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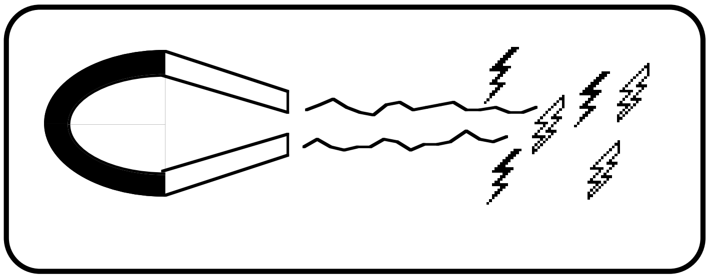
UNIT 26: ELECTRICITY AND MAGNETISM



Occasionally during these years . . .[Michael Faraday] thought of electrical problems. One of special interest was the question: Since magnetism can be produced from electricity, can electricity be produced from magnetism? Everything in nature is nicely balanced and symmetrical; in the words of Newton, there is action and there is reaction. Force will give motion; motion will give force. Heat will cause pressure; pressure will cause heat. Chemical action will produce electricity: electricity will produce chemical action. Then, since electricity will develop magnetism, will not magnetism develop electricity?

H.H.Skilling

OBJECTIVES

A. Magnetism from Electricity

1. To learn by direct observation about the direction of magnetic field lines produced by a current in a straight wire.

2. To learn about the potential health effects of magnetic fields induced by electrical currents in power lines and home appliances.

3. To understand how to use Ampère's law to calculate the magnetic field around a closed loop in the presence of electrical currents.

B. Electricity from Magnetism

4. To observe that an electric field can be produced by a changing magnetic field by a process known as induction.

5. To explore the mathematical properties of induction as expressed in Faraday's law and to verify Faraday's law experimentally.

OVERVIEW

In the last unit you observed that permanent magnets can exert forces both on freely moving charges and on electrical currents in conductors. We have postulated the existence of a mathematical entity called the magnetic field in order to introduce the Lorentz force law as a way of mathematically describing the nature of the force that a permanent magnet can exert on moving electrical charges. Newton's third law states that whenever one object exerts a force on another object, the latter object exerts an equal and opposite force on the former. Thus, if a magnet exerts a force on a current carrying wire, mustn't the wire exert an equal and opposite force back on the magnet? It seems plausible that the mysterious symmetry demanded by Newton's third law would lead us to hypothesize that if moving charges feel forces as they pass through magnetic fields, they should be capable of exerting forces on the sources of these magnetic fields. It is not unreasonable to speculate that currents and moving charges exert these forces by *producing magnetic fields themselves.* One of the agendas for this unit is to investigate the possibility that an electrical current can produce a magnetic field.

This line of argument, based on Newton's third law and its symmetry, can lead us into even deeper speculation. If charges have electric fields associated with them, then moving charges can be represented mathematically by changing electric fields. Thus, using the concept of "field" to describe forces that act at a distance, we can say that changing electric fields are the cause of magnetic fields. This leads inevitability to the question: If this is so, then, by symmetry, *can changing magnetic fields cause electric fields?*

This unit deals with two issues: (1) Does a long straight current-carrying wire produce a magnetic field? If so, what quantitative mathematical relationships can be used to describe the nature of such a field? This will lead us to present Ampère's law, which describes the magnetic field around a closed loop as a function of the current enclosed by the loop. (2) We will explore Faraday's law, which describes how changing magnetic fields produce electric fields and hence electrical currents.

Faraday's law lies at the absolute heart of the study of electricity and magnetism. It is one of the two or three most profound laws in classical physics. By using the Biot-Savart law along with Faraday's law, we can describe mathematically how electricity produces magnetism and how magnetism produces electricity. Thus, two seemingly different phenomena, electricity and magnetism, can be treated as aspects of the same phenomenon. At the end of this unit, we will peek briefly at the reformulation of some of the laws of electricity and magnetism which we have already learned into a famous set of four equations known as Maxwell's Equations.

SESSION ONE: MAGNETISM FROM ELECTRICITY

The Magnetic Field Near a Current Carrying Wire

In 1819, the Danish physicist H.C. Oersted placed a current-carrying wire near a compass needle during a lecture demonstration before a group of students. Although he predicted that the current would cause a force on the compass needle, the details of the results surprised him. What do you predict will happen?

✍ Activity 26-1: Magnetic Fields from Currents?

(a) Do you expect to see a magnetic field in the vicinity of a straight current-carrying wire? Why?

(b) If your answer to part (a) is yes, do you expect the magnitude of the field to increase, decrease, or stay the same as the distance from the wire increases? Why?

(c) Here's a tough one. In which direction do you think the magnetic field will point near the wire? What do you think will happen to the direction of the magnetic field if the direction of the current is reversed?

(d) What to you think will happen to the magnitude of the magnetic field if the current is reduced?

Let's repeat some of Oersted's observations and study the pattern of magnetic field lines in a plane perpendicular to a long straight conductor that is carrying current. To do this you will need:

- 4 D-cell batteries
- 3 40 cm lengths of wire w/ alligator clip leads
- A switch
- A ring stand w/ clamps and rods (non-magnetic)
- A 1 kg mass to hold down the wire at the bottom
- An 8" X 8" piece of cardboard (taped to a clamp)
- 6 small compasses

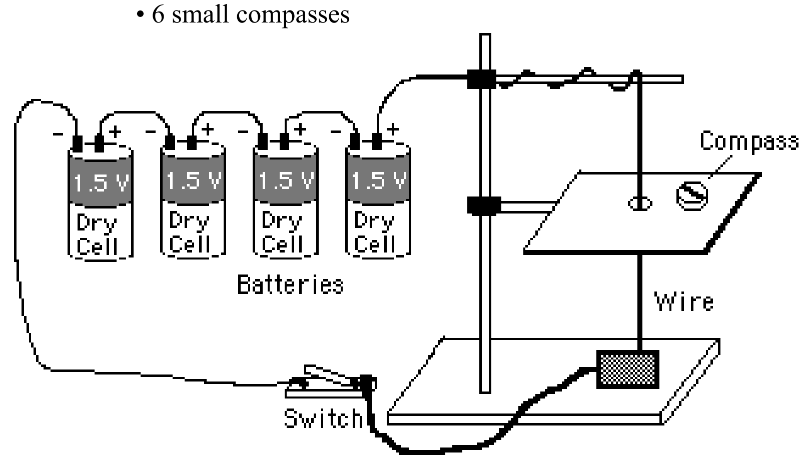


Figure 26-1 (a): Apparatus for Repeating Oersted's Observations on the Magnetic Field Produced by a Current.

Wire the battery, switch, and wires in series. The centre wire can be poked through a hole in a piece of cardboard with the plane of the cardboard lying perpendicular to the wire. *Turn the current on only when you are making observations;* it saves the batteries.

Check your small compass before starting, as sometimes the needles get stuck. Make sure it's pointing north and swinging freely before the current is switched on.

