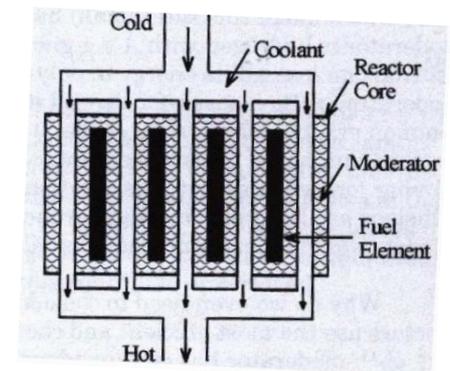
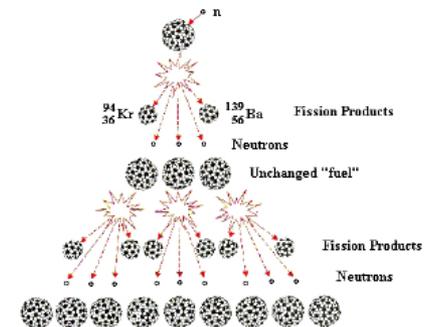
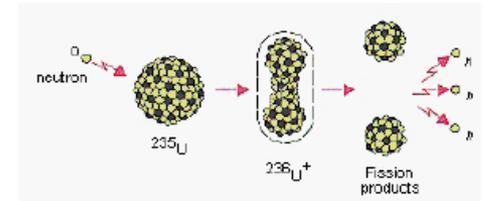


Nuclear reactor basic principles:

1. Neutron induced fission releases energy plus extra “fast” neutrons.
2. “Fast” neutrons are slowed down by a “moderator” such as water or graphite, allowing chain reaction to take place (rapid increase in neutron population). In water reactors, the coolant is also the moderator.
3. Chain reaction is controlled by controlling the condition of the moderator, or by use of neutron absorbing materials (e.g. cadmium control rods)
4. Heat is removed by some form of heat exchanger where it is used to run a heat engine.



Controlling the chain reaction



Each fission liberates 2-3 neutrons for a net increase of 1-2 neutrons per fission
When these neutrons are slowed down by the moderator they can cause more fissions

Chain reaction:

If N = number of neutrons:

Initially, dN/dt is proportional to $N \Rightarrow$ exponential growth of neutrons

However there are neutron loss mechanisms:

- Neutrons can escape from the reactor
- Neutrons can be absorbed by non fissionable isotopes

Loss mechanisms oppose the strong increase in neutron population

Reactor Criticality

Neutron population can be approximated as:

$$\frac{dN}{dt} = (k - 1) \frac{N}{\tau_D}$$

where N = neutron population ,

τ_D =neutron diffusion time ~ 0.1 s for a conventional “thermal reactor” using H_2O

k = “neutron multiplication factor” and depends on several factors:

- the probability of neutron generation by fission (increase)
- the probability of escape from the core (loss)
- the probability of absorption by other than fuel (loss)

Important Limiting Cases:

$k > 1$: exponential growth (not good)

$k = 1$: steady state population, $N = \text{constant}$ (good)

$k < 1$: exponential decay (shut down mode)

reactor is supercritical

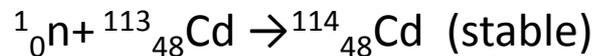
reactor is critical

reactor is subcritical

Shutdown Mechanisms

Reactors need some mechanism for rapidly controlling the concentration of neutrons e.g. during emergency shutdown

Cadmium (Cd) has a very high cross section for neutron absorption:

