacitance when your two capacitors are wired in series. Report your actual data. Is the result compatible with the predicted value?

(c) Guess a general equation for the equivalent capacitance of a series network as a function of C_1 and C_2.

$C_{eq} =$

(d) How do the mathematical relationships for series capacitors compare to those of resistors? Do series capacitors combine more like series resistors or parallel resistors? Explain.

SESSION TWO: CHARGE BUILD UP AND DECAY: QUANTITATIVE

RC Circuits

In this section, you will measure what happens to the voltage across a charged capacitor when it is placed in series with a resistor in a direct current circuit. Before making these measurements you should make some qualitative observations of capacitor behaviour so that you can explain what is happening as charge decays off a capacitor. For the observations in this session you will need:

- 3 D-cell batteries with holder
- 1 small low resistance light bulb
- 1 10 Ω resistor
- 1 digital multimeter
- 2 capacitors (.47 F)
- 6 alligator clip wires
- 1 SPST switch
- 1 small breadboard
- Assortment of small lengths of #22 wire

Qualitative Observations

By using a small light bulb as a resistor and one or more of the amazing new capacitors that have capacitances up to about a farad in a tiny container, you can "see" what happens to the current flowing through a resistor (i.e. the bulb) when a capacitor is charged by a battery and when it is discharged.

✍ Activity 24-6: Capacitors, Batteries and Bulbs

(a) Using the breadboard for the capacitor, connect the bulb in series with the 0.47 F capacitor, a switch, and 3 D-cell batteries. Describe what happens when you close the switch. Draw a circuit diagram of your setup.

(b) Now, can you make the bulb light again without the batteries in the circuit? Mess around and see what happens. Describe your observations and draw a circuit diagram showing the setup when the bulb lights without batteries.

(c) Draw a sketch of the approximate brightness of the bulb as a function of time when it is placed across a charged capacitor *without the battery present*. Let $t = 0$ when the bulb is first placed in the circuit with the charged capacitor. **Note**: Another way to examine the change in current is to wire an ammeter in series with the bulb.

(d) Explain what is happening. Is there any evidence that charge is flowing between the "plates" of the capacitor as it is charged by the battery with the resistor (i.e. the bulb) in the circuit, or as it discharges through the resistor? Is there any evidence that charge is *not* flowing through the capacitor? **Hints:** 1) You may want to repeat the observations described in (a) and (b) several times; placing the voltmeter across the capacitor or placing an ammeter in series with the capacitor and bulb in the two circuits you have devised might aid you in your observations. 2) Theoretically, how should the voltage across the capacitor be related to the magnitude of the charge on each of its conductors at any given point in time?

(e) What happens when *more capacitance* is put in the circuit? When *more resistance* is put in the circuit? (You can add the resistor to the circuit to get more resistance.). **Hint:** Be careful how you wire the extra capacitance and resistance in the circuit. Does more capacitance result when capacitors are wired in parallel or in series? How should you wire resistors to get more resistance?

Quantitative Measurements on an RC System

The next task is to do a more quantitative study of your "RC" system. We will do this in two ways.

The first involves measuring the voltage across a charged capacitor as a function of time when a carbon resistor has been placed in a circuit with it. A computer-based voltage logger setup can be used to obtain data and view the trace of potential difference vs. time in graphical form as the capacitor discharges. The goal here is to figure out the mathematical relationship between the potential difference across the capacitor and time which best describes the potential difference change as the capacitor discharges.

For the activities in this section you will need:

- 4 D-cell batteries with holder
- 1 capacitor, approx 3300 μF
- 1 SPDT switch
- 1 resistor, 1.0 kΩ
- 1 LabPro interface
- 1 voltage probe
- 1 digital multimeter
- 6 alligator clip wires
- 1 capacitance meter (for C ≥ 3000μF)

A bulb is not a good constant value resistor as its resistance is temperature dependent and rises when it is heated up by the current. For these more quantitative studies you should use a 1.0 kΩ resistor in place of the bulb while attempting to charge a 3300 μF capacitor. Wire up the circuit shown below in Figure 24-11 with a two position switch in it. The switch will allow you to flip from a situation in which the battery is charging the capacitor to one in which the capacitor is allowed to discharge through the resistor. The voltmeter and voltage probe leads should be placed in parallel with the capacitor — this allows you to measure the voltage across it.