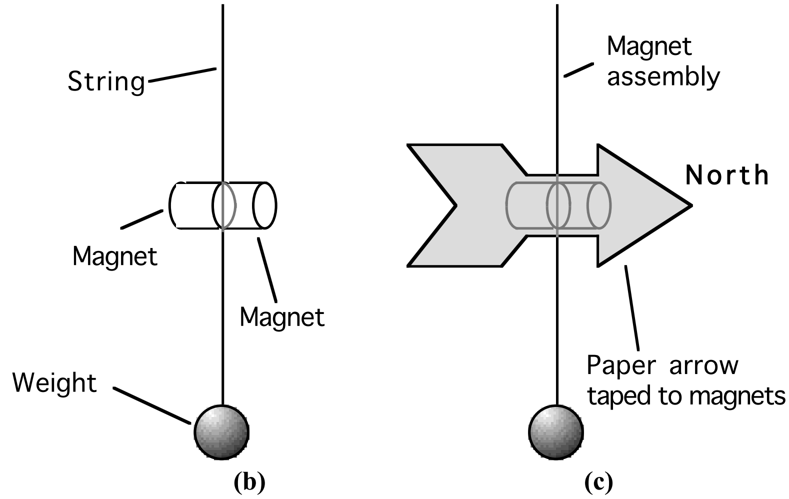
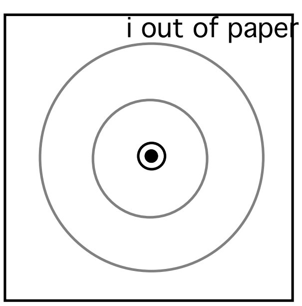


Figure 26-1: (b) Alternative to compass. A string is clamped between two small, cylindrical neodymium magnets. A small weight is suspended from the string. **(c)** A paper arrow is taped to the magnets.

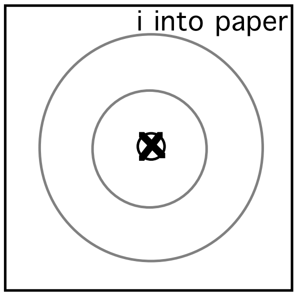
If you prefer, you can use the assembly shown in Figures 1(b) and 1(c) in place of a compass – this assembly is more sensitive than the compass and will show the field at greater distances from the wire. Allow two small neodymium magnets to come together with a string between them. Suspend a small weight from the string (the top of a soda bottle will do nicely). Cut an arrow (approximately 2 inches long) out of stiff paper (such as a manila file folder) and tape it to the magnets, taking care not to get the tape tangled in the string. Hold the assembly by the unweighted end of the string and use it like a compass. (It will help if you attach the arrow so that it points north!) If you use this method, you needn't thread the wire through cardboard resting on a clamp stand.

✍ Activity 26-2: The Magnetic Field Near a Wire

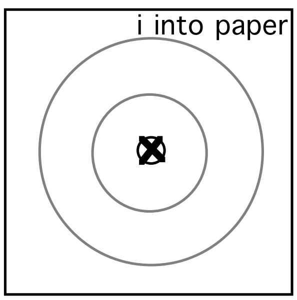
(a) First, wire the battery so that positive current is passing through the wire from bottom to top. Use the space below to map out the magnetic field directions with arrows. Move the compass slowly in a small imaginary circle centred on the wire and record the direction of the needle.



(b) Fiddle some more. What happens when the direction of the current is reversed? Wrap either your left or your right hand around the wire with your thumb in the direction of the current, and figure out a rule for predicting the direction of the magnetic field surrounding the wire. Hint: (1) Review the definition of magnetic field direction from the last unit and (2) pay attention to the direction of your fingers!



(c) What happens as the compass is moved around a circle that is further away from the wire? Does the strength of the magnetic field stay the same? Increase? Decrease?



(d) Take one of these batteries out of the circuit to reduce the current in the circuit. Hold the switch down for a *short* amount of time. What happens to the apparent strength of the magnetic field at a given distance when the current in the wire is decreased?

(e) How good were the predictions you made in Activity 26-1? In particular, did anything surprise you about the actual observation? If so, what?

Do Magnetic Field Sources Superimpose?
You should have established that a current-carrying wire has a magnetic field associated with it. Now, how can we determine the influence of *combinations* of current-carrying wires? Do the principles of superposition, which work when we combine the electric fields associated with static charges, also work for currents?

Examine the different configurations of the wire in the circuits below and predict what the relative strengths of the magnetic field might be at various locations near the circuits.

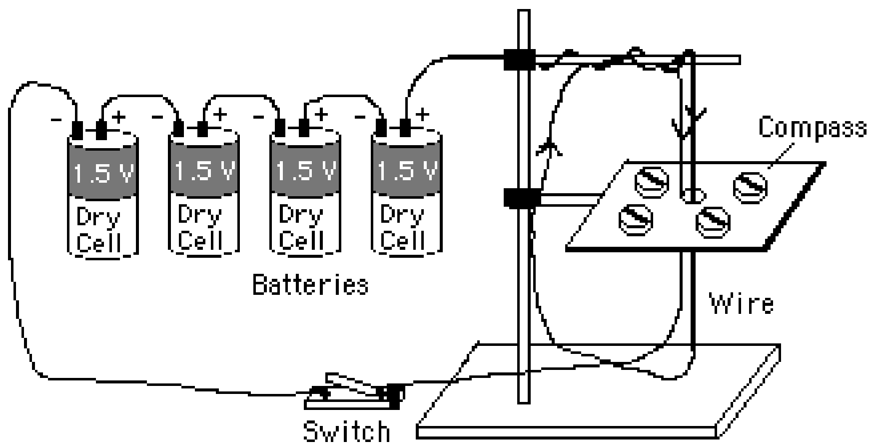


Figure 26-2 (a): Wires paired that carry current in the same direction.

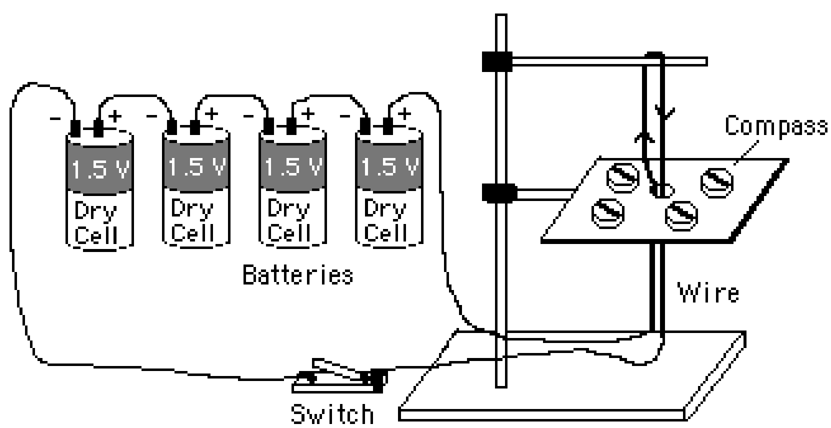


Figure 26-2 (b): Wires paired that carry current in opposite directions.

✍ Activity 26-3: Magnetic Fields from Different Wiring Arrangements

(a) How do you predict the strength of the magnetic field due to two wires carrying current in the same direction (see Figure 26.2(a)) will compare to the strength due to one wire carrying the same current? Explain the reasons for your prediction.

(b) How do you predict the strength of the magnetic field due to two wires carrying current in opposite directions (see Figure 26.2(b)) will compare to the strength due to one wire carrying the same current? Explain the reasons for your prediction.

(c) Adapt the circuit in **Activity 26-2** so that two lengths of wire run very close to each other and carry current in the *same* direction (as in Figure 26-2(a), above). Compare the strength of the magnetic field arising from this configuration to that of the field arising from a single wire. Is it weaker, stronger, or the same? How does this observation compare with your prediction?

