Name ______________________ Date(YY/MM/DD) ______/_________/_______
St.No. __ __ __ __ __-__ __ __ __ Section__________ Group #_____

UNIT 28: ELECTROMAGNETIC WAVES AND POLARIZATION

Hey diddle diddle, what kind of riddle
Is this nature of light?
Sometimes it's a wave,
Other times particle...
But which answer will be marked right?

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OBJECTIVES

1. To understand electromagnetic waves and how they propagate.

2. To investigate different ways to polarize light including linear and circular polarization.

 Adapted from PHYS 131 Optics Lab #3.

OVERVIEW

In this unit you will learn about the behaviour of electromagnetic waves. Maxwell predicted the existence of electromagnetic waves in 1864, a long time before anyone was able to create or detect them. It was Maxwell's four famous equations that led him to this prediction which was finally confirmed experimentally by Hertz in 1887.

Maxwell argued that if a changing magnetic field could create a changing electric field then the changing electric field would in turn create another changing magnetic field. He predicted that these changing fields would continuously generate each other and so propagate and carry energy with them. The mathematics involved further indicated that these fields would propagate as waves. When one solves for the wave speed one gets:

$$\textit{wave speed:}\quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{28.1}$$

Where ε_0 and μ_0 are the electrical constants we have already been introduced to. The magnitude of this speed led Maxwell to further hypothesize that light is an electromagnetic wave.

Once you have learned a few things about the nature of electromagnetic waves, you will go on to investigate the phenomenon of polarization.

SESSION 1 — ELECTROMAGNETIC WAVES

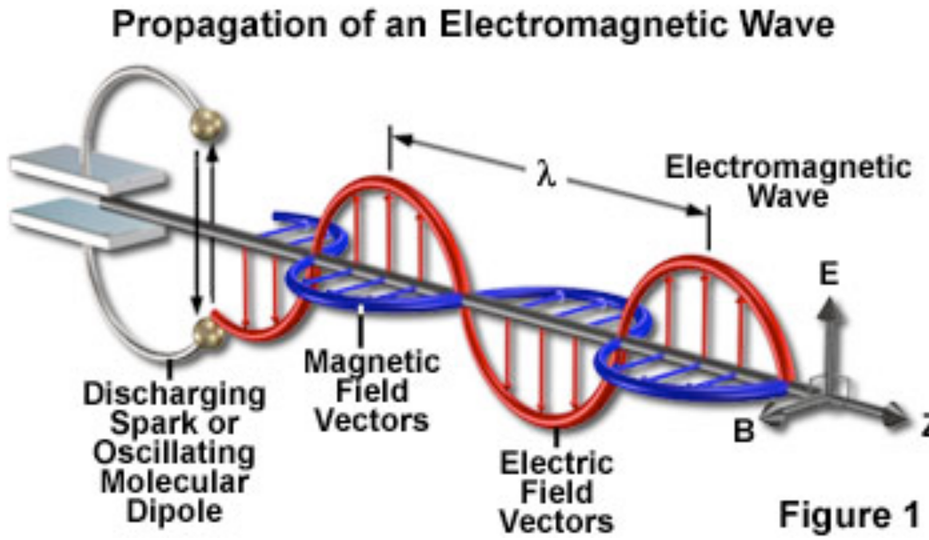

Figure 28.1: An electromagnetic wave.

An electromagnetic wave that is travelling in the positive z-direction with its electric field oscillating parallel to the x-axis and its magnetic field oscillating parallel to the y-axis (as shown in Figure 28.1) can be represented mathematically using two sinusoidal functions of position (z) and time (t):

$$\vec{E} = \vec{E}^{max} \sin(kz - \omega t) \quad (28.2a)$$

$$\vec{B} = \vec{B}^{max} \sin(kz - \omega t) \quad (28.2b)$$

where $\vec{E}^{max}$ and $\vec{B}^{max}$ are the amplitudes of the fields, ω is the angular frequency which is equal to $2\pi f$, where f is the frequency of the wave, and k is the wave number, which is equal to 2π/λ, where λ is the wavelength.

It is important to note that Figure 28.1 is an abstract representation of an electromagnetic wave that represents the magnitude and direction of the electric and magnetic fields at points along the z-axis. In a real electromagnetic wave travelling through space, for each line parallel to the z-axis there is a similar picture. These arrows, which represent field vectors, do not indicate a sideways displacement of anything. Also, as we'll soon see, there can be many axes along which the fields oscillate in one electromagnetic wave.

