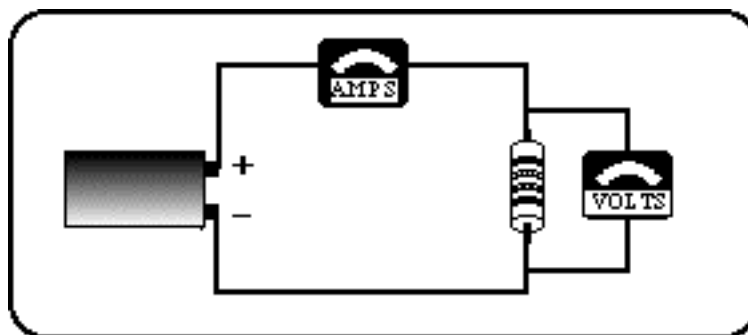


Name _____ Section _____ Date _____

Unit 23: DIRECT CURRENT CIRCUITS*

Estimated classroom time: Two 100 minute sessions



I have a strong resistance to understanding the relationship between voltage and current.

Anonymous Introductory Physics Student

OBJECTIVES

1. To learn to apply the concept of potential difference (voltage) to explain the action of a battery in circuits.
2. To understand the distribution of potential difference (voltage) in all parts of a series circuit.
3. To understand the distribution of potential difference (voltage) in all parts of a parallel circuit.
4. To understand and apply the relationship between potential difference and current for a resistor with negligible temperature dependence (Ohm's law).
5. To find a mathematical description of the flow of electric current through different elements in direct current circuits (Kirchhoff's laws).
6. To gain experience with basic electronic equipment and the process of constructing useful circuits while reviewing the application of Kirchhoff's laws.

* Portions of this unit are based on research by Lillian C. McDermott & Peter S. Shaffer published in AJP 60, 994-1012 (1992).

OVERVIEW

5 min

In the last two units you explored currents at different points in series and parallel circuits. You saw that in a series circuit, *the current is the same through all elements*. You also saw that in a parallel circuit, *the current divides among the branches so that the total current through the battery equals the sum of the currents in each branch*.

You have also observed that making a change in one branch of a parallel circuit does not affect the current flowing in the other branch (or branches), while *changing one part of a series circuit changes the current in all parts of the circuit*.

In carrying out these observations of series and parallel circuits, you have seen that certain connections of light bulbs result in a larger resistance to current flow and therefore a smaller current, while others result in a smaller resistance and larger current.

In this unit, you will first examine the role of the battery in causing a current to flow in a circuit. You will then compare the potential differences (voltages) across different parts of series and parallel circuits.

Based on your previous observations, you probably associate a larger resistance connected to a battery with a smaller current, and a smaller resistance with a larger current. In the last part of the first session you will explore quantitatively the relationship between the current through a resistor and the potential difference (voltage) across the resistor; this relationship is known as Ohm's law.

In addition, you will measure the effective resistance of carbon resistors when they are wired in series and in parallel. Finally you will formulate the rules for the calculation of the electric current in different parts of complex electric circuits consisting of many resistors and/or batteries wired in series and parallel. These rules are known as Kirchhoff's laws. To test your understanding of Kirchhoff's laws, you will learn to use a protoboard to wire complex electric circuits. By measuring the current in different parts of your circuit you and verify that your theoretical application actually describes "reality."

SESSION ONE: BATTERIES AND VOLTAGES IN SERIES CIRCUITS

Voltage and Potential Difference

So far you have developed a current model and the concept of resistance to explain the relative brightness of bulbs in simple circuits. Your model says that when a battery is connected to a complete circuit, a current flows. For a given battery, the magnitude of the current depends on the total resistance of the circuit. In the following activities you will explore batteries and voltages (potential differences) in circuits.

In order to do this you will need the following items:

- 2 fresh 1.5 volt alkaline batteries
- 6 wires with alligator clip leads
- 4 #14 bulbs in sockets
- A knife switch
- An MBL Current/Voltage Measuring System

You have already seen what happens to the brightness of the bulb in circuit 23-1 (a) if you add a second bulb in series as in circuit 23-1 (b). The two bulbs are less bright than the original bulb because the resistance of the circuit is larger, resulting in less current flowing through the bulbs.

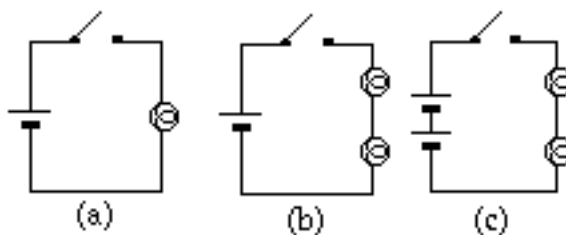


Figure 23-1: Series circuits with (a) one battery and one bulb, (b) one battery and two bulbs and (c) two batteries and two bulbs.

Notes:

Activity 23-1: Adding a Second Battery and Bulb

(a) What do you predict would happen to the brightness of the bulbs in Figure 23-1 if you connected a second battery in series with the first at the same time you added the second bulb (as in Figure 23-1 (c))? How would the brightness of the bulb in circuit 23-1 (a) compare to each bulb in circuit 23-1 (c)?

(b) Connect the circuit in Figure 23-1 (a). Then connect the circuit in 23-1 (c). (*Be sure that the batteries are connected in series – the positive terminal of one must be connected to the negative terminal of the other.*) Compare the brightness of each of the bulbs in 23-1(c) to the brightness of the single bulb in 23-1(a).

(c) What do you conclude about the current in the two bulb, two battery circuit as compared to the single bulb, single battery circuit?

(d) What happens to the resistance of a circuit as more bulbs are added in series? What must you do to keep the current from decreasing?

Let's compare the brightness of the bulb in the circuit below (Figure 23-2) to the brightness of the bulb in Figure 23-1 (a).

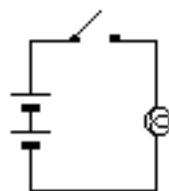


Figure 23-2: Series circuit with two batteries and one bulb.

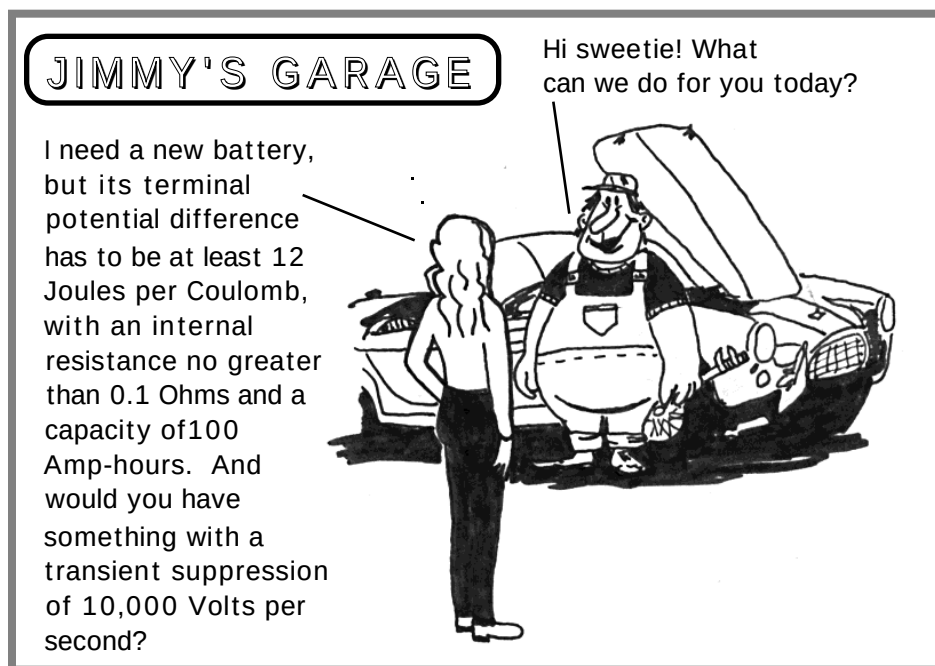
 Activity 23-2: Adding a Second Battery

(a) What do you predict will happen to the brightness of the bulb if a second battery is added? Explain the reasons for your prediction.

