

# Competing orders in quantum materials

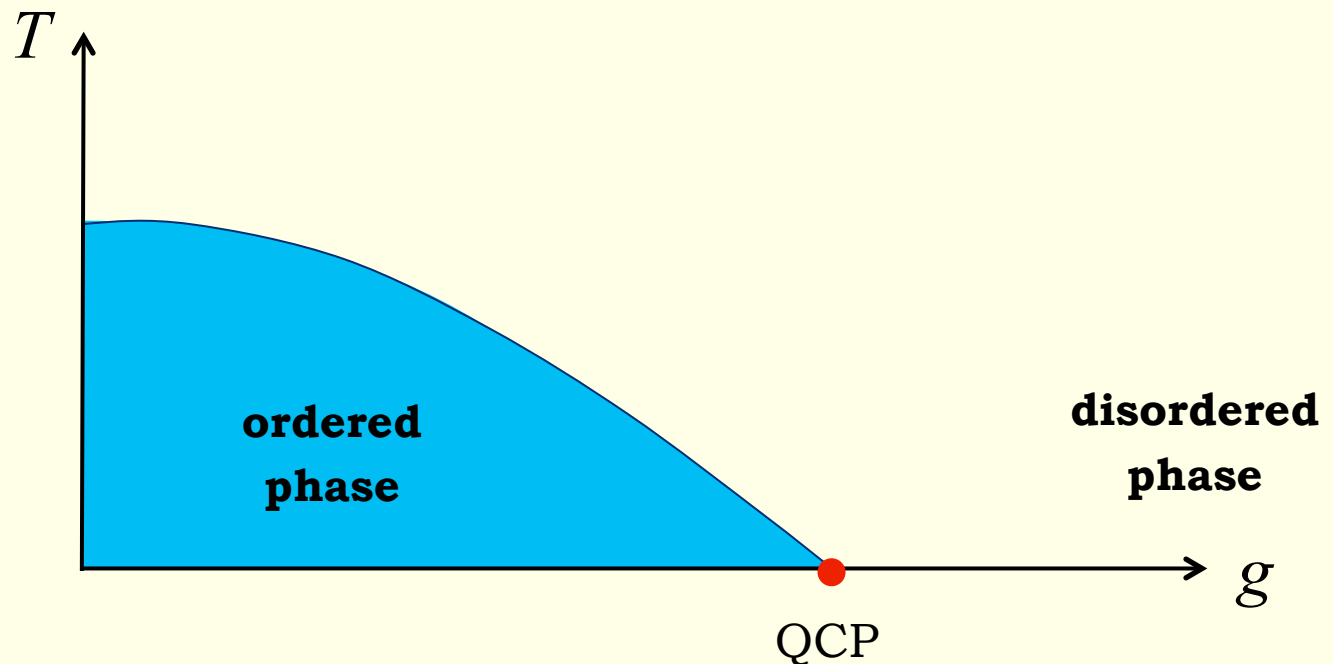
*Rafael M. Fernandes*

University of Minnesota

<http://homepages.spa.umn.edu/~rfernand/>

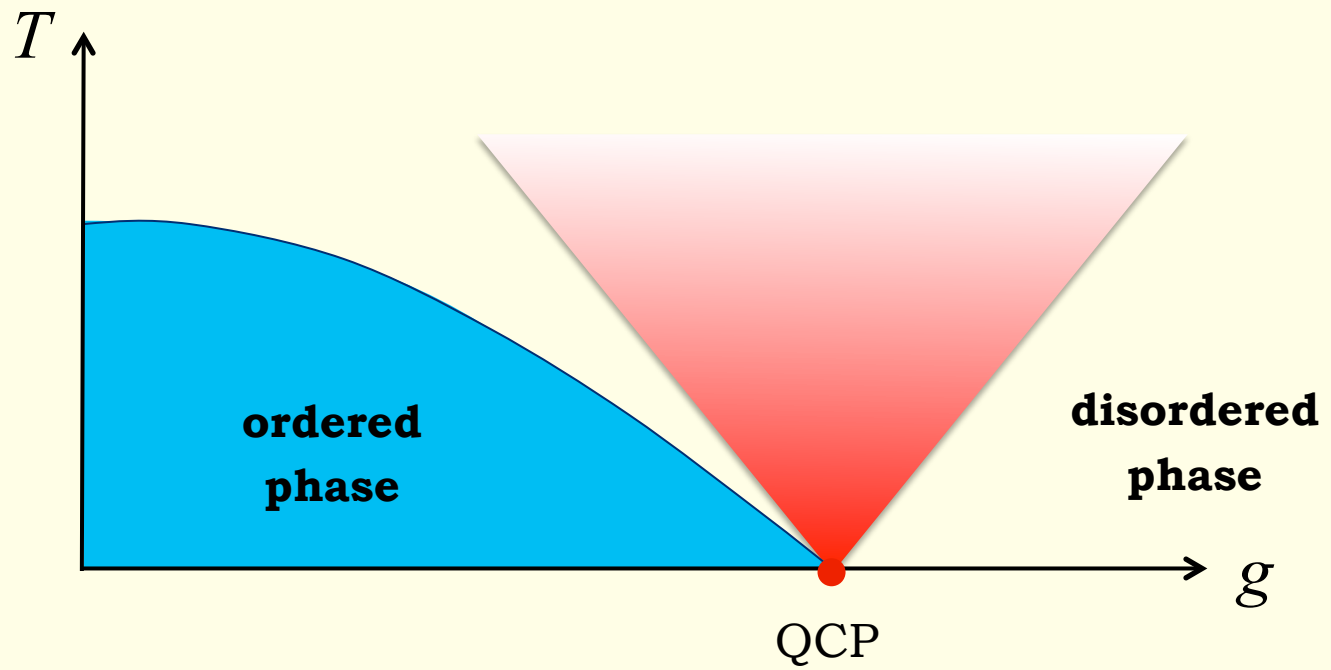
# Quantum phase transitions and critical fluctuations

- Tuning of a continuous phase transition down to  $T=0$



# Quantum phase transitions and critical fluctuations

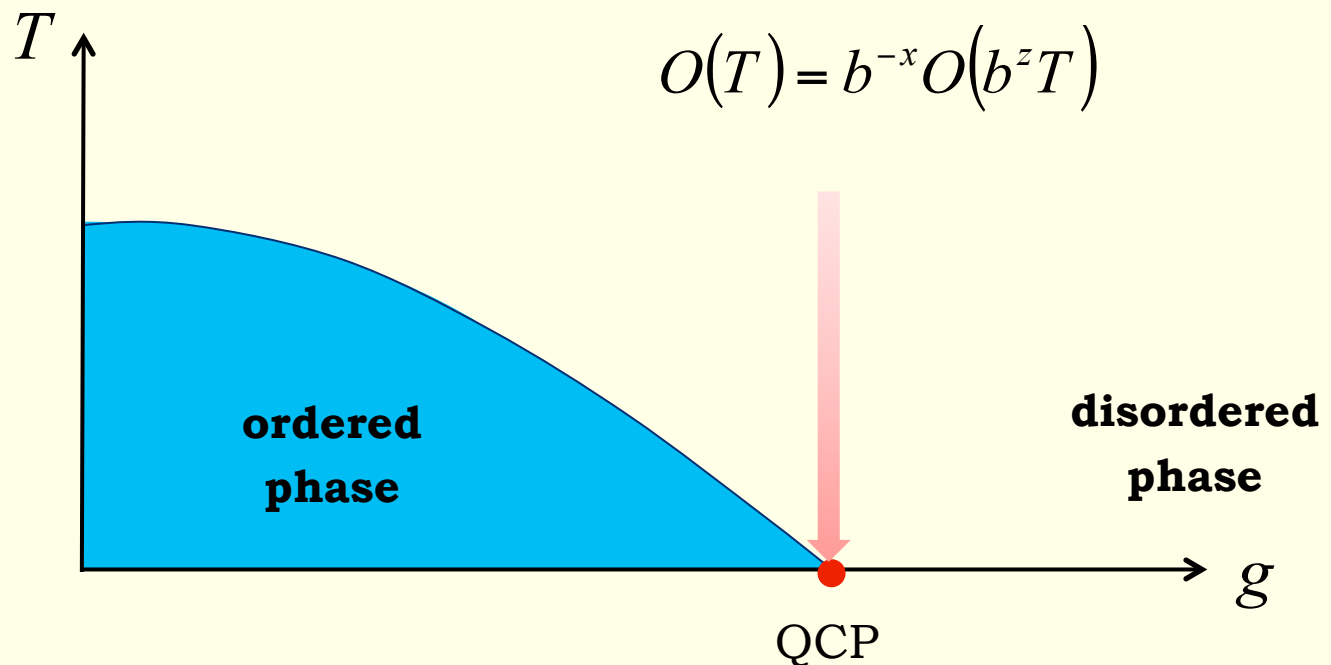
- Tuning of a continuous phase transition down to  $T=0$



*critical fluctuations around the quantum critical point*

# Quantum phase transitions and critical fluctuations

- Tuning of a continuous phase transition down to  $T=0$

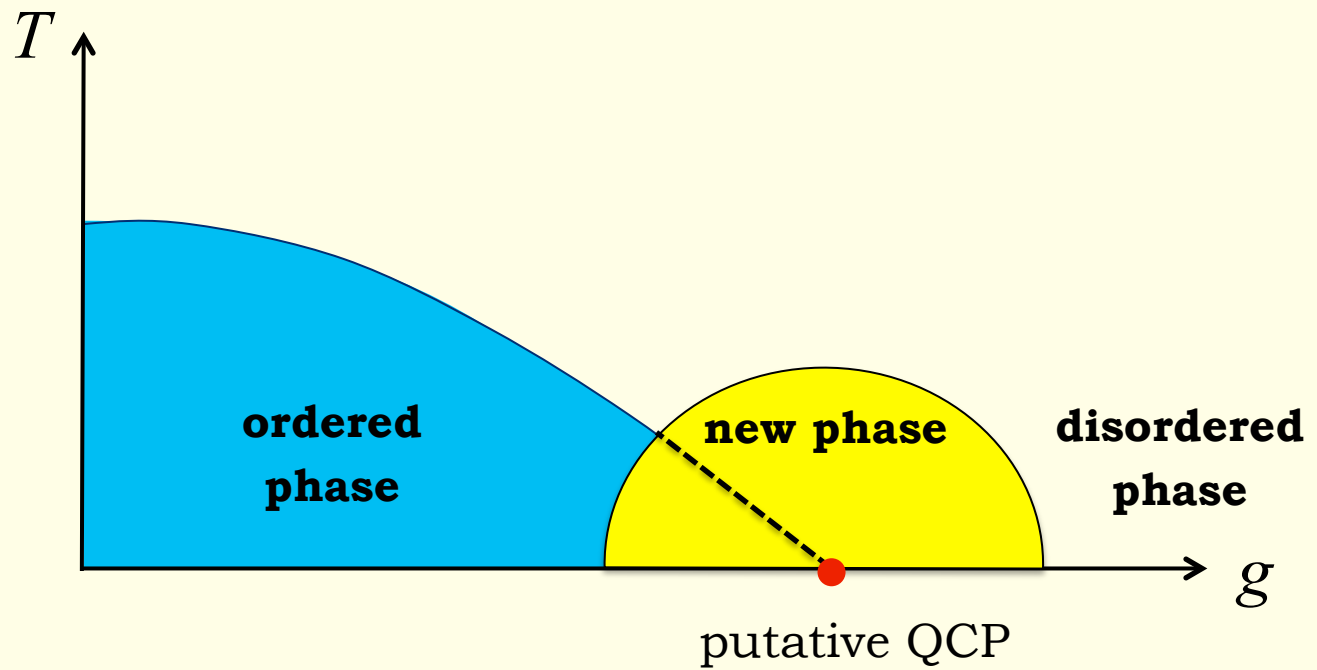


*scale invariance: critical exponents*



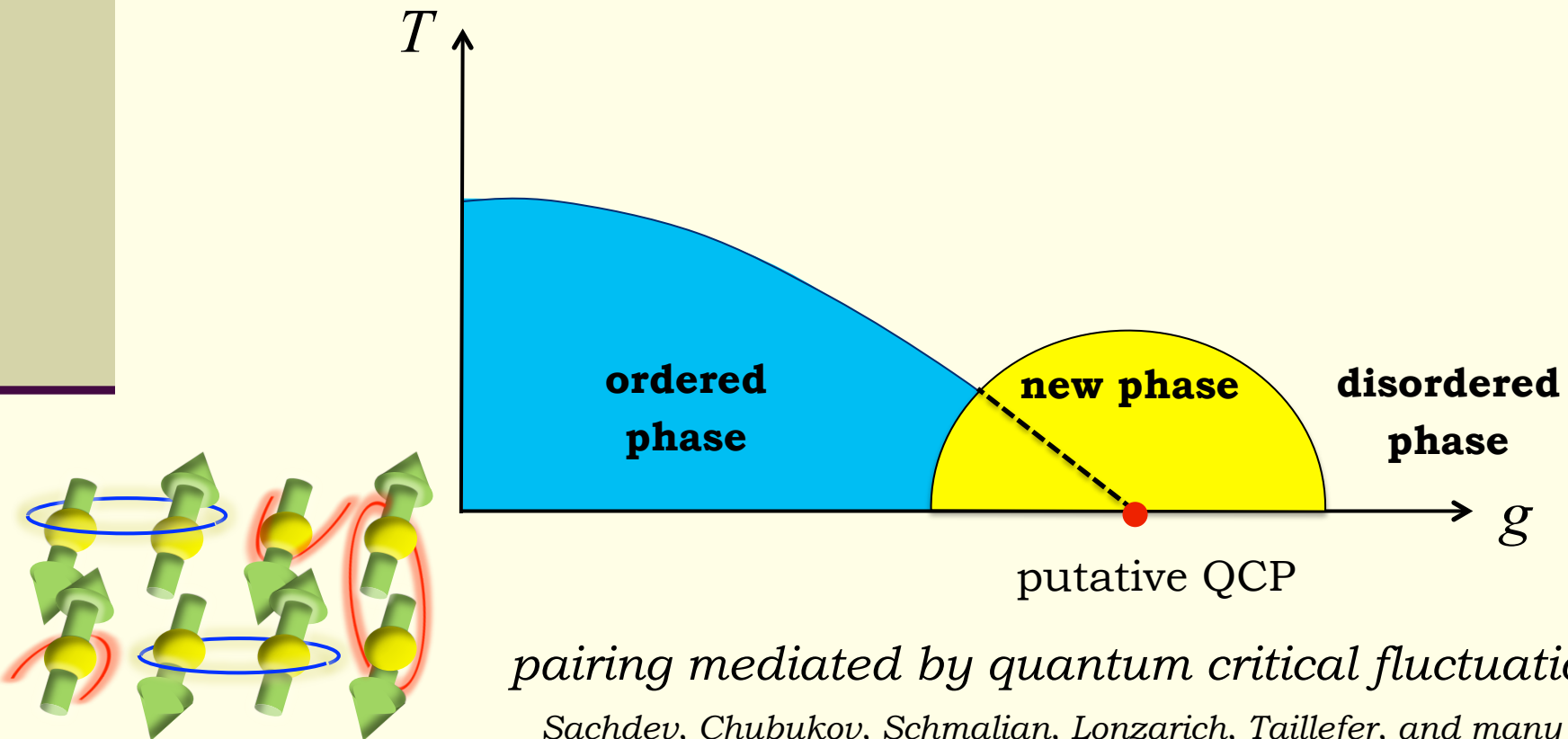
# Quantum phase transitions in metals

- Unusual behavior near the onset of putative QCPs: non-Fermi liquid behavior, **unconventional SC**,...



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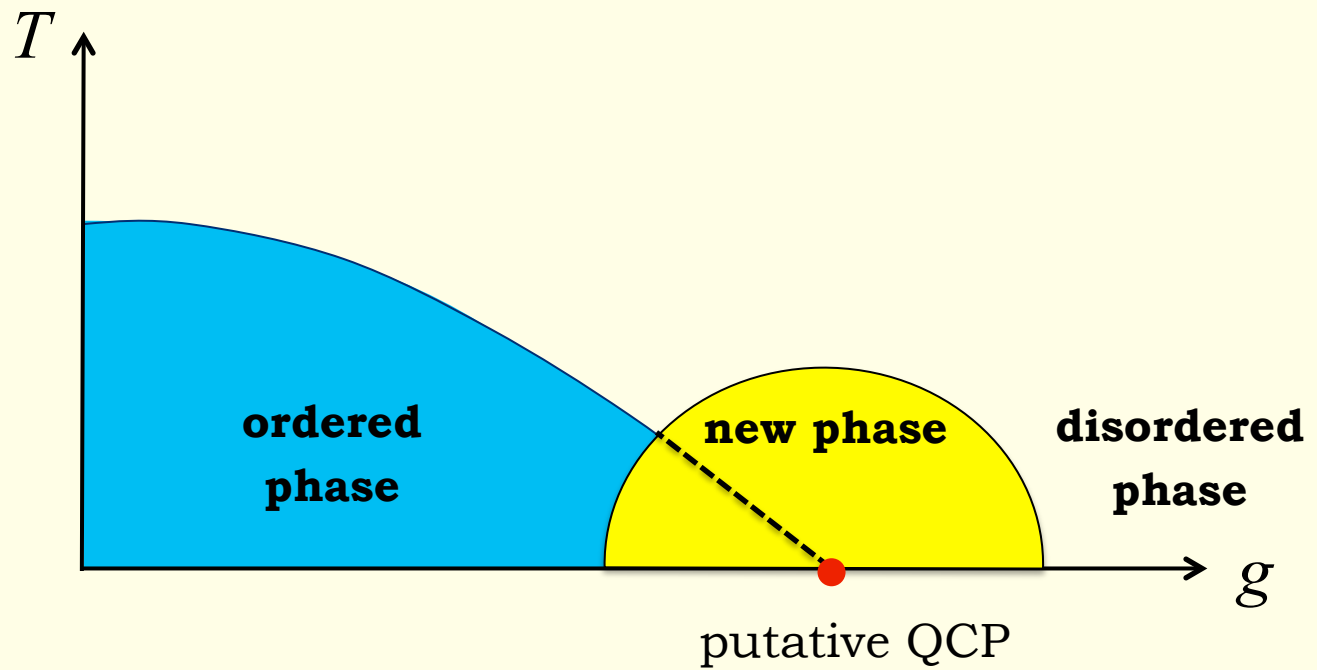


*pairing mediated by quantum critical fluctuations?*

*Sachdev, Chubukov, Schmalian, Lonzarich, Taillefer, and many others...*

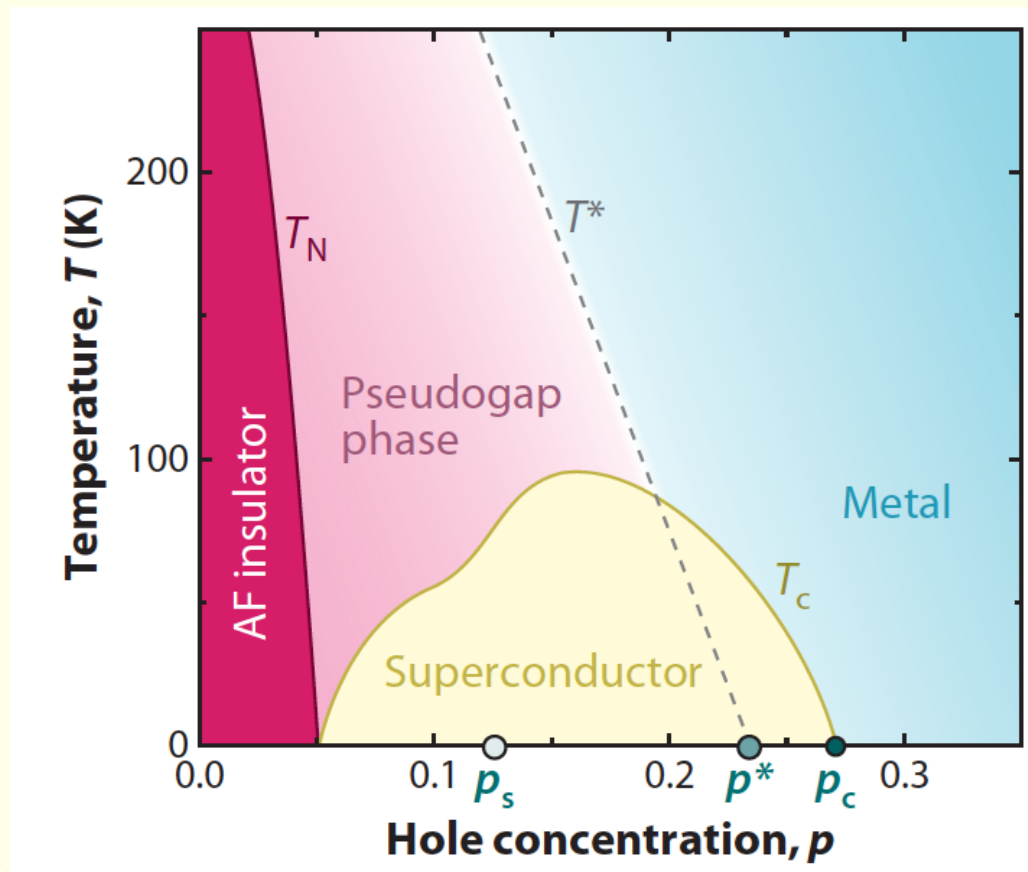
# Quantum phase transitions in metals

- However, the phases are usually associated with different broken symmetries and **compete** with each other