✍ **Activity 28-1: Wave Speed**

(a) Plug in the values for ε_0, the permittivity of free space and μ_0, the permeability of free space, into the following equation and verify that the wave speed of electromagnetic waves that you get is consistent with the speed of light in a vacuum, c = 299 792 458 m/s:

$$\textit{wave speed}: \quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{28.1}$$

The speed of a wave is equal to a product of its wavelength λ and its frequency *f*. So. for an electromagnetic wave we get:

$$c = \lambda f \tag{28.3}$$

This means there is an inverse relationship between wavelength and frequency for electromagnetic waves, i.e. the longer the wavelength of a wave, the lower the frequency.

Electromagnetic (EM) waves are often sorted into what's know as the Electromagnetic Spectrum, where the types of electromagnetic waves are sorted in order of their wavelength or frequency. One example of an electromagnetic spectrum where the waves are sorted by wavelength is shown below in Figure 28.2. As you know, modern technology takes advantage of the existence of many of these electromagnetic waves, which all travel at the speed of light, *c*.

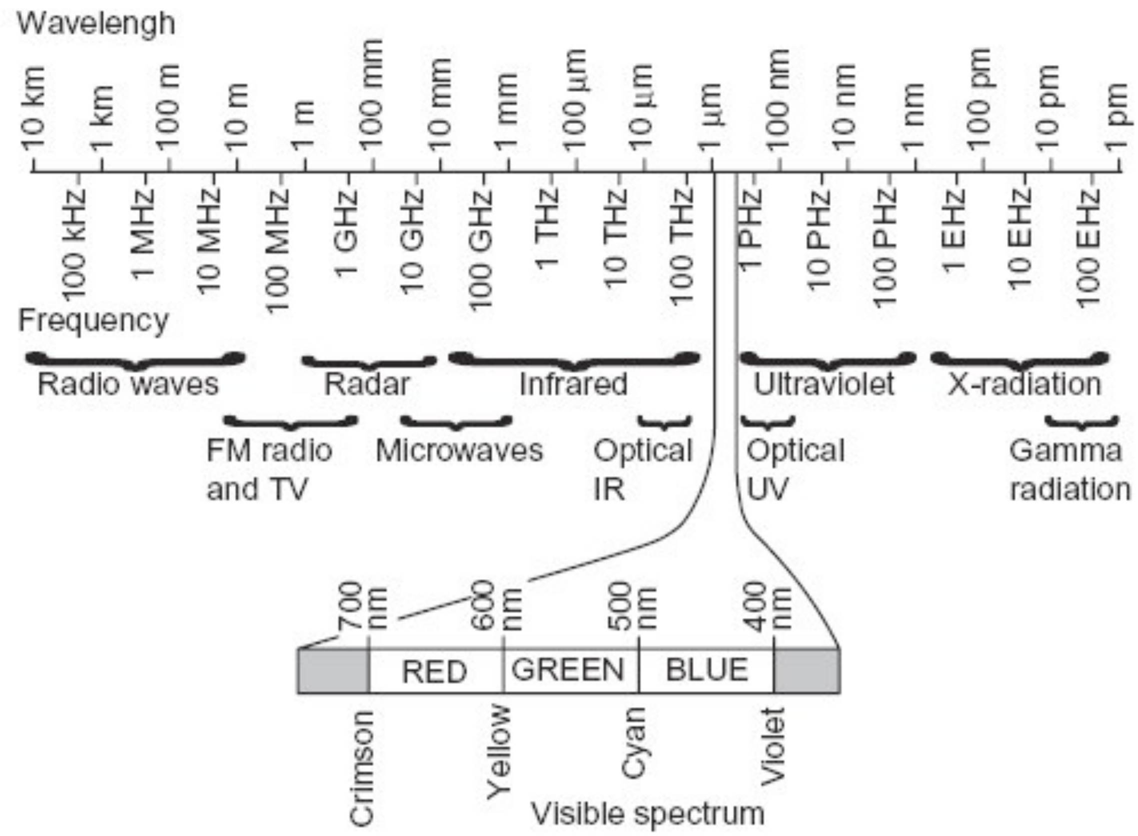

Figure 28.2: The Electromagnetic Spectrum.