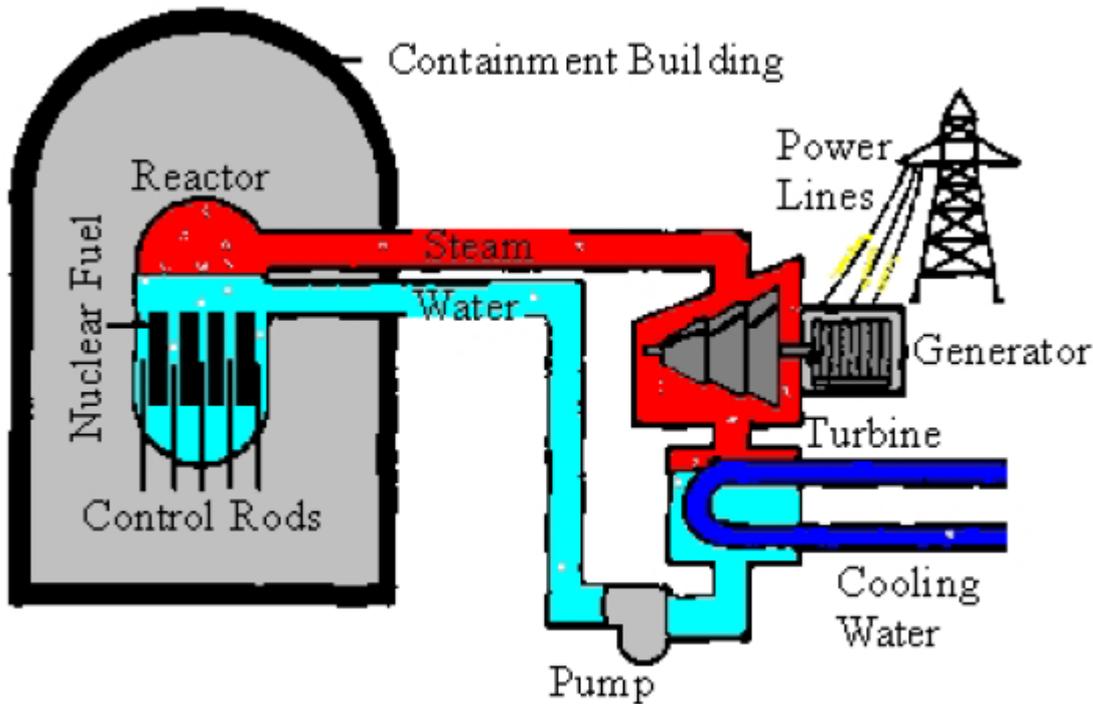


${}^{113}_{48}\text{Cd}$ neutron cross section: 2×10^4 Barns

compare with ${}^{235}_{92}\text{U}$: 582 Barns or ${}^1_1\text{H}$: 0.332 Barns

- Neutron population can be controlled by inserting or removing Cd control rods (shutdown/ startup)
- Note: Power level fine control is usually by means of coolant flow (more later)

Boiling Water Reactor (BWR)



