Effects of Air Resistance

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During the past few years microcomputer software and hardware have become available that allow the study of fast changing complex physical phenomena by recording and analyzing videotapes. We have analyzed several commercially available video clips showing the effects of air resistance on one-and two-dimensional motion. The positions of fast-moving bodies were fitted to specific models describing air resistance effects and the drag coefficients were obtained in several cases. The results presented here can form the basis of interesting classroom activities or laboratory sessions in either introductory or advanced physics classes. These activities can be performed within an hour and are appropriate for both high-school and college-level students.

Videopoint™ is a video-analysis software that allows students to measure the change in the position of bodies in fast mechanical systems. Experimental data are collected by using a frame-by-frame analysis of short video clips that are loaded from a CD-ROM.[1] This collection of more than 200 video clips covers a great variety of mostly mechanical systems.The students click directly on the computer screen and the software scales the data correctly by using objects of known lengths on the computer screen. The data collected can be easily exported to a spreadsheet for further analysis.

We describe here the numerical method used to model the motion of the bodies in air and show the results for a variety of objects of different masses and shapes moving in one and two dimensions under the influence of air resistance forces.

Numerical Method for Obtaining Trajectories

Two recent *TPT* articles discussed a numerical method for calculating the motion of bodies under the influence of air drag. In the first article a TI-82 graphics calculator was used to calculate the effect of air drag forces on the range of a variety of projectiles.[2] In the second article the physics of skydiving was discussed in detail.[3] We use the same numerical method to calculate the trajectories x(t) and y(t) of projectiles moving in one and two dimensions; these trajectories are fitted to a specific air drag model.

The articles in *The Physics Teacher* assumed that the drag force is proportional to the square of the speed and is given by

$$F = kv^2 \qquad (1)$$

Proportionality constant k is usually determined by a combination of theory and experimental work and is written as

$$k = \frac{1}{2}C\rho A \qquad (2)$$

Here A represents the cross-sectional area of the projectile, ρ is the density of air, and C is the drag coefficient (a dimensionless quantity depending on the physical characteristics of the projectile's surface). Even though the values of C are typically between 0.5 and 1.0 for a variety of simple systems, C is best determined *experimentally* for each system considered.

When the drag force becomes equal to the weight of the falling object, the object reaches terminal velocity V_t such that

$$mg = kV_t^2 \qquad (3)$$

From this equation, the coefficient k/m is given in terms of V_t by the expression:

$$k/m = g/V_t^2 \qquad (4)$$

Successive values of the position and velocity of a projectile are calculated according to the approximate equations

$$x' = x + v_x\Delta t \qquad y' = y + v_y\Delta t$$
$$v_x' = v_x + a_x\Delta t \qquad v_y' = v_y + a_y\Delta t \qquad (5)$$

where x', x, y', y are the successive positions of the body along the x- and y-axis correspondingly, and v_x', v_x, v_y', v_y are the corresponding velocity components; Δt is the small interval of time between successive frames. The video clips in the *VideoPoint* CD-ROM are recorded at a standard video rate of 30 frames per second, so the time interval used in all our calculations is $\Delta t = 1/30 = 0.0333$ s.

Parameters a_x, a_y in Eq. (5) represent the acceleration components along x, y axes and are given by:[2]