(b) Connect the circuit in Figure 23-2. *Only close the switch for a moment to observe the brightness of the bulb--otherwise, you will burn out the bulb.* Compare the brightness of the bulb to the single bulb circuit with only one battery (23-1(a)).

(c) How does increasing the number of batteries connected in series affect the current in a series circuit?

(d) What characteristic of the battery determines the bulb brightnesses?



Potential Difference and Voltage

When a battery is fresh, the voltage marked on it is actually a measure of the electrical *potential difference* between its terminals. *Voltage* is an informal term for potential difference. If you want to talk to physicists you should refer to potential difference. Communicating with a sales person at the local Radio Shack store is another story. There you would probably refer to voltage. We will use the two terms interchangeably.

Let's explore potential differences in series and parallel circuits, and see if you can develop rules to describe its behavior as we did earlier for currents.

How do the potential differences of batteries add when the batteries are connected in series or parallel? Figure 23-3 shows a single battery, two batteries identical to it connected in series, and then two batteries identical to it connected in parallel.

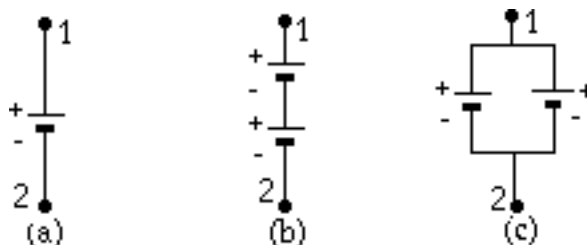


Figure 23-3: Identical batteries: (a) single, (b) two connected in series and (c) two connected in parallel.

You can measure potential differences with voltage probes connected as shown in Figure 23-4.

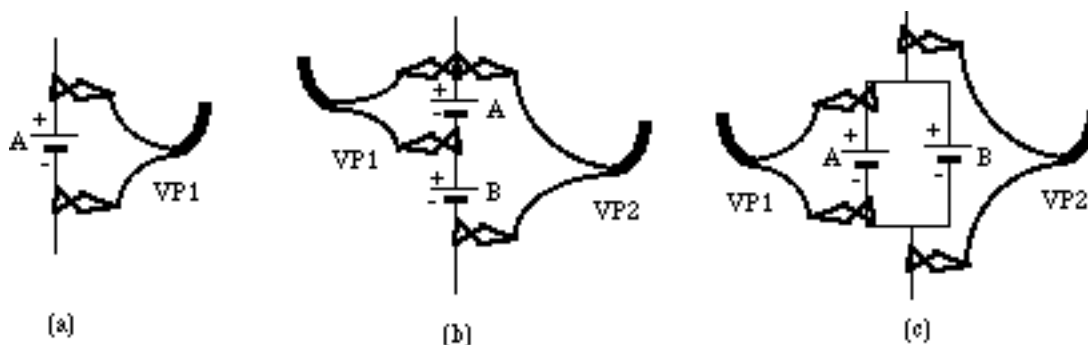


Figure 23-4: Voltage probes connected to measure the potential difference across (a) a single battery, (b) a single battery and two batteries connected in series, and (c) a single battery and two batteries connected in parallel.

 Activity 23-3: Batteries in Series and Parallel

(a) If the potential difference between points 1 and 2 in Figure 23-3(a) is known to be V , predict the potential difference between points 1 and 2 in 23-3(b) (series connection) and in 23-3(c) (parallel connection). Explain your reasoning.

(b) Connect Voltage Probe 1 across a single battery (as in Figure 23-4(a)), and Voltage Probe 2 across the other identical battery. Open the experiment Unit23 Two Voltages. Record the voltage measured for each battery below.

Voltage of battery A:

Voltage of battery B:

(c) How do your measured values agree with those marked on the batteries?

(d) Now connect the batteries in series (as in Figure 23-4(b)) and connect Probe 1 to measure the potential difference across battery A and Probe 2 to measure the potential difference across the series combination of the two batteries. Record your measured values below.

Voltage of battery A:

Voltage of A and B in series:

(e) How do your measured values agree with your predictions?

(f) Now connect the batteries in parallel as in Figure 23-4(c), and connect Probe 1 to measure the potential difference across battery A and Probe 2 to measure the potential difference across the parallel combination of the two batteries. Record your measured values on the top of the next page.

Voltage of battery A:

Voltage of A and B in parallel:

(g) How do your measured values agree with your predictions?

Potential Differences in Series Circuits

You can now explore the potential difference across different parts of a simple series circuit. Let's begin with the circuit with two bulbs in series with a battery which you looked at before in Unit 22. It is shown in Figure 23-5(a).

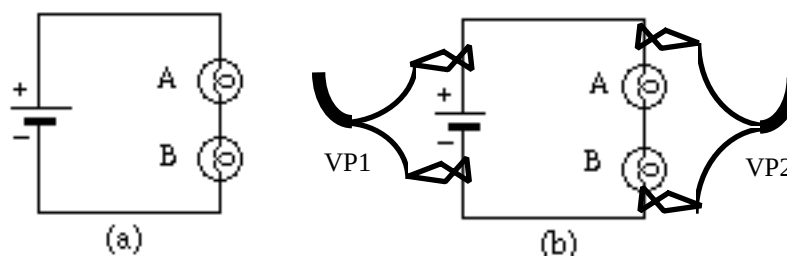


Figure 23-5: (a) A series circuit with one battery and two bulbs, and (b) the same circuit with Voltage Probe 1 connected to measure the potential difference across the battery and Probe 2 connected to measure the potential difference across the series combination of bulbs A and B.

Activity 23-4: Voltages in Series Circuits

(a) If bulbs A and B are identical, how do you predict that the potential difference across bulb A will compare to the potential difference (voltage) across the battery? How about the potential difference across bulb B? How will the potential difference across the series combination of bulbs A and B compare to potential difference across the battery?

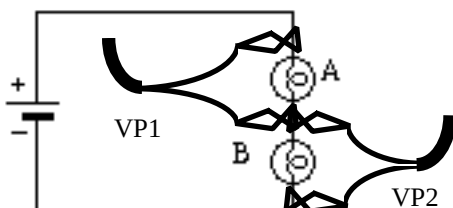
(b) Test your predictions by connecting the circuit and voltage probes as shown in Figure 23-5(b). Record your readings below.

Potential difference across the battery:

Potential difference across bulbs A and B in series:

(c) How do the two potential differences compare? Did your observations agree with your predictions?

(d) Connect the voltage probes as in Figure 23-6 to measure the potential differences across bulbs A and B individually. Record your measurements below.