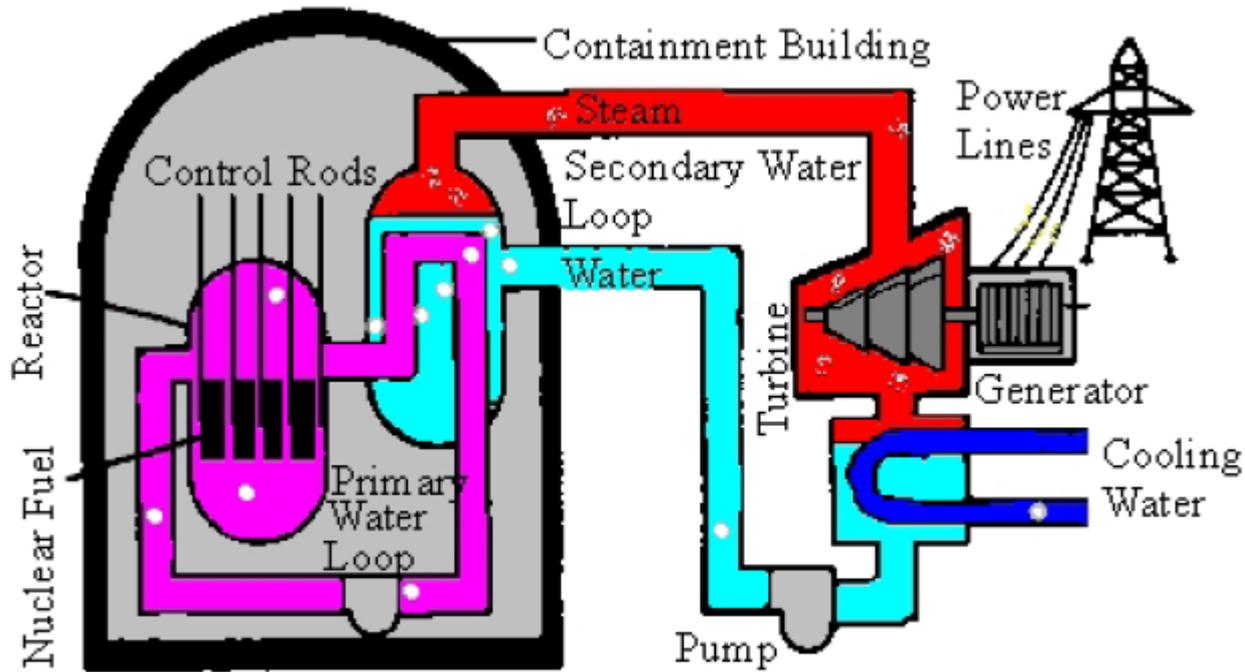
e.g. Fukushima Daichi

Power control method:
Coolant flow: low flow increases boiling which decreases moderation which decreases neutron population

- Moderate pressure (7MPa or 70atm)
- Boiling point : $T_H \sim 286\text{C}$ at 7MPa
- Same water used for both coolant and turbine steam
- Turbine is potentially exposed to radioactive materials (water)
- Heat exchanger to remove heat using non radioactive water

Pressurized Water Reactor (PWR)

e.g. Three Mile Island



Power control method:
Boric acid (high neutron cross section) is injected into coolant or removed from coolant (primary).

- Increase water pressure to 15MPa (~155 atm)
- $T_H \sim 345^\circ\text{C}$ pure liquid phase (no boiling in primary loop)
- Higher operating temperatures (greater thermal efficiency)
- Secondary coolant loop keeps radioactive products from turbine loop
- Higher operating pressures/temperatures places stringent requirements on materials