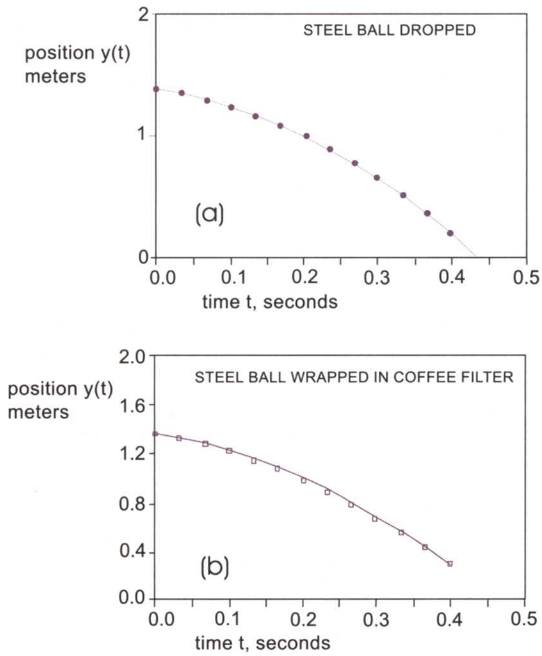
$$a_x = -\frac{k}{m}vv_x \quad a_y = -\frac{k}{m}vv_y - g \qquad (6)$$

where v is the speed of the projectile given by

$$v = \sqrt{v_x^2 + v_y^2}$$

Numerical calculations based on Eqs. (5) and (6) yield trajectories x(t) and y(t) of the body moving under the influence of air resistance. The only parameters required for the calculations are the initial positions x_0,y_0 and the initial speeds v_{0x}, v_{0y} of the projectile. These calculations can easily be done on a calculator or a spreadsheet. As an alternative, students can write their own programs on a microcomputer to calculate and also plot the trajectories x(t) and y(t).

The computer programs to perform the numerical calculations were written by our students using BASIC language.[4] The user inputs the initial



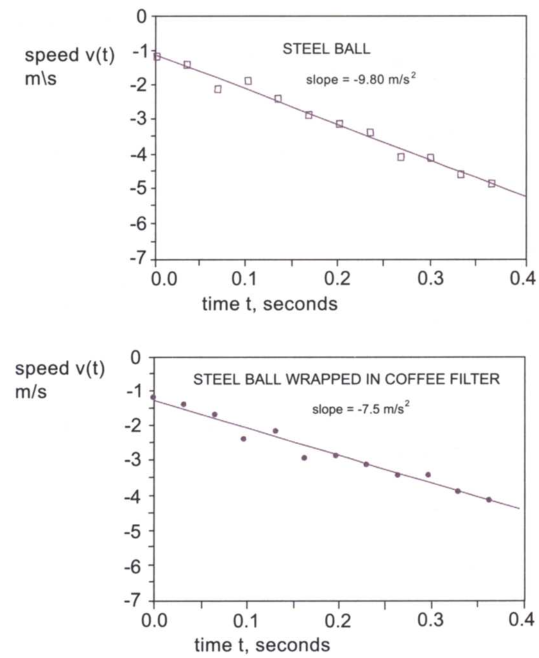


Fig. 1. Position* y(t) *for* (a) *a steel ball and* (b) *a steel ball wrapped in a coffee filter. Dots and squares show experimental values. Solid line shows calculated position* y(t) *using the air-drag model with* (a) *$k = 0$ (no air drag) and* (b) *$k/m = 0.4$ m^{-1}.

Fig. 2. Calculated speed* v(t) *for* (a) *a steel ball and* (b) *the steel ball wrapped in a coffee filter. Slope of lines gives acceleration of the two balls. First ball has a free fall with negligible air drag; motion of second ball shows significant air-drag effects.

position and velocity of the projectile in the x- and y-axes, the mass of the projectile m, and the air resistance coefficient k; the program calculates and plots the trajectories x(t) and y(t) under the influence of air drag.

Experimental Results

(a) Free-falling bodies under the influence of air drag

Figure 1 shows the results of analyzing a video clip showing two balls dropped simultaneously. The first ball is a steel ball of 1-in diameter and 65.9-g mass. The second ball is an identical steel ball wrapped in a coffee filter. The irregular shape of the coffee filter adds enough air resistance so that those watching the computer screen see the first ball clearly overtake the second ball, even though it is dropped from a higher initial position on the y-axis.