Potential difference across bulb A:

Potential difference across bulb B:

Figure 23-6: Connection of voltage probes to measure the potential difference across bulb A and across bulb B.

(e) Did your measurements agree with your predictions?

(f) Formulate a rule for how potential differences across individual bulbs in a series connection combine to give the total potential difference across the series combination of the bulbs. How is this related to the potential difference of the battery?

Parallel Circuits Revisited

You can also explore the potential differences across different parts of a simple parallel circuit. Let's begin with the circuit with two bulbs in parallel with a battery which you looked at before in Unit 22. This circuit is shown in Figure 23-7 (a) below.

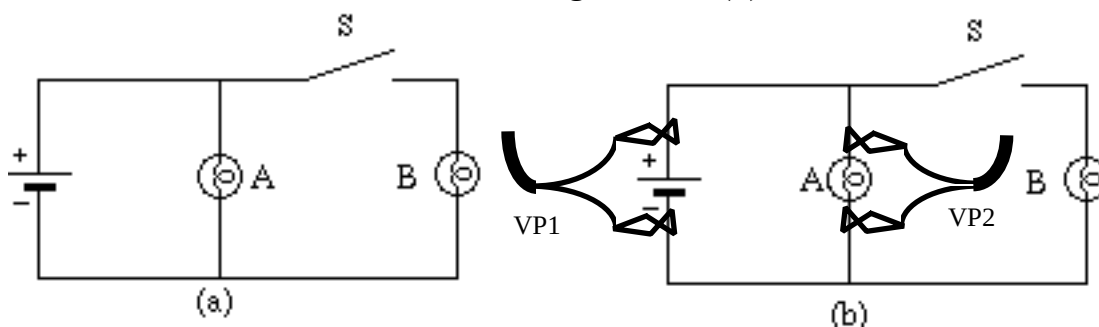


Figure 23-7: (a) Parallel circuit with two bulbs and a battery, and (b) same circuit with Voltage Probe 1 connected to measure the potential difference across the battery and Probe 2 connected to measure the potential difference across bulb A.

Activity 23-5: Voltages in a Parallel Circuit

(a) What do you predict will happen to the potential difference across the battery when you close the switch in Figure 23-7 (a)? Will it increase, decrease or remain essentially the same?

(b) How will the potential difference across bulb A compare to the voltage of the battery? How will the potential difference across bulb B compare to the voltage of the battery?

(c) Connect the circuit and voltage probes as in Figure 23-7 (b). Collect data while opening and closing the switch as you've done before. *Print your graph and affix it below.* Read the voltages using Analyze Data A.

Switch open:

Voltage across battery:

Voltage across bulb A:

Switch closed:

Voltage across battery:

Voltage across bulb A:

(d) Did your measurements agree with your predictions? Did closing and opening the switch significantly affect the voltage across the battery? The voltage across bulb A?

(e) Now connect the voltage probes as shown below in Figure 23-8, and graph and measure the voltages across bulbs A and B. Again close and open the switch while graphing. Affix the graph on the next page. Record your measurements using Analyze Data A.

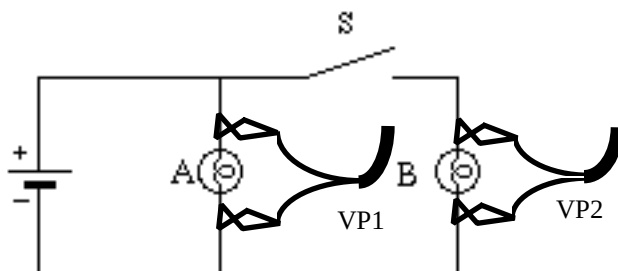


Figure 23-8: Voltage probes connected to measure the potential differences across bulbs A and B.

Switch open:

Voltage across bulb A:

Voltage across bulb B:

Switch closed:

Voltage across bulb A:

Voltage across bulb B:

(f) Did your measurements agree with your predictions? Did closing and opening the switch significantly affect the voltage across bulb A?

(g) Did closing and opening the switch significantly affect the voltage across bulb B? Under what circumstances is there a potential difference across a bulb?

(h) Based on your observations, formulate a rule for the potential differences across the branches of a parallel circuit. How are these related to the voltage across the battery?

(i) Based on your observations in this and the last two activities, is the potential difference across a battery significantly affected by the circuit connected to it?

(j) Is a battery a constant current source (delivering a fixed amount of current regardless of the circuit connected to it) or a constant voltage source (applying a fixed potential difference regardless of the circuit connected to it), or neither? Explain based on your observations in this and previous units.

Notes:

Return to a complex circuit.

In Unit 22 you explored what happened to the brightness of the bulbs in the circuit shown below when the switch was closed, i.e., when bulb C was added in parallel with bulb B.

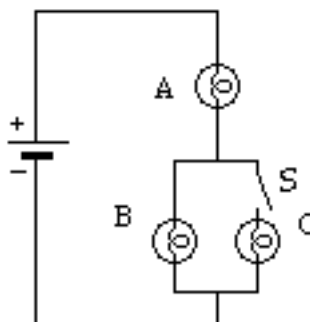


Figure 23-9: Circuit which has: (i) bulbs A and B in series when the switch is open; (ii) bulb A in series with the parallel combination of bulbs B and C when the switch is closed.

You were previously asked to rank the brightness of bulbs A, B and C after the switch was closed. The question now is *what happens to the brightness of bulb B when the switch is closed*. Does it increase, decrease, or remain the same?

 Activity 23-6: Applying Your Current and Voltage Models

(a) Based on the current and voltage models you have developed, *carefully* predict what will happen to the current through bulb B (and therefore its brightness) when bulb C is added in parallel to it – will it increase, decrease or remain the same? Explain the reasons for your answer.

(b) Connect the circuit in Figure 23-9 and make observations. Describe what happens to the brightness of bulb B when the switch is closed.

(c) Did your observations agree with your prediction? If not, use the current and voltage models to explain your observations.

Relationship Between Current and Potential Difference

You have already seen on several occasions that there is only a potential difference across a bulb when there is a current flowing through the bulb. The next question is how does the potential difference depend on the current? In order to explore this, you will need the following:

- A 1.5 V alkaline battery
- 6 wires with alligator clip leads
- A #14 bulb in a socket
- A 2 m length of 26 gauge nichrome wire
- A 10 Ω resistor
- An MBL Current/Voltage Measuring System

Examine the circuit shown in Figure 23-10. The arrow on the right represents an alligator clip which may be connected at various points along the piece of nichrome wire. Nichrome wire is special wire that has a large resistance compared to the connecting wires. Thus by connecting the alligator clip in different positions along the nichrome wire, you can vary the resistance by including more or less of the nichrome wire in the circuit.

Notes:

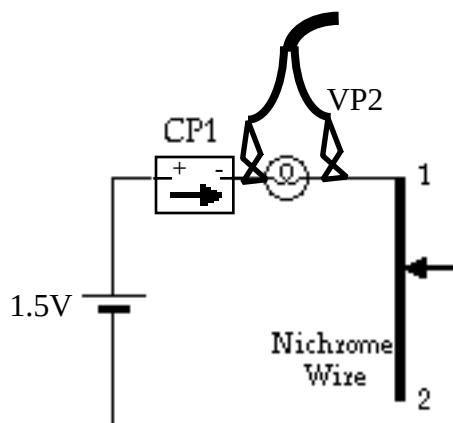


Figure 23-10: Circuit with a variable resistance to explore the relationship between current and potential difference for a light bulb.