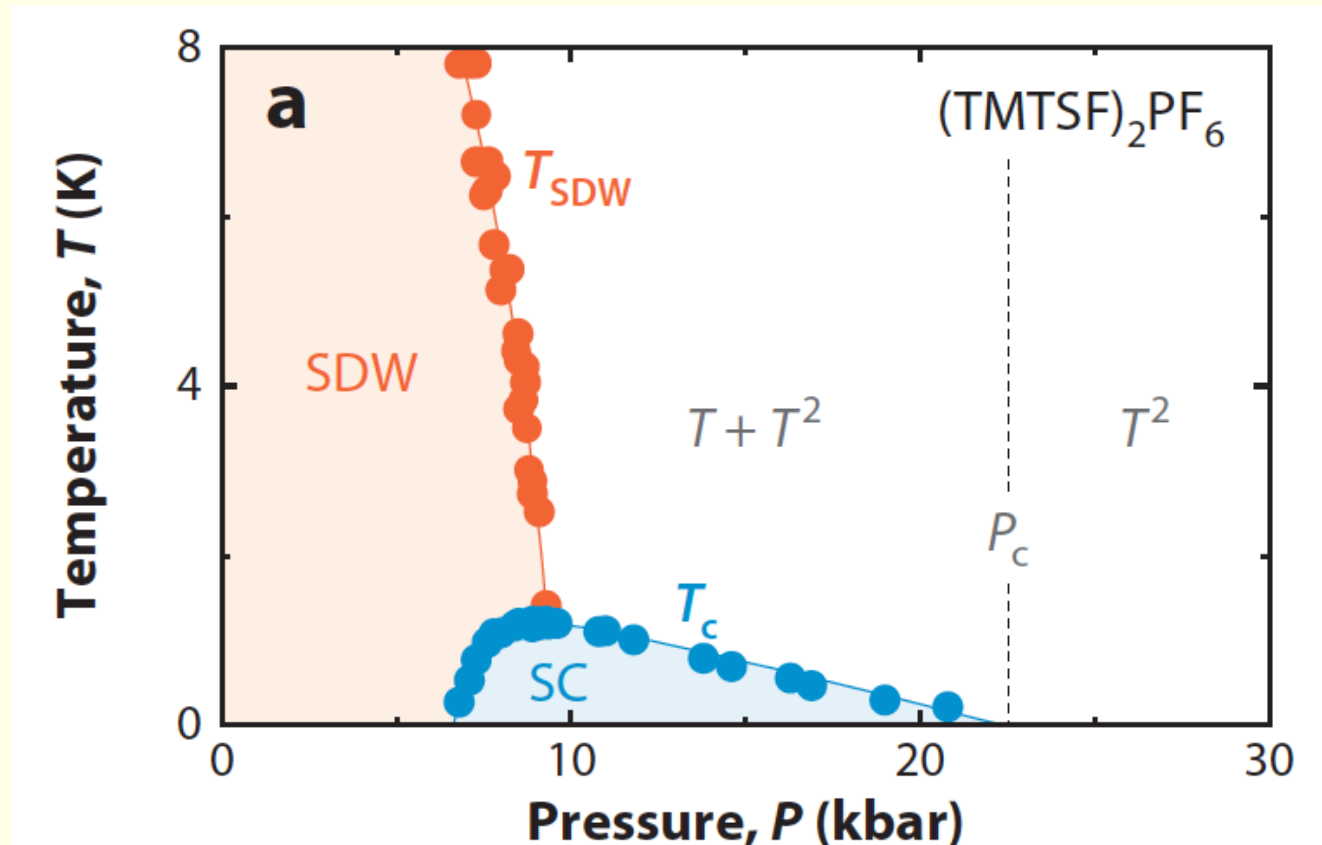
# Quantum phase transitions in metals

- Competing phases: examples in real materials
  - cuprates



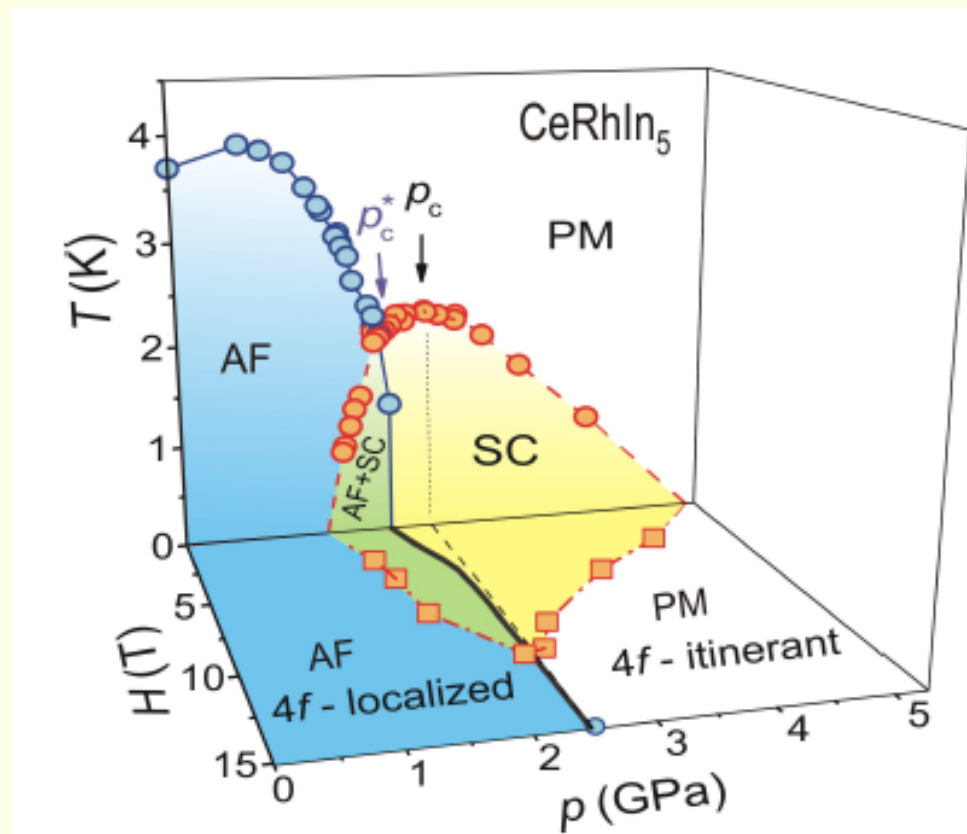
# Quantum phase transitions in metals

- Competing phases: examples in real materials
  - organics



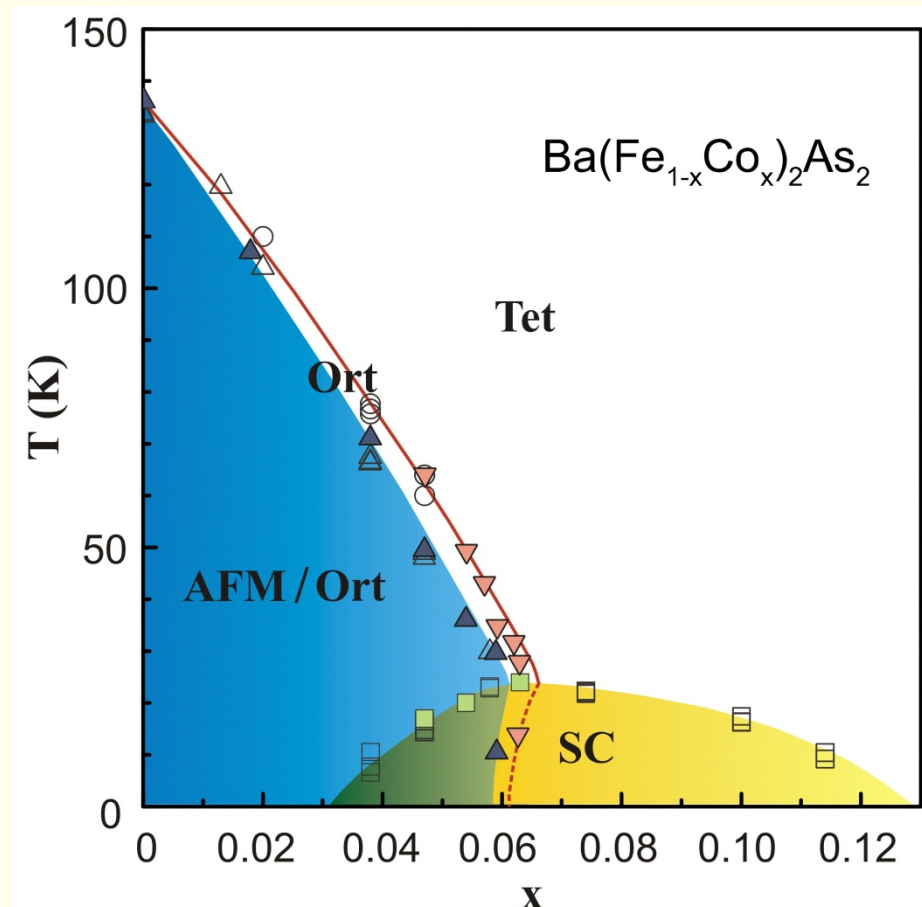
# Quantum phase transitions in metals

- Competing phases: examples in real materials
  - heavy fermions



# Quantum phase transitions in metals

- Competing phases: examples in real materials
  - iron pnictides



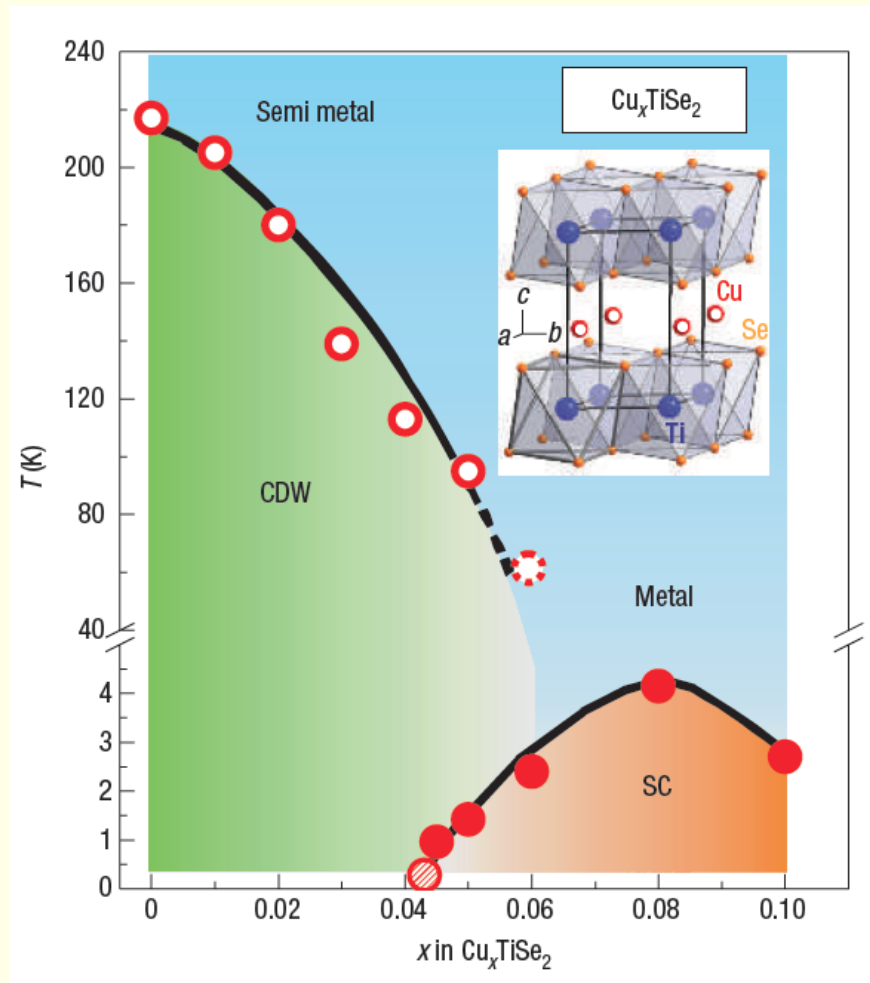
*Ni et al, PRB (2008)*

*RMF et al, PRB (2010)*

*Nandi et al, PRL (2010)*

# Quantum phase transitions in metals

- Competing phases: examples in real materials
  - dichalcogenides





# Outline

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- **Brief introduction to the iron pnictides**
  - experimental evidence for competing phases
- **Competition between magnetism and superconductivity**
  - symmetry of the Cooper pair wave function
- **Competition between nematicity and superconductivity**
  - indirect competition mediated by magnetism

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  - experimental evidence for competing phases
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## Collaborators:



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(Karlsruhe)



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(Madison)



Ilya Eremin  
(Bochum)



Elihu Abrahams  
(UCLA)



Saurabh Maiti  
(Madison)



Andy Millis  
(Columbia)

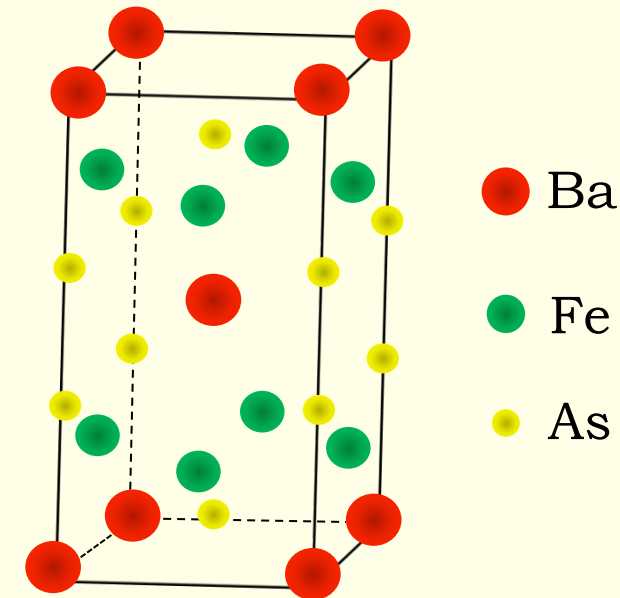
# Iron pnictides

- Layered materials: transition metal (**Fe**) plus pnictogen (nitrogen group, such as **As**)

### Superconducting Elements

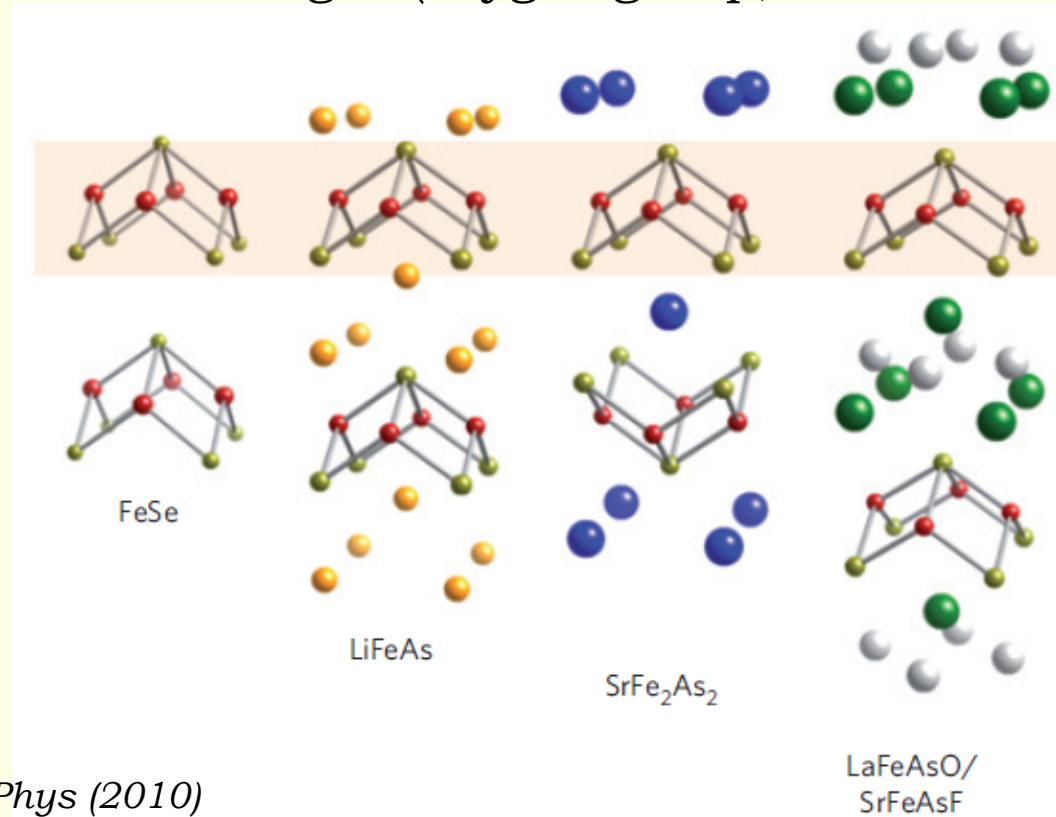
■ In Bulk at Ambient Pressure  
■ At High Pressure  
■ In Modified Form

1	H																	2	He																
3	Li	4	Be																	10	Ne														
11	Na	12	Mg																	18	Ar														
19	K	20	Ca	21	Sc	22	Ti	23	V	24	Cr	25	Mn	26	Fe	27	Co	28	Ni	29	Cu	30	Zn	31	Ga	32	Ge	33	As	34	Se	35	Br	36	Kr
37	Rb	38	Sr	39	Y	40	Zr	41	Nb	42	Mo	43	Tc	44	Ru	45	Rh	46	Pd	47	Ag	48	Cd	49	In	50	Sn	51	Sb	52	Te	53	I	54	Xe
55	Cs	56	Ba	57	La	58	Ce	59	Pr	60	Nd	61	Pm	62	Sm	63	Eu	64	Gd	65	Tb	66	Dy	67	Ho	68	Er	69	Tm	70	Yb	71	Lu		
87	Fr	88	Ra	89	Ac	90	Th	91	Pa	92	U	93	Np	94	Pu	95	Am	96	Cm	97	Bk	98	Cf	99	Es	100	Fm	101	Md	102	No	103	Lr		



# Iron pnictides

- Layered materials: transition metal (**Fe**) plus pnictogen (nitrogen group, such as **As**)
  - several families
  - also with chalcogen (oxygen group, such as **Se**).



- Rise of the iron pnictides

**J|A|C|S**  
COMMUNICATIONS

Published on Web 02/23/2008

**Iron-Based Layered Superconductor  $\text{La}[\text{O}_{1-x}\text{F}_x]\text{FeAs}$  ( $x = 0.05\text{--}0.12$ )  
with  $T_c = 26\text{ K}$**

Yoichi Kamihara,<sup>\*,†</sup> Takumi Watanabe,<sup>‡</sup> Masahiro Hirano,<sup>†,§</sup> and Hideo Hosono<sup>†,‡,§</sup>

- Rise of the iron pnictides

**J|A|C|S**  
COMMUNICATIONS

nature

Vol 453 | 15 April 2008 | doi:10.1038/nature06972

LETTERS

2)

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**Superconductivity at 43 K in an iron-based layered compound  $\text{LaO}_{1-x}\text{F}_x\text{FeAs}$**

Hiroki Takahashi<sup>1</sup>, Kazumi Igawa<sup>1</sup>, Kazunobu Arii<sup>1</sup>, Yoichi Kamihara<sup>2</sup>, Masahiro Hirano<sup>2,3</sup> & Hideo Hosono<sup>2,3</sup>

- Rise of the iron pnictides

**J|A|C|S**  
COMMUNICATIONS

nature

Vol 453|15 April 2008|doi:10.1038/nature06972

LETTERS

2)

CHIN.PHYS.LETT.

Vol. 25, No. 6 (2008) 2215

**Superconductivity at 55 K in Iron-Based F-Doped Layered Quaternary  
Compound  $\text{Sm}[\text{O}_{1-x}\text{F}_x]\text{FeAs}$  \***

REN Zhi-An(任治安)\*\*, LU Wei(陆伟), YANG Jie(杨杰), YI Wei(衣玮), SHEN Xiao-Li(慎晓丽),  
LI Zheng-Cai(李正才), CHE Guang-Can(车广灿), DONG Xiao-Li(董晓莉), SUN Li-Ling(孙力玲),  
ZHOU Fang(周放), ZHAO Zhong-Xian(赵忠贤)\*\*\*



- Rise of the iron pnictides

**J|A|C|S**  
COMMUNICATIONS

nature

Vol 453|15 April 2008|doi:10.1038/nature06972

LETTERS

2)

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A LETTERS JOURNAL EXPLORING  
THE FRONTIERS OF PHYSICS

September 2008

EPL, 83 (2008) 67006

[www.epljournal.org](http://www.epljournal.org)

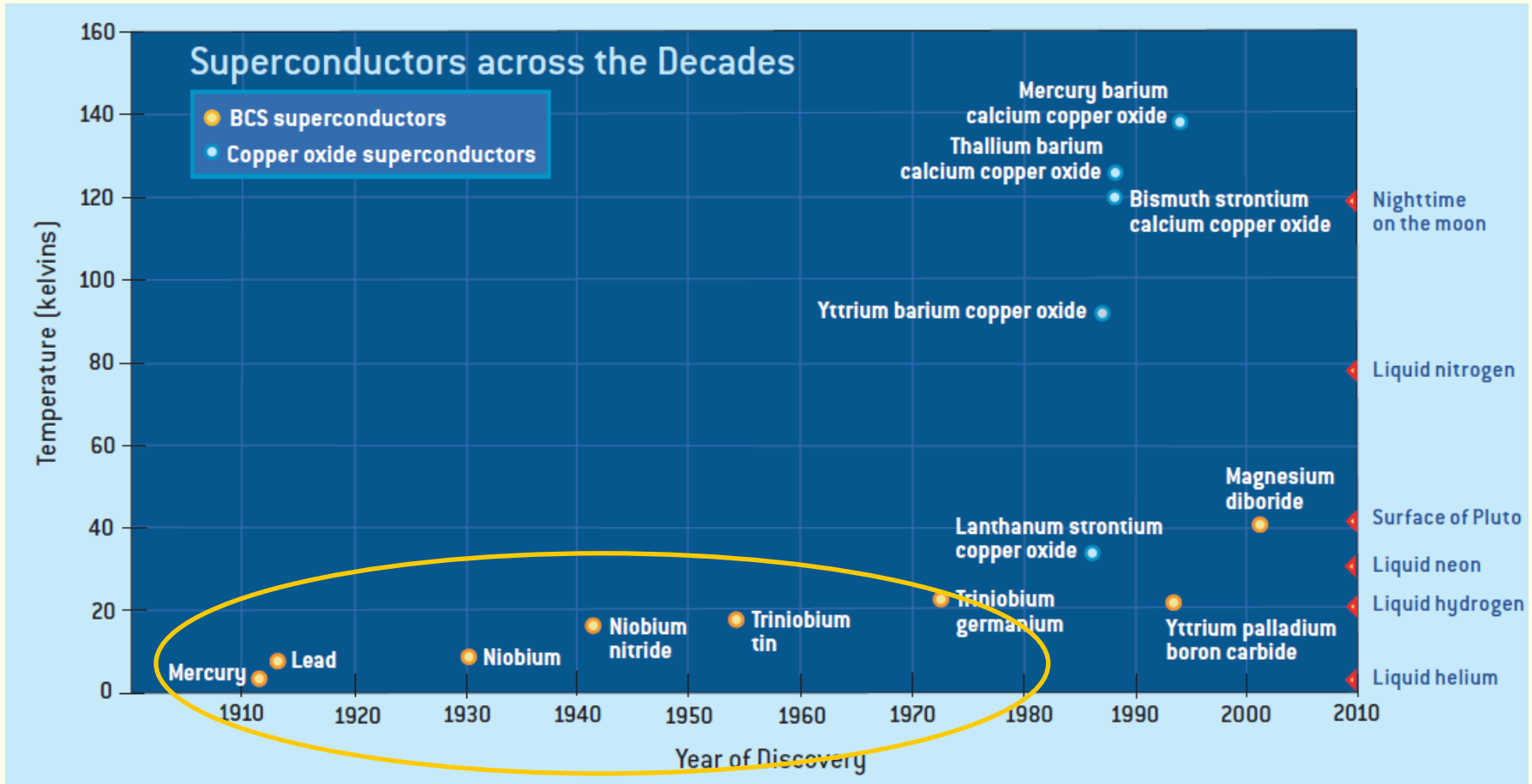
doi: 10.1209/0295-5075/83/67006

## Thorium-doping–induced superconductivity **up to 56 K** in $\text{Gd}_{1-x}\text{Th}_x\text{FeAsO}$

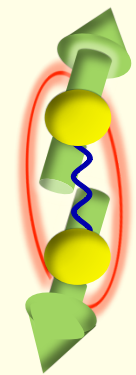
CAO WANG, LINJUN LI, SHUN CHI, ZENGWEI ZHU, ZHI REN, YUKE LI, YUETAO WANG, XIAO LIN,  
YONGKANG LUO, SHUAI JIANG, XIANGFAN XU, GUANGHAN CAO<sup>(a)</sup> and ZHU'AN XU<sup>(b)</sup>