(d) Adapt the circuit in **Activity 26-2** so that two lengths of wire run very close to each other but carry current in the *opposite* direction (as in Figure 26-2(b), above). Compare the strength of the magnetic field arising from this configuration to that of the field arising from a single wire. Is it weaker, stronger, or the same? How does this observation compare with your prediction?

Ampère's Law– A Mathematical Expression for $\vec{B}$

A particularly useful law was proposed by the French physicist André Marie Ampère, who became so excited by Oersted's observations on the magnetic behavior of current-carrying wires that he immediately devoted a great deal of time to making careful observations of electro-magnetic phenomena. These observations enabled Ampère to develop his own mathematical equation describing the relationship between current in a wire and the resulting magnetic field produced by the current. Conceptually, Ampère's law is a two-dimensional analog to Gauss' law because it relates the line integral of the magnetic field around a closed loop to the current enclosed by that loop. Ampère's law is given by:

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 I_{encl}$$

where μ_0 is a constant called the *permeability of free space* and I_{encl} is the net electric current passing through the loop. The funny integral sign with the circle in the middle tells the reader that the integral is a line integral around a closed loop. The loop is broken up into an infinite number of little vectors $d\vec{s}$ lying along an arbitrary closed loop that surrounds a current. For each of the $d\vec{s}$ vectors the component of the magnetic field $\vec{B}$ that lies parallel to $d\vec{s}$ is found. Thus, $\vec{B} \cdot d\vec{s} = Bds\cos\theta$. Each of these pieces is then added up around a complete loop. This is shown in Figure 26-3 below.

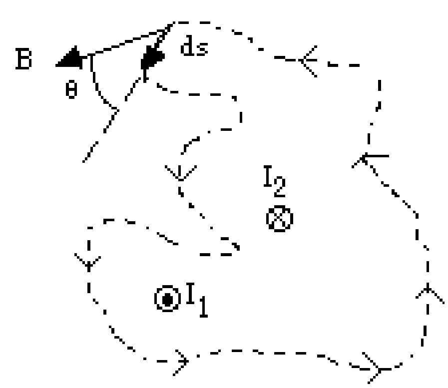


Figure 26-3: A General Ampèrian Loop w/ Net Current $I = I_1 - I_2$. The dotted line represents one of an infinite number of possible closed loops that enclose the two wires carrying currents I_1 and I_2. **Note**: The dotted loop represents an imaginary path, not a wire!

As is the case with Gauss' law, Ampère's law works best for symmetric geometries. For example, let's use Ampère's law to find the magnetic field caused by a current *I* flowing through a long cylindrical straight wire. We can draw an imaginary Ampèrian loop around the wire as shown in Figure 26-4 below.

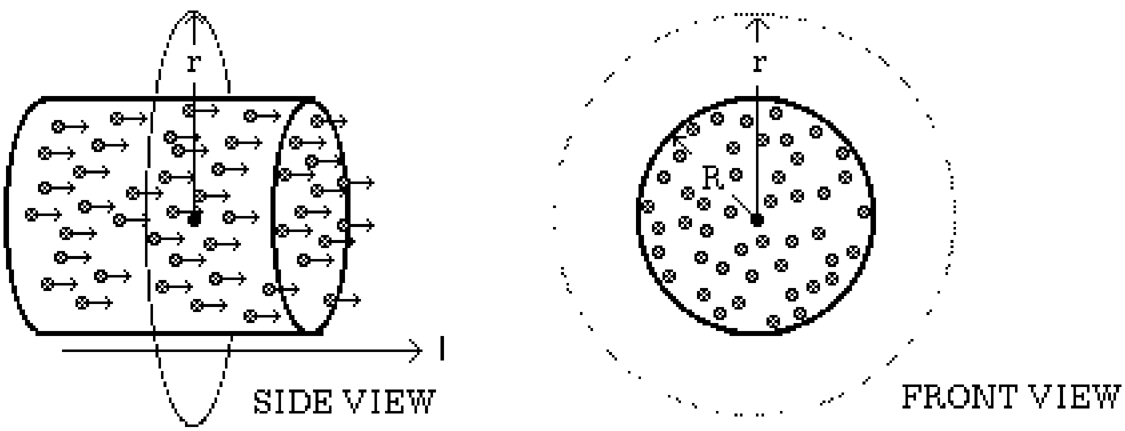


Figure 26-4: An imaginary circular Ampèrian loop of radius r constructed outside a conductor of radius *R* which carries a current *I*. The moving charges are indicated by the small gray dots.

✍ Activity 26-4: The Magnetic Field Outside a Wire

(a) In the front view of Figure 26-4, assume that the current is coming out of the page. Use your previous observations and the right hand rule to sketch the direction of $\vec{B}$ along the outer circle in the diagram.

(b) What is the angle between $\vec{B}$ and $d\vec{s}$ in degrees as you make a complete loop around the circle? Does it vary?

(c) Why is the magnitude of $\vec{B}$ the same at all points on the circle?

(d) Show mathematically that $\oint \vec{B} \bullet d\vec{s} = 2\pi r B$ for an imaginary circular loop that can be constructed around a straight wire.

(e) Using Ampère's law, show mathematically that, for a circular loop outside the conductor (e.g. for $r > R$), the magnitude of the magnetic field is given by

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r}$$

Are Current-Carrying Wires a Health Hazard?

There is some indication from epidemiological studies that individuals who live near high power transmission lines or who make regular use of devices such as electric blankets, heating pads, hair dryers, or water beds are at increased risk of developing cancer. It is believed that the biological damage is due to changing magnetic fields associated with the currents carried by wires.

Most of the electrical energy transmitted and used in contemporary homes and industries does not involve steady currents. Instead, the currents in wires typically alternate in direction 60 times each second. Since a current-carrying wire produces a magnetic field, wires carrying 60 hertz alternating currents also have 60 hertz alternating magnetic fields surrounding them. Biologists are conducting laboratory studies that expose single cells, groups of cells, organs, and small animals to low level 60 hertz magnetic fields. There is evidence that weak magnetic fields can interact with receptor molecules on cell surfaces that trigger changes within cells. Such changes include the rates of production of hormones, enzymes, and other proteins.

Although none of the epidemiological studies of cancer rates in human populations or the laboratory studies on animals are conclusive, many scientists are concerned about exposures various people have to magnetic fields.

No study of the health effects of magnetic fields can be conducted without having a way of measuring the magnetic fields associated with various electric devices and of estimating the *doses* of magnetic energy to which people are exposed. Let's take as a case study a hypothetical Simon Fraser University student who uses an electric blanket during a winter in British Columbia. By measuring the maximum magnetic field associated with a typical electric blanket and making some approximations we could provide scientists with valuable dose estimates.*

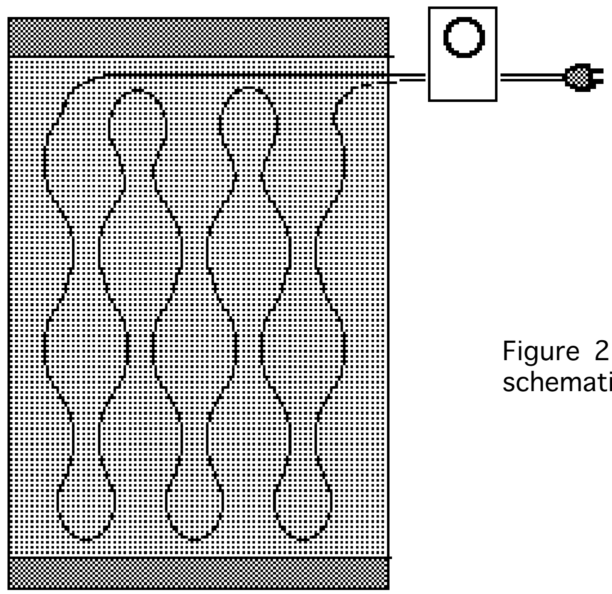


Figure 27-5: Approximate wiring schematic of an electric blanket.