Activity 23-7 Current and Potential Difference for a Light Bulb

a) If the battery has a voltage of 1.5 V, what will be the potential difference across the bulb when the alligator clip is connected at position 1, i.e., when the length of nichrome wire hooked into the circuit is zero? Explain.

b) What do you predict will happen to the brightness of the bulb as you move the alligator clip toward position 2 (as you increase the length of nichrome wire in series with the bulb)? Explain.

c) What do you predict will happen to the current through the bulb as you move the alligator clip from position 1 to position 2?

d) What do you predict will happen to the potential difference across the bulb as you move the alligator clip from position 1 to position 2?

e) Connect the circuit. *To save the battery, only make the connections to the battery when you are ready to make observations.* Note that the current probe is connected to measure the current through the bulb, and the voltage probe is connected to measure the potential difference across the bulb. Open the experiment Unit23 Voltage and Current. Beginning with the alligator clip connected to the nichrome wire in position 1, observe the brightness of the bulb and record the current and potential difference. Then repeat for several other positions of the alligator clip between position 1 and position 2. *Be sure that the nichrome wire does not bend back and make contact with itself.* Record your data below.

	<u>current through bulb</u>	<u>potential difference</u>
--	-----------------------------	-----------------------------

position 1		
------------	--	--

second position		
-----------------	--	--

third position		
----------------	--	--

position 2		
------------	--	--

f) What happened to the brightness of the bulb as more and more of the nichrome wire was connected in series with the bulb? Did this agree with your prediction?

g) What happened to the current in the circuit? Did this agree with your prediction?

h) How did the potential difference across the bulb change as the current through the bulb changed? Did this agree with your prediction?

i) How is the brightness of the bulb related to the potential difference across the bulb? To the current through the bulb?

A More Quantitative Look

In the last activity you explored *semi-quantitatively* the relationship between current through a light bulb and potential difference across the bulb. The actual *quantitative* relationship for a light bulb is rather complicated because the resistance of the bulb changes as the current flowing through it changes the temperature of the filament. Instead of a light bulb, you will explore the simpler relationship between current and potential difference for a device called a *resistor*.

In the circuit you used in Activity 23-7, replace the light bulb by the $10\ \Omega$ resistor. (Note that the zig-zag line in Figure 23-11 is the symbol for a resistor.)

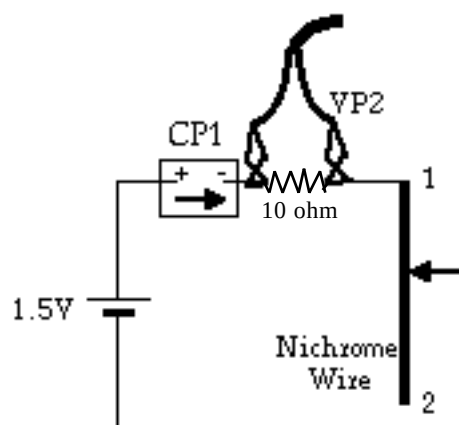


Figure 23-11: Variable resistance circuit to explore the quantitative relationship between the current and potential difference for a resistor.

Activity 23-8 Relationship Between Current and Potential Difference for a Resistor

a) Prepare to graph current vs. time with Probe 1 and voltage vs. time with Probe 2. Set the data collection rate to 5 per second. (Use Data Rate . . . on the Collect menu.) Set both time axes to 30 seconds. Start graphing with the alligator clip in position 1. Slowly slide the alligator clip to position 2 while graphing, *being sure that the clip is always in contact with the nichrome wire, and that the wire does not bend back and make contact with itself.*

What happened to the current through the resistor and the potential difference across the resistor as more of the nichrome wire was placed in series with the resistor? Is this what you expected from your observations with the bulb in Activity 23-7?

b) You can display a graph of potential difference vs. current by changing the horizontal axis on the bottom graph to Current (amps). (Point the mouse to the horizontal axis label, click, point to Current (amps) and release.) What appears to be the relationship between the potential difference across a resistor and the current through the resistor?

c) You can explore the relationship more quantitatively using Fit . . . on the Analyze menu. Find the equation which represents the relationship between the potential difference across the resistor and the current through it. Print the graph and affix it below.

d) In words, what is the mathematical relationship between potential difference and current for a resistor? Explain based on your graph.

e) Using the symbol I for current and V for voltage, write a mathematical relationship between V and I which represents the relationship you have observed in the graph. You may include a constant of proportionality in your relationship.

Ohm's Law and Resistance

The relationship between potential difference and current which you have observed for a resistor is known as Ohm's law. To put this law in its normal form, we must now define the quantity known as resistance. Resistance is defined by:

$$R \equiv \frac{\Delta V}{I}$$

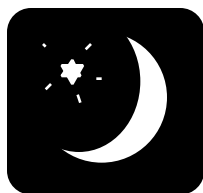
If potential difference is measured in volts and current is measured in amperes, then the unit of resistance is the ohm, which is usually represented by the Greek letter Ω , "omega."

Activity 23-9: Statement of Ohm's Law

a) State the mathematical relationship found in Activity 23-8 between potential difference and current for a resistor in terms of V, I, and R.

b) Based on your graph, what can you say about the value of R for a resistor – is it constant or does it change as the current through the resistor changes? Explain.

c) From the slope of your graph, what is the experimentally determined value of the resistance of your resistor in ohms? How does this agree with the value written on the resistor?



d) Complete the famous pre-exam rhyme used by countless introductory physics students throughout the English speaking world:

Twinkle, twinkle little star, ΔV equals _____ times _____

Note: Some circuit elements do not obey Ohm's law. The definition for resistance is still the same, but, as with the light bulb, the resistance changes due to temperature changes resulting from the flow of current. Circuit elements which follow Ohm's law--like carbon resistors--are said to be <i>ohmic</i>, while circuit elements which do not--like a light bulb--are <i>nonohmic</i>.
--

SESSION TWO: KIRCHHOFF'S LAWS AND MULTI-LOOP CIRCUITS,

25 min

Using a Multimeter

A digital multimeter is a device that can be used to measure either current, voltage, or resistance depending on how it is set up. The following activity will introduce you to the digital multimeter and give you some practice in using it. You will need:

- A digital multimeter
- A 1.5 V. alkaline battery w/ holder
- A switch
- 4 alligator clip wires
- 1 $10\ \Omega$ resistor

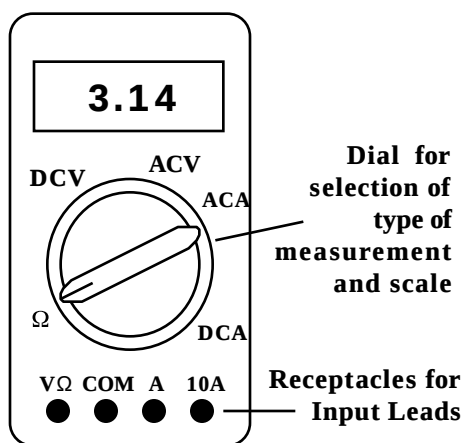


Figure 23-12: Diagram of a digital multimeter that can be used to measure resistances, currents, and voltages

By putting the input leads (red for positive, black for negative) into the proper receptacles and setting the dial correctly, you can measure resistances (Ω) as well as direct currents (DCA) and direct current voltages (DCV).