# Superconductivity reaches the iron age!

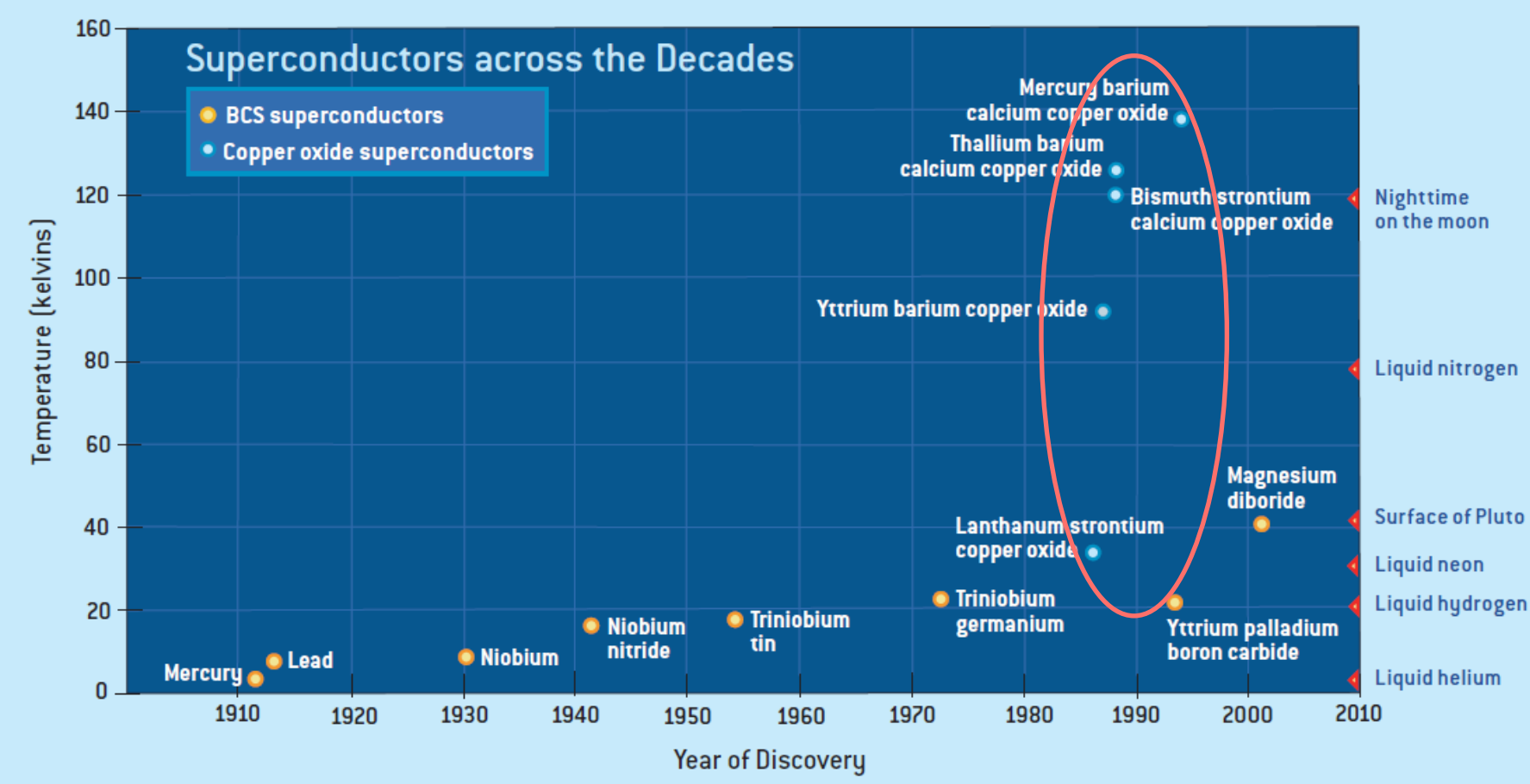
Canfield et al, Sci. Am. (2005)



stone age



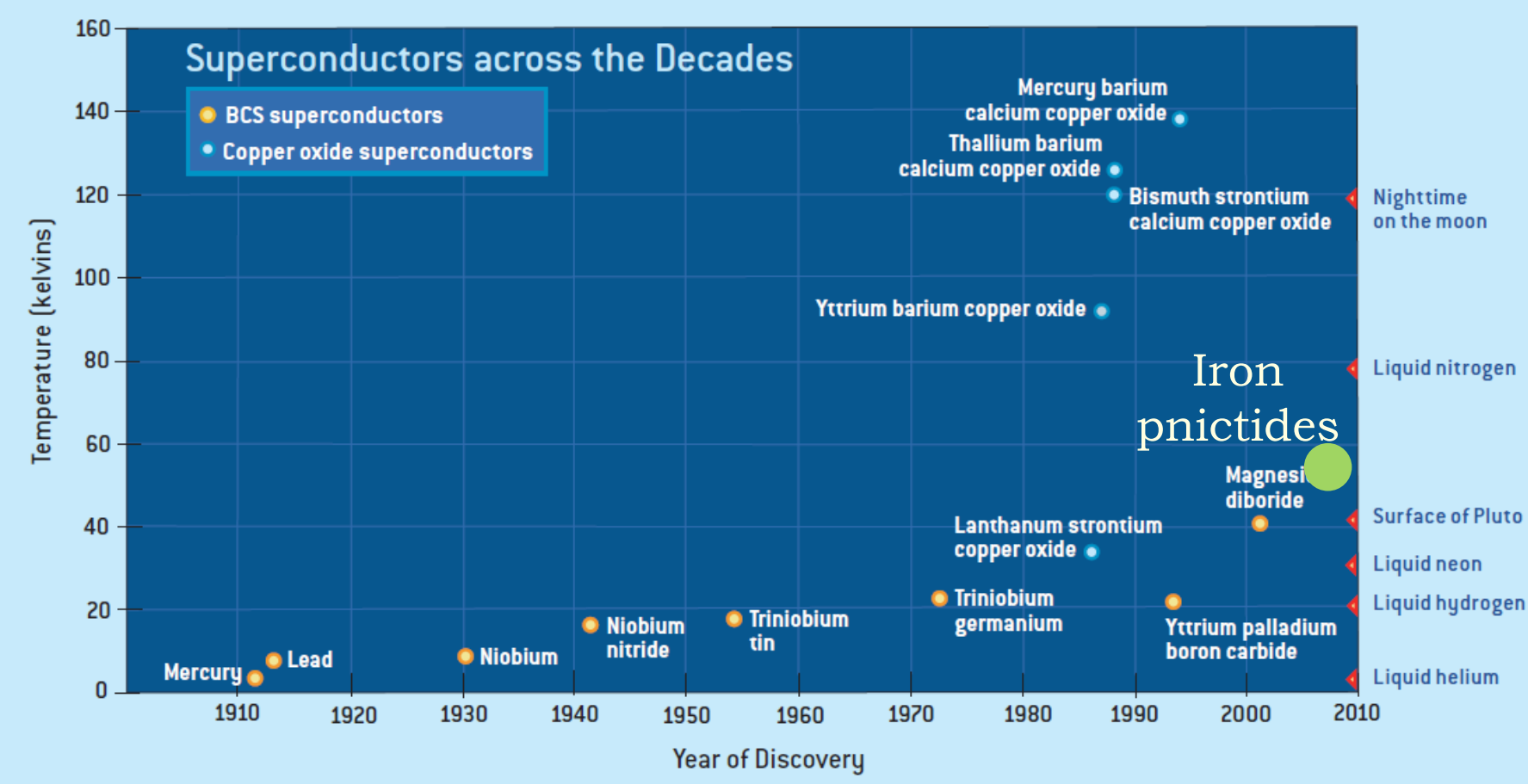
# Superconductivity reaches the iron age!



stone age → copper age



# Superconductivity reaches the iron age!



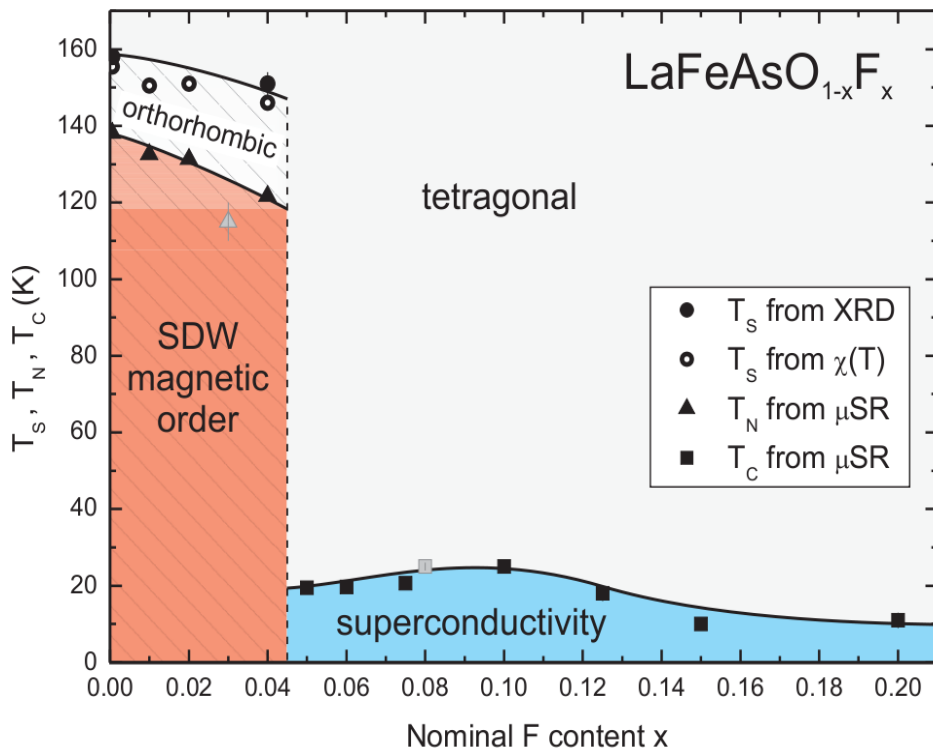
stone age → copper age → iron age

*first high-temperature superconductors since the cuprates!*

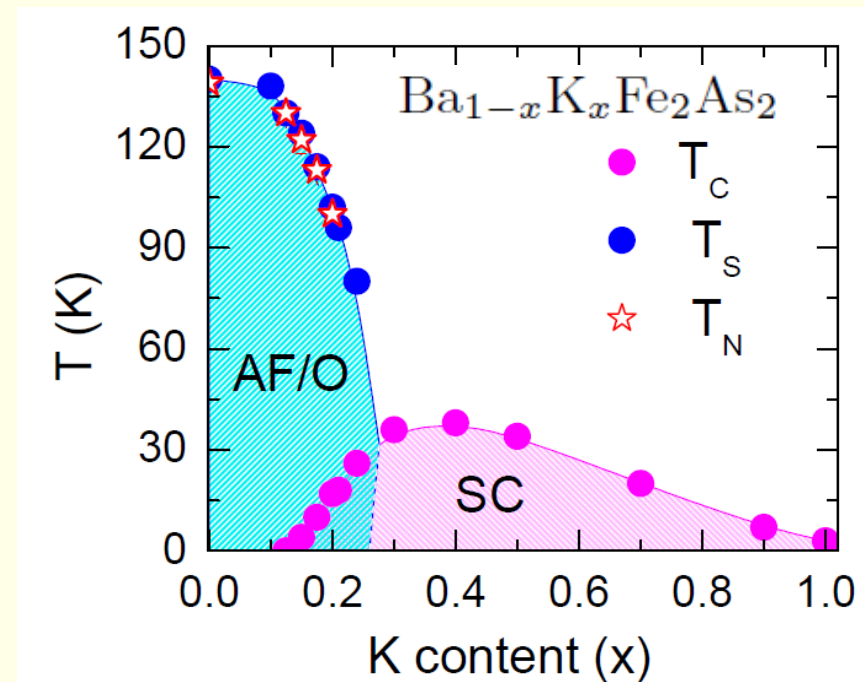


# Iron pnictides: phase diagrams

**Interplay between magnetic, superconducting and elastic degrees of freedom**



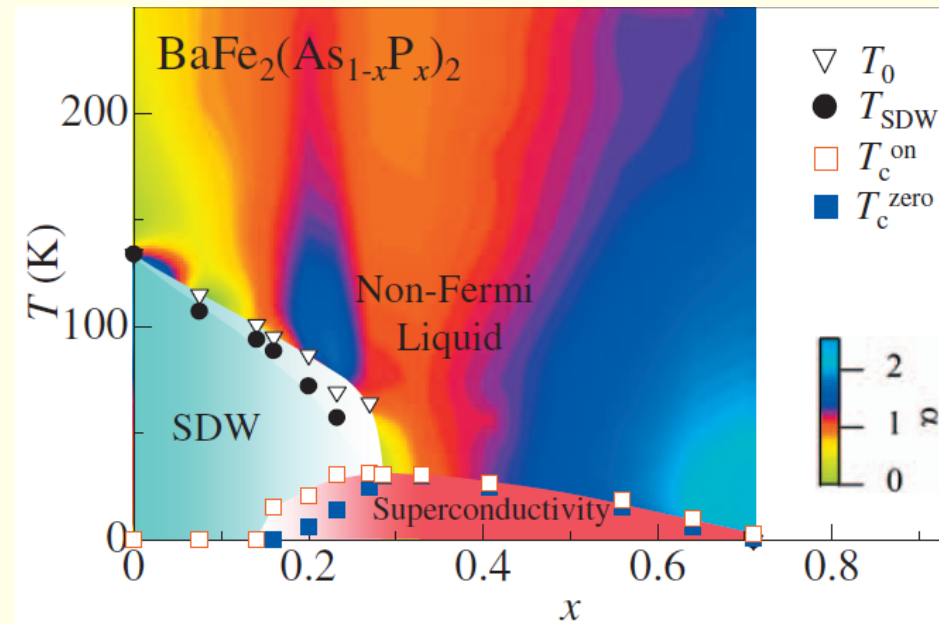
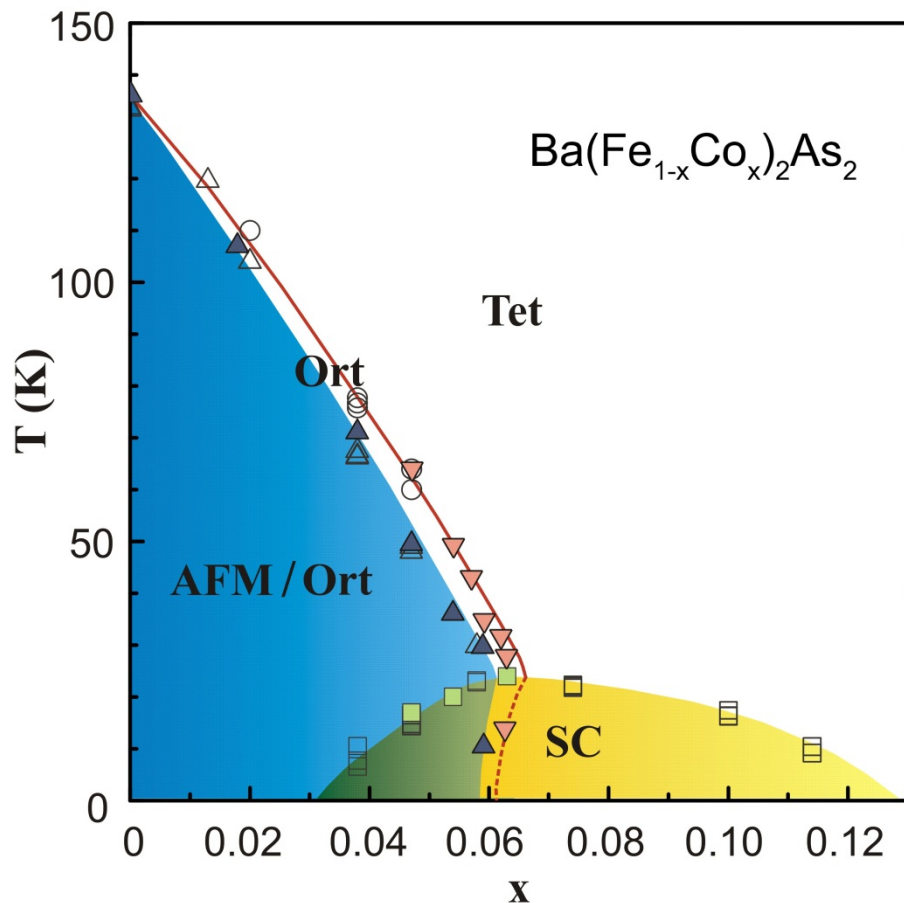
*Luetkens et al, Nature Mater. (2008)*



*Avci et al, PRB (2011)*

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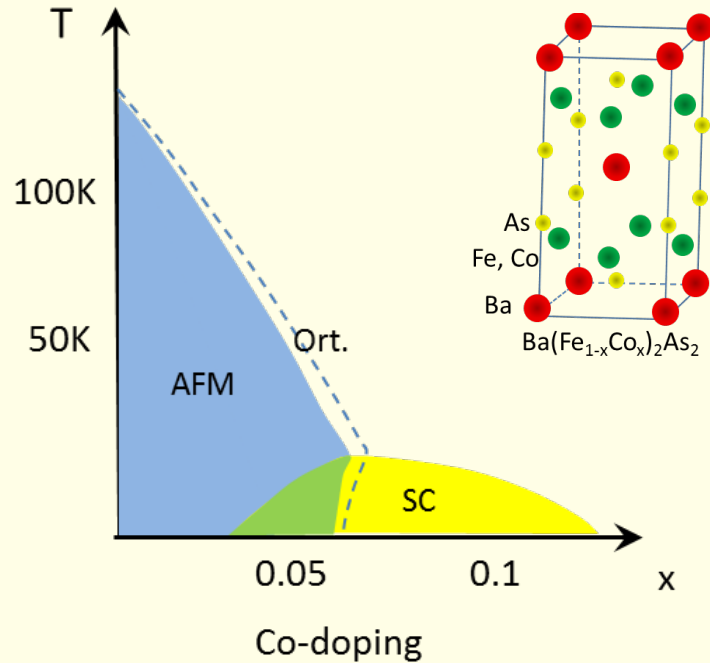


*Kasahara et al, PRB (2010)*

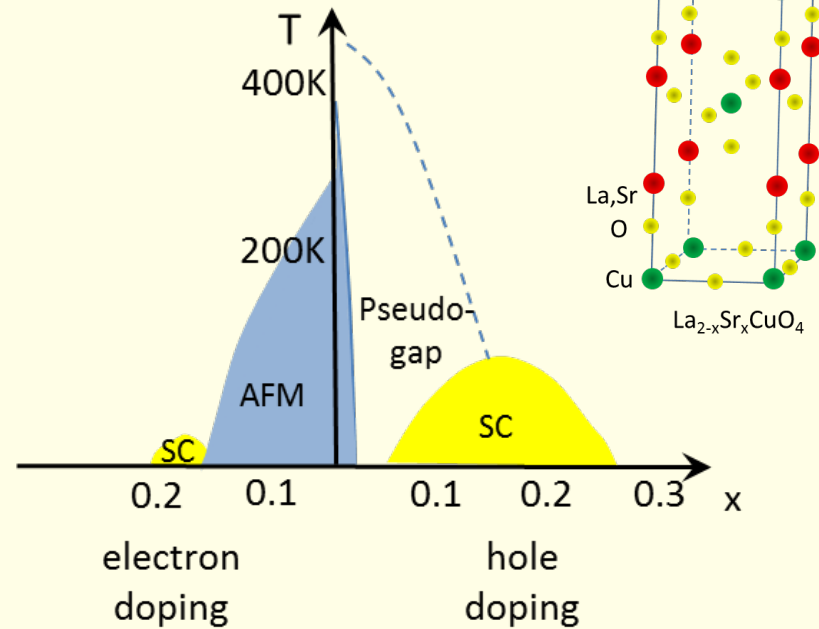
*Ni et al, PRB (2008) RMF et al, PRB (2010)*  
*Pratt et al, PRL (2009) Nandi et al, PRL (2010)*

# Iron arsenides vs cuprates

## Iron arsenides



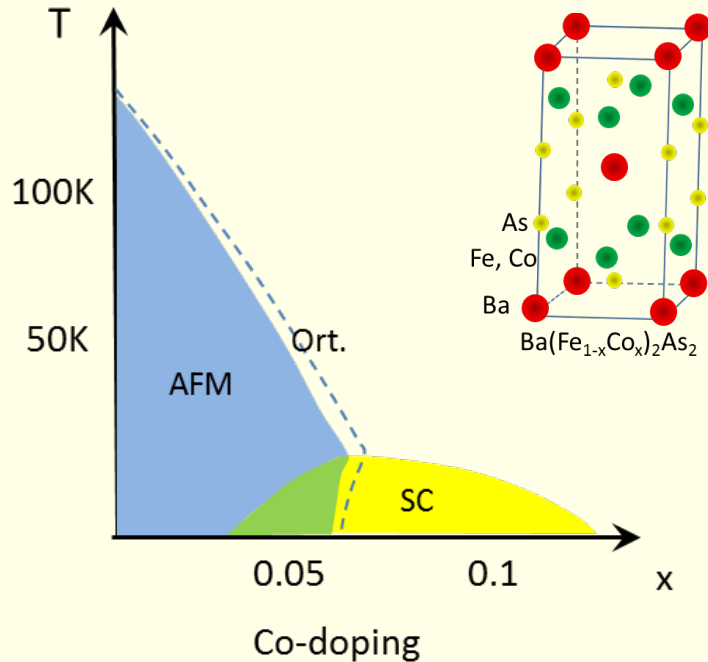
## Cuprates





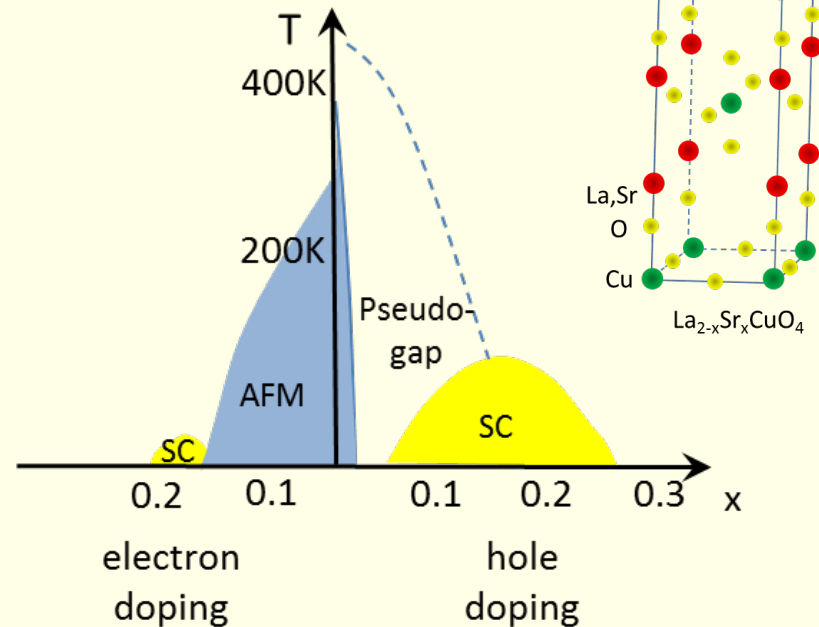
# Iron arsenides *vs* cuprates

## Iron arsenides



- 3d electrons
- layered materials
- high values of  $T_c$
- unconventional SC (?) close to AFM transition

## Cuprates

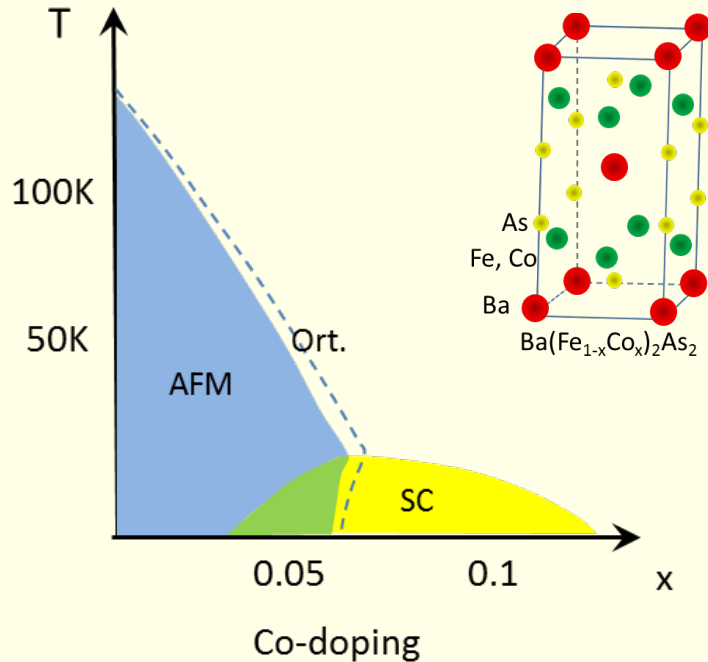


- 3d electrons
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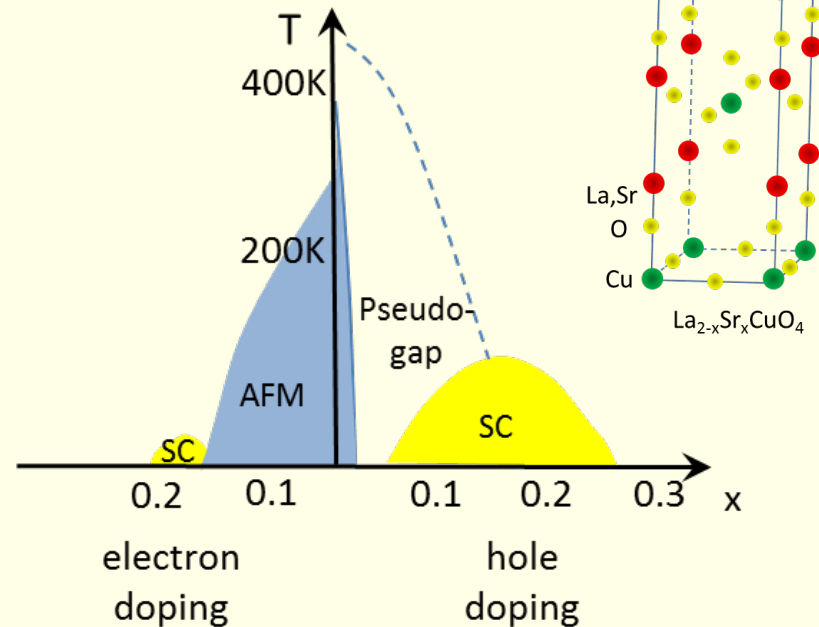
# Iron arsenides *vs* cuprates