Light water vs. Heavy water reactors

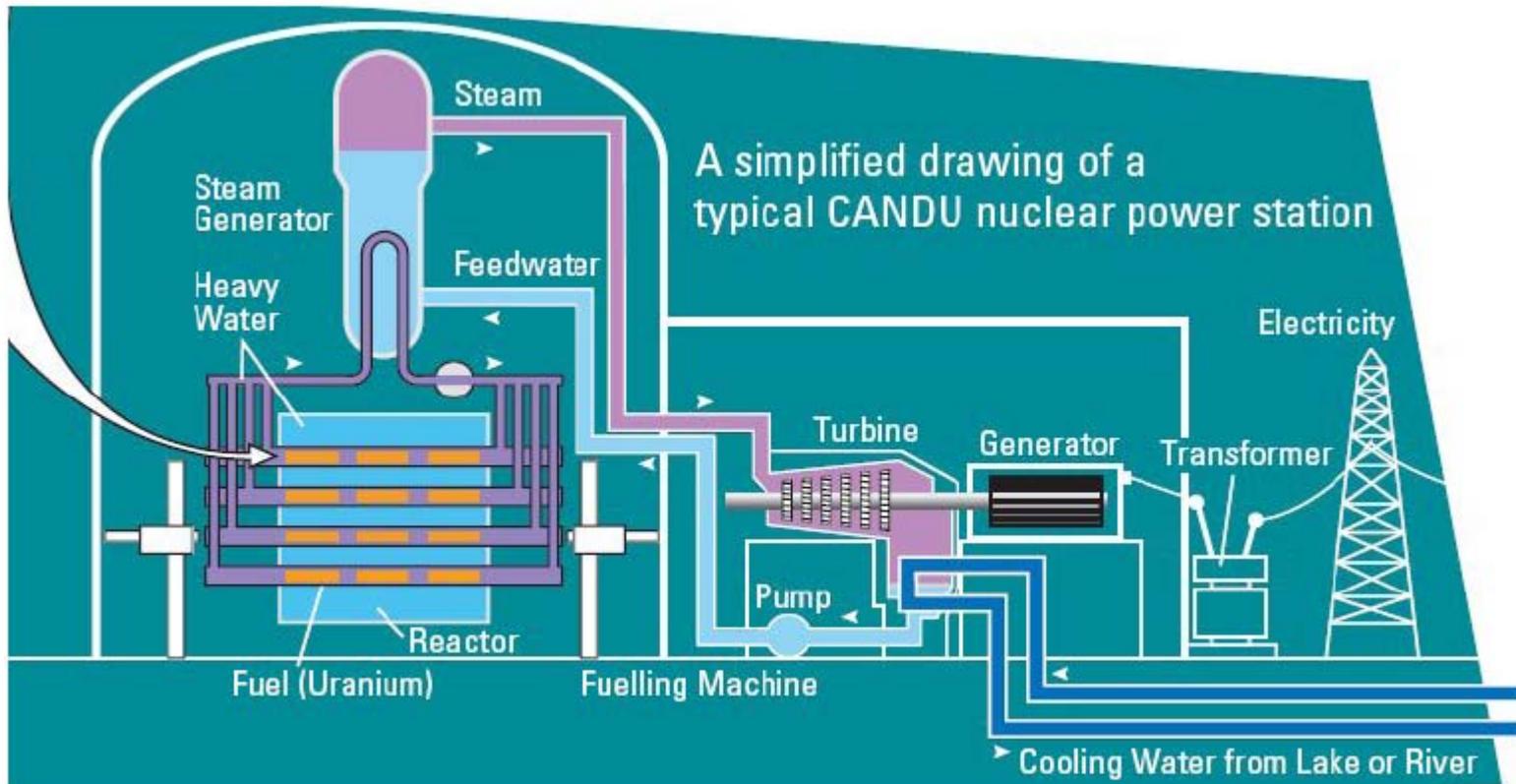
Light water reactors, LWR (most reactors):

- water moderator is effective at slowing neutrons 😊, but also absorbs neutrons strongly ($\sigma=0.33$ Barn), meaning fewer neutrons per fission 😞
- strong absorption of neutrons requires the use of enriched uranium: 3-5% ^{235}U 😞
- countries with enrichment facilities can potentially produce weapons grade U (typically greater than 85% ^{235}U) 😞

Heavy water reactors, HWR (Candu)

- D_2O is less effective as a moderator 😞 but has much lower neutron cross section ($\sigma=5.2 \times 10^{-4}$) i.e. more neutrons are available for fission. 😊
- Weaker absorption of neutrons allows the use of natural uranium (0.72% ^{235}U) 😊
- D_2O is expensive (~20% of cost of a reactor!) 😞
- But: D_2O enrichment is only required once (as opposed to ^{235}U enrichment for LWR) 😊
- heavy water reactors breed higher levels of ^{239}Pu making them useful sources of this material for weapons manufacture. 😞

CANDU Reactor (Pressurized heavy water reactor)



- natural uranium fuel
- uses 30%-40% less uranium than LWR
- full-power refueling
- can use waste fuel bundles from LWR as fuel

“There were 438 nuclear reactors in operation around the world in January 2002, 32 of them are of CANDU type.”
www.candu.org

Candu Issues:

Isotope	Neutron Cross section σ (Barns)
^{235}U	582
^{238}U	2720

The main fuel burned in a Candu is initially $^{235}_{92}\text{U}$, however as time passes, $^{239}_{94}\text{Pu}$ is generated by:



$^{238}_{92}\text{U}$ is both much more abundant and has a higher neutron capture cross section

After one year of fuel life, more heat is actually generated via $^{239}_{94}\text{Pu}$ fission than $^{235}_{92}\text{U}$

CANDUs spent fuel is high in ^{239}Pu , making it useful for ^{239}Pu extraction for weapons

Tritium is generated in the heavy water through $^1_0\text{n} + ^2_1\text{H} \rightarrow ^3_1\text{H}$ (t1/2=12.3 years)