Figure 23-13 shows a simple circuit which you can use to practice taking readings with the multimeter.

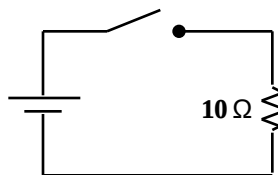


Figure 23-13: Simple circuit for testing multimeter

 Activity 23-10: Using a Multimeter

(a) Figure out what settings you need to use to measure the actual resistance of the " 10Ω " resistor. Record your measured value below.

(b) Figure out what settings you need to measure the potential difference across a resistor in series with a battery. Connect the circuit in Figure 23-13. Close the switch and measure the voltage across the resistor. Record the voltage below.

(c) Reconnect the circuit, this time using the multimeter as an ammeter. Close the switch again and measure the current *through* the resistor. Record the current below.

Notes:

20 min

Carbon Resistors in Parallel and Series

Carbon resistors are the most standard sources of resistance used in electrical circuits for two reasons. A light bulb has a resistance which increases with temperature and current and thus doesn't make a good circuit element when quantitative attributes are important. The resistance of carbon resistors doesn't vary with the amount of current passing through them. A second important characteristic of carbon resistors is that they are inexpensive to manufacture.

A typical carbon resistor contains a form of carbon, known as graphite, suspended in a hard glue binder. It usually is surrounded by a plastic case with a color code painted on it. It is instructive to look at samples of carbon resistors that have been cut down the middle as shown in the diagram below.

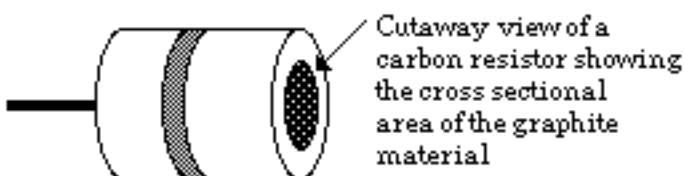


Figure 23-14: A cutaway view of a carbon resistor

Several identical resistors can be wired in *series* to increase their effective length and in *parallel* to increase their effective cross sectional area as shown in the next diagram.

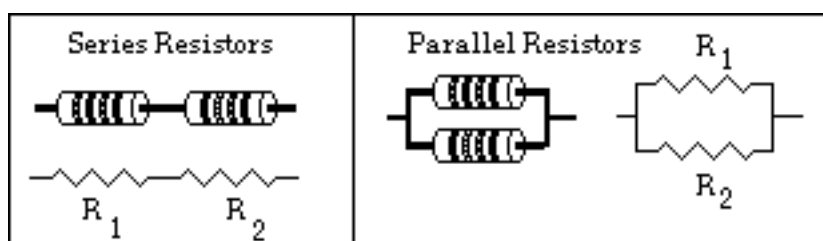


Figure 23-15: Carbon resistors wired in series and in parallel

Not all resistors are made of carbon. Since the resistance of a wire with a uniform cross-sectional area is directly proportional to length, it is possible to control the R -value of a wire fairly precisely. Thus, precision resistors with good temperature stability are often made of windings of fine wire.

In order to test your predictions and do some further exploration of equivalent resistances of different combinations of resistors you will need the following

- 6 identical carbon resistors
(for example 75Ω or 100Ω)
- 6 other identical carbon resistors
(for example 120Ω or 220Ω)
- A digital multimeter
- Connecting wires with alligator clips (optional)

Using the items listed above, devise a way to measure the equivalent resistance when three or more resistors are wired in series. Explain what you did and summarize the results of your measurements in the space below.

 Activity 23-11: Resistances for Series Wiring

(a) If you have three different carbon resistors, what do you think the equivalent resistance to the flow of electrical current will be if the resistors are wired in *series*? Explain the reasons for your prediction based on your previous observations with batteries and bulbs.

(b) Compare the calculated and measured values of equivalent resistance of the series network as follows:

Write down the *measured* values of each of the three resistors:

R_1 _____ Ω R_2 _____ Ω R_3 _____ Ω

Describe the method you are using to predict the equivalent resistance and calculate the predicted R value:

R_{pred} _____ Ω

(c) Draw a diagram for the resistance network for the three *different* resistors *wired in series*. Mark the measured values of the three resistances on your diagram.

(d) Measure the actual resistance of the series resistor network and record the value:

$$R_{\text{meas}} = \text{_____} \Omega$$

How does this value compare with the one you calculated?

(e) On the basis of your experimental results, devise a *general* mathematical equation that describes the equivalent resistance when n resistors are wired in series. Use the notation R_{eq} to represent the equivalent resistance and $R_1, R_2, R_3, \dots, R_n$ to represent the values of the individual resistors.

Devise a way to measure the equivalent resistance when two or more resistors are wired in *parallel*. Explain what you did and summarize the results of your measurements in the space below. Draw a symbolic diagram for each of the wiring configurations you use.

 Activity 23-12: Resistances for Parallel Wiring

(a) If you have two *identical* carbon resistors what do you think the resistance to the flow of electrical current will be if the resistors are wired in *parallel*? Explain the reasons for your prediction.

(b) Pick out two carbon resistors with an identical color code and draw a diagram for these two resistors *wired in parallel*. Label the diagram with the measured values $R_{1\text{meas}}$ and $R_{2\text{meas}}$. Predict the equivalent resistance of the parallel circuit and record your prediction below. Measure the value of the equivalent resistance of the network. Explain your reasoning and show your calculations in the space below.

Predicted value: R_{eq} _____ Ω

Measured value: R_{eq} _____ Ω

(c) Pick out three *different* resistors and draw a diagram for these three resistors *wired in parallel*. Label the diagram with the measured values $R_{1\text{meas}}$, $R_{2\text{meas}}$ and $R_{3\text{meas}}$. Measure the value of the equivalent resistance of the network and record it below.

Measured Value of the equivalent resistance of the network:

R_{eq} _____ Ω

(d) Use the notation R_{eq} to represent the equivalent resistance and $R_1, R_2, R_3, \dots$, etc. to represent the values of the individual resistors. Show that, within the limits of experimental uncertainty, the results of the measurements you made with parallel resistors are the same as those calculated using the equation:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \text{ etc}$$

1. For the two identical resistors wired in parallel:

Calculated Value: $R_{eq} =$

Measured Value: $R_{eq} =$

2. For the three resistors wired in parallel:

Calculated Value: $R_{eq} =$

Measured Value: $R_{eq} =$

(e) Show mathematically that if

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ then } R_{eq} = \frac{R_1 R_2}{(R_1 + R_2)}$$

25 min

Equivalent Resistances for Networks

Now that you know the basic equations to calculate equivalent resistance for series and parallel resistances, you can tackle the question of how to find the equivalent resistances for complex networks of resistors. The trick is to be able to calculate the equivalent resistance of each segment of the complex network and use that in calculations of the next segment. For example, in the network shown below there are two resistance values R_1 and R_2 . A series of simplifications is shown in the diagram below.