✍ **Activity 28-2: The Electromagnetic Spectrum**
(a) Using Figure 28.2 roughly estimate the average wavelength and frequency of the following EM waves:

1) Gamma rays

2) Ultraviolet waves

3) Microwaves

4) Radio waves

(b) On the grid below, graph log(frequency) vs. log(wavelength) for these 4 EM waves.

(c) Does your graph show the relationship between frequency and wavelength that you expect? What is this relationship? Why do you think it was necessary to take logarithms of both quantities?

The Wave Equation

Using the definitions of ω and k, $\omega = 2\pi f$ and $k = 2\pi/\lambda$, we can rewrite 28.3 as:

$$c = \lambda f = \left(\frac{2\pi}{k}\right)\left(\frac{\omega}{2\pi}\right) = \frac{\omega}{k} \tag{28.4}$$

Using this result we can rewrite our two wave equations as:

$$\vec{E}(z,t) = \vec{E}_0 \sin\left[\omega\left(\frac{z}{c} - t\right)\right] \tag{28.5a}$$

$$\vec{B}(z,t) = \vec{B}_0 \sin\left[\omega\left(\frac{z}{c} - t\right)\right] \tag{28.5b}$$

One can use Faraday's Law to determine the ratio of the electric and magnetic fields. This ratio is:

$$\frac{\left|\vec{E}^{\max}\right|}{\left|\vec{B}^{\max}\right|} = \frac{\left|\vec{E}\right|}{\left|\vec{B}\right|} = c \tag{28.6}$$

Thus if you know the magnitude of one of the two fields you can easily determine the magnitude of the other in an electromagnetic wave.

✍ **Activity 28-3: The Wave Equation**

Suppose you are given the following wave equation:

$$E_x = (4.0\mathrm{V/m})\sin\left[\left(\tfrac{\pi}{5}\times 10^{15}\ \mathrm{s}^{-1}\right)\left(\frac{z}{c} - t\right)\right],\ E_y = 0,\ E_z = 0$$

Determine the following quantities for this electromagnetic wave.

(a) ω = ?

(b) f = ?

(c) $\lambda = ?$

(d) $k = ?$

(e) Write expressions for the three components (x,y,z) of the magnetic field of this wave below.

(f) What type of electromagnetic wave do these equations describe? (radio wave?, x-ray? etc...)

Energy Transport by Electromagnetic Waves

We know that the energy per unit volume or *energy density* stored in an electric field of magnitude E is:

$$u^{elec} = \frac{1}{2}\varepsilon_0 E^2 \tag{28.7}$$

We also know that the energy per unit volume stored in a magnetic field of magnitude B is:

$$u^{mag} = \frac{B^2}{2\mu_0} \tag{28.8}$$

Using the fact that $E = cB$ for an electromagnetic wave, it is easy to show that these two energy densities are equal for em waves:

$$u^{elec} = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (cB)^2 = \frac{1}{2}\varepsilon_0 c^2 B^2$$
$$= \frac{1}{2}\varepsilon_0 c^2 2\mu_0 u^{mag} = c^2\left(\frac{1}{c^2}\right)u^{mag} = u^{mag}$$

where we have used Equation 28.1 to replace $\varepsilon_0\ \mu_0$ with $1/c^2$. This equality of the two energy densities is true everywhere along an electromagnetic wave.

The total energy density for an electromagnetic wave is the sum of the two energy densities:

$$u^{total} = u^{elec} + u^{mag} = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0} \tag{28.9}$$

Because the two densities are equal, one can also write:

$$u^{total} = 2u^{elec} = \varepsilon_0 E^2 \tag{28.10}$$

$$u^{total} = 2u^{mag} = \frac{B^2}{\mu_0} \tag{28.11}$$

These two equations are useful if you only know one of the two field strengths.

One can use these expressions to determine the rate of energy transport per unit area for an em wave. The instantaneous energy flow rate is given as:

$$S = \frac{1}{c\mu_0} E^2 = \varepsilon_0 c E^2 \tag{28.12}$$

✍ **Activity 28-4: Electromagnetic Wave Intensity**

(a) If intensity *I* is defined to be the time-average of *S,* show that for an em wave travelling in the positive *z*-direction:

$$I = \langle S \rangle = \frac{1}{c\mu_0} \left\langle (E^{\max})^2 \sin^2(kz - \omega t) \right\rangle$$

(b) Rewrite the intensity in terms of the root-mean square value of the magnitude of the electric field $E^{rms} = E^{max}/\sqrt{2}$ and using the fact that the time-average of $\sin^2(kz - \omega t)$ in this case equals 1/2.