## Iron arsenides



- parent compound is metallic
- five iron bands with  $n = 6$
- itinerant magnetism

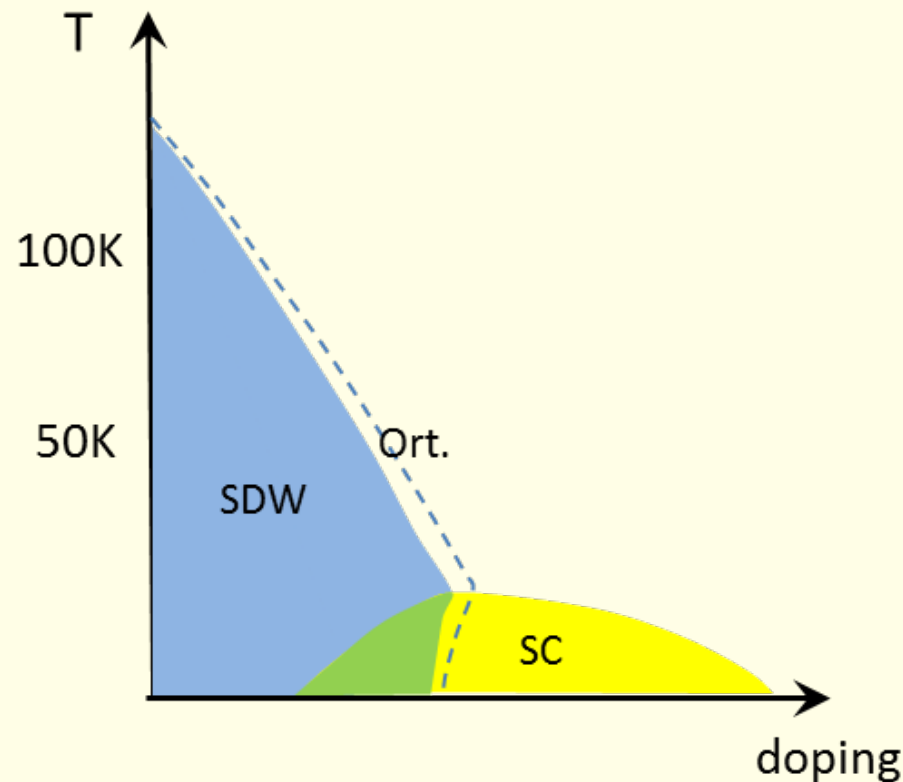
## Cuprates



- parent compound is a Mott insulator
- one copper band with  $n = 1$
- localized magnetism

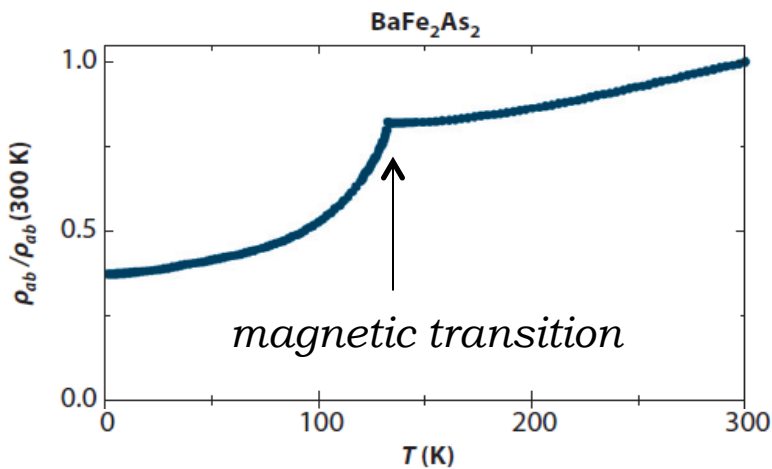
# Iron pnictides: typical phase diagram

*magnetic, structural, and superconducting order*

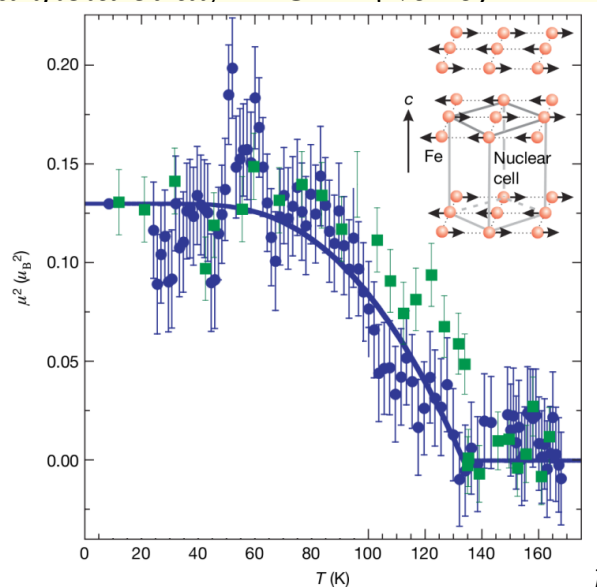


- How these different ordered states interact with each other?
- Is there a primary degree of freedom?

# Iron pnictides: magnetic order

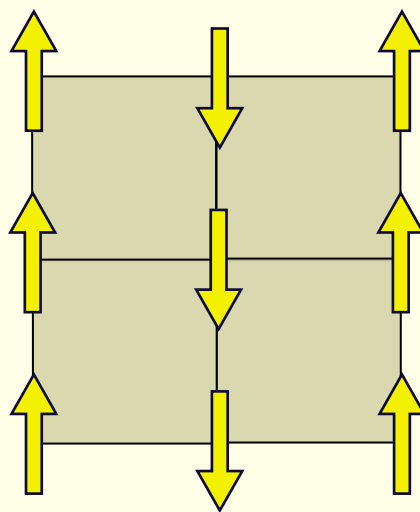


Canfield et al, ARCOMP (2010)

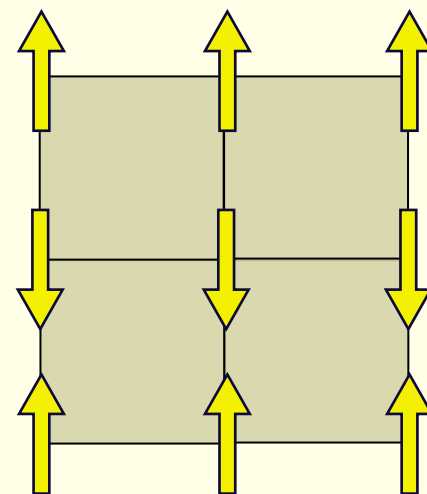


system remains metallic

**two magnetic  
stripe states**



$$Q_1 = (\pi, 0)$$

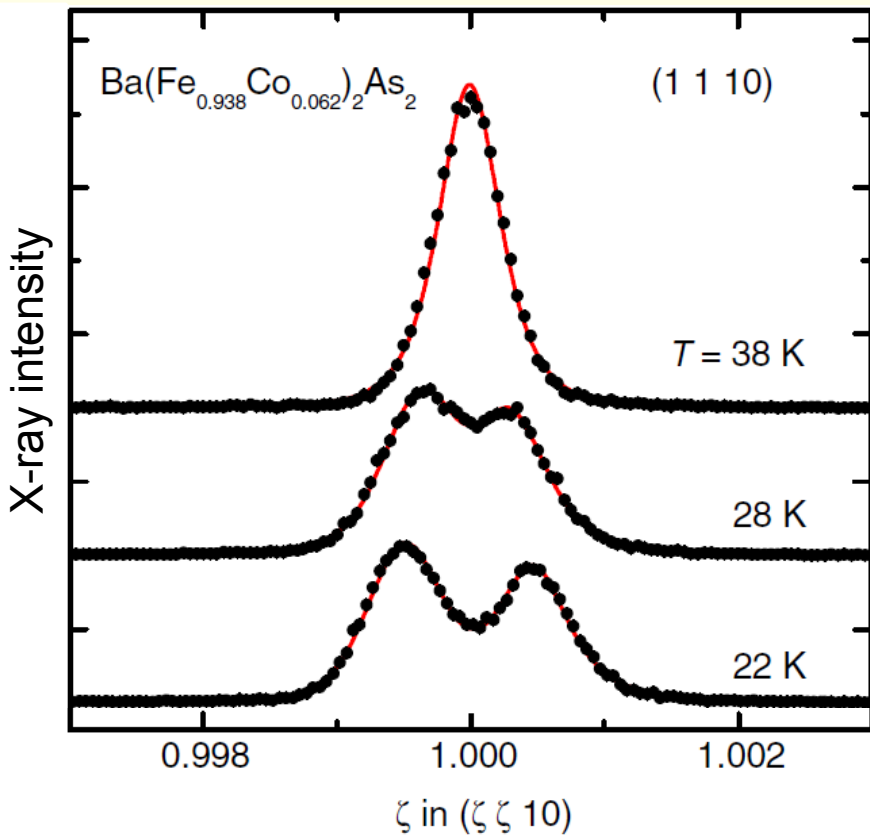


$$Q_2 = (0, \pi)$$

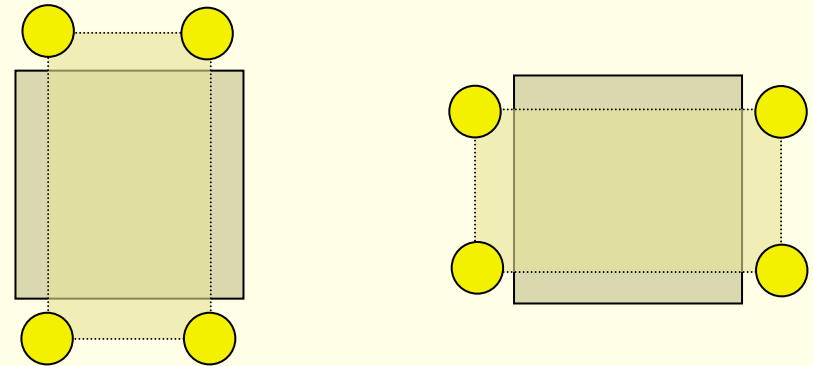
De la Cruz et al, Nature (2008)

# Iron pnictides: structural order

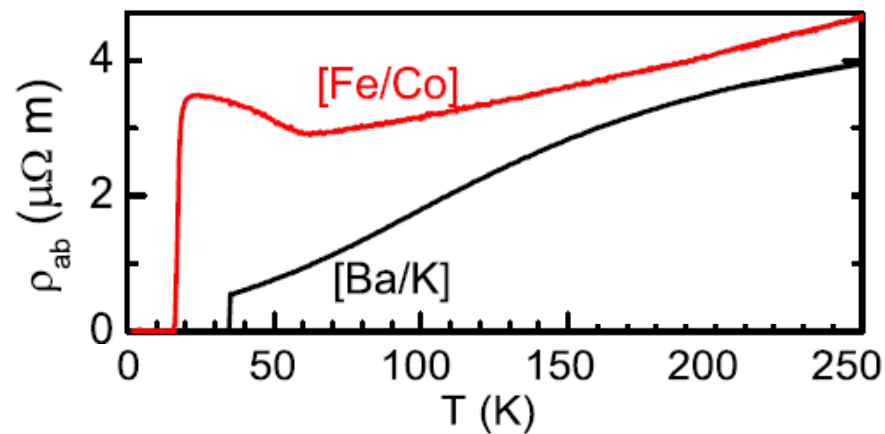
tetragonal to orthorhombic transition



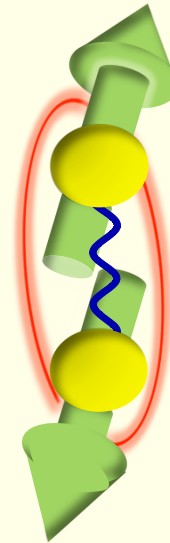
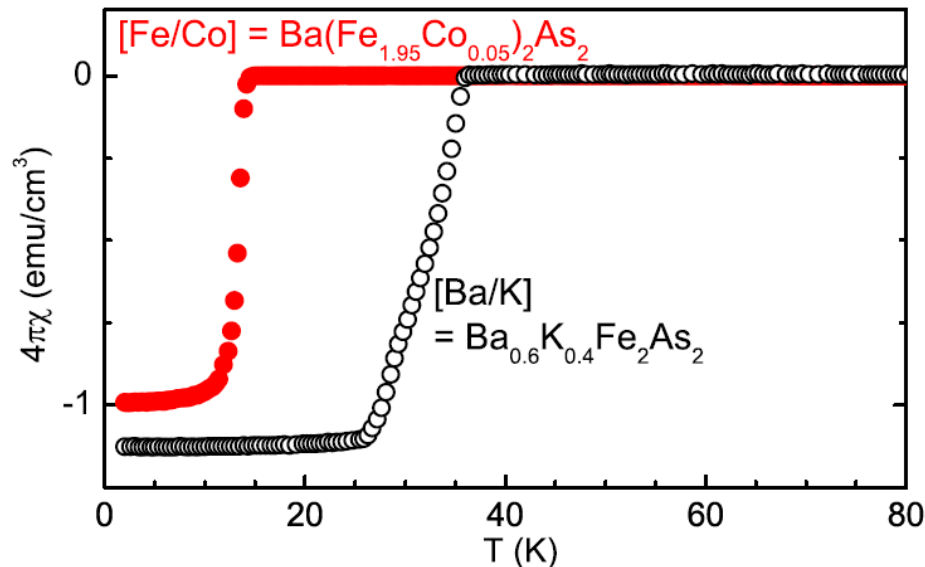
**twin domains**



# Iron pnictides: superconducting order

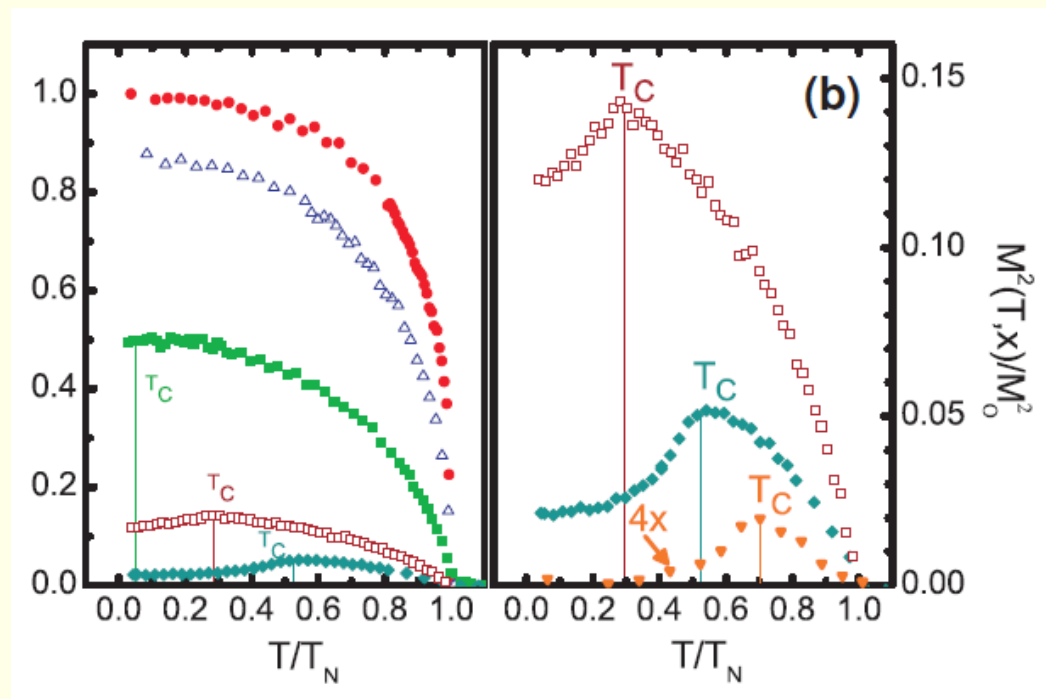
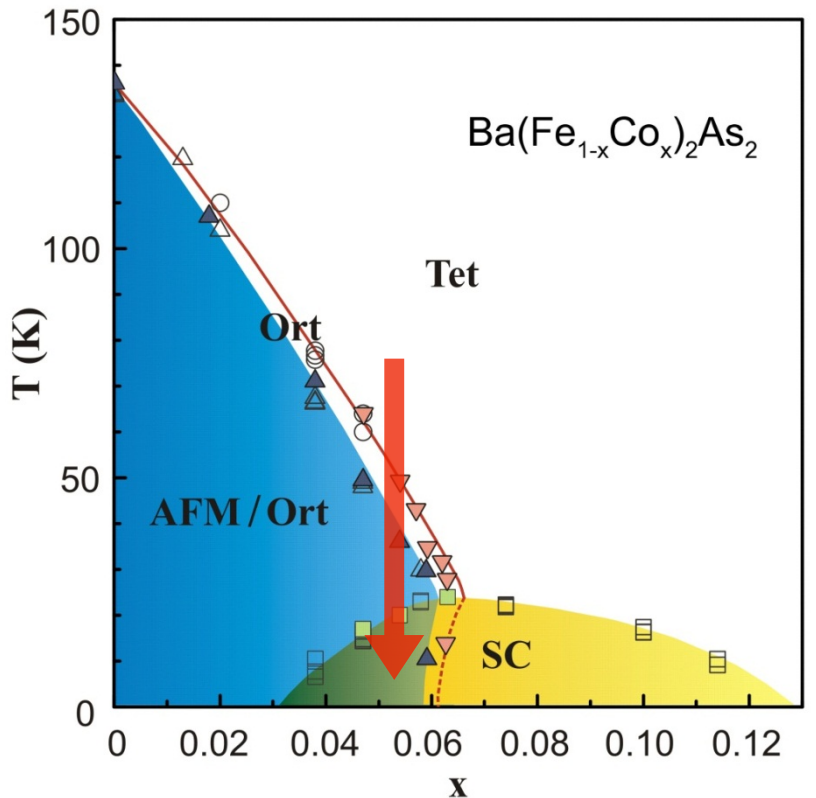


**pairing: conventional (phonons)  
or unconventional (electronic)?**



# Competing phases: experimental observations

- Neutron diffraction: suppression of the **magnetic order parameter** below  $T_c$

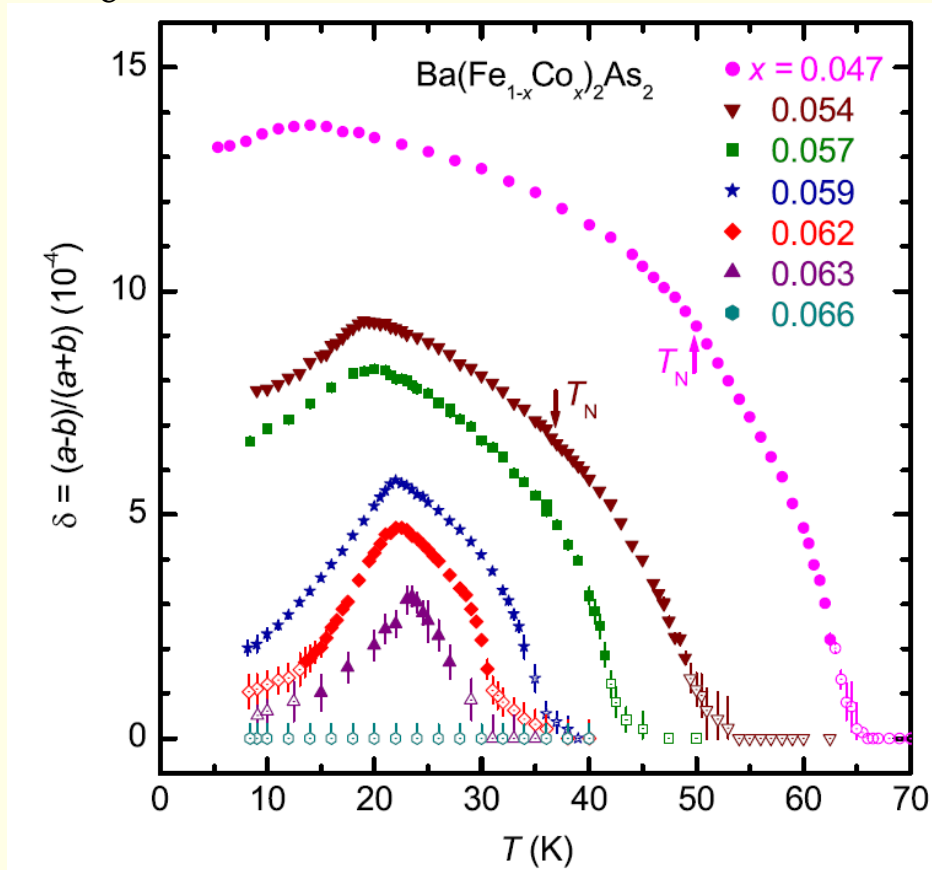
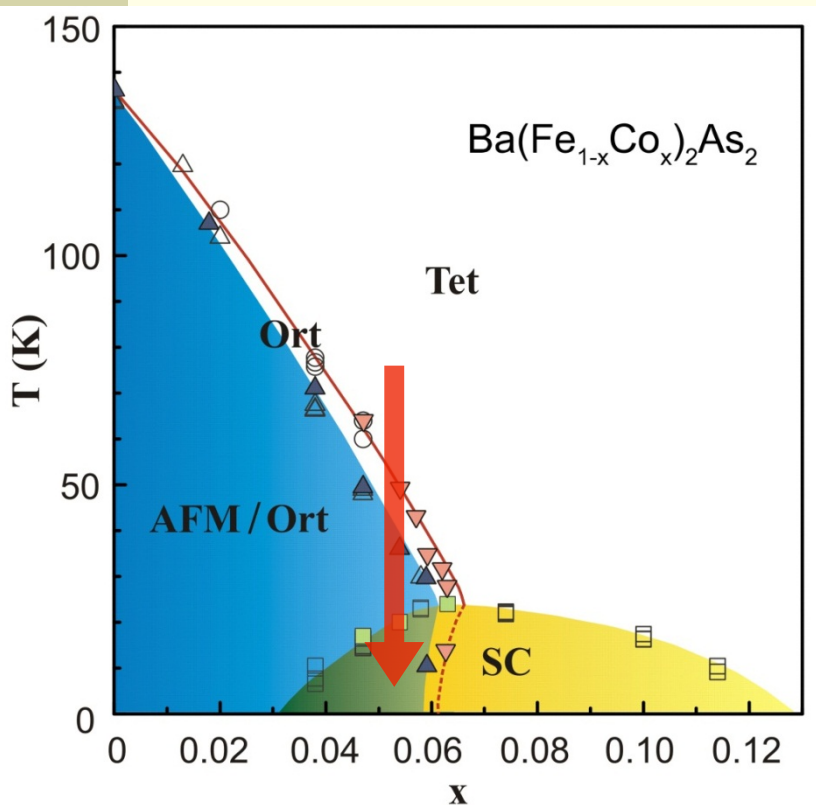