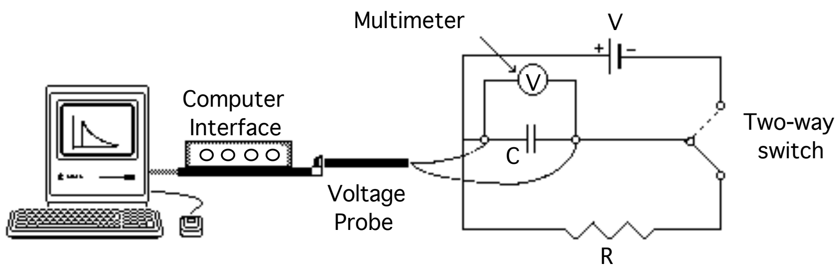


Figure 24-11: RC circuit with potential difference measurements across a discharging capacitor

---

✍ Activity 24-7: The Decrease of Voltage in an RC Circuit

(a) Assume that the capacitor has been charged by the battery. What do you predict will happen to the potential difference across the capacitor when the two position switch is flipped so that the battery is removed from the circuit? Explain the reasons for your prediction.

(b) As soon as the switch is flipped from a battery terminal to a terminal of the capacitor, you can start measuring the voltage across the capacitor at least every one or two seconds until the voltage across the capacitor is about 1/100th of its initial value. Graph your data and guess the functional form. **Hint:** The *V* vs. *t* curve cannot be fit using a simple power or root of *V*. Thus, you should try some simple functions to model *V* such as: $k/t, \ln k, e^{-kt}$ and so on where *k* is a constant.

Note: ***Be sure to save your data, as you will need to use it again in Activity 24-9.***

(c) How did your observations fit with the prediction you made in part (a)?

The Theoretical RC Decay Curve :

If you made careful measurements of *V* vs. *t* for a capacitor, *C*, discharging through a resistor, *R*, you should have been able to plot what is known as an *exponential decay curve*. Mathematical reasoning based on the application of Ohm's law as well as the definition of current and capacitance can be used to predict exponential decay given by:

$$V = V_0 e^{(-t/RC)} \qquad \text{[Eq. 24-2]}$$

where V_o is the initial voltage across the capacitor and t represents the time elapsed since the switch was thrown to cut the battery out of the circuit.

✍ Activity 24-8: Derivation of the Theoretical Decay Curve

(a) What is the equation for the potential difference across the capacitor in Figure 24-12 in terms of the charge q on the capacitor and C? **Hint**: What is the *definition* of C?

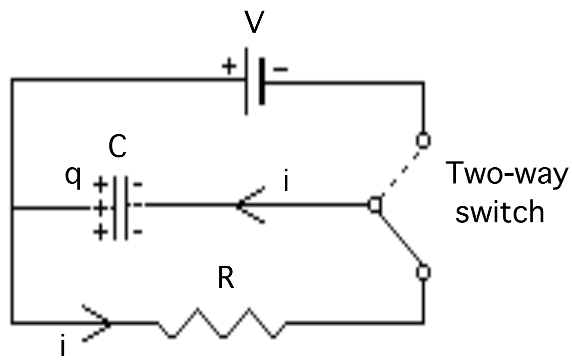


Figure 24-12

(b) What is the equation for the potential difference across the resistor in terms of the current, i , flowing through it (due to the discharge of the capacitor) and its resistance, R. **Hint:** Recall that voltage *drops* in the direction of current flow.

(c) Assume that the switch is in the position shown in Figure 24-12 and that q represents the charge on the capacitor. Show that at all times while the capacitor is discharging,

$$\frac{q}{C} = iR$$

Note: As the capacitor discharges, q gets smaller and smaller.