* Sometime in the early 1990s a number of electric blanket manufacturers redesigned their products to minimize the magnetic fields surrounding the wires. The activity which follows will only work properly with older blankets.

✍ Activity 26-5: Annual B-Field Exposure Estimates for Electric Blanket Users in British Columbia

(a) If the average power rating of a typical electric blanket is 120 Watts when it is plugged into a standard household voltage source of V_{rms} = 120 volts A.C. at 60 Hz, what is the rms magnitude of the current (I_{rms}) through the wires in the blanket?

(b) What is the rms magnetic field Brms in milli-teslas (mT) at a distance of one centimeter from a single wire? At 10 centimeters?

(c) *Very approximately*, if a portion of the skin of a person is about 1.0 cm from a wire, what magnetic field is the skin exposed to?

(d) Approximately how many hours a year will a Simon Fraser student who uses an electric blanket be sleeping under that blanket? (You can report a range of hours if you like.). Explain your reasoning!!!

(e) How many milli-tesla-hours of exposure is there to the skin of a typical Simon Fraser electric blanket user in one year?

You can measure the 60 Hz magnetic field near an electric blanket using a *Hall effect* sensor. The Hall effect and its use in measuring magnetic fields are explained in many introductory physics textbooks. To measure the average B-field near an electric blanket you will need:

- A LabPro system
- An MG-BTA Vernier Magnetic Field Sensor
- An "old" electric blanket*

*(about 1992 some manufacturers started rewiring their blankets to minimize B fields)

Biologists suspect that rapidly changing magnetic fields are potentially harmful, while steady ones such as the Earth's magnetic field are not. Typically, the current passing through a household appliance such as an electric blanket changes its direction 50 or 60 times each second. Thus we are interested in having you measure the difference between the Earth's steady magnetic field and the changing magnetic field of the electric blanket produced by alternating household currents.

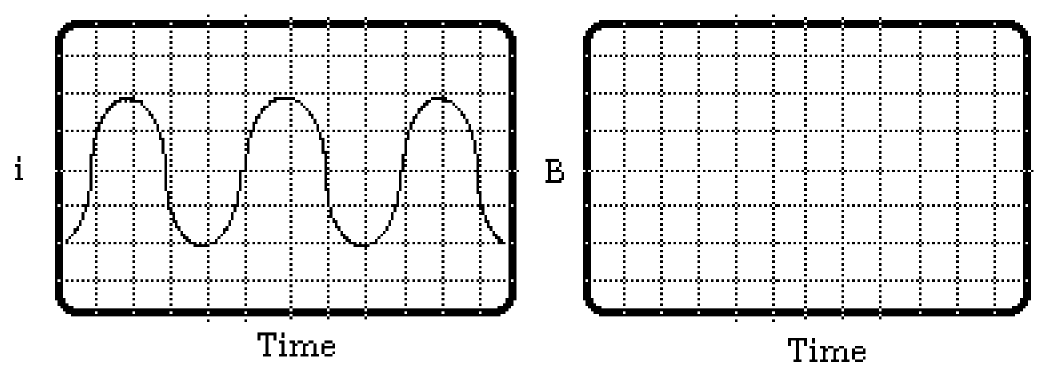
Set up the Logger Pro software to display the difference of the magnetic field when it is on, and when its is off. The magnetic field sensor you will use consists of an SS94A1 Hall effect sensor attached to an amplifier which has two settings. You should set the amplifier to high amplification (X200). At the X200 amplification the magnetic field in gauss is given by

High Amplification: B (mT) = voltage difference/0.625

where the voltage difference is the difference between the sensor output voltages with the appliance on and off.

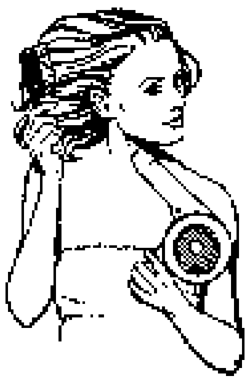
✍ Activity 26-6: Measuring the B-Field Near an Electric Blanket Wire

(a) If a graph of the alternating current through the electric blanket varies as shown in the diagram below, what should the shape of the graph of the magnetic field as a function of time look like? Describe the shape in words and sketch it below.



(b) Set the data logger software to a high data collection rate. We'd suggest 1000 data points per second. Set the magnetic field sensor to maximum sensitivity. The Hall effect sensor reacts to a magnetic field that is *perpendicular* to its flat area. How should the sensor be placed relative to the orientation of a wire inside an electric blanket to get the maximum B-field measurement? Draw a sketch, if needed.

(c) After some practice with the sensor, use the Logger pro software file L270502 to obtain a graph of the B-field change as a function of time near a wire in an electric blanket. Start by placing the sensitive area of the Hall effect sensor about 1 cm or less from a stretch of blanket wire. Next zero the sensor with the blanket turned off. *Then, without moving the sensor at all,* turn on the blanket and measure the change in magnetic field for 0.1 seconds.

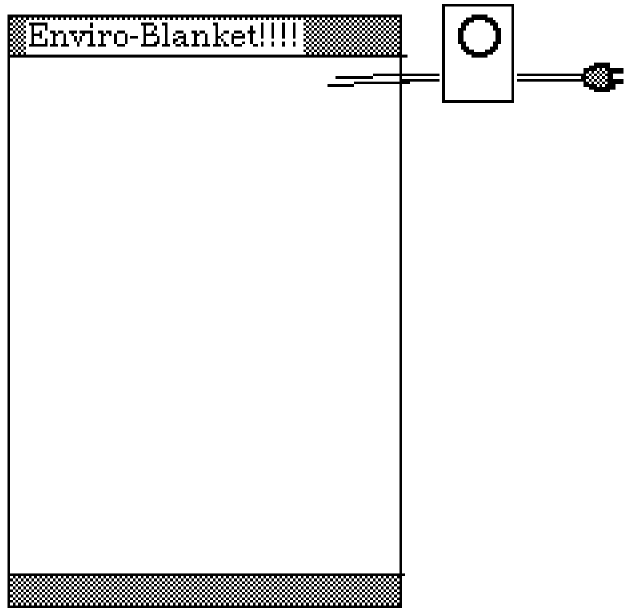


(d) By using the "analyze" feature find the period and frequency of oscillation of the magnetic field produced by the blanket wire. *Show your calculations*. Explain what you did.

(e) Use the "analyze" feature to find the **maximum change** in magnetic field that the sensor detects at a distance of about 1.0 cm from the blanket wire. Find B_{rms} from this peak-to-peak value. Record the result below.

(f) How does your measured result for the rms magnetic field at about 1 cm from a blanket wire compare with the value you estimated in **Activity 26-5(b)**?

(g) Suppose you were a blanket manufacturer under pressure to produce a much safer blanket. Use the principle of superposition to design a wiring scheme for your blanket that is safe. Sketch your wiring scheme in the space below. Explain why your design is safer! Write an ad for it if you like!