Our students first measure the positions of each ball y(t) as a function of time by direct measurements on the computer screen. These data are saved as a text file, ready to be accessed by graphing programs or spreadsheets. From this set of data (usually 10 to 15 experimental points) the students then calculate the velocity of the falling balls using

$$v' = \frac{x' - x}{\Delta t}$$

where x, x' are successive data for the position of each ball. The resulting velocity, v, is plotted as a function of time t in Fig. 2a, b. These linear curves show that the velocity of the balls is varying linearly with time, even in the presence of air drag. Slopes of the graphs in Fig. 2a, b yield the acceleration of the balls. The acceleration of the steel ball is equal to $g(-9.80 \text{ m/s}^2)$ and that for the steel-ball/coffee-filter combination is significantly less, with a value of -7.5 m/s^2.

To fit our numerical model to the experimental data y(t), we need the initial positions and speeds of the falling bodies. Initial positions of the two balls are obtained by direct measurements on the screen. Initial speeds of the balls are obtained from the y-intercept of the v(t) graphs in Fig. 2. The resulting parameters are $y_0 = 1.386$ m, $v_0 = -1.2$ m/s and $y_0 = 1.354$ m, $v_0 = -1.17$ m/s for the steel ball and steel-ball/filter combination correspondingly. We note that the initial speeds of the balls are not zero; this is because we skipped the first few video frames, which are difficult to measure accurately on the computer screen.

Students calculate the trajectories of the balls, y(t), using these initial values in the numerical model. They quickly find out that data for the steel ball fit very closely the free-fall equation with $k = 0$, whereas data for the second ball clearly deviate from this model. They are then asked to plot a series of curves with various values of the parameter k/m until a good fit to the data is obtained; the "best fit" obtained in this manner is shown in Fig. lb. It yields a value of $k/m = 0.4$ m^{-1} for the steel-ball/filter combination.

We estimated an average diameter

Table I. Summary of Drag Coefficients.

Object	Drag Coefficient C	Area (m^2)	Proportionalty Constant k (kg/m)
Steel ball w/ filter	9.4	0.005	0.027
Styrofoam™ ball	0.56	0.0005	0.00017
Nested coffee filters	1.02	0.024	0.0145

of the ball/filter system from the computer screen to be 2R = 0.078 m. By using this value for the diameter of the system, its mass 0.068 kg, and the density of air (1.20 kg/m^3 at 20°C) in Eq. (2), we obtained a value for drag coefficient C of 9.4. This high value of the drag coefficient is due primarily to the irregular shape of the system.

(b) A ball thrown vertically upwards

Figure 3 shows the results of analyzing a video clip showing a student at Dickinson College throwing a ball vertically upwards. The initial position of the ball is measured directly on the computer screen and the initial speed of the ball is approximated by using

$$v_0 = \frac{v_1 - v_0}{\Delta t}$$

where y_1, y_0 are the first two positions of the ball and Δt is the time interval between frames, equal to 1/30 s. By using the initial values y_0 = 1.427m, v_0 = 2.648 m/s, and no air drag (k/m=0), the computer program produces the curve shown as a dashed line in Fig. 3. By plotting a series of curves with different air-resistance coefficients, k/m, students obtain a good fit to the experimental data when k/m = 0.08 m^{-1}. This is shown as a solid line in Fig. 3. The drag coefficient in this case cannot be calculated due to the lack of data on the size and mass of the ball. This example illustrates that all relevant information must be given for the video clips before they are analyzed by the students.

(c) Projectile motion of a Styrofoam ball

Figure 4 shows the results of analyzing a video clip showing a 0.25-g Styrofoam ball fired at an angle of 30°. The diameter of the ball is one inch.

Initial positions x_0, y_0 of the ball are obtained by direct measurements on the screen. Initial speed of the ball on the x-axis (v_{0x}) is obtained from the initial linear part of the graph x(t) as shown in the figure. In this example v_{0x} = 3.60 m/s. Initial speed on the y-axis (v_{0y}) is obtained by using the known angle, θ = 30°, of the projectile and the value of v_{0x} = 3.60 m/s by using the equation: $v_{0y} = v_{0x}\tan(0)$ = 3.60 tan(30) = 2.08 m/s.