(c) Now write the intensity I in terms of the root-mean square of the magnetic field: B^{rms}.

The Poynting Vector

One can define a vector quantity $\vec{S}$, called the Poynting vector after John Henry Poynting, as the following:

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \tag{28.13}$$

This vector has a magnitude equal to the energy transport rate and points in the direction that the energy travels i.e. the wave propagation direction.

✍ **Activity 28-5: The Poynting Vector**

(a) Show that the magnitude of the vector $\vec{S}$ given above in Equ. 28.13 is equal to the quantity S given in Equ. 28.12. (**Hint:** Don't forget to deal with the angle between the two field vectors.)

(b) Using Equ. 28.13 determine the direction of the Poynting vector for the EM wave shown in Figure 28.1. Is this direction consistent with the direction the wave is shown to be travelling?

SESSION 2 — POLARIZATION

Theory of Polarization

(This section must be read before coming to class or you will not get finished in time.)

To describe light one must specify its frequency, its direction of propagation and its state of polarization. Our interest in this session is with polarization, so let us assume that we have monochromatic light propagating along the +z direction of a right-handed co-ordinate system. Light is a transverse electromagnetic wave—the electric field is always perpendicular to the direction of propagation. Because the direction of propagation is along the +z axis, the electric field vector $\vec{E}$ must lie in the plane formed by the x and y axes. This can be expressed mathematically as follows:

$$\vec{E}(x,y,z,t) = E_x(z,t)\hat{i} + E_y(z,t)\hat{j} \tag{28.14}$$

The components of the electric field E_x and E_y do not depend on x and y because we assume that the wave is a plane wave propagating along the +z direction.

Light is linearly polarized if the electric field vector $\vec{E}$ is always parallel to the same line which is perpendicular to the direction of propagation. Mathematically:

$$\vec{E}(x,y,z,t) = A_x\hat{i}\cos(\omega t - kz) + A_y\hat{j}\cos(\omega t - kz) \tag{28.15}$$

The amplitudes A_x and A_y are real constants. The intensity of the light is proportional to the square of the amplitude. Figure 28.3 shows the electric field vectors at a fixed time along a line in the direction of propagation. Figure 28.3a illustrates light polarized along the x direction ($A_x \neq 0$, $A_y = 0$) and 28.3b shows polarization along the y direction ($A_x = 0$, $A_y \neq 0$). As time goes on the entire pattern moves in the +z direction. Light polarized in an arbitrary plane is a superposition of these two independent possibilities ($A_x \neq 0$, $A_y \neq 0$). The plane of polarization is determined by the relative magnitudes of A_x and A_y.

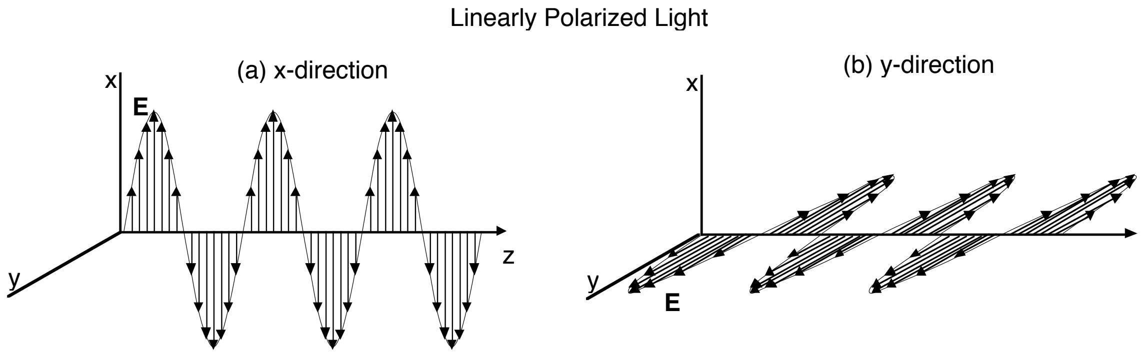

Figure 28.3: The electric field vectors of linearly polarized light.