Enviro-Blanket!!!!

SESSION TWO: BIOT-SAVART LAW AND FARADAY'S LAW

Biot-Savart Law; The Magnetic Field at the Center of a Current Loop

During this session you will explore some effects of changing magnetic fields. One way to produce a changing magnetic field is by varying the current in a loop of wire. Let's predict and observe the direction and relative magnitude of the magnetic field inside a coil consisting of one or more circular loops of wire as shown in the diagram below.

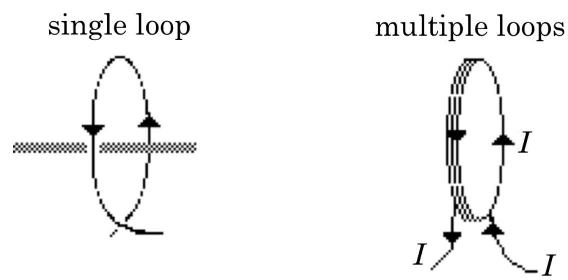


Figure 26-6: A wire loop carrying a current *I*.

In order to predict the *direction* of the magnetic field at the centre of the coil due to the current in one of its loops you can use the rule you devised and explained in **Activity 26-2(b)**.

For the prediction and investigation of the magnetic field in the centre of a current loop, you'll need the following:

- An insulated wire, 1m long (16 AWG w/ thermoplastic insulation)
- A large, flat 200-turn coil (known as a "field coil")
- A 1.5 V D-cell battery plus holder
- A SPST switch
- 2 alligator clip leads, > 10 cm
- A small compass
- A LabPro system
- An MG-BTA Vernier Magnetic Field Sensor
- An ammeter, 1A
- A Lucite holder or 3” length of wooden dowel

✍ Activity 26-7: The Magnetic Field in a Loop (Record answers to questions and all results in your logbook)

(a) On the basis of your observation of the magnetic field surrounding a straight wire, what direction do you think the magnetic field will be in the centre of the single loop shown in Figure 26-6 above? How do you expect the magnitude of the field at the centre of the loop to change if you make two loops? Three loops? Cite evidence from previous observations to support your prediction.

(b) Wrap the wire once around the dowel or Lucite holder, making a single loop and slip the loop off the dowel. Set up a current through the loop in the direction shown in Figure 26-6 above. Use a compass to determine the direction of the magnetic field at the center of the loop and sketch the direction. How does it compare with your prediction?

(c) To eliminate the effects of the Earth's magnetic field, open the L270601 experiment file in LoggerPro and zero the magnetic field sensor in place with the current turned off. Then turn on the current to measure the magnetic field change due to one loop. Next measure the magnitude of the field when you coil the single wire into more loops. Use the ammeter to measure the current through the wire. Take these measurements for at least five values of N (N = # of loops) and record them in your logbook. Make a graph in your logbook of B vs. N. Put error bars on your points representing the uncertainty in your B field readings. One way to estimate the B uncertainty is use the half-range of several readings. Do your observations agree with your prediction in part (a) above?

Note: (1) Tape the sensor to the table so it does not move as you add loops. (2) Use the lower amplification setting (higher range setting) on the magnetic field sensor. (3) DO NOT leave the current in the wire for more than a few seconds at a time or the battery will wear out.

(d) In your circuit, replace the wire with a 200-turn field coil. Measure the current through the coil and the magnetic field at the centre of the coil and record these values. *You may need these values in the next session.* What is the ratio of B to NI? Do error analysis and compare the theoretical value to the experimental value.

(Record these values in your logbook)

I_{field} = ±

$B_{field(measured)}$ = ±

$B_{field(measured)}/NI$ = ±

R_{field} (this is the radius of the field coil) = ±

$B_{field(theory)}/NI = \mu_0/2R_{field}$ = ±

Compare the measured values with the theoretical value. Do they agree within experimental error?

A very useful result of the formal mathematical calculation for a circular coil of wire is that *the magnetic field at the centre of the coil is proportional to the current flowing through its windings and to the number of turns of wire in the coil.* Thus, we will be using the expression

$$B \propto NI$$

in the next session as we explore Faraday's law.

Michael Faraday's Quest

In the nineteenth century, the production of a magnetic field by a current-carrying wire was regarded as the creation of magnetism from electricity. This led investigators to a related question: Can magnetism create an electric field capable of causing current to flow in a wire? Michael Faraday, thought by many to be the greatest experimental physicist of the nineteenth century, attempted numerous times to produce electricity from magnetism. He reportedly put a wire that was connected to a galvanometer near a strong magnet, but no current flowed in the wire. Faraday realized that getting current to flow would involve a kind of perpetual motion unless the magnet were to lose some of its magnetism in the process. Although the law of conservation of energy had not yet been formulated, Faraday had an intuitive feeling that the process of placing a wire near a magnet should not lead to the production of electrical current.

Faraday fiddled with this problem off and on for ten years before discovering that he could produce a current in a coil of wire with a *changing* magnetic field. This seemingly small feat has had a profound impact on civilization. Most of the electrical energy that has been produced since the early nineteenth century has been produced by changing magnetic fields! This process has come to be known as *induction*.

To make some qualitative observations on electric field "induction" and associated currents, you'll need:

- A galvanometer
- 3-4 assorted wire coils (with different areas and numbers of turns)
- 2 alligator clip leads
- 2 bar magnets
- A strong horseshoe magnet

The goal of these observations is two-fold – first, to get a feel for what induction is like, and second, to discover what factors influence the amount of current induced in the coil. To start your observations you should wire one of the coils in series with a galvanometer and fiddle around with the bar magnet in the vicinity of the coil.

✍ Activity 26-8: Current from a Coil and Magnet

(a) Play around with the coils and magnet and make a list of as many factors as possible that will determine the maximum current that can be induced in the coil. Each coil has a characteristic resistance and whenever a current is induced in it there is a potential difference created across the coil. This magnetically generated potential difference is called an *electromotive force* or emf.

(b) Is it possible to have a current or emf in the coil when the magnetic field is not changing in the centre of the coil? If necessary, make more observations and explain your results.

This is a good time to make more careful observations on the relationship between various factors that influence the magnitude of the emf induced in a coil. Pick a factor from the list above that can be observed directly and make more detailed observations on its effects. See if you can hypothesize a *simple* mathematical relationship for your factor. For example, you might find that the emf increases with the cross sectional area of the coil. This could lead you to the intelligent guess (that's what's meant by a hypothesis) that the emf induced in a coil is proportional to its area ($emf \propto A$), and so on.

✍ Activity 26-9: Describing an Induced EMF

(a) How do you think the emf induced in a coil depends on the *rate* at which the magnetic field changes in it? **Hint**: Is there any emf induced whenever the magnet and coil are at rest relative to each other?

(b) How do you predict the emf induced in a coil depends on the area of the coil?

(c) How do you predict the emf induced in a coil depends on the number of turns in the coil?

(d) Are there any other factors that you think might influence the emf?

(c) Check with some of your classmates and find out what relationships they are hypothesizing for other factors. Write down a trial equation that describes the induction of an emf as a function of the factors you think are important.