The four parameters v_{0x}, v_{0y}, x_0, y_0 are entered into the program and the students are asked again to vary the parameter k/m until a good fit to both the x(t) and the y(t) data is obtained. The "best fit" obtained in this manner is shown in Figure 5. It yields a value of k/m=0.69 m^{-1}. By using the values of the diameter of the sphere (1 in), its mass (0.25 g), and the density of air (1.20 kg/m^3 at 20° C) in Eq. (2), we obtain a value for the drag coefficient C of 0.56. The drag coefficient in this case is influenced by the constitution of the surface of the Styrofoam ball.

(d) Dropping a series of nested coffee filters

Figure 6 shows the results of analyzing a video clip showing several nested coffee filters dropped from a certain height. The *VideoPoint* CD-ROM shows groups of 1, 2, 4, 6, 9, and 13 nested coffee filters being dropped. The mass of each coffee filter is 1.64 g. By observing the shape of the graphs y(t) in Fig. 6a, students realize that the coffee filters quickly reach terminal velocity because they have small weight and a shape that causes a large air drag.

By calculating the slope of the graphs y(t) at large values of time t, as in Fig. 6a, students find the value of the terminal velocity v_t = 2.80 m/s. This value of the terminal velocity is entered into Eq. (4) and gives the value of the coefficient, k/m, as 1.25 m^{-1}. The initial velocity of the coffee filters is assumed to be v_0 = 0. Finally, the numerical model with x_0 = 1.54 m, v_0 = 0 m/s, and k/m = 1.25 m^{-1} is used to calculate the curves y(t) as shown in Fig. 6b.

By studying the terminal velocity of several combinations of air filters students can get a good grasp of the effects of air drag. By using the terminal velocity and the mass of the

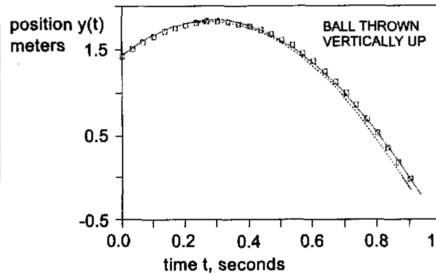


Fig. 3. Position y(t) for a ball thrown vertically upwards. Solid line shows calculated y(t) using numerical model with k/m = 0.08 m^{-1}. Squares show experimental data. Dashed line shows y(t) calculated with k/m = 0 (no air drag).

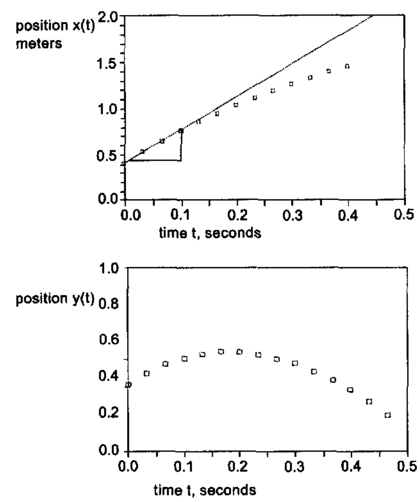


Fig. 4. Positions x(t) and y(t) for a soft Styrofoam ball fired at a 30° angle. Solid line shows initial slope of x(t), which yields component of velocity at time t = 0 along x-axis: v_{0x} = 3.60 m/s.