(d) Using the definition of the instantaneous electric current passing *through* the resistor, explain why

$$i = -\frac{dq}{dt}$$

where q represents the charge on the capacitor (as opposed to the charge flowing through the resistor). **Hints:** 1) What is the source of the charge flowing through the resistor? 2) What is the relationship between the rate of flow of charge through the resistor, dq_R/dt, and the rate at which excess charge flows off the capacitor plates, dq/dt ?

(e) Use the answers given above to show that

$$\frac{dq}{dt} = -\frac{q}{RC}$$

in the circuit under consideration.

(f) Show that the equation $q = q_0 e^{(-t/RC)}$, where q_0 is a constant representing the initial charge on the capacitor, satisfies the condition that:

$$\frac{dq}{dt} = -\frac{q}{RC}$$

Hint: Take the derivative of q with respect to t and replace $q_0 e^{(-t/RC)}$ with q.

(g) Use the definition of capacitance once again to show that theoretically we should expect that the voltage across the capacitor will be given by

$$V = V_0 e^{(-t/RC)}$$

where V_0 is a constant representing the initial voltage across the capacitor.

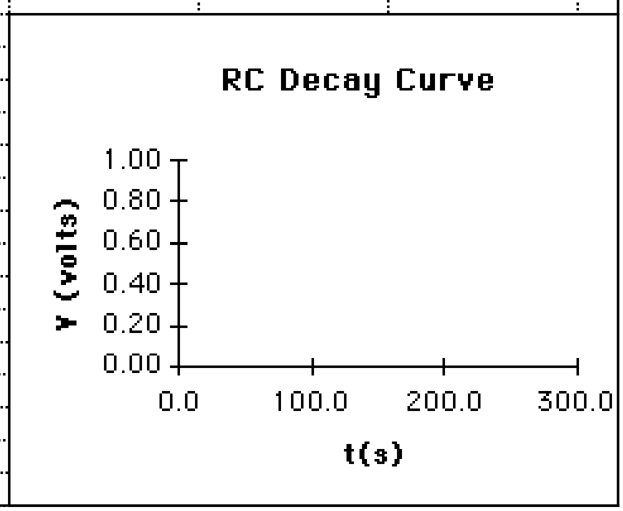
Now comes the acid test: the comparison of the experimentally determined rate of the capacitor discharge to the theoretically predicted rate.

✍ Activity 24-9: Does the Observed Decay Curve Fit Theory?

(a) Carefully measure the R and C of the resistor and capacitor you used in Activity 24-7 using a digital multimeter and a capacitance meter. List these values below (with proper units, of course). Also list the value of V_0 from that experiment.

(b) Develop a Modeling Worksheet. Set R, C, and V_0 as absolute parameters, using your measured values from part (a), and calculate the theoretical V vs. t for the circuit using the results of Activity 24-8. Create an overlay plot of the theoretical and experimental values for V vs. t, using the experimental data you obtained in Activity 24-7. Submit a copy of your worksheet online. Your worksheet should appear roughly as follows:

	R(Ω)=	1000
	C(F)=	5.00E-03
	Vo(V)=	4.5
RC Decay Curve		
V (volts)		
t(s)	V-exp	V-theory
0.0		
1.0		
2.0		
3.0		
4.0		
5.0		
6.0		
7.0		
8.0		
9.0		
10.0		
etc.		



(c) How well does theory match with experiment in this case?

(d) What do you think would happen to the decay time if R were doubled? If C were doubled?

A Qualitative Summary of RC Decay: Let's consider the process of discharging a capacitor that is in series with a resistor one more time.

✍ **Activity 24-10: Explaining Discharging Qualitatively**

(a) Assume that the capacitor is fully charged. When the switch is first closed *so the battery is no longer in the circuit*, how much charge is on the capacitor C? What is the potential difference V_o across the plates?