Light is circularly polarized if the electric field vector moves in a circle. As there are two senses of rotation, there are again two independent polarization states, left and right:

$$\begin{aligned} &\textit{left}: && \vec{E}(x,y,z,t) = A_L[\hat{i}\cos(\omega t - kz) + \hat{j}\sin(\omega t - kz)] \\ &\textit{right}: && \vec{E}(x,y,z,t) = A_R[\hat{i}\cos(\omega t - kz) - \hat{j}\sin(\omega t - kz)] \end{aligned} \tag{28.16}$$

Figure 28.4 shows the electric field vector at a fixed time for the two circular polarization states. As time goes on the pattern moves in the $+z$ direction. If we look into the oncoming beam and track the electric field vector at any point, it moves in a circle with constant radius. By convention, clockwise rotation is called right circularly polarized light because the helical path traced by the direction of electric field at any fixed time follows the threads of a right-handed screw.

The most general polarization state is a superposition of x and y linearly polarized light with arbitrary amplitudes and phases. This general polarization state can also be considered as a superposition of left and right circularly polarized light with arbitrary amplitudes and phases of the two circular components.

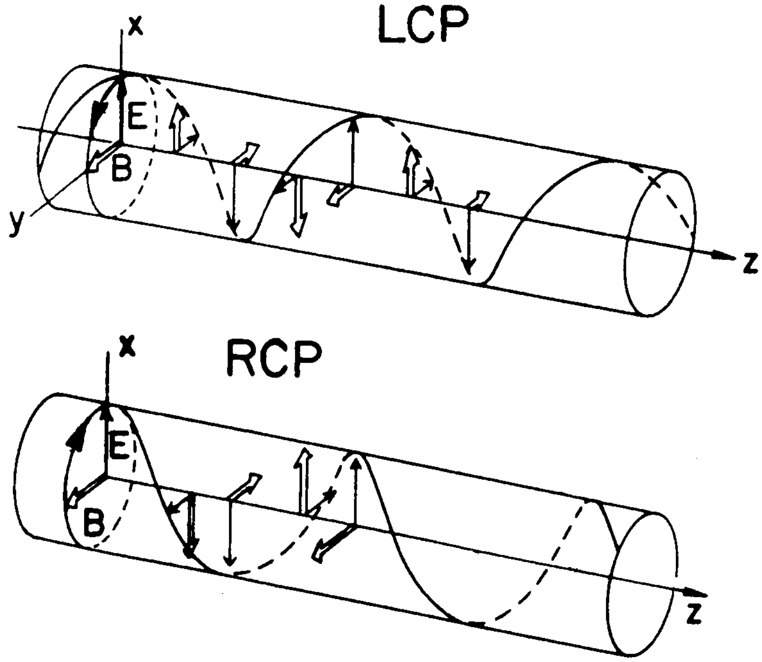

Figure 28.4: Left and right circular polarization.

Most light sources do not produce polarized light. Individual atoms in the source radiate independently. Although at any instant in time light received from a radiating atom in the source has a definite state of polarization the state of polarization changes rapidly with time. You may think of unpolarized light as having two polarization components (left/right or x/y) which are radiated independently and randomly. Between polarized light which has a fixed relation in time between the amplitude and phase of the two polarization components and unpolarized light which has a random relation in time between the two polarization components, is partially polarized light. Partially polarized light is a mixture of polarized and unpolarized light.

Polarization Filters and Retardation Plates

A linear polarizer produces polarized light by selectively absorbing light polarized perpendicular to the polarization axis and transmitting light parallel to the transmission axis. Our HN22 linear polarizers absorb more than 99.99% of the perpendicular component and transmit 44% of the parallel component. Thus the linear polarizer transmits 22% of incident unpolarized light. A perfect polarizer would transmit 50% of incident unpolarized light.

Consider unpolarized light falling on a polarizer which only transmits the components parallel to the x-axis. The filtered light has amplitude A and intensity $I = A^2$. Let this filtered light fall on a second polarizer whose polarization axis x' is at an angle θ with respect to the x-axis. The incident light has components $A \cos \theta$ and $-A \sin \theta$ along the x' and y' axes. If both polarizers are perfect, the intensity after the second polarizer would be $|A \cos \theta|^2$ but for the non-ideal polarizers the transmitted intensity is $0.44\ |A \cos \theta|^2$.

