of three (a mix of different abilities, genders, ethnicities, and personalities) to solve problems. The instructor tours among the different groups, answers questions, and guides student understanding of the physics topic of that week.

Textbooks are often selected by the teachers themselves, and instructors may skip some chapters and present others very carefully, depending on needs and program requirements. Teachers may pay more attention to the implementation of principles than to their deduction and derivation. Students are expected to understand the physics concepts rather than memorizing. Individual instruction is sometimes given.

Examinations are given not only as a means for testing, but also as a measure of the consolidation of knowledge by students. Consequently, American teachers give many tests, often with the goal of both showing the students how they are doing and giving the instructor a gauge of how the method of instruction is succeeding. At CSCC there would be a quiz once a week, three midterms, and a final. It is obvious that in American colleges much emphasis is put on satisfying individual needs because of quite different backgrounds and goals.

Observations

Until recently, physics courses in Chinese colleges have been based on a unified syllabus, long course hours, mandated textbooks, and students with a solid foundation of basic knowledge. However, China's economy is moving toward a market economy, and therefore two-year vocational colleges are needed. In these institutions, students will no longer be homogeneous in age or academic background. There must be varied course hours, syllabuses, textbooks, and exams in a number of different programs. It is necessary for teachers in the Chinese system to learn diversity and flexibility in physics teaching.

Further Reading

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Modeling Air Drag

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In most introductory physics courses, the treatment of projectile motion is based on a simplified model in which air drag is neglected. This model predicts that a Ping-Pong ball tossed in the air will follow a parabolic path and that the horizontal component of the ball's velocity will remain constant while it is in flight. Experience tells us that this approximation is limited, particularly when the projectile is relatively light for its size. Therefore I decided to take up the topic of projectile flight, including drag effects, as a special project with some students in my advanced-placement physics class. The catalyst for this activity was a discussion on computer modeling of baseball trajectories that appears in Hugh Young's *University Physics.*[1]

Many introductory physics texts state (without support) that the drag force on an object moving at a relatively high speed through the air is approximately proportional to the square of its speed. Because this result is central to our modeling operation, I felt compelled to give my students some idea as to how this result comes about. I was able to do so based, in part, on my recollection of a "loose" derivation that George Amann shared at an AAPT meeting several years ago as part of an unpublished talk on the physics of tennis. We consider an elastic collision between a tennis ball, mass M, moving with speed v and an air molecule, mass m, initially at rest. The momentum lost by the ball is

$$\Delta p = m\left(\frac{2M}{M+m}\right)v \qquad (1)$$

which reduces to $\Delta p = 2mv$ since $M >> m$.

As the ball moves during a small time interval, Δt, it sweeps out a volume of air $V = Av\Delta t$ where A represents the cross-sectional area of the ball. The mass of the air volume swept out can be expressed as a product of air density and volume, $\rho(Av\Delta t)$. Consequently, the average force applied to the ball due to air molecule collisions during the small interval Δt is

$$F_d = \frac{\Delta p}{\Delta t} \qquad (2)$$

$$= \frac{2\rho\,(Av\Delta t)\,v}{\Delta t} = kv^2$$

And so the drag force is proportional to the square of the ball's speed. The resulting acceleration caused by air drag is given by

$$a_d = \frac{k}{M}v^2 \qquad (3)$$

The proportionality constant, k, can be determined through a combination of theory and experimentation.

$$k = (1/2)\,C\rho A \qquad (4)$$

where C is a dimensionless quantity called the drag coefficient, which depends on the size, shape, and texture of the projectile's surface. Typically, values of C range from 0.5 to 1.0.

The essence of Young's discussion of computer modeling for baseball trajectories is based on the notion that the horizontal and vertical acceleration components for a projectile in flight are modified when air resistance is considered. When the drag force is neglected, the acceleration components are given as $a_x = 0$ and $a_y = -g$.

With air drag included, the x-component of acceleration becomes

$$a_x = \frac{k}{M}v^2\,(\cos\alpha) \qquad (5)$$

where α is the angle between $\vec{v}$ and the positive x-axis. Or, since $v\,(\cos\alpha) = v_x$, simply

$$a_x = -\frac{k}{M}v\,v_x \qquad (6)$$

The negative sign is consistent with the notion that the drag acceleration is directed opposite the corresponding velocity component.