*RMF et al, PRB (2010)*

*Pratt et al, PRL (2009) Christianson et al, PRL (2009)*

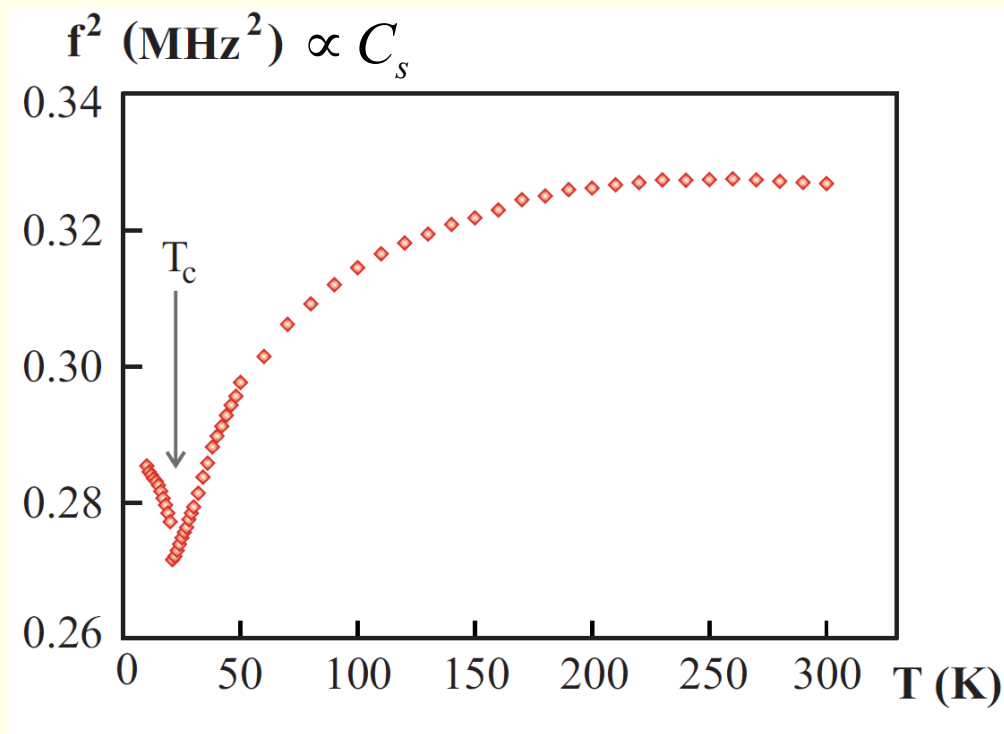
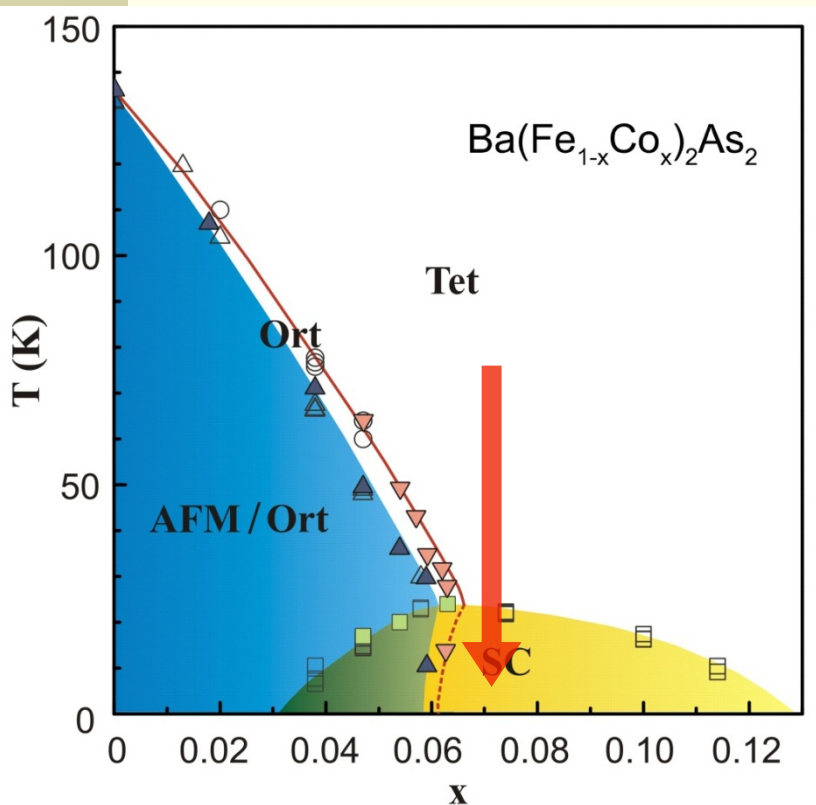
# Competing phases: experimental observations

- X-ray diffraction: suppression of the **orthorhombic order parameter** below  $T_c$



# Competing phases: experimental observations

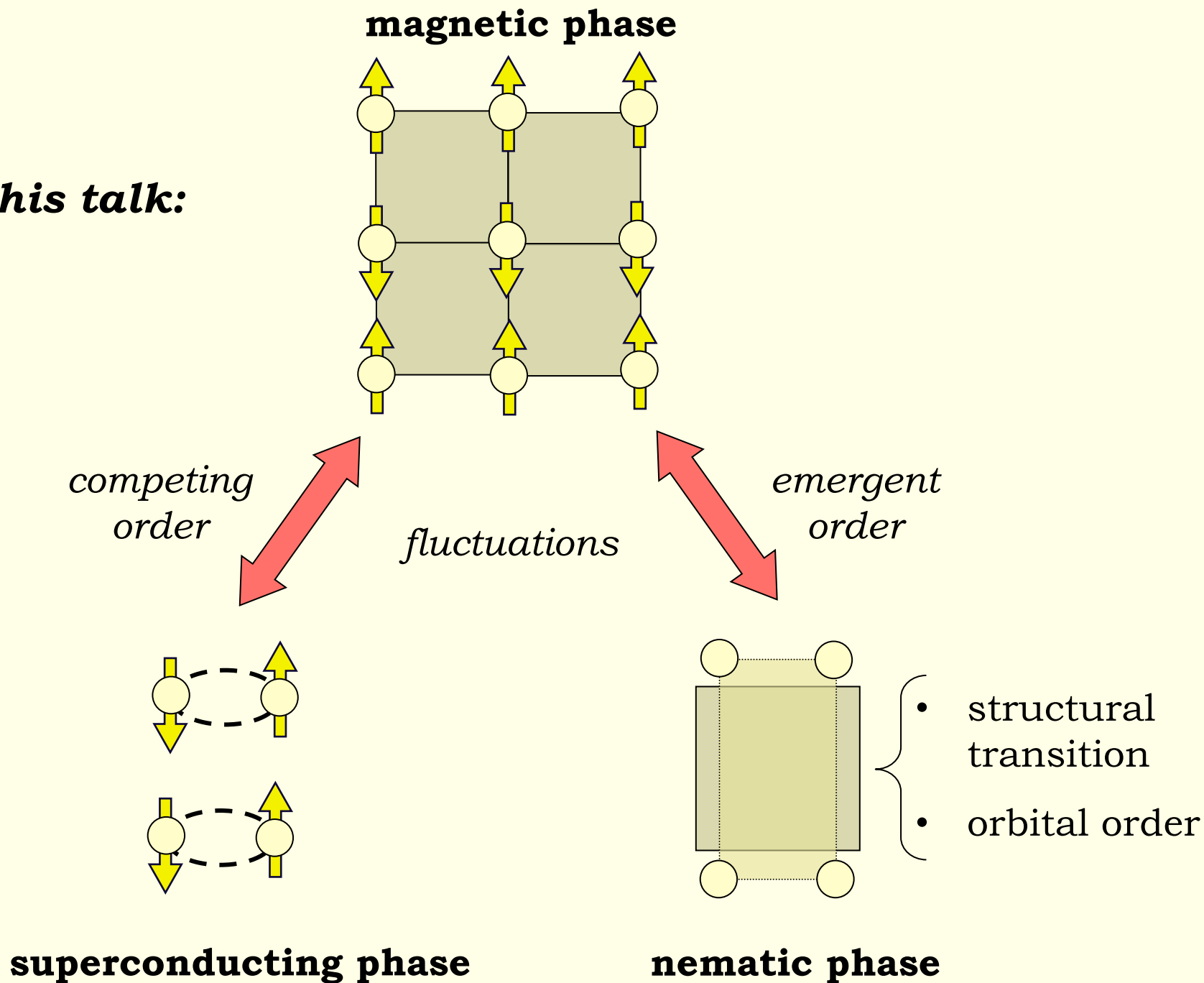
- Ultrasound spectroscopy: hardening of the **shear modulus** below  $T_c$



*RMF et al, PRL (2010)*



***In this talk:***

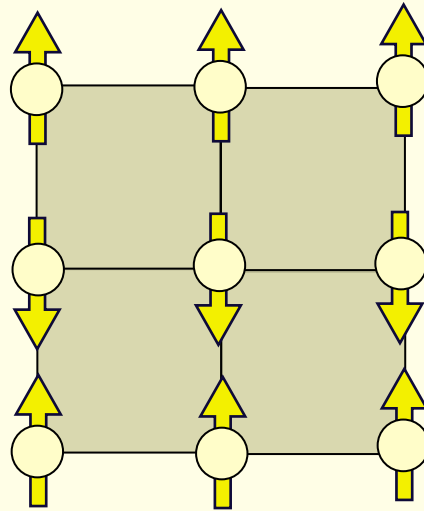


# Outline

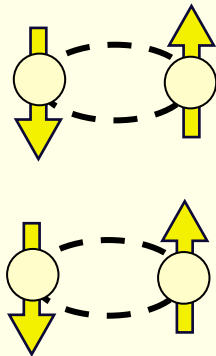
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- **Brief introduction to the iron pnictides**
  - experimental evidence for competing phases
- **Competition between magnetism and superconductivity**
  - symmetry of the Cooper pair wave function
- **Competition between nematicity and superconductivity**
  - indirect competition mediated by magnetism

# antiferromagnetic phase

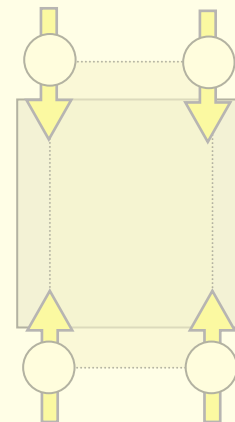
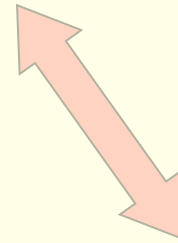


*competing  
order*



# superconducting phase

*emergent  
order*

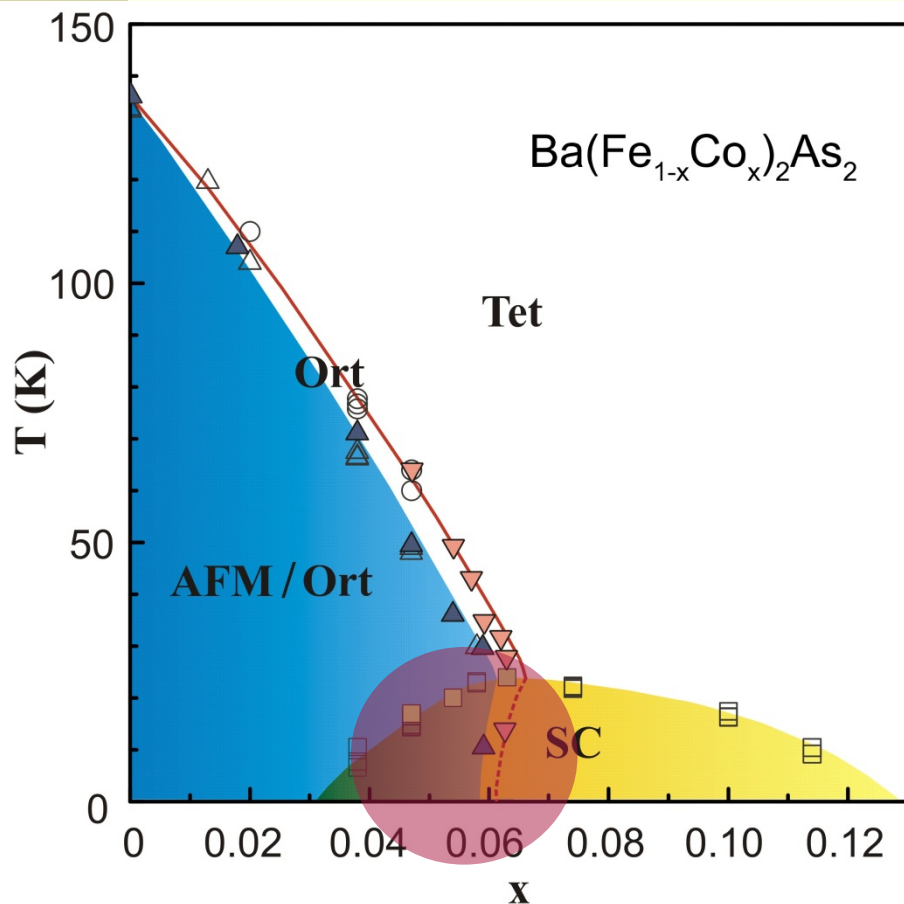


- structural transition
- orbital order

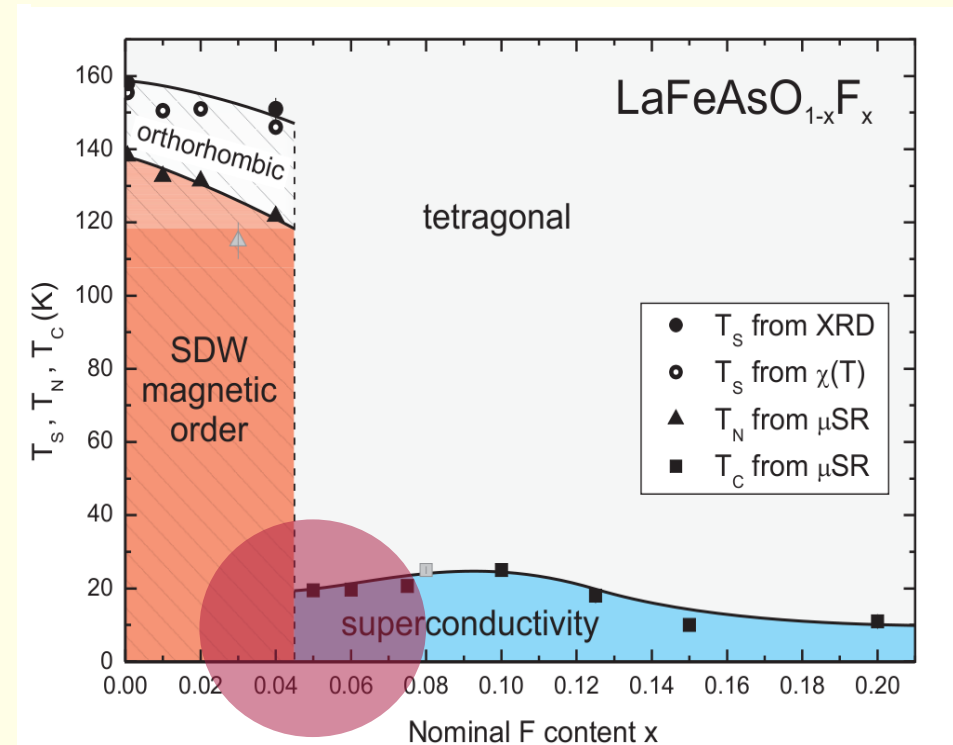
# nematic phase

# Competition between SDW and SC: coexistence or phase separation?

Local probe experiments:



*microscopic coexistence*



*phase separation*

# Competition between SDW and SC: coexistence or phase separation?

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- Bulk measurements and local probes: in some FeAs compounds there is microscopic coexistence and in others, mutual exclusion.

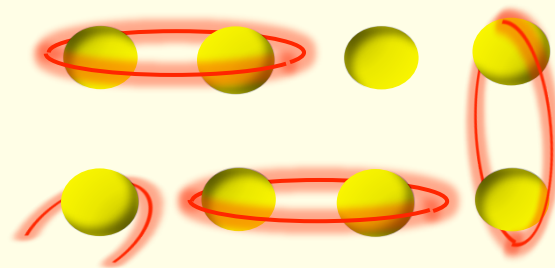
**Microscopic coexistence:**  $\text{Ba}(\text{Fe}_{1-x}\text{Co}_x)_2\text{As}_2$ ;  $\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$ ;  $(\text{Ba}_{1-x}\text{K}_x)\text{Fe}_2\text{As}_2$ ;  $\text{SmFeAs}(\text{O}_{1-x}\text{F}_x)$   $\longrightarrow$  unsettled

**Phase separation:**  $\text{LaFeAs}(\text{O}_{1-x}\text{F}_x)$ ;  $\text{PrFeAs}(\text{O}_{1-x}\text{F}_x)$ ;  $(\text{Sr}_{1-x}\text{Na}_x)\text{Fe}_2\text{As}_2$ ;  $\text{CaFe}_2\text{As}_2$  (pressure)

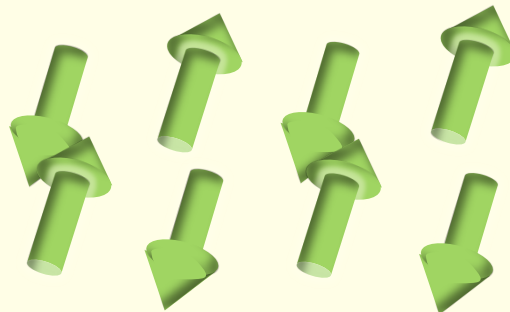
# Competition between SDW and SC: coexistence or phase separation?

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- In some conventional superconductors, magnetism can only coexist with superconductivity when the two phenomena involve *different* electrons



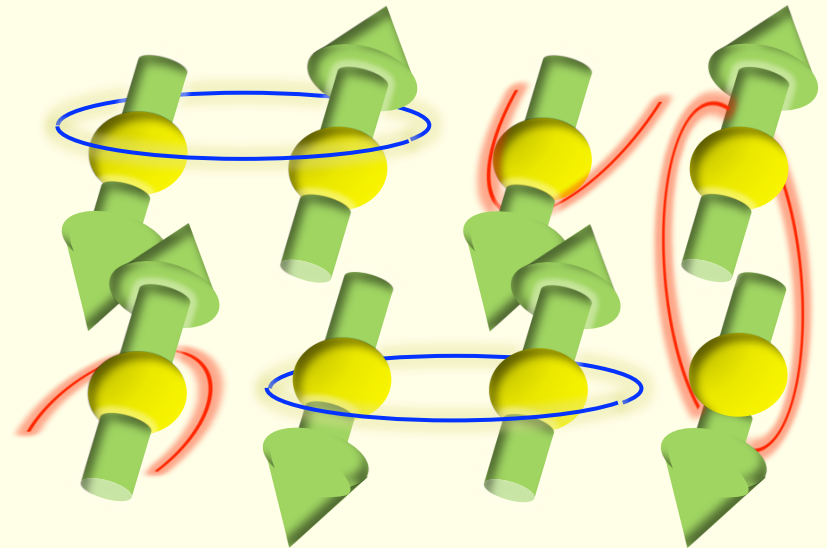
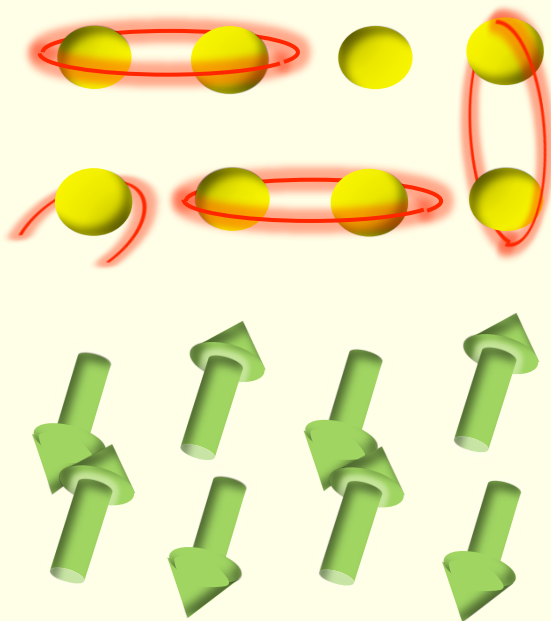
**conduction electrons  
(3d band)**



**localized spins  
(4f band)**

# Competition between SDW and SC: coexistence or phase separation?

- In some conventional superconductors, magnetism can only coexist with superconductivity when the two phenomena involve *different* electrons
  - here, the electrons that cause magnetism are the *same* that cause superconductivity



# Competition between SDW and SC: phenomenological model

---

$$F[M, \Delta] = \frac{a_m}{2} M^2 + \frac{u_m}{4} M^4 + \frac{a_s}{2} |\Delta|^2 + \frac{u_s}{4} |\Delta|^4 + \frac{\gamma}{2} |\Delta|^2 M^2$$



# Competition between SDW and SC: phenomenological model

$$F[M, \Delta] = \frac{a_m}{2} M^2 + \frac{u_m}{4} M^4 + \frac{a_s}{2} |\Delta|^2 + \frac{u_s}{4} |\Delta|^4 + \frac{\gamma}{2} |\Delta|^2 M^2$$

➤ Minimization with respect to  $M$  leads to

$$a_m + u_m M^2 = -\gamma |\Delta|^2$$

and we obtain the effective free energy

$$F[\Delta] = -\frac{a_m^2}{4u_m} + \frac{a_s}{2} \left( 1 - \frac{a_m \gamma}{a_s u_m} \right) |\Delta|^2 + \frac{u_s}{4} \left( 1 - \frac{\gamma^2}{u_s u_m} \right) |\Delta|^4$$

