Deflection of Electrons by Electric and Magnetic Fields

References


Introduction

An electron with charge $e$ in an external electric field $\vec{E}$ experiences a force $\vec{F}$

$$\vec{F} = -e\vec{E}.$$  \hspace{1cm} (1)

Note that the acceleration is in the direction opposite that of the field.

In an external magnetic field, $\vec{B}$, the force on the electron is given by

$$\vec{F} = -e\vec{v} \times \vec{B}$$ \hspace{1cm} (2)

where $\vec{v}$ is the velocity of the electron. Note that the acceleration depends on the magnitudes of the velocity and the magnetic field, the angle between them, and is perpendicular to both.

To analyze the motion of the electrons we will use a cathode ray tube (CRT). The CRT is the basic component of oscilloscopes and television sets. The principal parts of the CRT used in this experiment are shown in Fig. 1.

![Figure 1 Side View of the 5DEP1 cathode ray tube. The dimensions are in cm. The separation of the X deflection plates is unknown.](image-url)
The indirectly heated cathode has a coating of barium and strontium oxides which emit a high-density electron stream. These electrons are then accelerated and focussed by the anode and grid structure shown. After they have passed the second anode, the electrons are not subjected to any more force in the z direction since the screen is at the same potential as the anode and the electrons may be considered to be a "beam" which goes from the anode to the screen. The electrons will all have the same velocity, given by

\[ \frac{1}{2} m v^2 = |e|(V_{anode} - V_{cathode}) = |e|V_{acc} \]  

where \( V_{acc} \) is the "accelerating voltage". Eq. 3 may be obtained from the law of energy conservation, or, alternatively, from integrating Eq. 1.

A voltage applied to the X or Y deflection plates of the CRT produces an electric field between the plates which will exert a force on the electrons (Eq. 1) as they pass through. As a result the beam is deflected by this field. The "deflection sensitivity" is defined as the ratio of the deflection of the electron beam at the screen of the CRT to the deflection voltage, \( V_{defl} \).

Television sets use magnetic deflection as it has some practical advantages over electric deflection. A pair of coils is mounted on the side of the CRT. A current passing through the coils produces a magnetic field which exerts a force on the electrons (Eq. 2). For this situation the deflection sensitivity is defined as the ratio of the deflection of the electron beam at the screen of the CRT to the current in the deflection coils, \( I_{defl} \).

**Prelab Questions**

1. Derive a formula for the deflection sensitivity for deflection by an electric field. Using values of the dimensions given in Fig. 1 compute the deflection sensitivity for this apparatus. The flare on the plates makes an exact calculation of the deflection sensitivity difficult. A simple approximation is to assume that the electric field is equal to the voltage divided by the plate separation at all points between the plates. The average electric field over the whole 1.8 cm is then given by \( E = \frac{V_{defl}}{d_{eff}} \), where \( d_{eff} = 0.175 \) cm for the Y plates of the 5DEP1. We suggest you use this value.

2. Calculate the speed of an electron accelerated through a potential difference of 1000 volts. How long does it take this electron to travel from the second anode to the screen of the CRT? Calculate the transverse velocity this electron gains if the voltage on the Y plates is 50 volts.

3. The magnetic field produced by the deflection coils is not uniform along the tube axis. Thus the calculation of the deflection sensitivity is more difficult for magnetic as opposed to electric deflection. The deflection sensitivity for deflection by a spatially varying field is derived in the Appendix. The variation of the magnetic field along the axis of the CRT for this apparatus is given in Table 1. Calculate the deflection sensitivity by making a numerical integration of the magnetic field along the axis of the tube.

4. Verify Eq. 4, see below. You will need to assume some quantities are small compared to others, and to make an appropriate approximation.
**Apparatus**

- 5DEP1 cathode ray tube chassis
- DC power supply
- Digital multimeter
- Plastic ruler, vernier calipers

**Experiment**

1) Examine the sample cathode ray tube. Compare it with Fig. 1 and identify the component parts.

2) In any experiment it is a good idea to become familiar with your equipment. Spend some time finding out what the various controls (focus, accelerating potential, and brightness) do. Observe the effects of applying voltages to the X and Y deflection plates of the cathode ray tube.

3) Determine the deflection sensitivity due to an electric field applied across the Y deflection plates. Compare your calculated and experimental deflection sensitivities. Show how the deflection sensitivity varies with the accelerating voltage.