You use two retardation plates in this lab: a *quarter-wave* plate and a *half-wave* plate. Both plates are made of a clear plastic film whose index of refraction depends on the polarization direction of the light passing through it. The material has two perpendicular axes called the *fast axis* and the *slow axis*. If the light beam passing through is polarized along the material's fast axis, the index of refraction is n_f. Likewise if the light's axis of polarization is along the slow axis the index of refraction is n_s. The velocity of light in a material is c/n. Light polarized along the fast axis travels faster than that polarized along the slow axis implying $n_f < n_s$. Light polarized along an arbitrary direction can be resolved into components parallel to the fast axis and parallel to the slow axis. Because the faster travelling component emerges from the material first, the two polarization components which were in phase upon entering the film, are shifted out of phase when they emerge. The phase shift, δ, is given by:

$$\delta = 2\pi(n_s - n_f)\frac{d}{\lambda} \tag{28.17}$$

where d, is the thickness of the film and λ is the light's wavelength.

If $(n_s - n_f)\, d/\lambda = 1/4$ then the slower light component will emerge from the film 1/4 wavelength behind the faster component. The half-wave plates are designed so that the two polarization components are 1/2 wavelength out of phase after they have passed through the film: $(n_s - n_f)\, d/\lambda = 1/2$.

Both the quarter- and half-wave plates work ideally for only one wavelength in the middle of the visible spectrum. The phase shift for other wavelengths deviates from exactly one-quarter or exactly one-half.

Combining linear polarizers and retardation plates

Several interesting effects occur when polarizing filters are combined with either a quarter-wave plate or a half-wave plate.

1. *Linear polarizer followed by half-wave plate*

Consider a linear polarizer whose transmission axis is at an angle θ with respect to the fast axis of the half-wave plate. When the light emerges from the half-wave plate the component polarized along the slow axis will lag the fast component by 180° or π radians. Shifting a sine wave by 180° is the same as inverting it (multiplying by –1). Therefore the plane of polarization is changed from θ to –θ. See Figure 28.5.

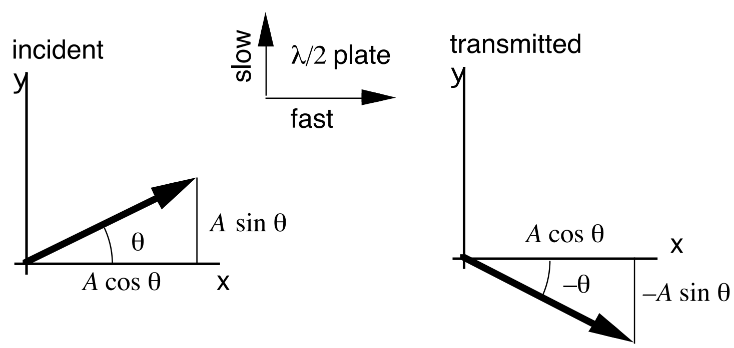

Figure 28.5: A half-wave plates changes the angle of linearly polarized light to its negative.

2. *Linear polarizer followed by quarter-wave plate*

Let the transmission axis of the linear polarizer be ±45° with respect to the fast axis. In this case the slow component is shifted by 90° or π/2 radians. Shifting a sine wave by π/2 changes it to a cosine wave. With the polarization at 45° to the fast axis, both fast and slow components will have the same amplitude but the 90° phase shift will change linear polarization to circular polarization. If θ is +45° the light emerges left-circularly polarized (Fig. 28.6); if θ is –45°, it is right-circularly polarized.

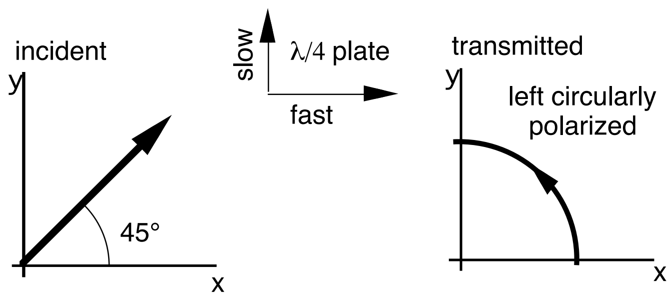

Figure 28.6: Linearly polarized light passing through a quarter-wave plate at 45° becomes circularly polarized.