Some Observations of Magnetic Induction

You should be convinced by now that: (1) currents can be induced in a conductor in the presence of a changing magnetic field; and (2) currents cause magnetic fields. Let's observe two phenomena that depend on one or both of these two facts using the following equipment. We have one set-up for the entire class that we will take turns with:

- A solenoid with an 110 VAC plug
- A pickup coil w/ a small bulb attached (w/ a larger diameter than the solenoid)
- A small metal ring
- A small metal ring with a vertical cut
- An aluminum tube
- A strong cylindrical dipole magnet that fits in the tube

Phenomenon #1: Magnet, Pickup Coil & Light Bulb
Suppose a 60 hertz alternating current is fed into a solenoid (which consists of a long wire wound into a series of circular wire loops) to create an electromagnet with a changing magnetic field with $dB/dt = A \sin(\omega t)$. What happens when a coil of wire, with a light bulb attached to it, is placed over the solenoid as shown in Figure 26-7 below? How about if a metal ring is placed over it and it is suddenly turned on?

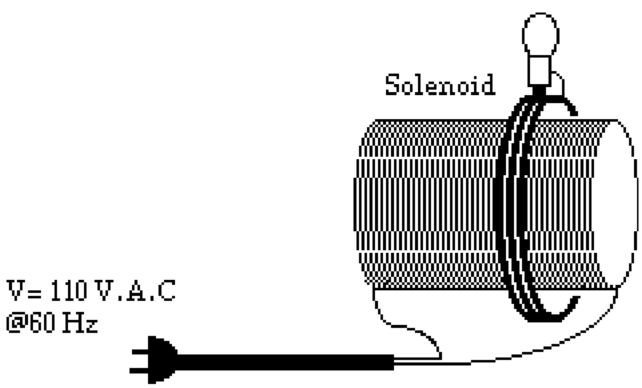


Figure 26-7: A pickup coil with light bulb attached in series surrounding a solenoid but not touching it.

Phenomenon #2: The Metal Tube and Cylinders
Suppose a non-magnetic cylindrical metal object is dropped through a metal tube. What will happen? How fast will it fall? Suppose a cylindrical magnet is dropped through the same metal tube. What might be different? Why?

✍ Activity 26-10: Induction Phenomena

(a) What did you predict for phenomenon #1, in which a coil with a bulb attached to it surrounds a changing magnetic field? What did you see? How about the ring(s)?

(b) Explain phenomenon #1.

(c) What did you predict for phenomenon #2, in which two objects are dropped down a conducting tube? What did you see?

(d) Explain phenomenon #2.

A Mathematical Representation of Faraday's Law

By performing a series of quantitative experiments on induction, it can be shown that the emf induced in a coil of wire is given by the equation

$$\varepsilon = -N\frac{d\Phi^{mag}}{dt}$$

where the magnetic flux through a single loop of the coil Φ^{mag} is given by the expression

$$\Phi^{mag} = \vec{B} \cdot \vec{A}$$

where $\vec{B}$ is the average magnetic field inside the coil, and $\vec{A}$ is a vector whose magnitude is the cross-sectional area of the coil and whose direction is given by the normal to that cross-section. N is the number of loops in the coil of wire. Thus, Faraday's Law relating emf to flux can be written in two alternate forms:

$$\varepsilon = -N\frac{d\Phi^{mag}}{dt} = -N\frac{d(\vec{B} \cdot \vec{A})}{dt}$$

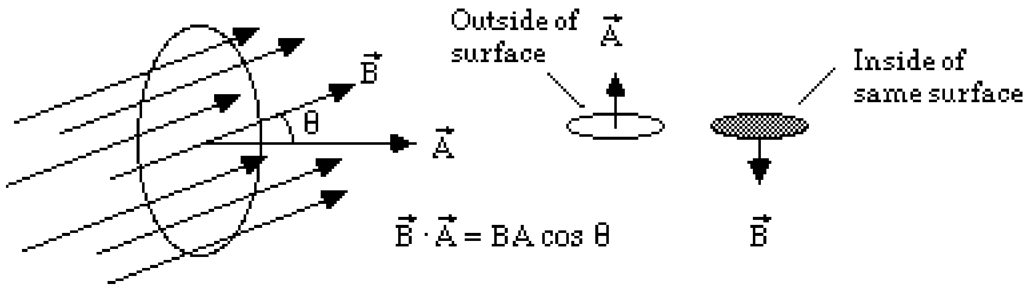


Figure 26-8: Magnetic flux through an area A is the dot product of the magnetic field vector and the vector normal to the area.

Whenever the plane of a coil of area A is perpendicular to the magnetic field vector, then the dot product can be dropped and the equation for the emf can be written in terms of vector components along a common axis (in this case called the z-axis) so that:

$$\varepsilon = -NA_z \frac{dB_z}{dt}$$

Computing Flux from Currents in a Loop as a Function of Time

A loop of wire known as a *pickup coil* has a radius R and N turns. Suppose it is placed perpendicular to a uniform magnetic field B that varies with time so that

$$B = B_0 \sin(\omega t)$$

where B_0 is a constant representing the amplitude of the magnetic field.

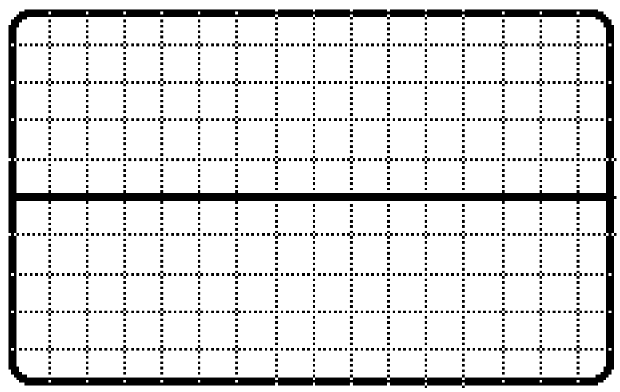
✍ Activity 26-11: Applying Faraday's Law to a Wire Loop

(a) What is the equation for the flux through a single loop of the coil in terms of R, B_0, ω and t?

(b) According to Faraday's law, what is the equation for the emf in the pickup coil in terms of N, R, B_0, ω and t? **Hint**: The cosine function is involved. Why?

(c) In the space below sketch two graphs – one showing the shape of the B vs. t graph for at least two complete cycles and one showing the shape of the emf vs. t graph for the same two cycles. Be careful to line the two graphs up properly!

B



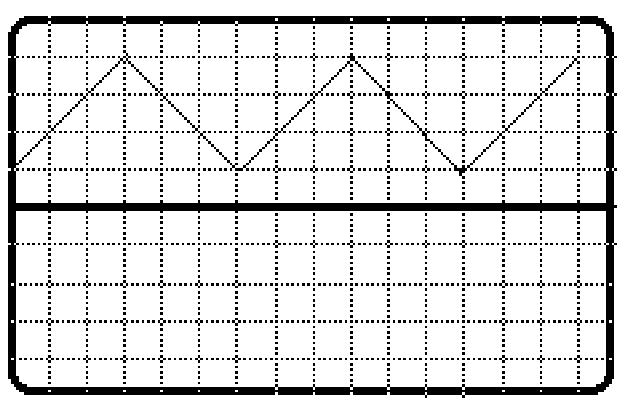
time

EMF

(d) Suppose that the magnetic field varies over time in a triangular fashion, as shown in the diagram below. Sketch the shape of the induced emf function in the space below.

Hints: (1) It is not the same as the emf in part (c). (2) Remember that the derivative of a function is its slope at each point in time.