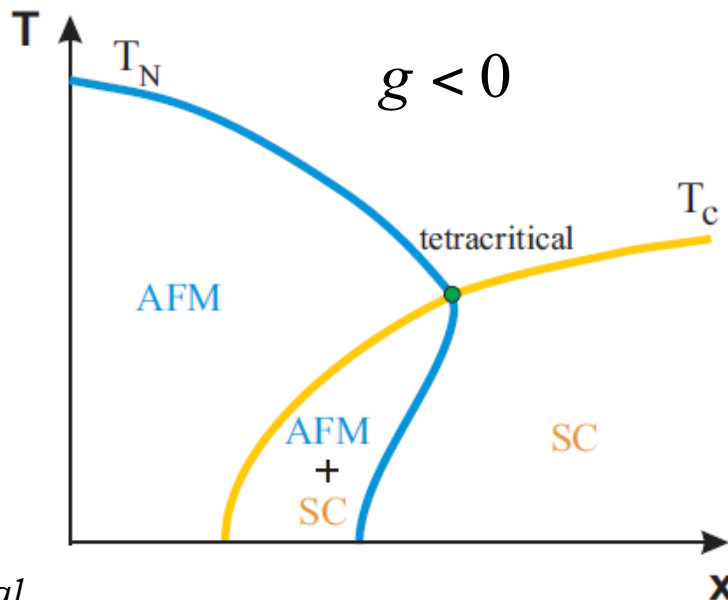
**<0 : first-order**

**>0 : second-order**

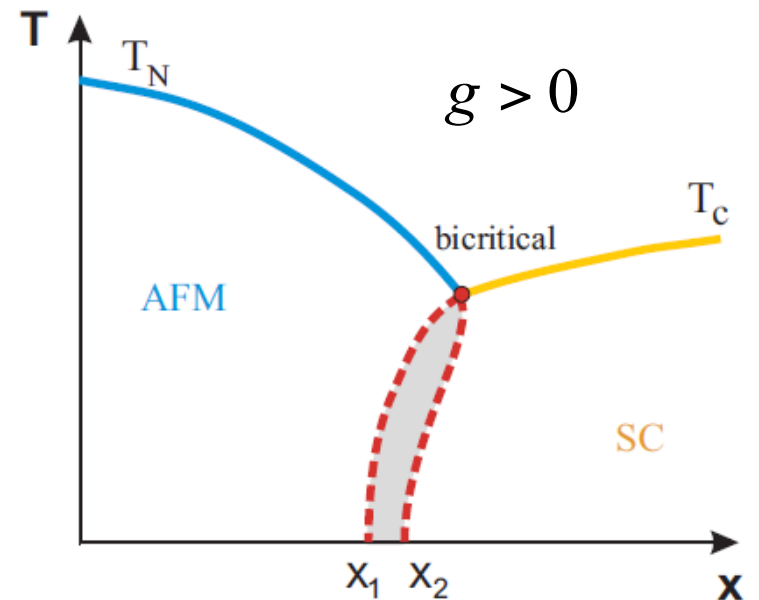
# Competition between SDW and SC: phenomenological model

$$F[M, \Delta] = \frac{a_m}{2} M^2 + \frac{u_m}{4} M^4 + \frac{a_s}{2} |\Delta|^2 + \frac{u_s}{4} |\Delta|^4 + \frac{\gamma}{2} |\Delta|^2 M^2$$

$$g = \frac{\gamma}{\sqrt{u_m u_s}} - 1$$



**coexistence**

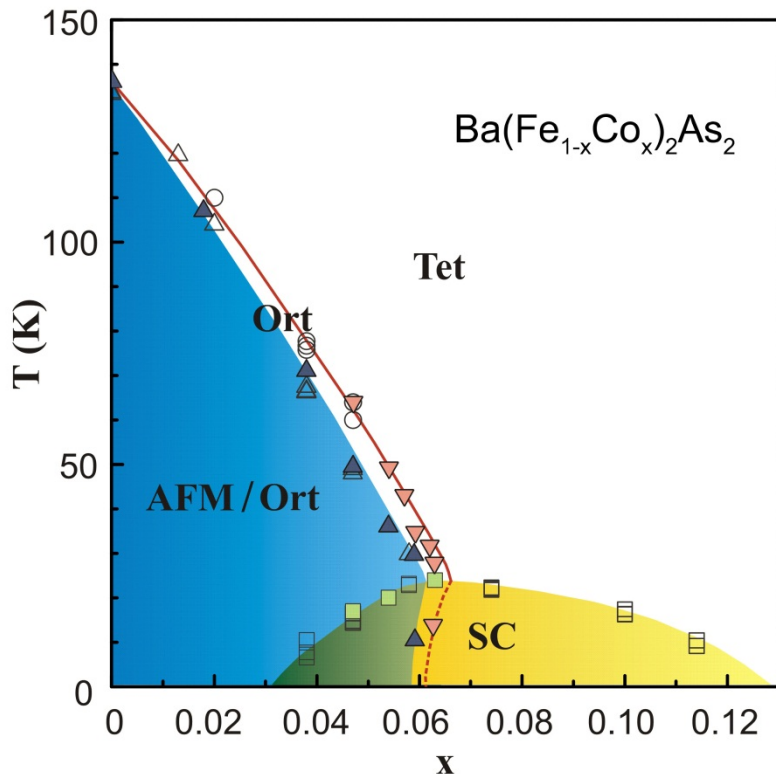


**phase separation**

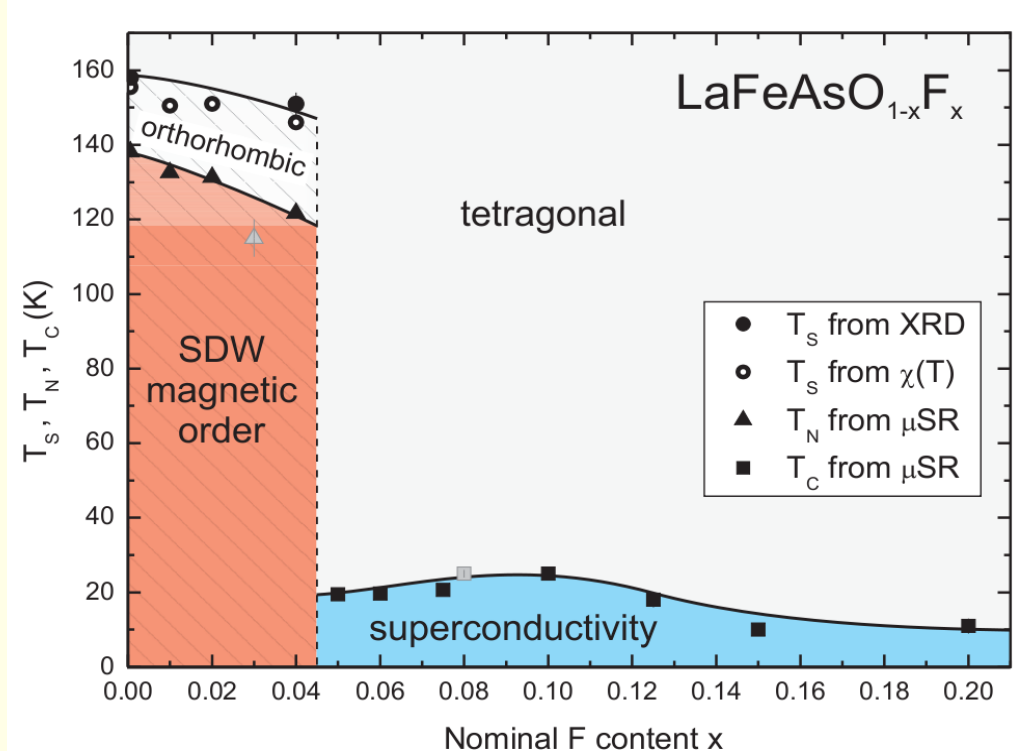
# Competition between SDW and SC: phenomenological model

$$F[M, \Delta] = \frac{a_m}{2} M^2 + \frac{u_m}{4} M^4 + \frac{a_s}{2} |\Delta|^2 + \frac{u_s}{4} |\Delta|^4 + \frac{\gamma}{2} |\Delta|^2 M^2$$

$g < 0$



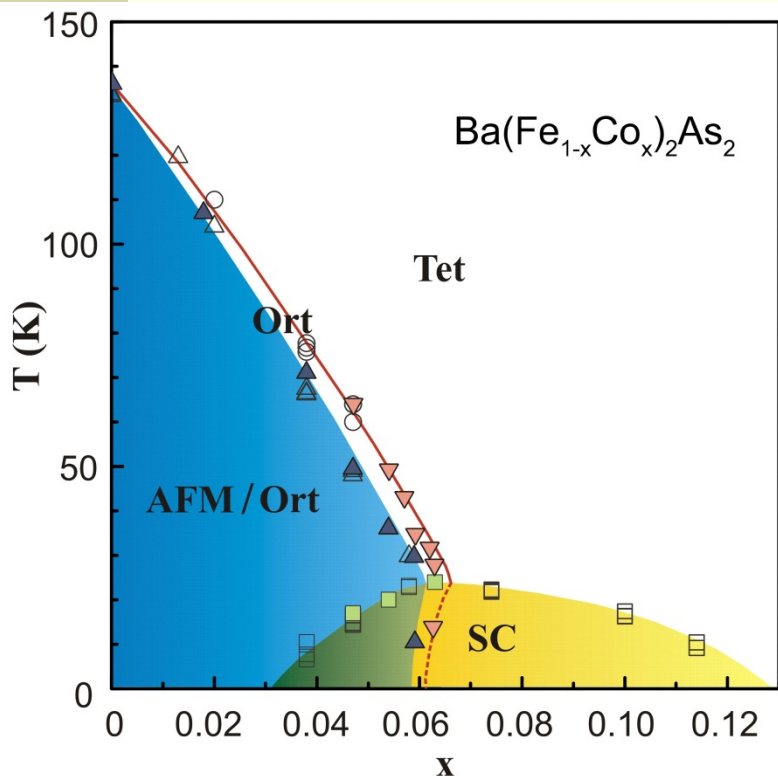
$g > 0$



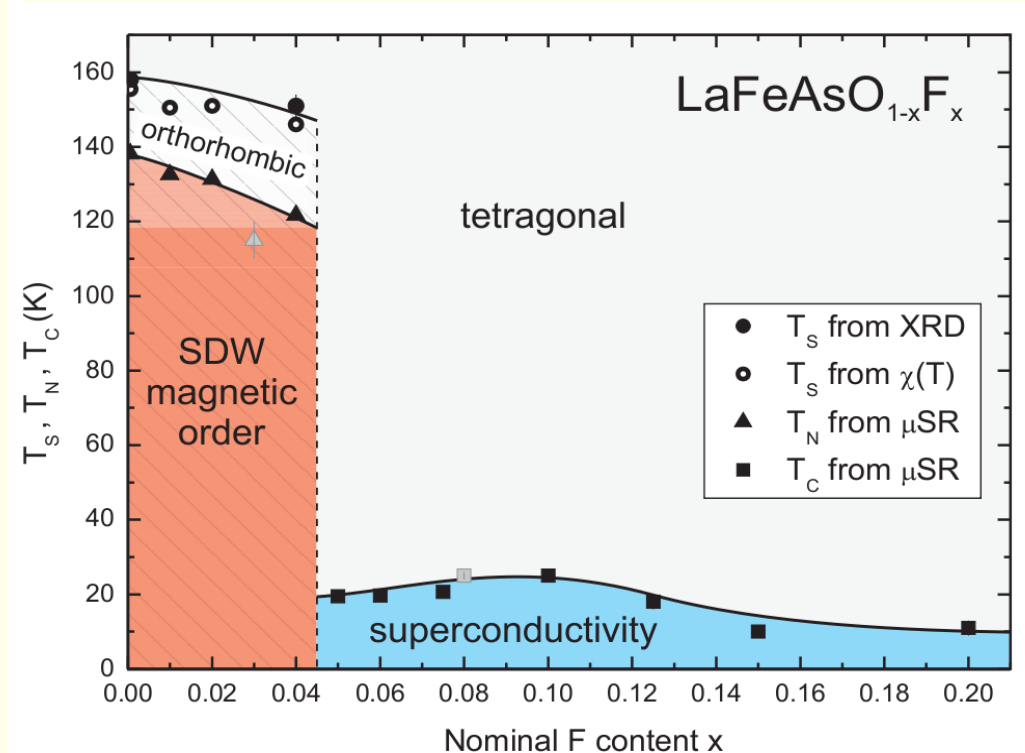
# Competition between SDW and SC: phenomenological model

How to describe this competition from a microscopic model?

$$g < 0$$

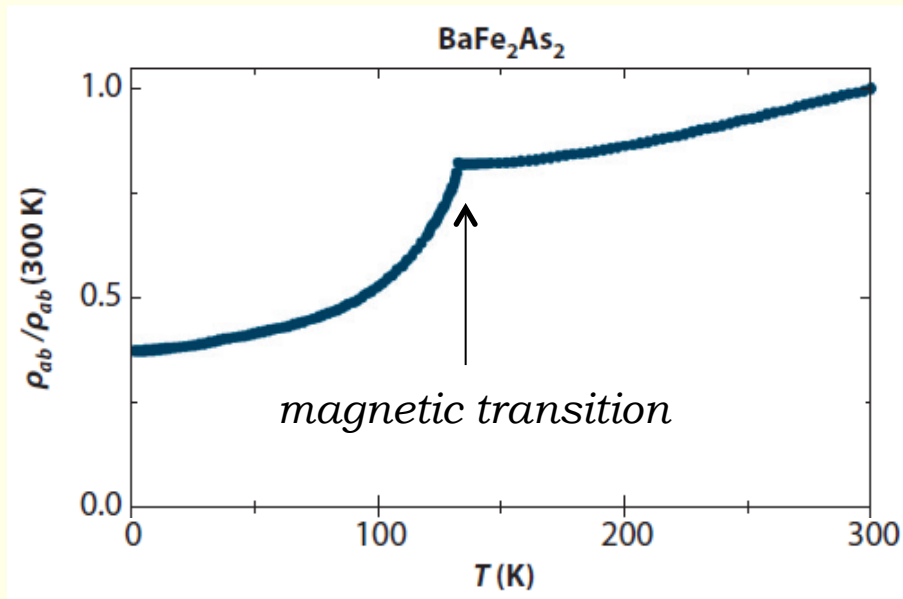


$$g > 0$$

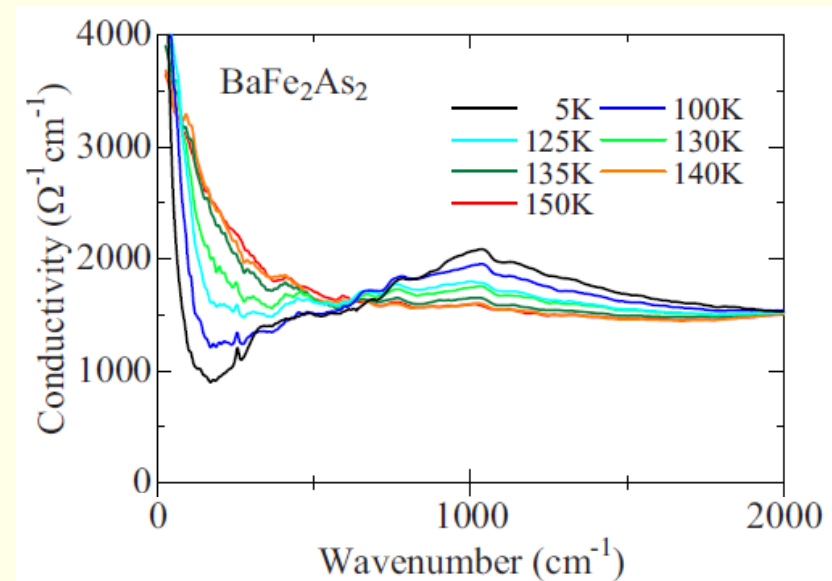


# Magnetic state: itinerant approach

- Experiments reveal that these systems are better described as metallic itinerant magnets



*Canfield et al, ARCMP (2010)*

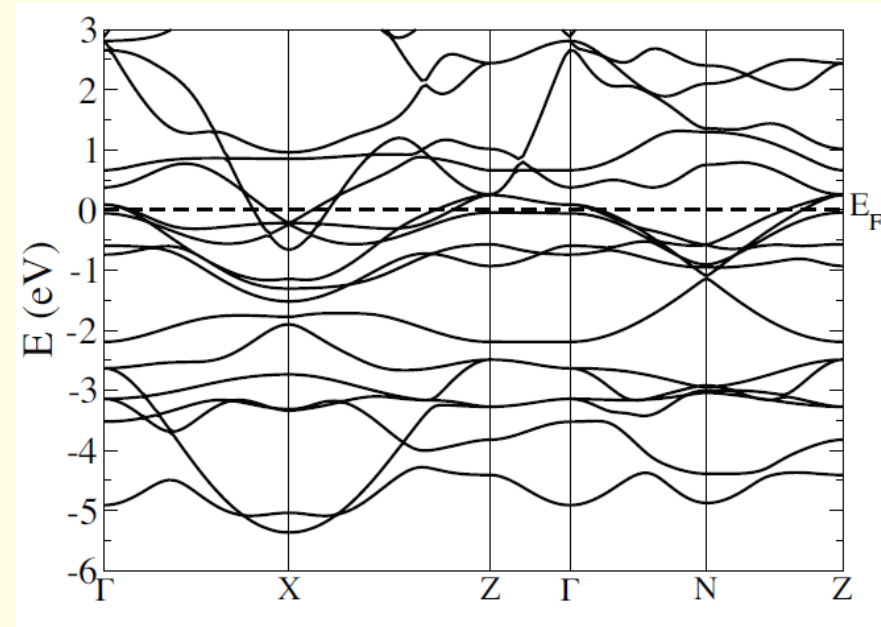
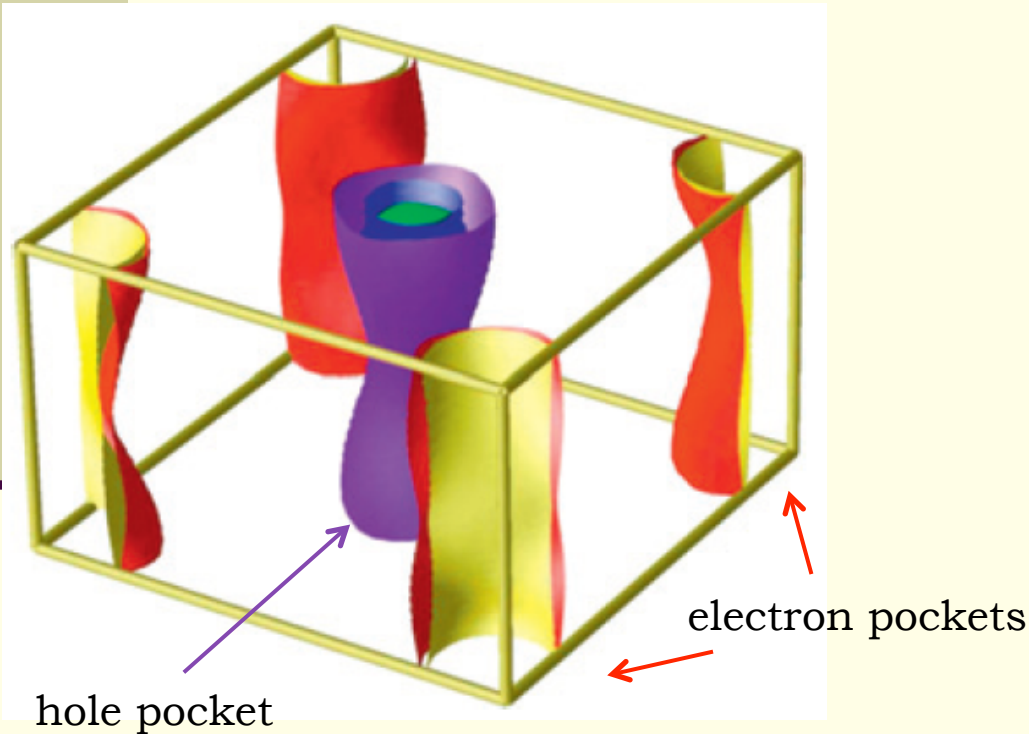


*Nakajima et al, PRB (2010)*

- conduction electrons are responsible for the magnetism

# Iron pnictides: band structure

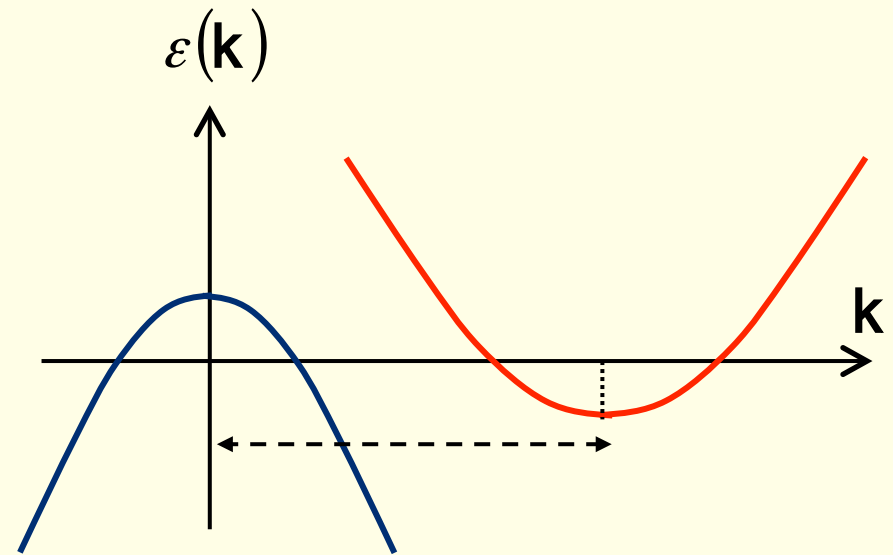
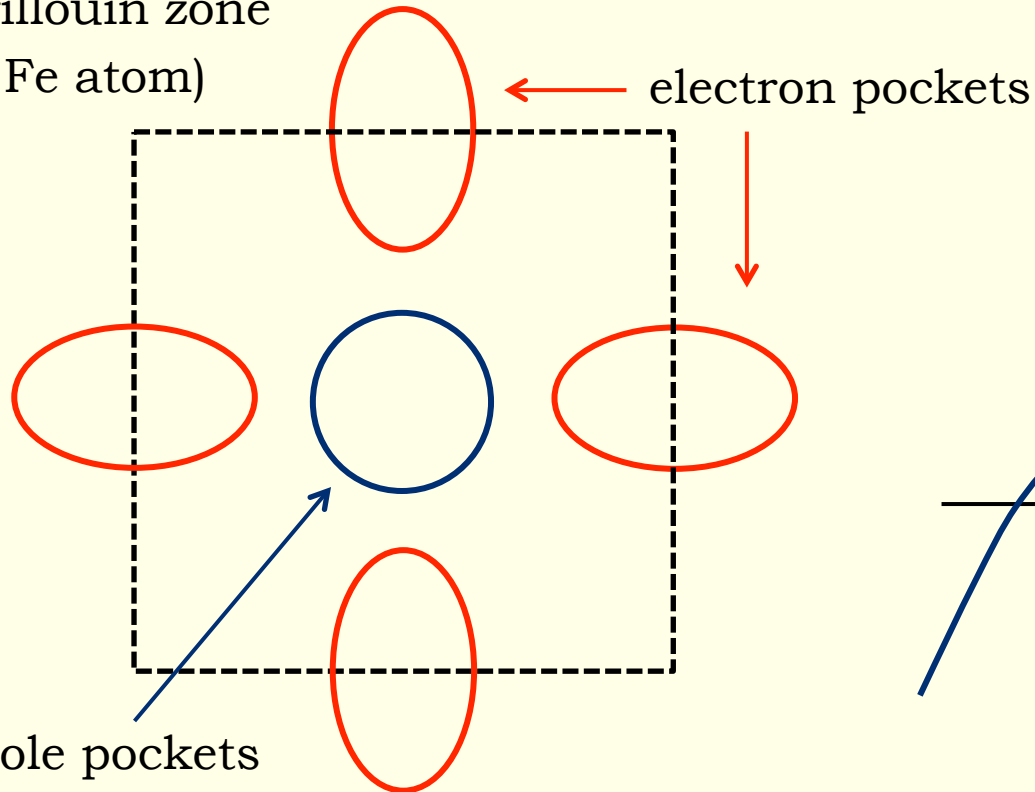
- DFT calculations: Fermi surface has multiple sheets
  - $3d^6$  configuration: several orbitals cross the Fermi level



# Iron pnictides: band structure

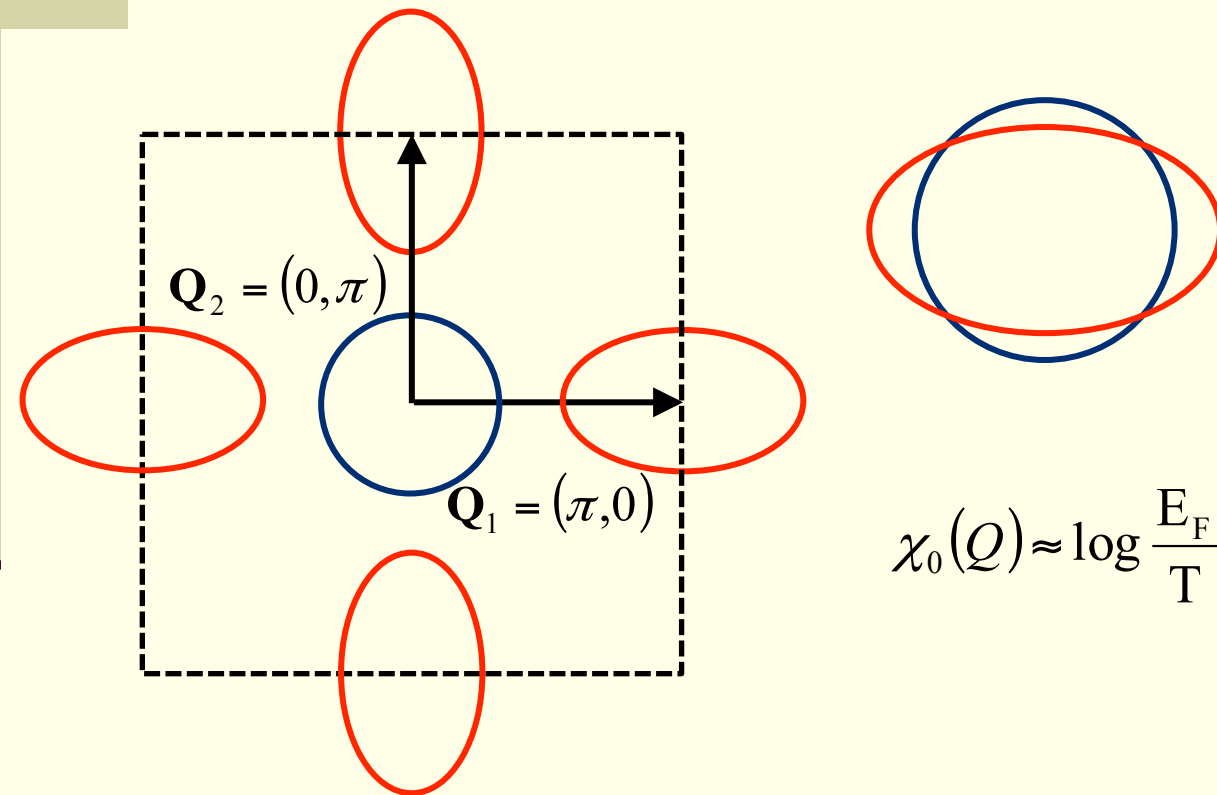
- Fermi surface of the iron pnictides (theorist view):