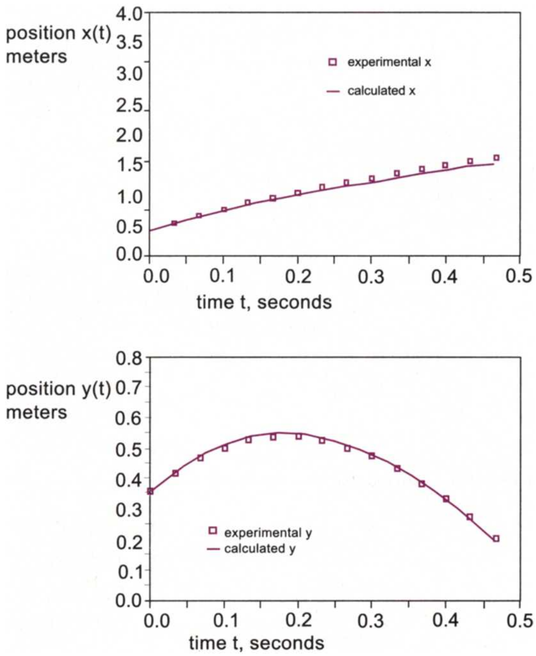


Fig. 5. Positions* x(t) *and* y(t) *for the soft Styrofoam ball fired at a 30° angle, using the numerical model with k/m = 0.69 m⁻¹.

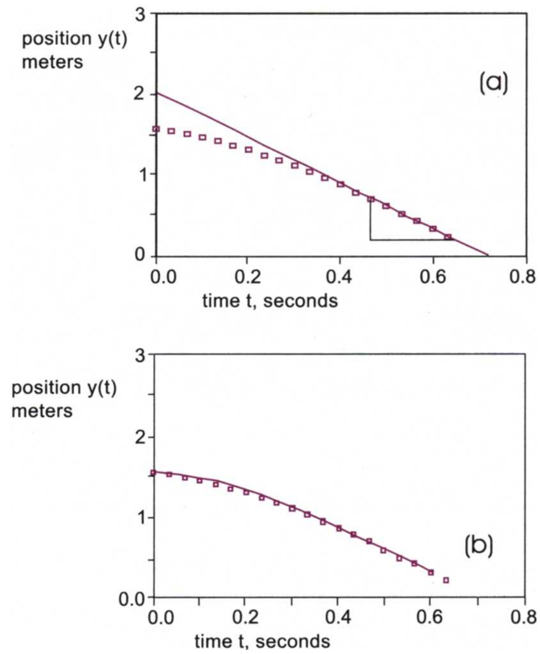


Fig. 6. Position* y(t) *for six nested coffee filters. Slope of curve* y(t) *at large times t shown in* (a) *is equal to the terminal velocity of the filters* v_t *= 2.80 m/s. This value is used to obtain the value of* k/m *= 1.25 m⁻¹. In* (b) *the solid line is calculated position* y(t) *using numerical model with k/m = 1.25 m⁻¹.

system they can obtain the value of k from Eq. (4). They find that the terminal velocity can vary from 1 m/s for a one-filter system to 3.6 m/s for a 13-filter system. Even though v_t changes dramatically for different masses of the system, the calculated k-values are almost constant, varying from k = 0.012 kg/m to 0.016 kg/m. The average value of $k = 0.0145 \pm 0.0017$ kg/m.

The effective diameter of the coffee-filter system can be measured directly on the computer screen. By using an average value of the diameter ($2R$ = 0.174 m), the density of air (ρ = 1.20 kg/m^3 at 20° C), and the average value of k (0.0145 kg/m), we obtain the value of the drag coefficient, C = 1.02.

Conclusions and Classroom Discussions

Analysis of the four video clips can be preceded and/or followed by a classroom discussion of the various factors affecting air resistance. It is instructive to summarize the drag coefficients (C), areas (A), and proportionality constants (k) as shown in Table I. By inspecting the values in this table, apparent misconceptions about the air-drag force can be explained.

The first concept is that the drag coefficient C, although unitless, is not a fraction and can have values greater than one. Many students on first inspection of Eq. (2) assume that the drag coefficient is a fraction ranging from 0 to 1, with maximum air resistance represented by C = 1.