B



time

EMF

SESSION THREE: VERIFICATION OF FARADAY'S LAW

Verifying Faraday's Law Quantitatively

Your mission, should you choose to accept it, is to do a quantitative investigation of the emf in a pickup coil as a function of the rate of change of the flux through it to see if

$$\varepsilon = -N\frac{d\Phi^{mag}}{dt}$$

In this project you can use one current-carrying wire to create a magnetic field that induces an EMF in a second coil. The first of these coils, called the *field coil*, can have a changing current from a function generator pushed through it. The magnetic field that is produced in the centre of the field coil also varies with time and is proportional to the current in the field coil. An inner coil, called the *pickup coil*, will have a current induced in it as a result of the time-varying magnetic field. A dual trace oscilloscope can be used to display both the current in the field coil and the emf induced in the pickup coil. For this activity and the next you will need the following equipment:

- A large flat 200 turn field coil
- A pickup coil
- A 100 Ω resistor
- A function generator
- An oscilloscope
- 3 female banana plug to male BNC connectors
- 6 banana plug leads
- Alligator clips
- A ruler
- A Vernier calliper
- A protractor

The experimental setup which is pictured below can be used to take measurements of the induced emf in the pickup coil as a function of the time rate of change of the magnetic flux in the central region of the pickup coil.

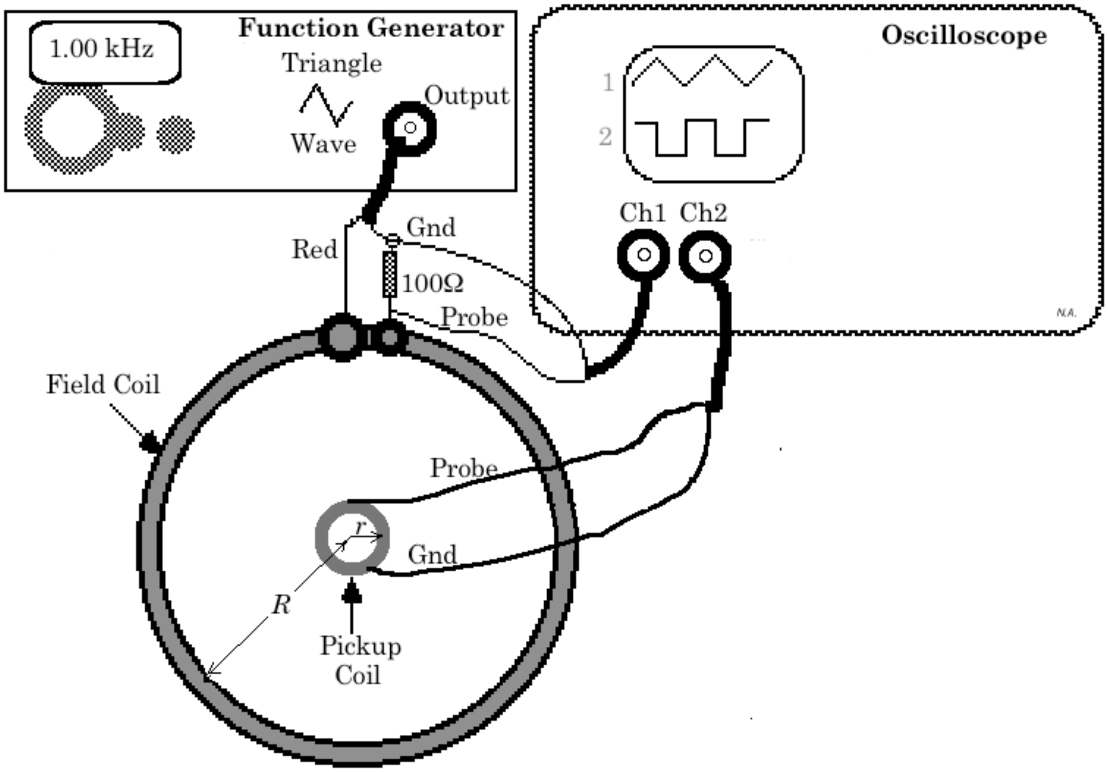


Figure 26-9: Faraday's Law Apparatus

In order to verify Faraday's law you need to use the standard equation for the magnitude of the magnetic field at the centre of the field coil. This equation, which is derived in most introductory physics texts, is

$$B_z = \frac{\mu_0 N_f I}{2R}$$

where R is the radius of the field coil and N_f is the number of turns in the field coil. If the normal vector of the pickup coil makes an angle θ with the magnetic field vector $\vec{B}$ then

$$\Phi^{mag} = \vec{B} \cdot \vec{A} = \left[\frac{\mu_0 N_f A\cos\theta}{2R}\right] I$$

Thus, for a fixed angle between $\vec{B}$ and $\vec{A}$, the flux, Φ^{mag}, through the pickup coil is directly proportional to the current I. So the change in flux $d\Phi^{mag}/dt$ will be directly proportional to dI/dt. *This is important!*

Thus, you need to generate a changing magnetic flux in the centre of the field coil by changing the current in the field coil. You can then see how the changing flux affects the emf that is induced in the pickup coil.

The first step is to connect the wave generator to the field coil and to the oscilloscope (as shown in the diagram above) and put a changing current (in the form of a 1000 hertz triangle wave) into the field coil from the wave generator. *Note that the voltage drop, V_1, across the 100Ω resistor, R_i, can be measured by the oscilloscope Ch1. Ohm's law can then be used to calculate the current, I, in the field coil.*

The rate of change of the magnetic flux through the pickup coil is proportional to the rate of change of the current in the field coil. Let's consider a plot of the triangle wave representing the change in current as a function of time as shown in Figure 26-10 below.

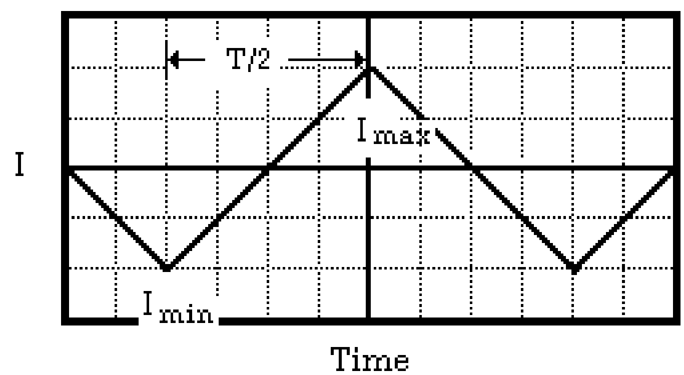


Figure 26-10: A graph of current vs. time in the field coil when a triangle wave form is fed into a field coil.

It can be seen from the plot above that, when the slope of the triangle wave is positive,

$$Slope = \frac{dI}{dt} = \frac{I^{max} - I^{min}}{T/2}$$

If the frequency of the wave is set at *f* on the wave generator, we can use the fact that *T= (1/f)* to find the slope in terms of *f*.

$$Slope = \frac{dI}{dt} = 2f(I^{max} - I^{min})$$

Noting that the negative slope has the same magnitude, it is clear that in general

$$Slope = \frac{dI}{dt} = \pm 2f(I^{\max} - I^{\min})$$

so that by differentiating the equation for the magnetic flux:

$$\Phi^{mag} = \vec{B} \cdot \vec{A} = \left[\frac{\mu_0 N_f A \cos\theta}{2R}\right] I$$

we can find the rate of change of magnetic flux through each single loop in the pickup coil:

$$\frac{d\Phi^{mag}}{dt} = \frac{d(\vec{B} \cdot \vec{A})}{dt} = \left[\frac{\mu_0 N_f A \cos\theta}{2R}\right] \frac{dI}{dt}$$

and finally for our situation with $\theta = 0$:

$$\frac{d\Phi^{mag}}{dt} = \frac{d(\vec{B} \cdot \vec{A})}{dt} = \pm\left[\frac{\mu_0 N_f A}{2R}\right] 2f(I^{\max} - I^{\min})$$

Where

N_f is the number of turns in the *field* coil
R is the radius of the *field* coil, and
A is the area of the *pickup* coil

To determine the emf induced in the pickup coil, you should connect the pickup coil to the oscilloscope as shown in Figure 26-9 above. (Use the scope probe on x1.)