The y-component of acceleration for a projectile includes the effect of gravity as well

$$a_y = -\frac{k}{M}v\,v_y - g \qquad (7)$$

Successive velocity components are computed based on the idea that $a = \Delta v/\Delta t$ for small intervals of time

$$v_x' = v_x + a_x\,\Delta t \qquad (8)$$

$$v_y' = v_y + a_y\,\Delta t$$

Similarly, the projectile's coordinates are calculated assuming that $v = \Delta s/\Delta t$ for small values of Δt

$$x' = x + v_x\,\Delta t \qquad (9)$$

$$y' = y + v_y\,\Delta t$$

We wrote a short and simple program (about 12 lines) for the TI-82 graphics calculator[2] in which the projectile's coordinates are computed and plotted over a number of iterations according to the relations just described. In the program, the user is prompted for the initial conditions, the projectile's mass, and its average cross-sectional area.

Naturally my students wanted to test our calculator model against real-life data, so they designed an experiment in which plastic, wooden, cork, and Styrofoam™ slugs were projected from a tube. They measured the resulting ranges to see how well the calculator model predicted the slug's actual range.

The experimental setup is shown in Fig. 1. The projectile launcher consists

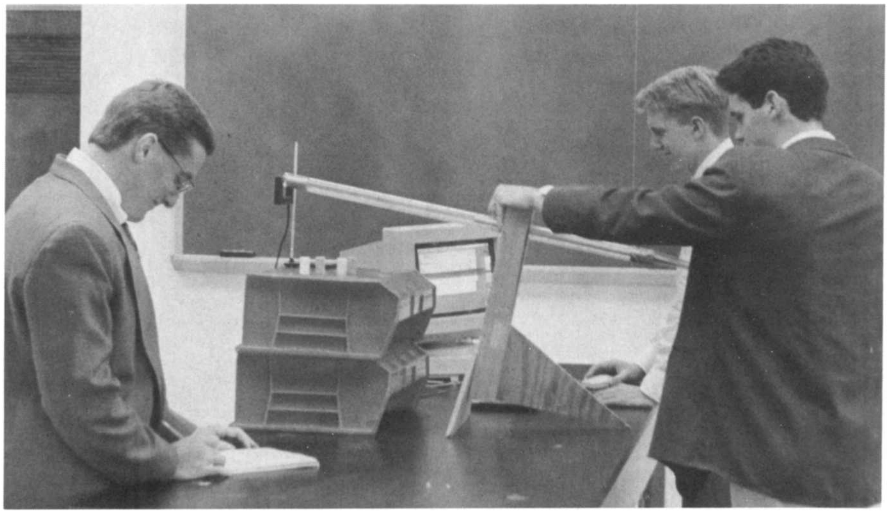

Fig. 1. Experimental arrangement.

Table I. Experimental data for a variety of slug types.

	plastic	wood	cork	foam
1. mass (g)	17.06	7.59	2.92	0.27
2. average initial speed (m/s)	10.56	11.69	12.07	8.54
3. calculated range (m) without drag	7.97	9.14	9.54	6.07
4. calculated range (m) with drag	7.49	7.82	6.61	1.70
5. average experimental range (m)	7.37	7.60	6.26	2.07
6. % discrepancy between (4) and (5)	1.55	2.92	5.53	19.5