Brillouin zone  
(1 Fe atom)

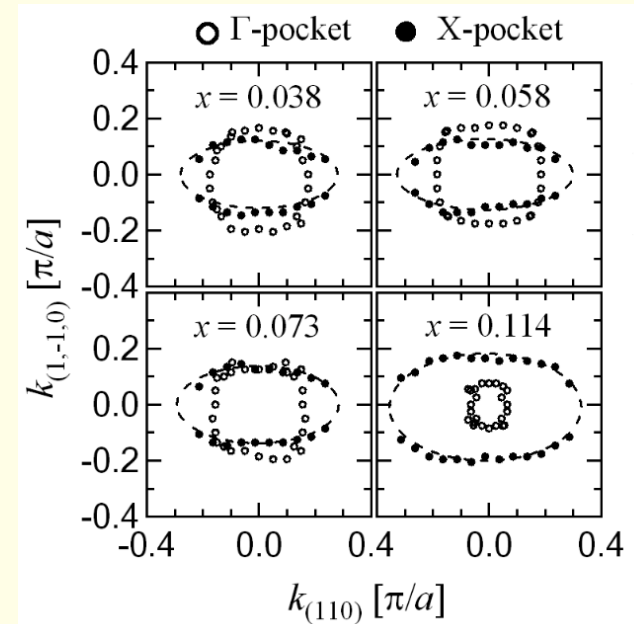


# Iron pnictides: itinerant magnetism

- Bands have good nesting features: Stoner magnetism



$$\chi_0(Q) \approx \log \frac{E_F}{T}$$

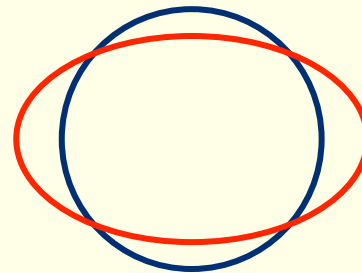
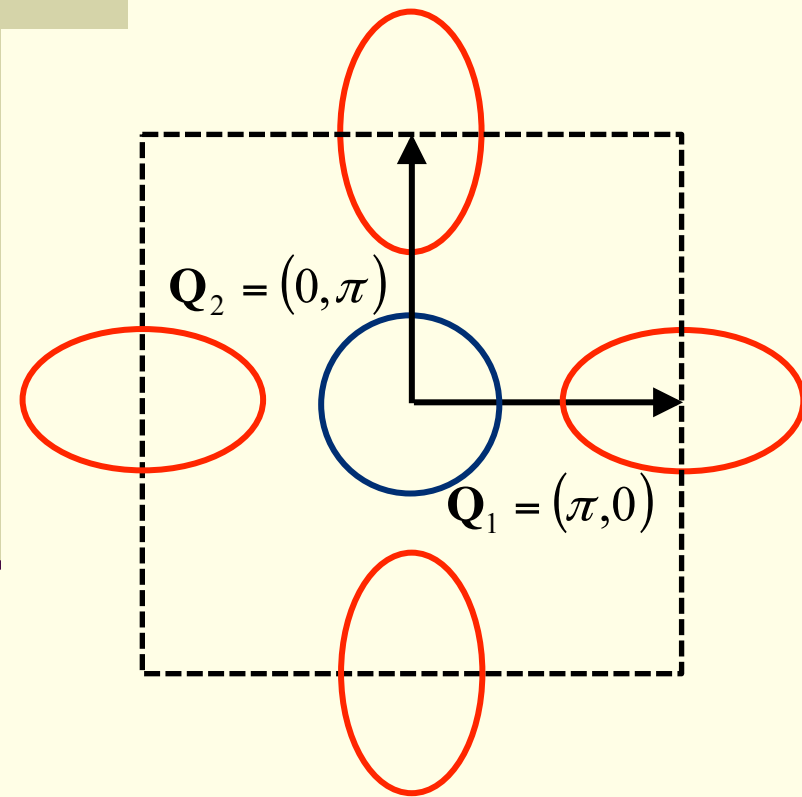


**ARPES data**



# Iron pnictides: itinerant magnetism

- Bands have good nesting features: Stoner magnetism



$$\chi_0(Q) \approx \log \frac{E_F}{T}$$

*include interaction:*

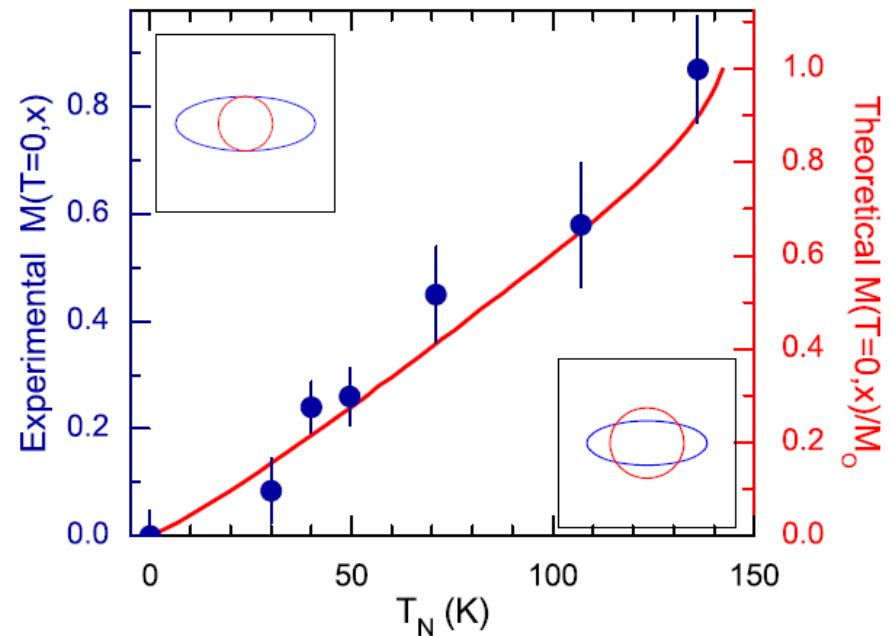
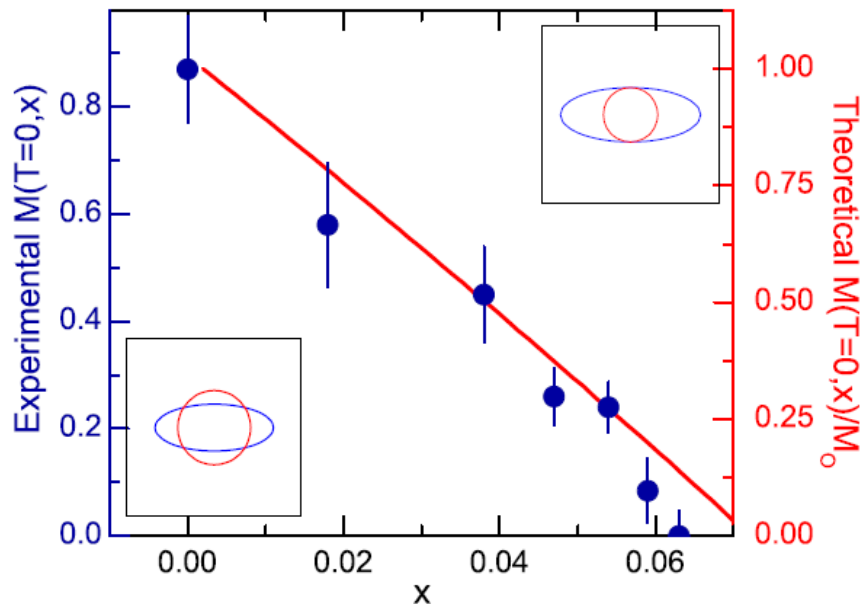
$$\chi(Q) = \frac{\chi_0(Q)}{1 - U \chi_0(Q)}$$

**small electronic interaction can lead to a magnetically ordered state (SDW) with ordering vector  $Q_i$**

# Iron pnictides: itinerant magnetism

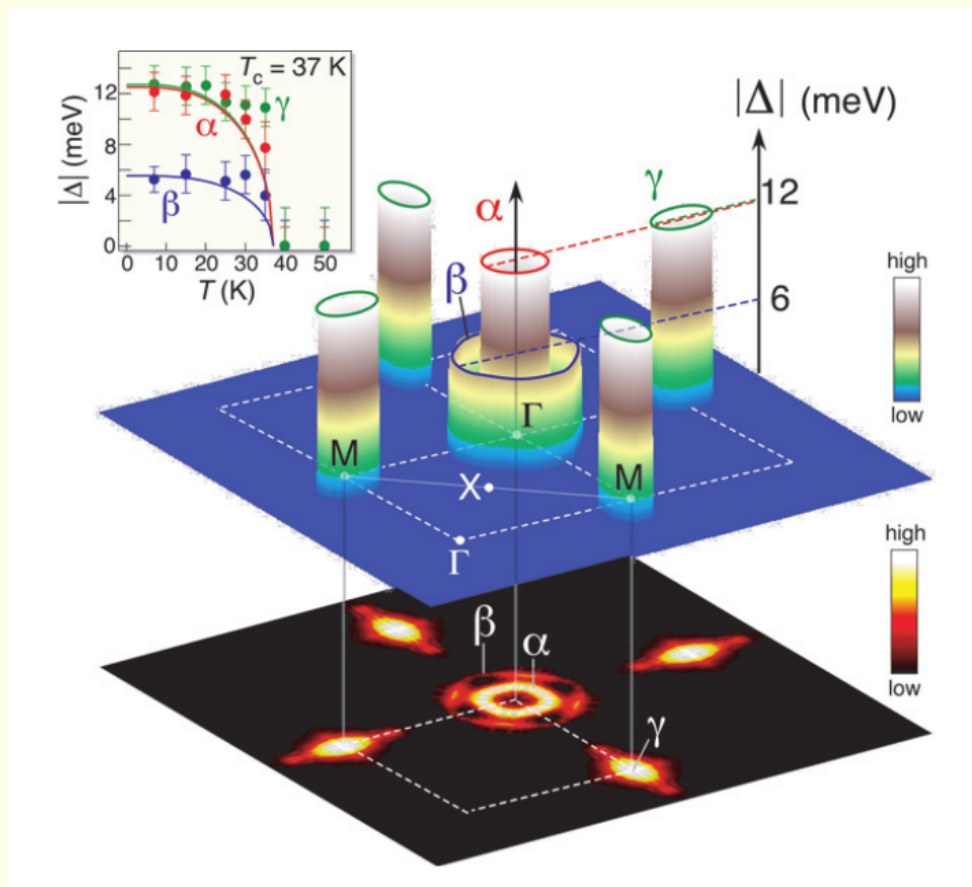
- Quantitative agreement with experimental data

$$H_{\text{SDW}} = - \sum_{\mathbf{k}, \sigma} \sigma M (c_{\mathbf{k}+\mathbf{q}\sigma}^+ d_{\mathbf{k}\sigma} + d_{\mathbf{k}\sigma}^+ c_{\mathbf{k}+\mathbf{q}\sigma})$$

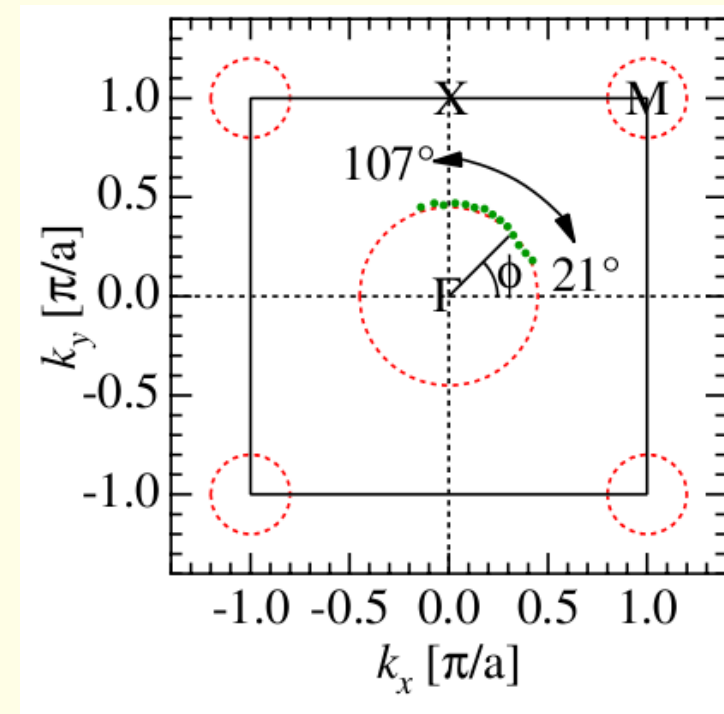


# Iron pnictides: superconducting state

- ARPES: amplitude of the gaps, but not their relative sign



Ding et al, EPL (2008)

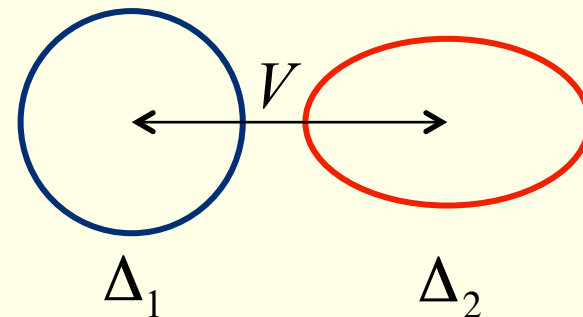
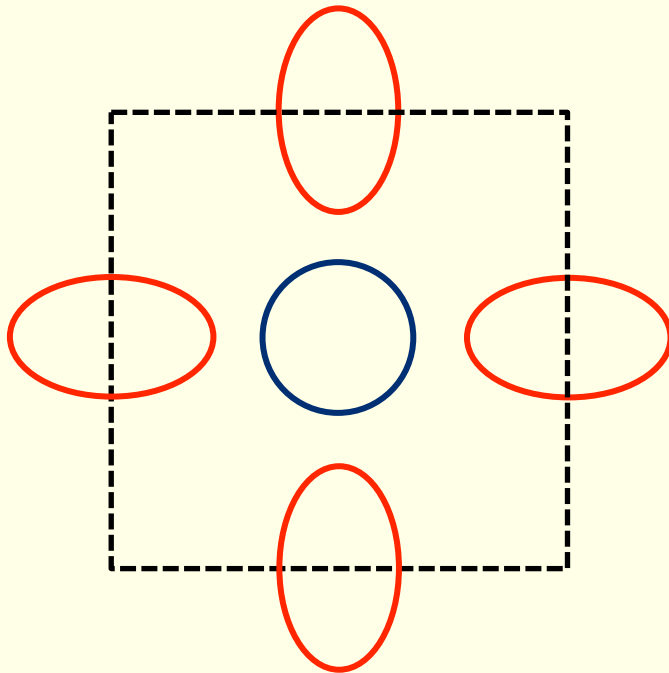


Kondo et al, PRL (2008)

# Iron pnictides: candidates for the SC state

- Pairing problem in a simplified two-band model

$$H_{\text{SC}} = - \sum_{\mathbf{k}+\mathbf{Q}} \Delta_1 (c_{\mathbf{k}\uparrow}^+ c_{-\mathbf{k}\downarrow}^+ + c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}) - \sum_{\mathbf{k}} \Delta_2 (d_{\mathbf{k}\uparrow}^+ d_{-\mathbf{k}\downarrow}^+ + d_{-\mathbf{k}\downarrow} d_{\mathbf{k}\uparrow})$$



$$\begin{cases} \Delta_1 = -VN_2\Delta_2 \ln\left(\frac{\Lambda}{T_c}\right) \\ \Delta_2 = -VN_1\Delta_1 \ln\left(\frac{\Lambda}{T_c}\right) \end{cases}$$

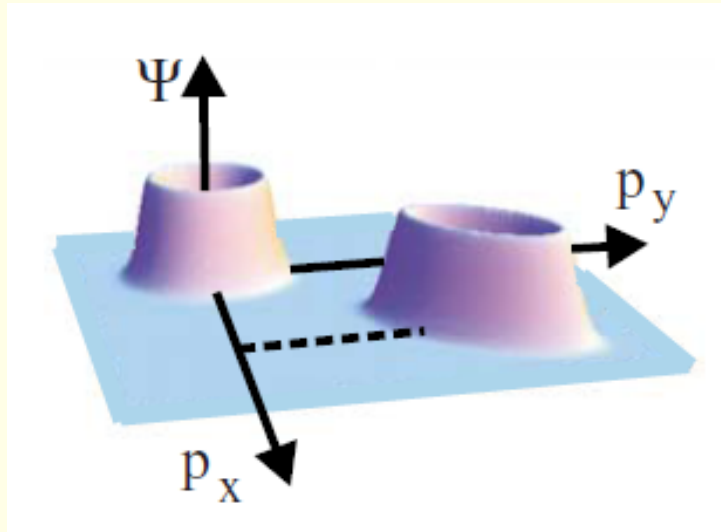
# Iron pnictides: candidates for the SC state

- Pairing due to conventional attractive electron-phonon interaction (enhanced by charge/orbital fluctuations)

$$V < 0$$

*Kontani & Onari, PRL (2010)*

$S^{++}$  state



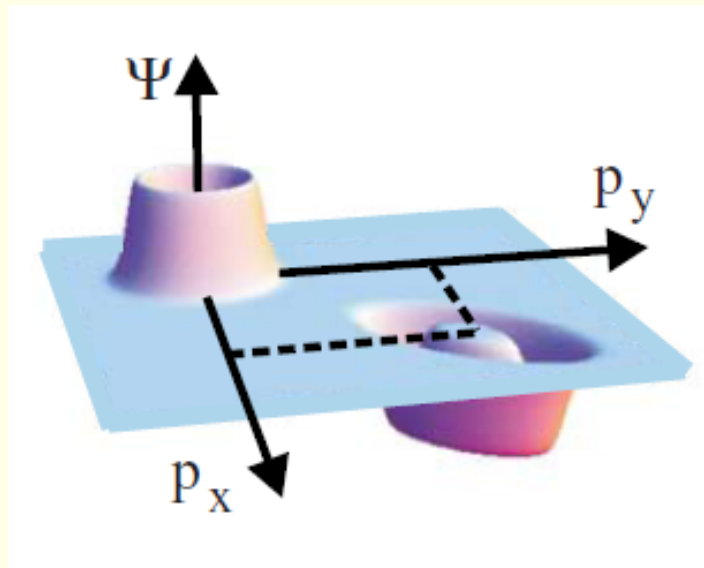
$$\begin{cases} \Delta_1 > 0 \\ \Delta_2 > 0 \end{cases}$$

# Iron pnictides: candidates for the SC state

- Pairing due to purely repulsive electronic interaction (enhanced by spin fluctuations)

$$V > 0$$

$S^{+-}$  state

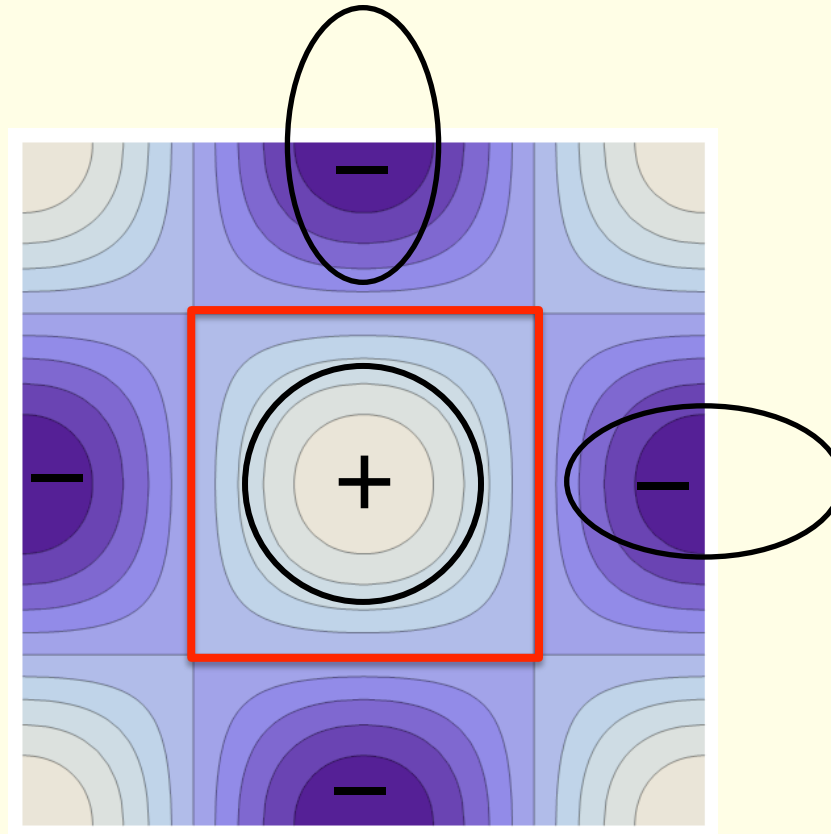


$$\begin{cases} \Delta_1 > 0 \\ \Delta_2 < 0 \end{cases}$$

*Mazin et al, PRL (2008)*  
*Kuroki et al, PRL (2008)*  
*Chubukov et al, PRB (2008)*  
*Cvetkovic et al, EPL (2009)*  
*Wang et al, PRL (2009)*  
*Sknepnek et al, PRB (2009)*  
*Graser et al, NJP (2009)*

# Iron pnictides: candidates for the SC state

- In the  $S^{+-}$  state, the position of the gap function zeros is not enforced by symmetry



# Competition between SDW and SC: microscopic model

- Hertz-Millis approach to the two-band model

$$H = H_0 + H_{\text{SDW}} + H_{\text{SC}}$$

$$\left\{ \begin{array}{l} H_0 = \sum_{\mathbf{k}, \sigma} (\varepsilon_{1, \mathbf{k}+\mathbf{Q}} - \mu) c_{\mathbf{k}+\mathbf{Q}\sigma}^+ c_{\mathbf{k}+\mathbf{Q}\sigma} + \sum_{\mathbf{k}, \sigma} (\varepsilon_{2, \mathbf{k}} - \mu) d_{\mathbf{k}\sigma}^+ d_{\mathbf{k}\sigma} \\ H_{\text{SDW}} = - \sum_{\mathbf{k}, \sigma} \sigma M (c_{\mathbf{k}+\mathbf{q}\sigma}^+ d_{\mathbf{k}\sigma} + d_{\mathbf{k}\sigma}^+ c_{\mathbf{k}+\mathbf{q}\sigma}) \\ H_{\text{SC}} = - \sum_{\mathbf{k}+\mathbf{Q}} \Delta_1 (c_{\mathbf{k}\uparrow}^+ c_{-\mathbf{k}\downarrow}^+ + c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}) - \sum_{\mathbf{k}} \Delta_2 (d_{\mathbf{k}\uparrow}^+ d_{-\mathbf{k}\downarrow}^+ + d_{-\mathbf{k}\downarrow} d_{\mathbf{k}\uparrow}) \end{array} \right.$$