3. *Circular polarized light falls on a half-wave plate*

Circular polarization results when one polarization component lags the other by 90°. A half wave plate will add an additional 180° lag to one component giving a total 270° or $3\pi/2$ radians. A $3\pi/2$ lag (phase shift = $-3\pi/2$) appears the same as a $\pi/2$ lead (phase shift $=+\pi/2$). Thus right circular polarization is transformed to left circular polarization or left to right.

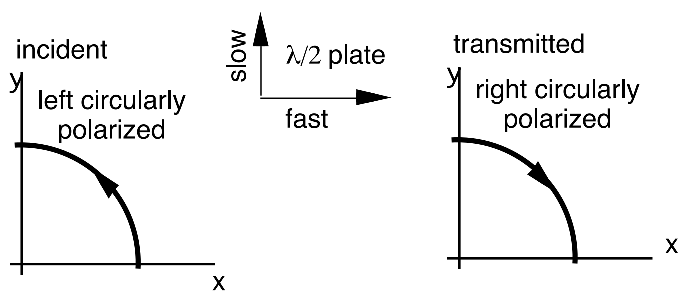

Figure 28.7: A half-wave plate changes the handedness of circularly polarized light.

For the following experiments, mount the light source and filters on the optical bench as shown in Figure 28.8.

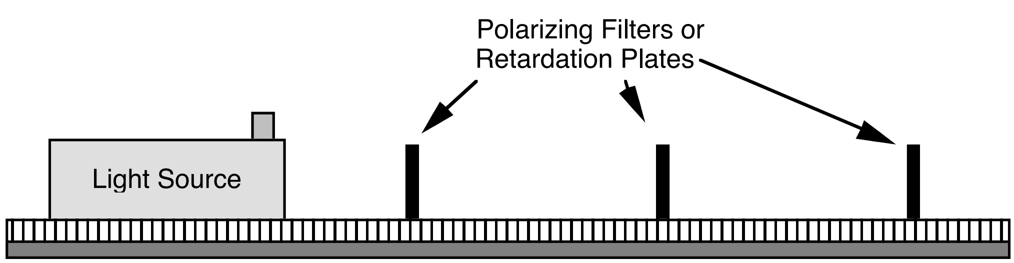

Figure 28.8: Experimental set-up for exploring polarization.

For the next few activities you will need the following equipment:

From the Optics Kit:

- Optical Bench
- Light Source
- All component holders
- Polarization Filters
- Viewing Screen

Also:

- Left circular polarizers: 2
- Right circular polarizers: 2
- Quarter-wave plate
- Half-wave plate

✍ **Activity 28-6: Linear Polarization**
(a) Look at the light source through a linear polarizer while rotating the polarizer through 360°. Try to determine if the light source is polarized. Describe your findings below.

(b) Now produce linearly polarized light by placing a linear polarizer in front of your light source. Look at this light through a second linear polarizer, called the analyser. For the angle θ between the two polarizers' axes given in the table below describe how the intensity of the output light compares to the intensity at θ = 0. (i.e. brighter, a lot brighter, dimmer, a lot dimmer, about the same)

θ (degrees)	Intensity comparison
45	
90	
135	
180	
225	
270	
315	

Malus' Law

Malus' Law States that the intensity of the light exiting the analyser I depends on θ , the angle between the axes of the first polarizer and the analyser (the second polarizer), according to the following formula:

$$I = I_0 \cos^2 \theta \tag{28.18}$$

where I_0 is the intensity of the linearly polarized light entering the analyser. The cosine indicates that only the component of the field parallel to the polarization axis is allowed to pass through. The cosine-squared dependence arises because the intensity is proportional to the electric field strength squared. See Figure 28.9.