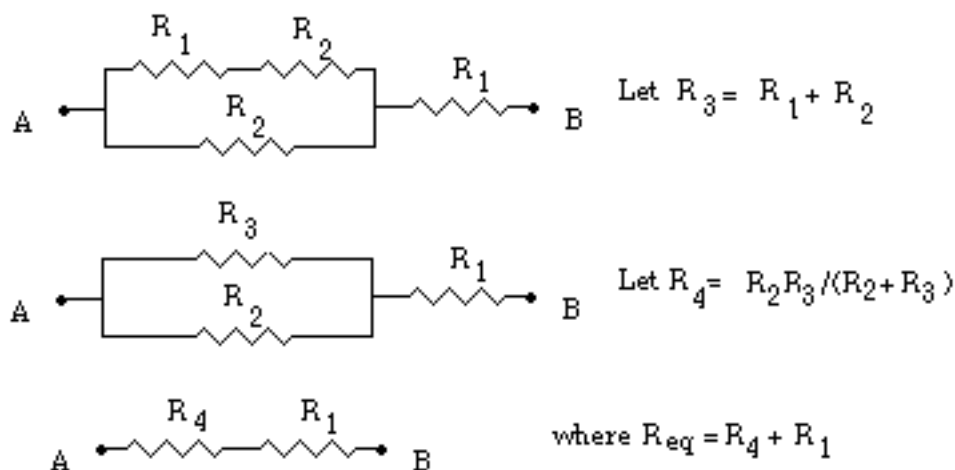


Figure 23-16: A sample resistor network

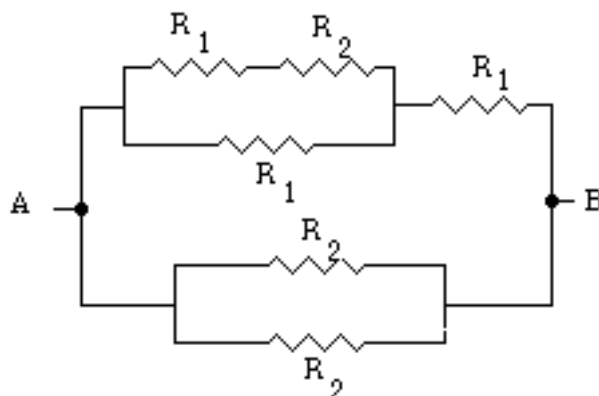
In order to complete the equivalent resistance activities you will need the following apparatus:

- 3 identical carbon resistors
- 3 more identical carbon resistors w/ approx.
twice the resistance of the first three resistors
- 1 digital multimeter

Notes:

Activity 23-13: The Equivalent Resistance for a Network

(a) Consider the sets of identical resistors you just used to explore parallel and series resistances. Use the color coded value for your lowest identical resistor for R_1 and the color coded value for your highest identical resistor for R_2 to calculate the equivalent resistance between points A and B for the network shown below. *You must show your calculations on a step-by-step basis.*



(b) Set up the network of resistors and check your calculation by measuring the equivalent resistance directly.

Calculated Value: $R_{eq} =$

Measured Value: $R_{eq} =$

50 min

Theoretical Application of Kirchhoff's Laws

Suppose we wish to calculate the currents in various branches of a circuit that has many components wired together in a complex array. In such circuits, simplification using series and parallel combinations is often impossible. Instead we can state and apply Kirchhoff's laws more formally to aid with the solution of such problems. These rules can be summarized as follows:

1. Junction (or node) Rule (based on charge conservation): The sum of all the currents entering any node or branch point of a circuit (i.e. where two or more wires merge) must equal the sum of all currents leaving the node.

2. Loop Rule (based on energy conservation): Around any closed loop in a circuit, the sum of all emfs and all the potential drops across resistors and other circuit elements must equal zero.

Steps for Applying Rules

1. Assign a current symbol to each branch of the circuit and label the current in each branch I_1 , I_2 , I_3 , etc.; then arbitrarily assign a direction to each current. (The direction chosen for the circuit for each branch doesn't matter. If you chose the "wrong" direction the value of the current will simply turn out to be negative.) Remember that the current flowing out of a battery is always the same as the current flowing into a battery.

2. Apply the loop rule to each of the loops by: (a) letting the potential drop across each resistor be the negative of the product of the resistance and the net current through that resistor (reverse the sign to "plus" if you are traversing a resistor in a direction opposite that of the current); (b) assigning a positive potential difference when the loop traverses from the – to the + terminal of a battery. (If you are going through a battery in the opposite direction assign a *negative* potential difference to the trip across the battery terminals.)

3. Find each of the junctions and apply the junction rule to it. You can place currents leaving the junction on one side of the equation and currents coming into the junction on the other side of the equation.

In order to illustrate the application of the rules, let's consider the circuit in Figure 23-17 below.

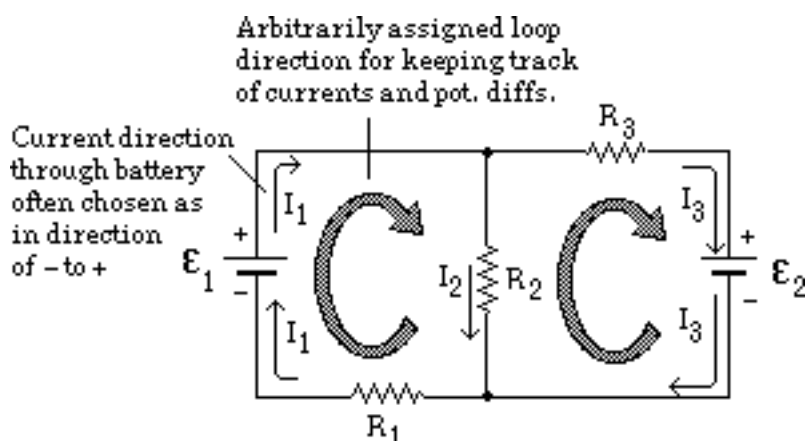


Figure 23-17: A complex circuit in which loops 1 and 2 share the resistor R_2 . The currents I_1 and I_3 flow through R_2 in opposite directions and the net current through R_2 is denoted by I_2 .

In Figure 23-17 the directions for the currents through the branches and for I_2 are assigned arbitrarily. *If we assume that the internal resistances of the batteries are negligible*, then by applying the loop and junction rules we find that

$$\text{Loop 1 Eq.:} \quad \mathcal{E}_1 - I_2 R_2 - I_1 R_1 = 0 \quad [\text{Eq. 23-1}]$$

$$\text{Loop 2 Eq.:} \quad -\mathcal{E}_2 + I_2 R_2 - I_3 R_3 = 0 \quad [\text{Eq. 23-2}]$$

$$\text{Node 1 Eq.:} \quad I_1 = I_2 + I_3 \quad [\text{Eq. 23-3}]$$

(current into junction = current out of junction)

It is not obvious that the loops and their directions can be chosen arbitrarily. Let's explore this assertion theoretically for a simple situation and then more concretely with some specific calculations. In order to do the following activity you'll need a couple of resistors and a multimeter as follows:

- Resistors (rated values of $39 \, \Omega$ and $75 \, \Omega$)
- A digital multimeter
- A 4.5 V battery
- A 1.5 V alkaline battery

Activity 23-14: Applying the Loop Rule Several Times

(a) Use the loop and node rule along with the new arbitrary direction for I_2 to rewrite the three equations relating values of battery emfs, resistance, and current in the circuit shown in Figure 23-18 below.