The second and more puzzling concept is that the drag coefficient is often larger for more massive objects. Prior to performing this exercise, most students would rank the drag coefficients of these objects in an order that reflects each object's terminal velocity. They believe that the object with the lowest terminal velocity must have the greatest drag coefficient. Thus, they are surprised to find out that the object with the highest terminal velocity (the steel ball wrapped with the coffee filter) has the largest drag coefficient. Similarly, the object with the smallest terminal velocity, one coffee filter, has the lowest drag coefficient.

What is even more surprising is that the steel ball wrapped with the coffee filter has a larger proportionality constant than the nested coffee filters, even though the latter system has the largest cross-sectional area (0.024 m^2). To explain this result, the inertia of the objects must be considered. Since the steel ball has a much larger inertia than either the Styrofoam ball or one coffee filter, a larger force is required to change its motion. So even a small change in the velocity of the steel ball requires a much larger force and, consequently, a larger proportionality constant.

The third concept, which is made clear upon analysis, is the relationship between the proportionality constant (k), the drag coefficient (C), and the area of an object. From Table I it is clear that even though the C values for the ball and the nested filters are comparable (0.56 and 1.02 respectively), the ball has a much smaller k (0.00017 vs 0.0145 kg/m). The determining factor in this case is the much larger area of the coffee filter. In a previous article in this magazine, it was reported that a sky diver with an open parachute has a drag coefficient of 1.42 in the vertical direction.[3] Although this does not seem large enough to slow down a human speeding towards the hard earth, the enormous area of the parachute gives a large value of k and, hence, a large air resistance. Overall, we believe that these exercises give students a better understanding of the subtle nature of the forces of air resistance.

References

1. *VideoPoint*™ requires a Mac running System 7.1 or newer or a PC running Windows 3.1 or Windows 95. This video-analysis software package is available from PASCO scientific (10101 Foothills Blvd., Roseville, CA 95678; 800-772-8700) as either a teacher's kit with manual and CD-ROM ($160) or a student's version with CD-ROM only ($50).
2. C. Brueningsen et al., "Modeling air drag," *Phys. Teach.* **32**, 439 (1994).
3. G. Wagner and R. Wood, "Skydiver survives death plunge (and the physics that helped)," *Phys. Teach.* **34**, 543 (1996).
4. A listing of the computer programs in BASIC is available by writing author Guerra.

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Only an External Force Shifts the Center-of-Mass

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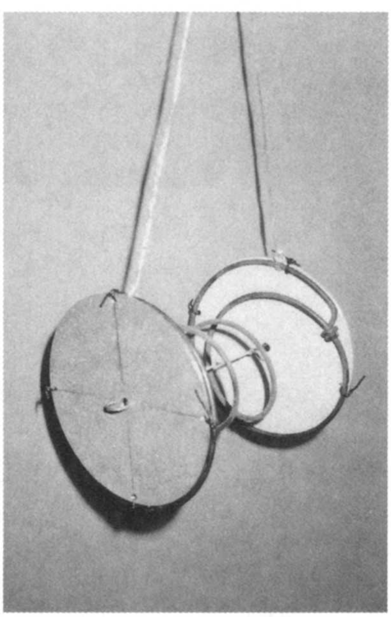

A spiral spring from an old sofa or mattress is useful in demonstrating that internal forces don't shift the center-of-mass, only unbalanced external forces do.

To both ends of the spring attach an identical aluminum disc with a hole in the center to form a cylindrical envelope (see figure). Compress the spring by pulling and tying a string stretched between the center holes. Suspend the system horizontally by strings as shown.

The center-of-mass is midway between the discs and is at rest in the lab. Now use a match flame to release the spring. The system oscillates vigorously but the center-of-mass stays at rest. The total momentum remains zero.

Repeat the experiment near a wall so that when the spring is released one end will hit the wall. In this case, the center-of-mass moves wildly. An unbalanced external force—the reaction of the wall—has been introduced and the total momentum is no longer conserved.