Reactor Stability

Stability refers to the ability of the system to recover from the effect of a small change in power output

H₂O and D₂O reactors tend to be inherently self stabilizing:

- Uncontrolled increase in fission rate leads to vaporization of coolant/moderator
- This results in a loss of moderation because of the sudden decrease in moderator density (liquid => gas)
- This tends to reduce the fission rate
- This mechanism is not available in graphite reactors such as Chernobyl

Liquid H₂O and D₂O based reactors are said to have a “negative void” coefficient

Graphite reactors have a “positive void coefficient”, making these systems more susceptible to uncontrolled output situations like Chernobyl (more later)

Thermodynamic efficiency of a nuclear reactor

Reactors are just heat engines using nuclear fuel

- The maximum operating temperature is lower compared with fossil fuel plants because of the extremely harsh materials environments in nuclear reactor components
- Coolant tubes must withstand high pressure, and radiation damage due to activation of the pipes by neutrons and the generation of structural defects.
- Therefore operating temperatures tend to be lower than for fossil fuel plants.
- Typical operating temperatures: $T_H \sim 285^\circ\text{C}$, $T_C \sim 100^\circ\text{C}$ (BWR)
- Max thermodynamic efficiency:

$$\eta_{\max} = \frac{T_H - T_C}{T_H} = \frac{185\text{K}}{523\text{K}} = 35\%$$

State of the art gas or coal plants can now approach 50% thermal efficiency

Estimating Uranium Usage:

How much natural uranium is required to fuel a 1GW reactor for one year?

$$1\text{GW-year} = (10^9\text{J/s-year}) \times (365\text{days/year}) \times (24\text{hours/day}) \times (3600\text{s/hour}) = \underline{3.15 \times 10^{16} \text{ J}}$$

Assuming 40% thermal efficiency this means we need

$$Q_H = W/\eta = (3.15 \times 10^{16} \text{ J})/0.40 = 7.88 \times 10^{16} \text{ J per year}$$

Previously we saw that natural U gives $5.8 \times 10^{11} \text{ J/kg}$ of heat energy

Therefore we need $(7.88 \times 10^{16} \text{ J}) / (5.8 \times 10^{11} \text{ J/kg}) = 1.36 \times 10^5 \text{ kg} = 136 \text{ tonnes per year}$

Known Uranium Resources

	million tons uranium
Australia	1.14
Kazakhstan	0.82
Canada	0.44
USA	0.34
South Africa	0.34
Namibia	0.28
Brazil	0.28
Russian Federation	0.17
Uzbekistan	0.12
World total (conventional reserves in the ground)	4.7
Phosphate deposits	22
Seawater	4 500

Table 24.2. Known recoverable resources of uranium. The top part of the table shows the “reasonable assured resources” and “inferred resources,” at cost less than \$130 per kg of uranium, as of 1 Jan 2005. These are the estimated resources in areas where exploration has taken place. There’s also 1.3 million tons of depleted uranium sitting around in stockpiles, a by-product of previous uranium activities.

McKay, pg 162

Estimating Uranium Resource Lifetime

Assume conventional U reserves of 5.4×10^6 tonnes (2009)

This gives $(5.4 \times 10^9 \text{ kg}) \times (5.8 \times 10^{11} \text{ J/kg}) = 3.1 \times 10^{21} \text{ J} = \underline{3132 \text{ EJ}}$ of heat

This gives $\sim 0.4 \times 3132 \text{ EJ} = \underline{1253 \text{ EJ}}$ of electrical energy

World annual electricity consumption (2010) = 21,325 TWh =
 $(21,325 \times 10^{12} \text{ Wh}) \times (3600 \text{ s/h}) = 7.68 \times 10^{19} \text{ J} = \underline{76.8 \text{ EJ}}$

At this rate these reserves would last : $(1253 \text{ EJ}) / (76.8 \text{ EJ/year}) = 16.3 \text{ years}$

Note: Today's technology wastes most of the available ^{238}U

Future breeder reactors could recover most of this giving and almost 100X increase in energy yield