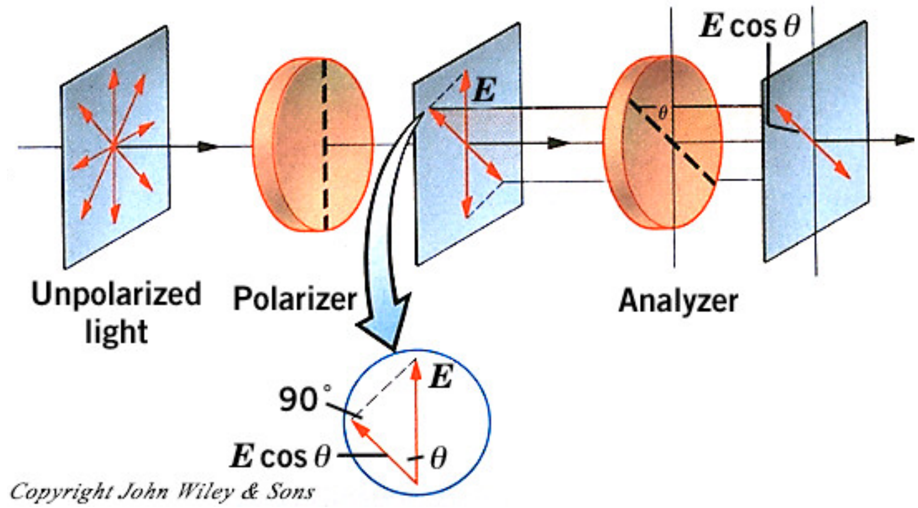

Figure 28.9: Unpolarized light travelling through two linear polarizers.

✍ **Activity 28-7: Malus' Law**
(a) Fill in the table below with the calculated value of the ratio of the outgoing intensity to the incident intensity: I/I_0 for the given angles.

θ (degrees)	I/i_0
0	
45	
90	
135	
180	
225	
270	
315	

(b) Compare the numerical values for the intensity ratios in the table above with the qualitative intensity comparisons you did in the previous activity. Do the two agree with each other for all of the angles?

✍ **Activity 28-8: Linear Polarization Puzzler**

(a) Orient the first polarizer and the analyser so that a minimum amount of intensity exits the analyser. (This should mean that the polarization axes are oriented at a 90 degree angle with respect to each other.) Take a third linear polarizer and place it in between the first polarizer and the analyser. Start this middle polarizer oriented with its axis aligned parallel to the first polarizer. Now rotate it through 360 degrees and record the relative intensity of the light emitted by the analyser (as compared to the intensity seen at 0 degrees) in the table below.

θ (degrees)	Intensity comparison
45	
90	
135	
180	
225	
270	
315	

(b) Can you think of an explanation for what you see. Why does adding a third linear polarizer between the other two allow more light to pass when very little light is emitted when there are only two? (**Hint:** Think about Malus' Law and what happens to the angles of orientation when you add the third polarizer. Look at Figure 28.9 again.)

(c) If we call the intensity leaving the first polarizer I_0, the intensity leaving the second I and the intensity leaving the third I', show that:

$$I' = I_0 \cos^2\theta \cos^2(\theta' - \theta)$$

where θ is the angle between the axes of the first and second polarizers and θ' is the angle between the axes of the first and third polarizers.

(d) If θ' = 90 degrees, calculate the ratio of I' to I_0 for θ = 45, 90, 135 and 180 degrees.

(e) Are your results in (d) consistent with your observations in (a)? Explain.

✍ **Activity 28-9: Circular Polarization**

(a) Look at the light source through a left and a right circular polarizer. Record what you see. What happens when the polarizer is rotated through 360°? Is the light from the source circularly polarized?

(b) Produce left circularly polarized light by allowing light from the light source to exit the polarizer through the face marked "left circular polarizer." Look at this light through a left and then a right circular polarizer run backwards as a circular analyser, i.e., through the faces marked "left circular analyser" and "right circular analyser." What do you see? What happens when you rotate the analysers? Explain your results based on what you just learned about linear polarizers.

✍ **Activity 28-10: Polarizers and Quarter and Half Wave Plates**

(a) Mount two linear polarizers some distance apart with their polarization axes oriented at 90° with respect to one another. Observe that no light is transmitted through the pair. Mount the half-wave plate between the two polarizers. Observe and record what happens when the half-wave plate is rotated through 360 degrees. Is this what you expect based on the theory given in this unit? Explain.

(b) Mount a linear polarizer with its polarization axis at 45°. Next mount a quarter-wave plate with one of its axes vertical. For your convenience the quarter-wave plate has been cut so that its axes are parallel to its edges. Look at the light transmitted through the combination with a right and then by a left circular polarizer run backwards as a circular analyser. What can you conclude about the light that exits the quarter-wave plate? Explain.

(c) Rotate the quarter-wave plate by 90° and again look at the light transmitted through the combination through the two circular analysers. How does this compare to what you saw in (b)?

(d) Produce left (right) circularly polarized light. Next mount a half-wave plate. Look at the light transmitted through a right and a left circular polarizer run backwards as a circular analyser. What happens? Explain what the half-wave plate is doing to the circularly polarized light.