4) Determine the deflection sensitivity due to a magnetic field produced by a current in the magnetic deflection coils. Choose a single value of the accelerating voltage for this measurement. Explain your reasons for your choice of accelerating voltage. Compare your experimental observations with the result you calculated above.

5) You probably have noticed that the spot on the 5DEP1 moves horizontally when the accelerating potential is varied. This deflection is due to the motion of the electrons in the earth's magnetic field and is given by:

\[
x = \frac{B_v L^2 e}{2 \sqrt{2V_{acc} m}}
\]  

where \(x\) is the amount of horizontal deflection, \(B_v\) is the vertical component of the earth's field, \(L\) is the distance from the second anode to the screen, and \(V_{acc}\) is the accelerating potential.

Measure \(x\) as a function of \(V_{acc}\) and determine \(B_v\).

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Table 1.

The magnetic field in Gauss produced by a current of 100 mA in the magnetic deflection coils for Chassis #5. Values of the field are quoted as a function of distance from the second anode along the axis of the CRT. The direction of the magnetic field is perpendicular to the axis of the CRT.

Owing to small differences in construction and assembly the field produced by the coils on Chassis #2 is slightly smaller. Multiply the values for Chassis #5 by 0.91 to obtain the magnetic field produced by a current of 100 mA in the coils for Chassis #2.

<table>
<thead>
<tr>
<th>Distance from second anode (cm)</th>
<th>Magnetic field (Gauss) Chassis 5</th>
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<tr>
<td>0</td>
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<tr>
<td>1</td>
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<tr>
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Appendix: Calculation of Deflection Sensitivities

The aim of this appendix is to determine the deflection sensitivities of the CRT for both electric and magnetic fields. As illustrated below, the electrons leave the second anode with velocity $v$ in the $z$ direction and are subsequently deflected by a transverse force $F(z)$ where

$$F(z) = -e E(z) \quad \text{electric fields}$$

$$= -e v B(z) \quad \text{magnetic fields}$$

Let $u(z)$ be the transverse velocity and $\Delta(z)$ be the transverse deflection at $z$. We assume that as they leave the second anode exit (at $z = 0$), the electrons have both zero transverse velocity and deflection (i.e., $u(0) = 0$ and $\Delta(0) = 0$). Their subsequent motion is determined by Newton’s Second Law:

$$m \frac{du(z)}{dt} = F(z) \quad \text{(A1)}$$

The time dependence of $u$ is of no interest and so we use the chain rule to write

$$\frac{du(z)}{dt} = \frac{du(z)}{dz} \frac{dz}{dt} = v \frac{du(z)}{dz}$$

whereby Eqn. (A1) becomes

$$mv \frac{du(z)}{dz} = F(z) \quad \text{(A2)}$$

Integrating Eqn. (A2) and setting $u(0) = 0$, we obtain

$$u(z) = \frac{1}{mv} \int_0^z F(z')dz' \quad \text{(A3)}$$

The deflection $\Delta(z)$ can now be obtained from the relationship

$$u(z) = \frac{d\Delta(z)}{dt} = \frac{d\Delta(z)}{dz} \frac{dz}{dt} = v \frac{d\Delta(z)}{dz} \quad \text{(A4)}$$
Using Eqn. (A4) in Eqn. (A3), we integrate once more and set \( \Delta(0) = 0 \) to get

\[
\Delta(z) = \frac{1}{mv^2} \int_0^z \left( \int_0^{z'} F(z'')dz'' \right) dz'
\]

(A5)

where \( z' \) and \( z'' \) are dummy variables.

The integral in Eqn. (A5) is taken over the shaded areas in the \((z',z'')\) plane as shown in the diagram.

The student can show that by interchanging the order of integration (i.e., by integrating with respect to \( z' \) first and making use of the fact that \( F \) is a function of \( z'' \) only) Eqn. (A5) can be written as

\[
\Delta(z) = \frac{1}{mv^2} \int_0^z \left( \int_0^{z''} F(z'')dz'' \right) dz'
\]

which becomes

\[
\Delta(z) = \frac{1}{mv^2} \int_0^z (z - z'')F(z'')dz''
\]

(A6)

The quantity of interest is the total deflection at the screen. If \( L \) is the distance from the anode exit to the screen we then have for the total deflection

\[
\Delta(L) = \frac{1}{mv^2} \int_0^L (L - z)F(z)dz
\]

(A7)

If the force \( F(z) \) is known in either analytical form (as in the case of the electric field) or in numerical form (as in the case of the magnetic field) then Eqn. (A7) can form the basis for computing the deflection sensitivity.