Error Assignment #2

1. Quantities measured in the lab depend on a range of variables and the uncertainty in the measurement will depend on the uncertainties in the variables. Given a quantity $z = f(u, v, w...)$, show that the uncertainty in z, s_z, is given by

$$s_z{}^2 = s_u{}^2 \left(\frac{\partial z}{\partial u}\right)^2 + s^2{}_v \left(\frac{\partial z}{\partial v}\right)^2 + s^2{}_w \left(\frac{\partial z}{\partial w}\right)^2 ...$$

   Note:

   (a) The mean value of z is defined

   $$\bar{z} = f(\bar{u}, \bar{v}, \bar{w}...)$$

   (b) The definition of the standard deviation

   $$s_z{}^2 = \frac{1}{N-1} \sum (z_i - \bar{z})^2$$

   (c) You can use a Taylor expansion to find an expression for $z_i - \bar{z}$. To first order,

   $$z_i - \bar{z} = (u_i - \bar{u})\frac{\partial z}{\partial u} + (v_i - \bar{v})\frac{\partial z}{\partial v} + (w_i - \bar{w})\frac{\partial z}{\partial w}...$$

   (d) You may assume that the errors in the parameters are uncorrelated.

2. Use the definition of the mean

$$\bar{z} = \frac{1}{N} \sum z_i$$

   and the standard formula for the uncertainty in a function of several variables

$$s_z{}^2 = s_u{}^2 \left(\frac{\partial z}{\partial u}\right)^2 + s^2{}_v \left(\frac{\partial z}{\partial v}\right)^2 + s^2{}_w \left(\frac{\partial z}{\partial w}\right)^2 ...$$

   to show that the uncertainty of the mean value, or the Standard Error in the Mean, is given by

$$s_{\bar{z}}{}^2 = \frac{1}{N} s_z{}^2 \quad .$$