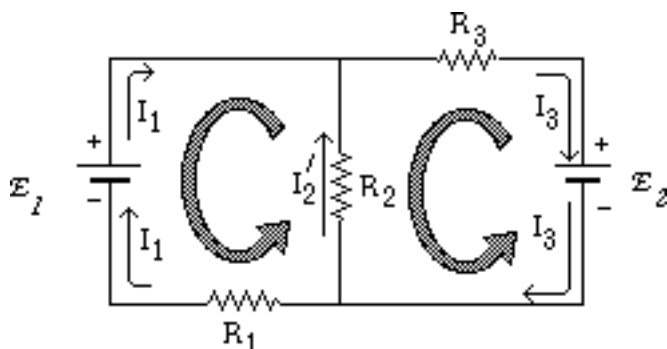


Figure 23-18: A similar complex circuit

(b) Show that if $I_2' = -I_2$ then the three equations you just constructed can be rearranged algebraically so they are *exactly the same* as Equations 23-1, 23-2, and 23-3.

(c) Suppose the values of each component for the circuit shown in Figure 23-17 shown above are rated as

$$\mathcal{E}_1 = 4.5 \text{ v.}, \mathcal{E}_2 = 1.5 \text{ v.}$$

$$\text{Rated Fixed Resistances: } R_1 = 75 \, \Omega, R_3 = 39 \, \Omega$$

$$\text{Variable Resistance: } R_2 = 100 \, \Omega$$

1. Since you are going to test your theoretical results for Kirchhoff's law calculations for this circuit experimentally, you should measure the actual values of the two fixed resistors (rated at 75Ω and 39Ω) and the two battery voltages with a multimeter. List the results below.

Measured value of the battery emf rated at 4.5 v $\mathcal{E}_1 = \underline{\hspace{2cm}}$

Measured value of the battery emf rated at 1.5 v $\mathcal{E}_2 = \underline{\hspace{2cm}}$

Measured value of the resistor rated at 75Ω $R_1 = \underline{\hspace{2cm}}$

Measured value of the resistor rated at 39Ω $R_3 = \underline{\hspace{2cm}}$

2. Carefully rewrite Equations 23-1, 23-2 and 23-3 with the appropriate *measured* (not rated) values for emf and resistances substituted into them. Use 100Ω for the value of R_2 in your calculation. You will be setting a variable resistor to that value soon.

(d) Solve these three equations for the three unknowns I_1 , I_2 , and I_3 in amps using one of the following methods: (1) substitution, (2) determinants, or (3) equation-solving computer or programmable calculator software (see Appendix H for details on computer or calculator solutions). Either show your calculations or name and describe the computer or calculator tool you used in the space below. If you use a computer tool include a printout of the command and the solution in the space below.

(e) Show by substitution that your solutions actually satisfy the equations.

50 min

Verifying Kirchhoff's Laws Experimentally Using a Protoboard

Since circuit elements have become smaller in the past 20 years or so, it is common to design and wire simple circuits on a device called a protoboard. A protoboard has hundreds of little plastic holes in it that can have small diameter wire poked into them. In the protoboard model shown in Figure 23-19 below, these holes are electrically connected in vertical columns of 5 near the middle. The top of the protoboard has two horizontal rows of 40 connected holes. There is a similar arrangement at the bottom.

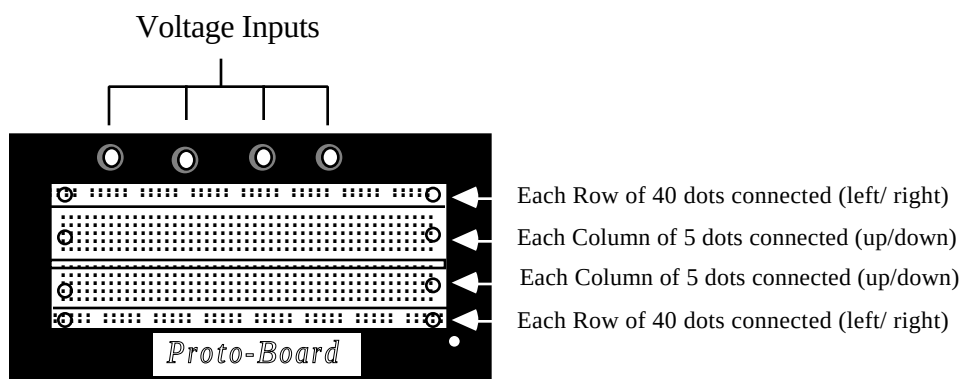


Figure 23-19: A protoboard

Usually, one connects the voltage inputs to the long rows of connected dots toward the outside of the circuit; these rows can then serve as power supplies. As part of the next project with the protoboard you will be using some simple circuit elements to design a tricky circuit with more than one battery and several branches in it! To design this circuit you will be using the following items:

- A pot (200 Ω DIP style)
- 2 resistors (rated at 39 Ω and 75 Ω)
- A 4.5 V battery
- A 1.5 V alkaline battery
- A protoboard (w/ the pot installed on it)
- A digital multimeter
- A small screw driver or paper clip

The word "pot" stands for potentiometer. It is a variable resistor. There is a 200 Ω pot already installed on your protoboard. The pot has three leads. The two outside leads are across the 200 Ω resistor while the center lead taps off part of the 200 Ω . The resistance between an outside lead and the center tap can be adjusted from 0 to 200 Ω with a screwdriver; the resistance between the two outside leads is always 200 Ω . The circuit symbol for the pot is shown below.

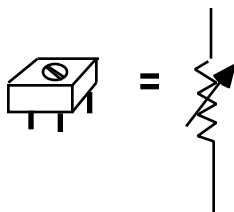


Figure 23-20: A protoboard pot

To wire up the circuit shown in Figure 23-17 on the protoboard, you will need to examine the details of how the protoboard is arranged, as shown in Figure 23-19. Although there are many legitimate ways to connect the leads, a possible configuration is shown in the figure below.

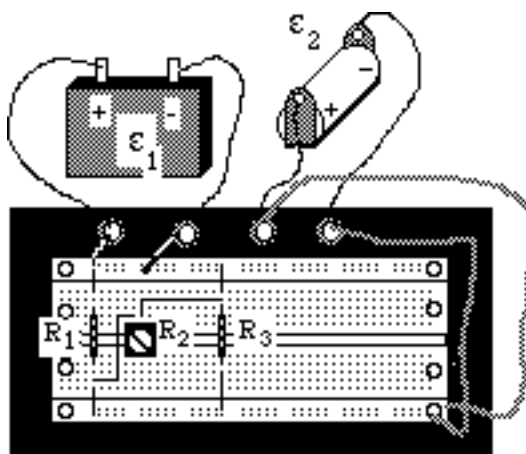


Figure 23-21: A possible wiring arrangement for the Kirchhoff's law circuit to be tested

Activity 23-15: Testing the Loop Rule with a Real Circuit

(a) Use the ohmmeter feature of the digital multimeter to measure the total resistance across a pot that is labeled 200Ω . Then measure the resistance between the center tap on the pot and one of the other taps. What happens to the ohmmeter reading as you use a paper clip or small screwdriver to change the setting on the pot?

(b) Set the pot so that there is 100Ω between the center tap and one of the other taps. Was it difficult?

(b) Wire up the circuit pictured in Figure 23-17 above; use a protoboard and the pot (set at 100Ω) as R_2 . Measure the current in each branch of the circuit and compare the measured and calculated values of the current by computing the % discrepancy in each case.