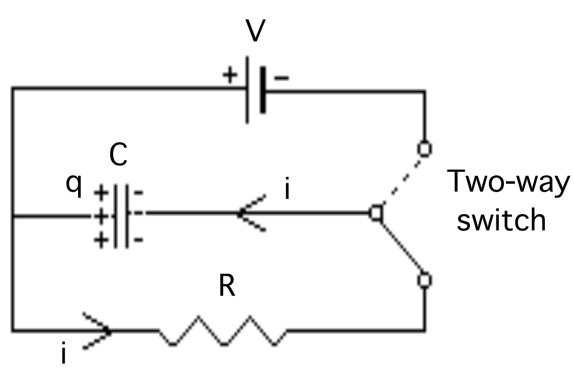


Figure 24-12: Again!

(b) Is the current in the circuit a maximum or a minimum right after the switch is closed? Does this current flow *through* the capacitor? Explain.

(c) How is the potential difference across the resistor related to that across the capacitor?

(d) What happens to the potential difference across the capacitor as charge drains away from it? Explain.

(e) What happens to the potential difference across the resistor at the same time? Explain.

(f) If the potential across the resistor starts to change what must happen to the current in the circuit? Explain.

(g) Why does the draining of charge from the capacitor eventually stop? Why does the current in the circuit go to zero?

Capacitor Charging

If the resistor, *R,* in Figure 24-12 is moved up next to the battery as shown in Figure 24-13, an uncharged capacitor, *C,* can be charged by the battery in the presence of the resistor. The qualitative and

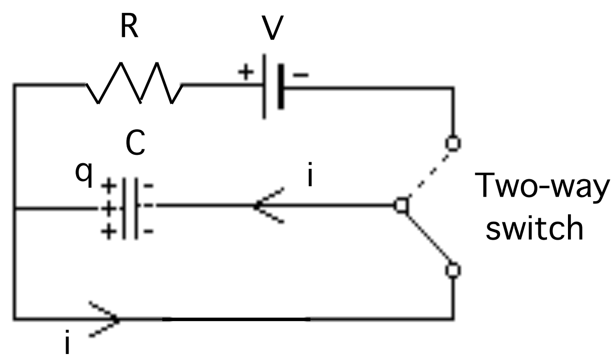


Figure 24-13

quantitative considerations of this situation are very analogous to that of capacitor decay. For example, the capacitor charges up more rapidly at first when there are no charges on either of the capacitor plates to repel each other. Also, after a while when the potential difference across the capacitor is equal to that across the battery, the charging stops completely. It can be shown that the potential difference, *V,* across the capacitor as it is charging is described by the equation

$$V = V_B(1 - e^{(-t/RC)}) \qquad \text{[Eq. 24-3]}$$

where V_B is the potential difference across the battery. This charging equation is used in the design of the flashing lights used at road construction sites and in the flash units used by photographers.

Exponential Decay

During the last several activities you were asked to derive an equation that describes the decay of voltage across a capacitor wired in series with a carbon resistor. Recall that the theoretical equation is given by

$$V = V_0 e^{(-t/RC)} \qquad \text{[Eq. 24-2]}$$

You were also asked to make measurements to verify the equation experimentally for only *one* resistor and capacitor combination. The exponential decay of potential difference across the capacitor is known as the *exponential relaxation of charge*. The product RC in a circuit has the units of a time and is called the *"RC time constant"* of the circuit because it determines the rate of decay of potential difference.

In the activity which follows you are to conduct a theoretical investigation of the relationship between R, C, and the time it takes a discharging capacitor to "relax."

✍ Activity 24-11: Theoretical RC Relaxation Times

(a) Show that in a time equal to RC the potential difference V across a capacitor drops to 36.8% of its initial value. **Hint**: Start with the capacitor decay equation and substitute RC for the time t.

(b) Another convenient equation is that describing the *half-life* of the capacitor decay as a function of the time constant *RC,* of the circuit. Show that the half-life is given by the equation

$$t_{1/2} = RC\ln 2 = 0.693RC \qquad \text{[Eq. 24-4]}$$

Hint: Start with the capacitor decay equation. Substitute $V_0/2$ for the voltage and $t_{1/2}$ for the time and take the natural logarithm of both sides of the equation.

Figure 24-14: The decay of charge from a capacitor in an RC circuit is sometimes called "relaxation".