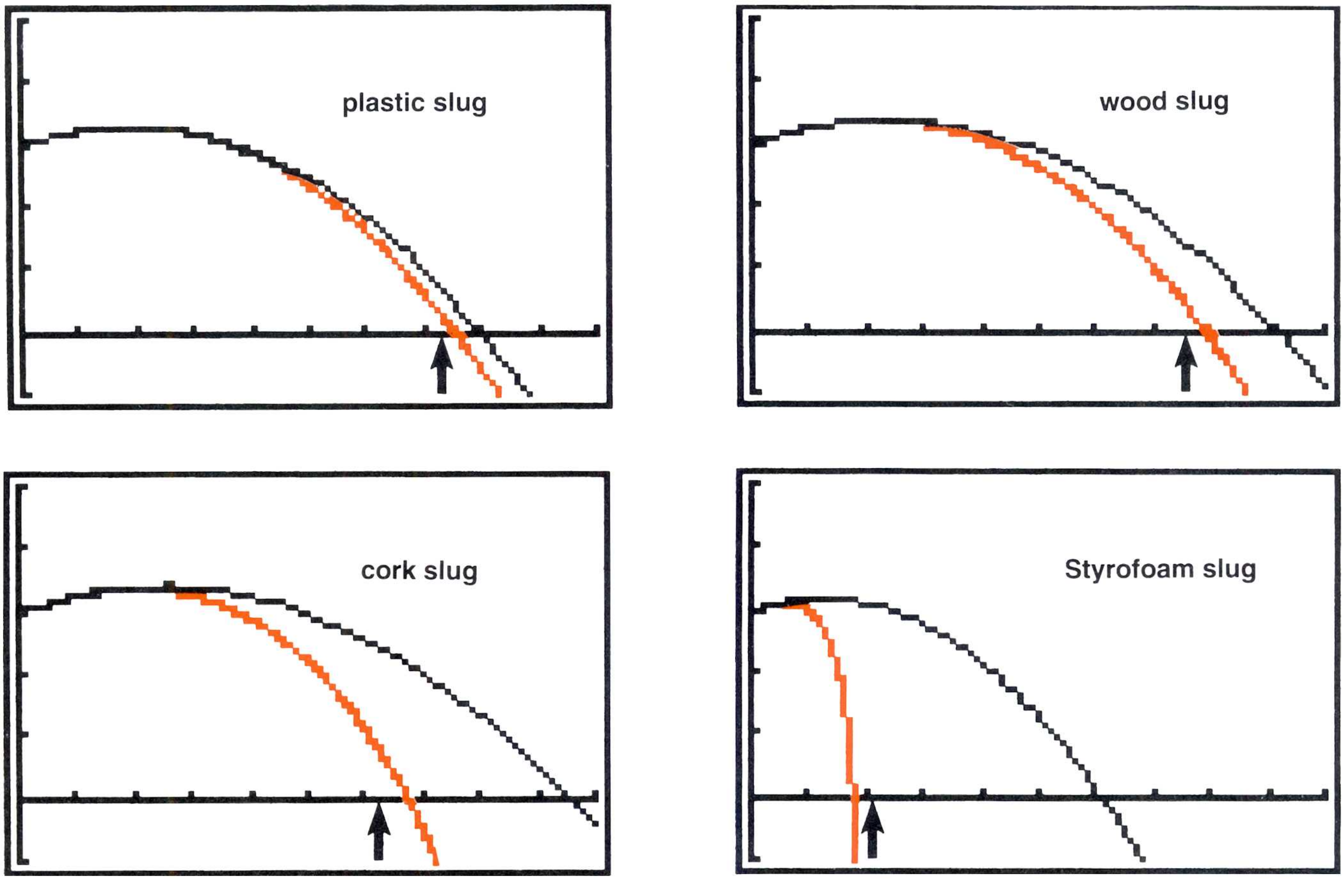

Fig. 2. Calculator-generated trajectories, with (colored line) and without (black line) air drag. Experimentally determined ranges are marked with arrows.

of a long, hollow tube mounted on a plywood base. Several rubber bands are fixed to a dowel and attached to the launch tube with duct tape. Cylindrical slugs are launched from the tube by pulling back the dowel and quickly releasing it. A photogate placed at the end of the tube allows the slug's initial speed to be computed.

Due to the rather crude construction of our launcher, it was nearly impossible to obtain consistent readings for initial speeds. For each slug, our results represent the average range value for three trials during which the projectile's initial speeds deviated by less than 5.0%. The angle of projection and initial height were held constant for all the trials. A summary of the calculated (predicted) ranges, both with and without air drag, and the actual (measured) ranges is given in Table I. The theoretical ranges with air resistance neglected were computed using traditional projectile equations found in any elementary physics text.

Figure 2 shows the paths of the slugs predicted with our calculator model with corresponding measured ranges noted. I was very surprised (more so than my students, at least) to see how reasonably the modeled results matched our experimental findings.

This exercise proved to be rich in physical concepts and very rewarding for both me and my students. The availability of the air drag program in hand-held calculator form led to a number of "what if" speculations in which outcomes for cases with and without air drag were considered. Students gained an appreciation for the role air drag plays in many seemingly straightforward projectile problems. Of particular interest to my students were problems involving the trajectories of eggs, snowballs, and water balloons. I have been promised that the experimental data for these projectiles is forthcoming.

References

1. H.D. Young, *University Physics* (Addison-Wesley, New York, 1992), pp. 74–76.
2. Readers can get a listing of the program by sending a self-addressed stamped envelope to the author.