Note: The most accurate way to measure current with a digital multimeter is to measure the potential different across each of the resistors and use Ohm's law to calculate I from ΔV and R .

	Measured $R (\Omega)$	Measured ΔV (volts)	Measured $I = \Delta V / R$ (amps)	Theoretical I (amps)	% Discrepancy
1					
2	100				
3					

(c) What do you predict will happen to each of the currents as the resistance on the pot is decreased? That is, will the currents I_1 , I_2 , and I_3 increase or decrease? Explain your predictions.

(d) What actually happens to each of the currents as you decrease R_2 ? How good were your predictions?

UNIT 23 HOMEWORK AFTER SESSION ONE

Before Wednesday, April 6th

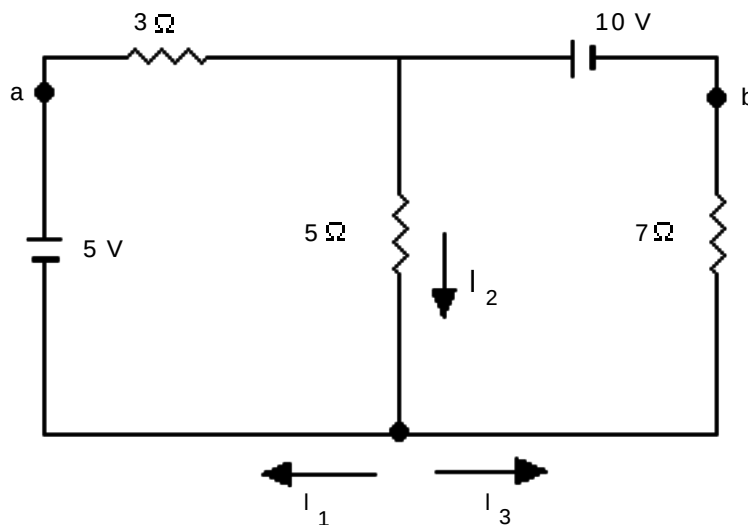
- *Do Homework 23 #1 on Many Element Circuits and Ohm's Law*
- *Do Homework 23 #2 on Qualitative Aspects of Kirchhoff's Laws*
- **Read Chapter 25 sections 25-1 through 25-3 in the textbook.**
- *Work Supplemental Problems SP23-1 through SP23-4* listed below.*

* Hint: The answers to SP23-4 are $I_1 = .141 \text{ A}$, $I_2 = .915 \text{ A}$, $I_3 = .774 \text{ A}$

SP23-1) (a) What is the current in a $5.6\text{-}\Omega$ resistor connected to a battery with an $0.2\text{-}\Omega$ internal resistance if the terminal voltage of the battery is 10 V ? (b) What is the emf of the battery?

SP23-2) What potential difference will be measured across an $18\text{-}\Omega$ load resistor when it is connected across a battery of emf 5 V and internal resistance $0.45 \text{ }\Omega$?

SP23-3) Find the potential difference between points a and b in the circuit in the figure below.



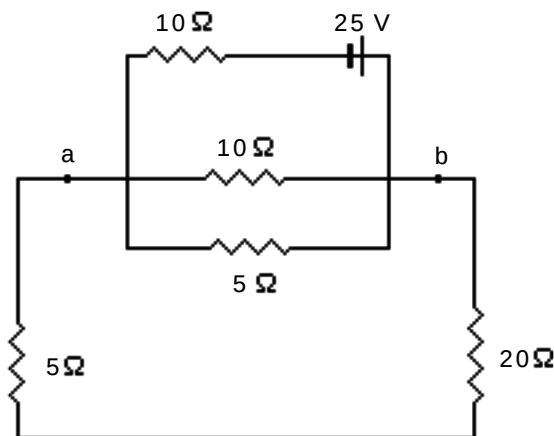
SP23-4) Find the currents I_1 , I_2 , and I_3 in the circuit shown in the figure above.

UNIT 23 HOMEWORK AFTER SESSION TWO

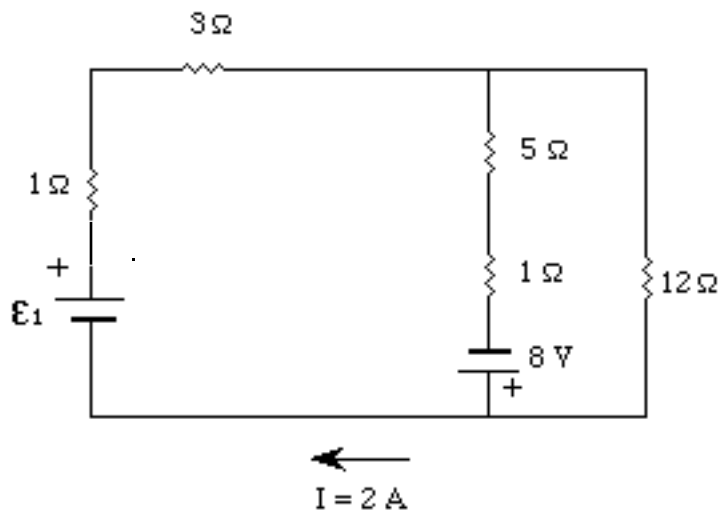
Before Friday, April 8th

- Read Chapter 24 section 24-3 (and reread Chapter 25 section 25-3 if needed)
- Work Supplemental Problems SP23-5 through SP23-9 (*Hint: If we were you, we'd use Maple to solve the equations in SP23-5 through SP23-9.*)
- Complete and hand in entries for the Unit 23 Activity Guide

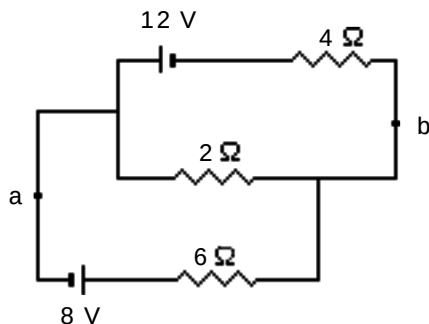
SP23-5) Consider the circuit shown below. Find (a) the current in the $20\text{-}\Omega$ resistor and (b) the potential difference between points a and b.



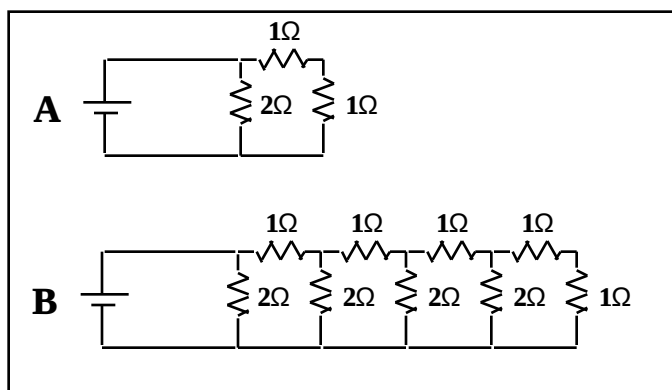
SP23-6) What is the emf of the battery in the circuit shown below?



SP23-7) For the circuit shown in figure below, calculate (a) the current in the $2\text{-}\Omega$ resistor and (b) the potential difference between points a and b.



SP 23-8) In the diagram below, which circuit draws more current? Circuit A? Circuit B? Or neither? Show your calculations and reasoning.



SP 23-9) Find the equivalent resistance of the following network. (All resistances in ohms.) Show your work!