If Faraday's Law holds, then the magnitude of measured electromotive force induced in the pickup coil should be equal to the calculated value of

$$emf = N_p \frac{d\Phi^{mag}}{dt}$$

where N_p is the number of turns in the pickup coil.

✍ Activity 26-12: Results: EMF vs. $d\Phi^{mag}/dt$

(Record answers to questions and results in your logbook.)

(a) The *emf* induced in the pickup coil is given by $0.5V_2$ where V_2 is the peak-to-peak voltage from the pickup coil as recorded on input Ch. 2 of the oscilloscope. Explain why $emf = 0.5V_2$ rather than V_2.

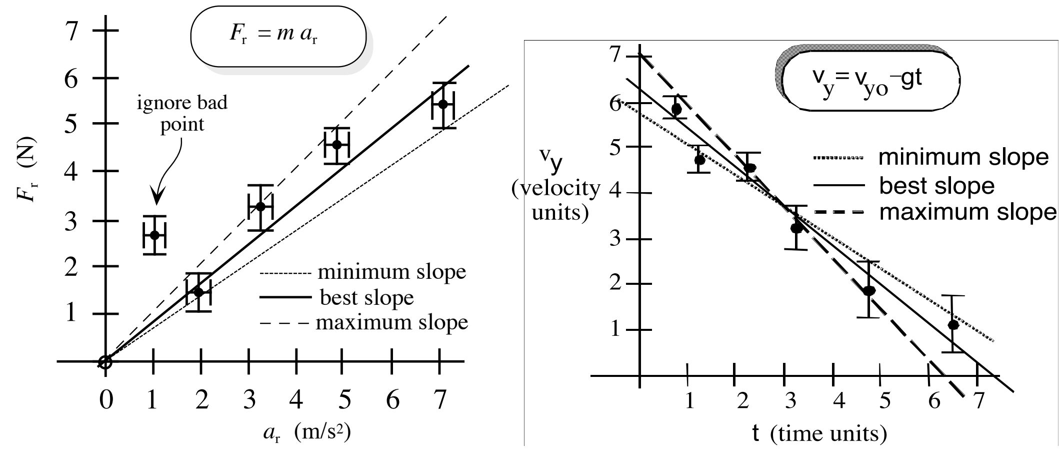
(b) Vary the output frequency of the triangular wave between about 200 and 1000 Hz. Create a data table in your logbook like the one below to record the value of *emf* as a function of frequency. Don't forget to add columns for the errors on the quantities.

measured calculated from measurements

f (Hz)	Emf (v)	$d\Phi^{mag}/dt$
200		
400		
600		
800		
1000		

(c) Use the value of (I^{max} - I^{min}) to calculate $d\Phi^{mag}/dt$ for each frequency. **Show a sample calculation in your logbook.** Plot *emf* vs. $d\Phi^{mag}/dt$ with error bars on both quantities in your logbook. Draw a best fit line on your graph. **Hint**: Use the peak-to-peak voltage of Ch. 1 V_1 and the value of the input resistor R_i to find (I^{max} - I^{min}). For the error bars on $d\Phi^{mag}/dt$ you will need to combine the errors on A, R, and V_1 using the error propagation rules that were introduced in Unit 23. **You should also show a sample error calculation in your logbook.**

(d) **Answer the following questions in your logbook:** What is the value of the slope of the graph? What is the error in the slope? (To get the error in the slope you need to draw two more "best fit" lines that fit within your error bars like the example shown below. Then: slope error = (max slope – min slope)/2.) What does the theory say the slope should be?



Graphs with error bars: The data on the left are modelled by a proportionality: only one parameter, the slope, is varied. The right-hand graph is a general linear relationship with two parameters: slope and intercept

(e) Does Faraday's law seem to hold? Explain why or why not.

Flux as a Function of Angle
So far you have concentrated on measuring emf as a function of the wave form that causes a time varying magnetic field at the site of the pickup coil. Suppose the normal vector for the pickup coil makes an angle θ with respect to the normal vector of the field coil. If you hold everything else the same, what happens to the maximum emf induced in the pickup coil?

✍ Activity 26-13: Experimental Results: EMF vs. θ

(Record answers to questions and results in your logbook)

(a) Sketch a graph of the *predicted* maximum *emf* as a function of the angle between the field coil and the pickup coil in the space below. Do this for angles between 0^{o} and 180^{o} and explain the theory behind your prediction. **Hint**: How does

$$\Phi_m = \vec{B} \cdot \vec{A}$$

depend on the angle? Please label the axes and specify units.

(b) Set up an experiment to measure the maximum emf of a changing magnetic field at the site of the pickup coil as a function of angle for at least six angles between 0^{o} and 180^{o}. Record your data and create a graph of *emf* vs *angle* in your logbook. Include error bars on this graph for both quantities. The angle error can be estimated based on the smallest division of the protractor and other factors.

(c) How did your results compare with your prediction?

Electricity and Magnetism – Maxwell's Equations

James Clerk Maxwell, a Scottish physicist, was in his prime when Faraday retired from active teaching and research. He had more of a mathematical bent than Faraday and pulled many of the basic equations describing electric and magnetic effects into a set of four very famous equations. These equations are shown below in simplified form for situations in which no dielectric or magnetic materials are present.

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q}{\varepsilon_0} \quad \text{(Gauss' Law in Electricity)}$$

$$\oint \vec{B} \bullet d\vec{A} = 0 \quad \text{(Gauss' Law in Magnetism)}$$

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_m}{dt} \quad \text{(Faraday's Law)}$$

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_e}{dt} \quad \text{(Ampere - Maxwell Law)}$$

If we add the Lorentz force

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

to Maxwell's equations, then *we can derive a complete description of all classical electromagnetic interactions* from this set of equations.

Perhaps the most exciting intellectual outcome of Maxwell's equations is their prediction of electromagnetic waves and our eventual understanding of the self-propagating nature of these waves. This picture of electromagnetic wave propagation was not fully appreciated until scientists abandoned the idea that all waves had to propagate through an elastic medium and accepted Einstein's theory of special relativity; these changes occurred in the early part of the twentieth century. To the vast majority of the world's population the practical consequences of Maxwell's formulation assume much more importance than its purely intellectual joys. Richard Feynman wrote the following:

Now we realize that the phenomena of chemical interaction and ultimately of life itself are to be understood in terms of electromagnetism. . .. The electrical forces, enormous as they are, can also be very tiny, and we can control them and use them in many ways. . . From a long view of the history of mankind – seen from, say, ten thousand years from now – there can be little doubt that the most significant event of the nineteenth century will be judged as Maxwell's discovery of the laws of electrodynamics. The American civil war will pale into provincial insignificance in comparison with this important scientific event of the same decade.