# Competition between SDW and SC: microscopic model

---

- We can then **derive** the Ginzburg-Landau coefficients

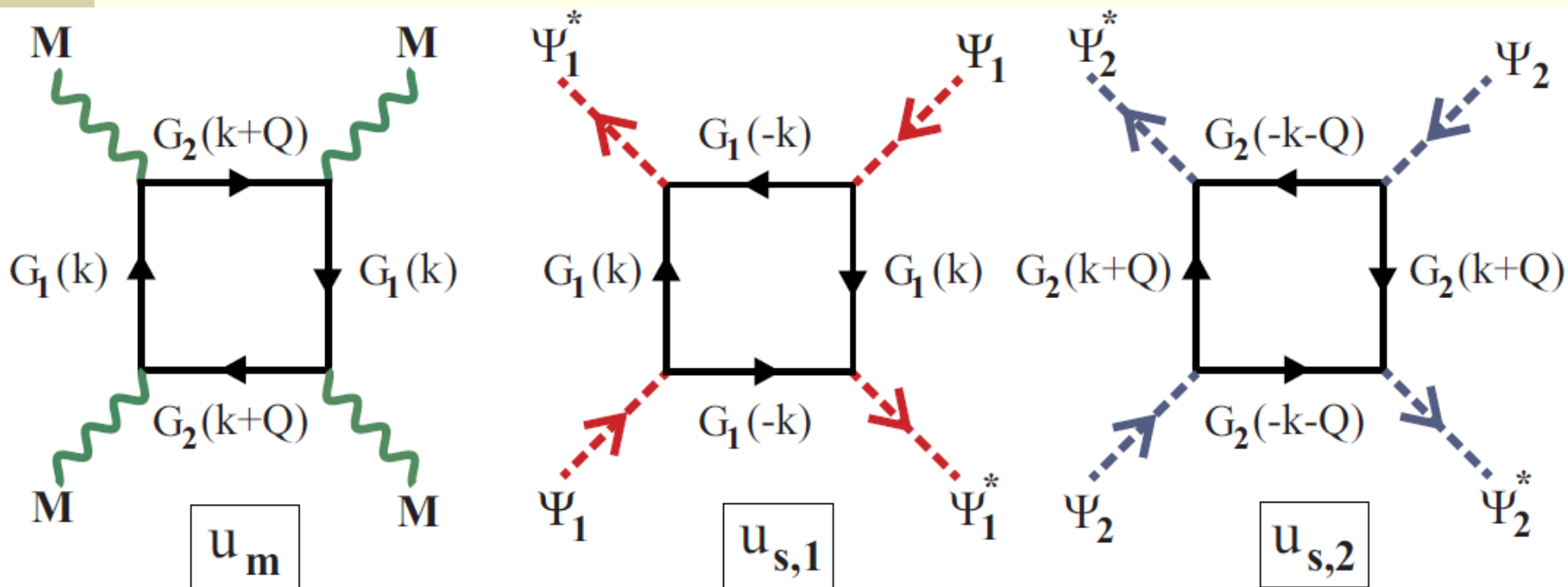
$$F[M, \Delta] = \frac{a_m}{2} M^2 + \frac{u_m}{4} M^4 + \frac{a_s}{2} |\Delta|^2 + \frac{u_s}{4} |\Delta|^4 + \frac{\gamma}{2} |\Delta|^2 M^2$$

$$g = \frac{\gamma}{\sqrt{u_m u_s}} - 1$$

# Competition between SDW and SC: microscopic model

- We can then **derive** the Ginzburg-Landau coefficients

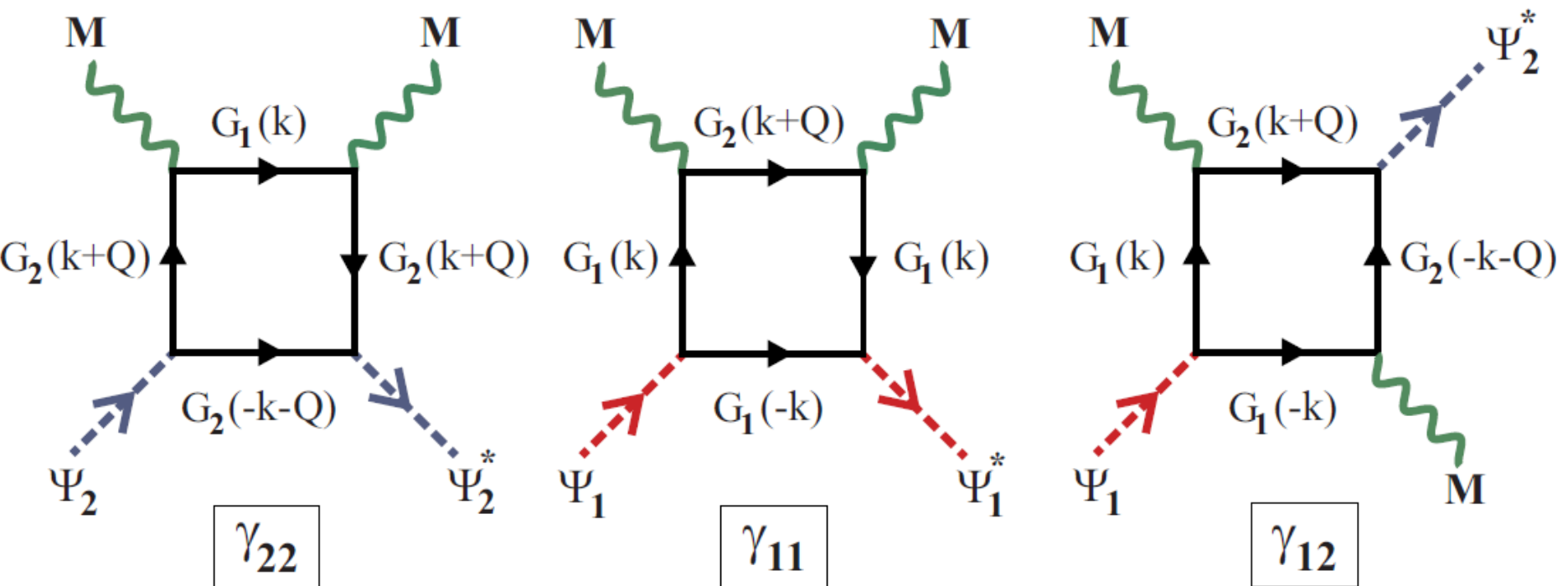
$$\longrightarrow \equiv G_i(\mathbf{k}, \omega_n) = \frac{1}{i\omega_n - \xi_{i,\mathbf{k}}}$$



# Competition between SDW and SC: microscopic model

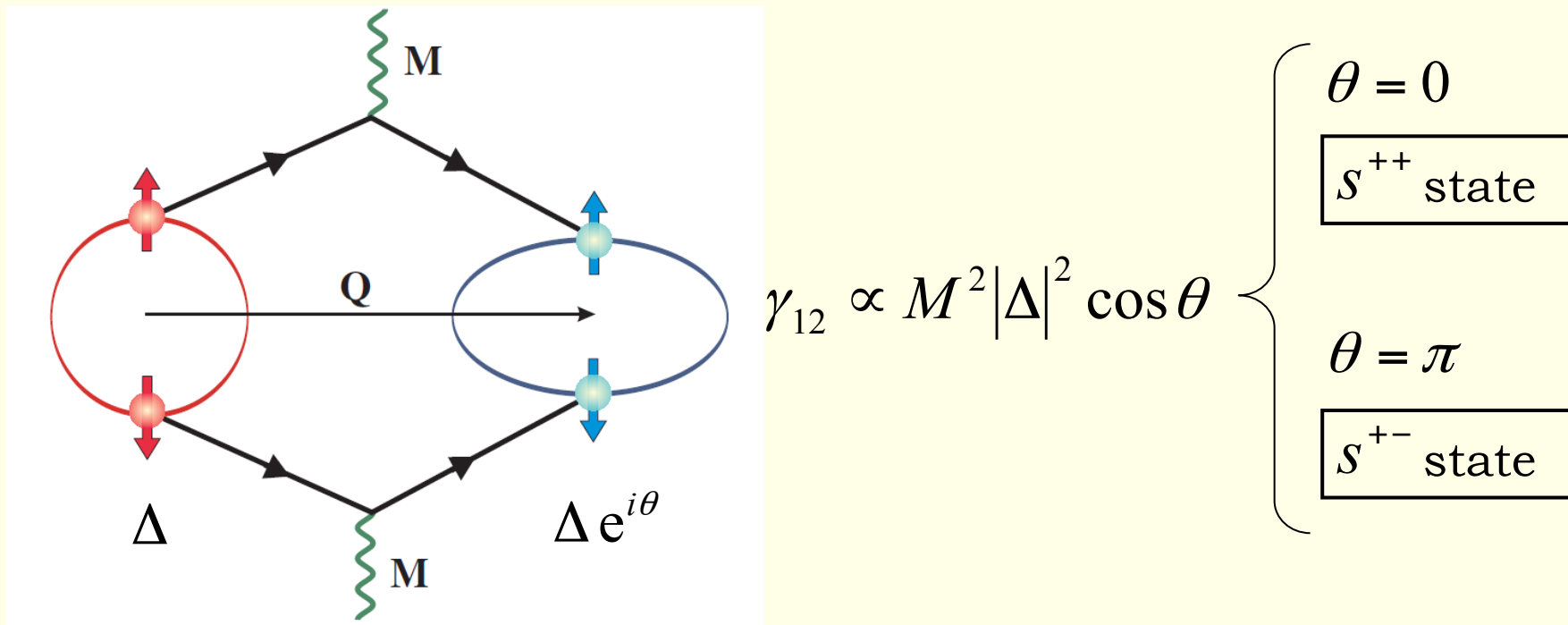
- We can then **derive** the Ginzburg-Landau coefficients

$$\longrightarrow \longrightarrow \equiv G_i(\mathbf{k}, \omega_n) = \frac{1}{i\omega_n - \xi_{i,\mathbf{k}}}$$



# Competition between SDW and SC: microscopic model

- Strength of the competition term depends on the symmetry of the superconducting state



# Coexistence between SDW and SC: perfect nesting

---

- For perfect nesting:

$$F = \frac{a}{2} (|\Delta|^2 + M^2) + \frac{u}{4} (|\Delta|^2 + M^2)^2 + g u |\Delta|^2 M^2$$

$$g = \frac{1 + \cos \theta}{2}$$

$s^{+-}$  state :  $g = 0$   
borderline

$s^{++}$  state :  $g = 1$   
phase separation

# Coexistence between SDW and SC: perfect nesting

- For perfect nesting:

$$F = \frac{a}{2} (|\Delta|^2 + M^2) + \frac{u}{4} (|\Delta|^2 + M^2)^2 + g u |\Delta|^2 M^2$$

$s^{+-}$  state :  $g = 0$   
borderline

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phase separation

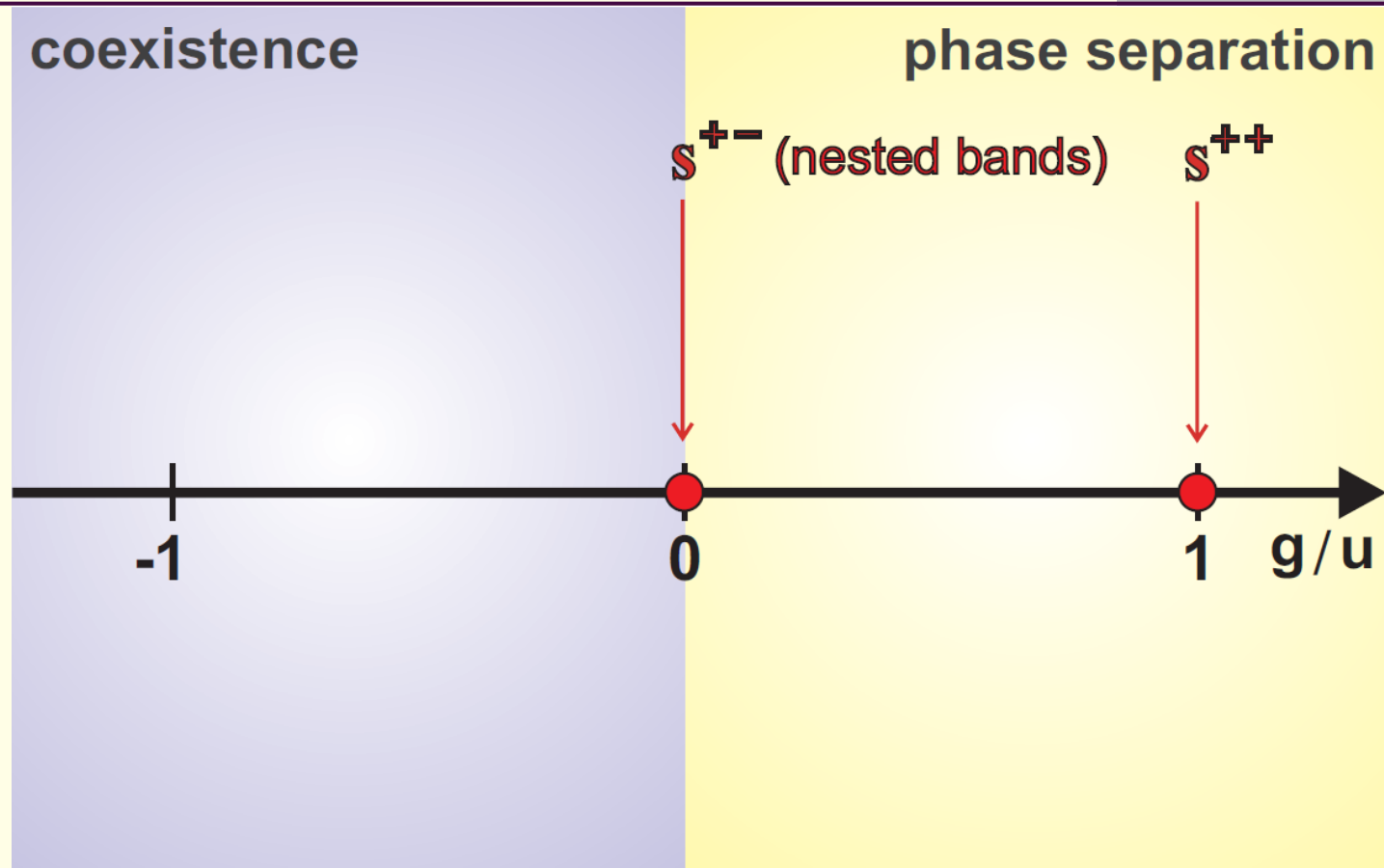
Note that  $g = 0 \Rightarrow$  emergent SO(5) symmetry

$$\vec{\mathbf{N}} = (\text{Re } \Delta, \text{Im } \Delta, \mathbf{M})$$

*Podolsky et al, EPL (2009)*

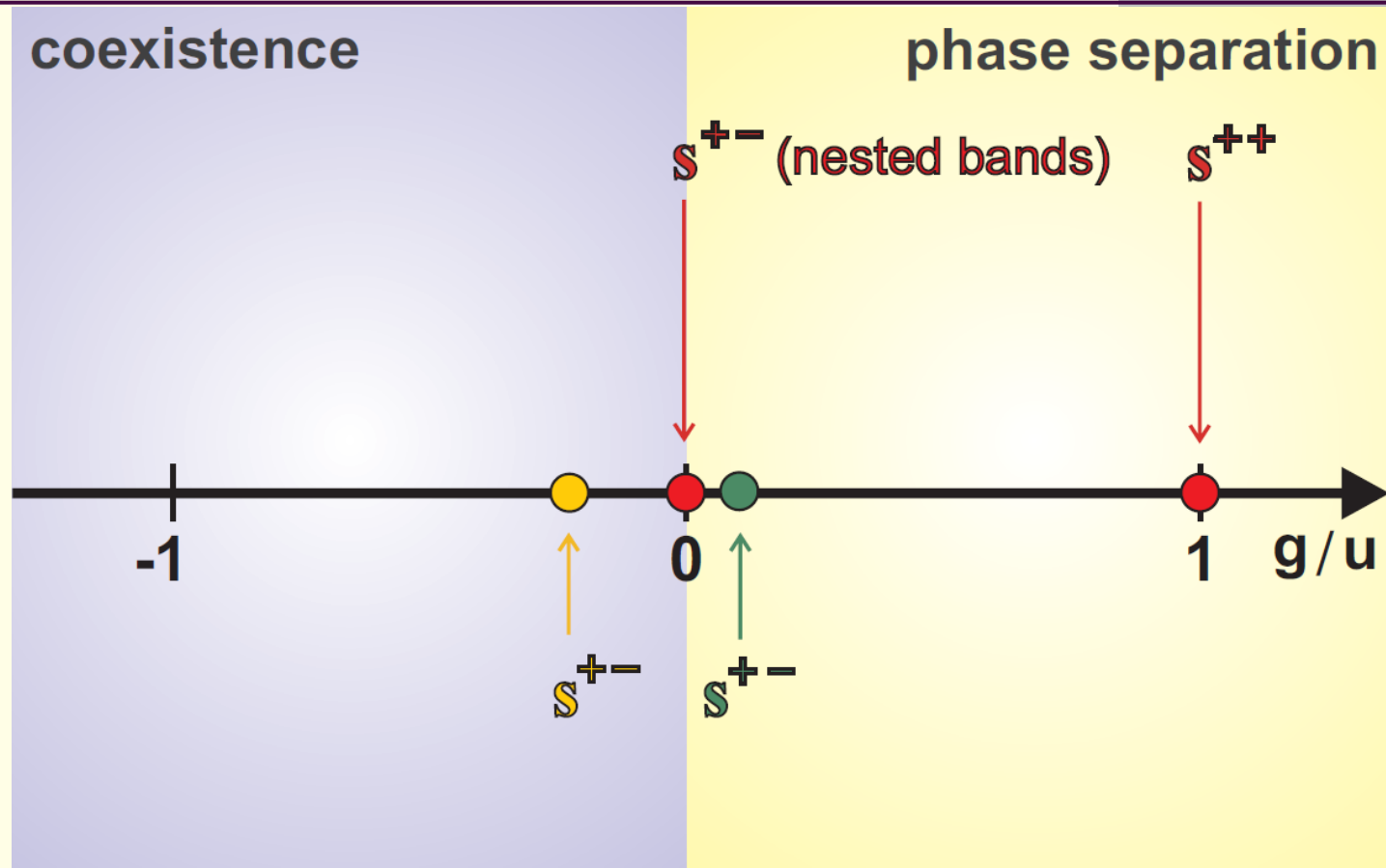
*Chubukov, Physica C (2008)*

# Competition between SDW and SC: coexistence



perfect nesting:  $g = \frac{1 + \cos \theta}{2}$

# Competition between SDW and SC: coexistence

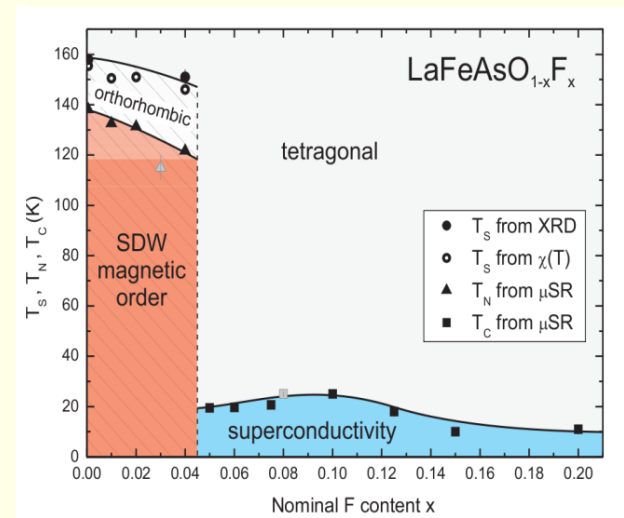
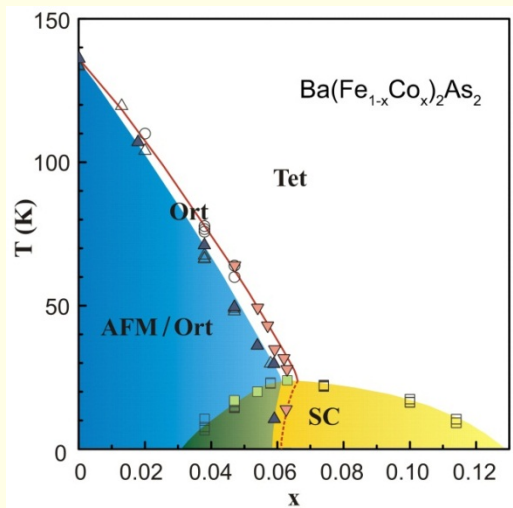


- $s^{++}$  cannot coexist with magnetism
- $s^{+-}$  may or may not coexist

*RMF et al, PRB (2010)*

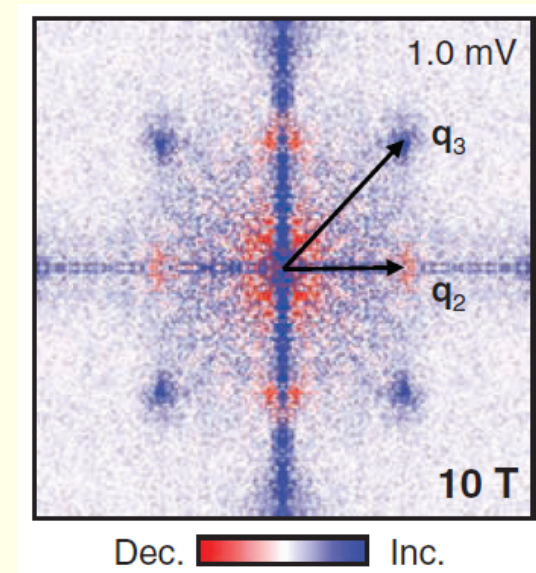
*RMF & Schmalian, PRB (2010)*





**Observation of microscopic coexistence in some iron arsenides rules out the possibility of an  $S^{++}$  state**

Phase sensitive STM measurements confirm that the SC state is  $S^{+-}$

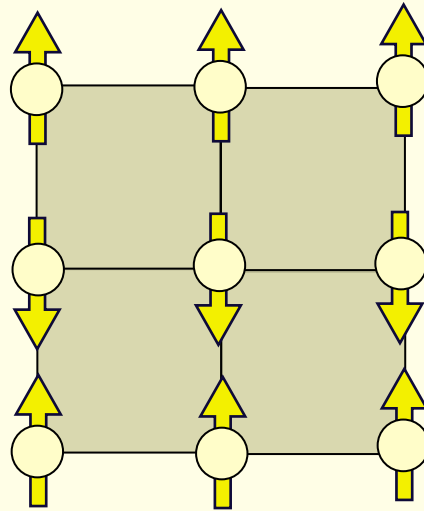


# Outline

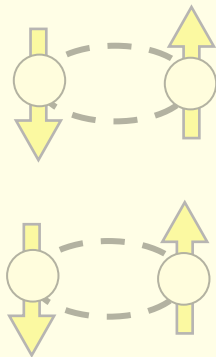
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- **Brief introduction to the iron pnictides**
  - experimental evidence for competing phases
- **Competition between magnetism and superconductivity**
  - symmetry of the Cooper pair wave function
- **Competition between nematicity and superconductivity**
  - indirect competition mediated by magnetism

**antiferromagnetic phase**

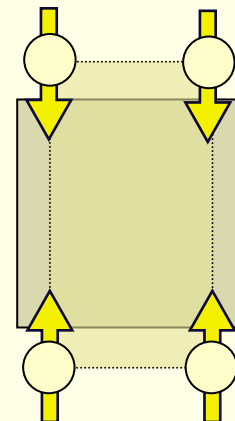
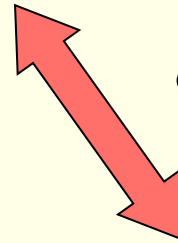


*competing  
order*



**superconducting phase**

*emergent  
order*

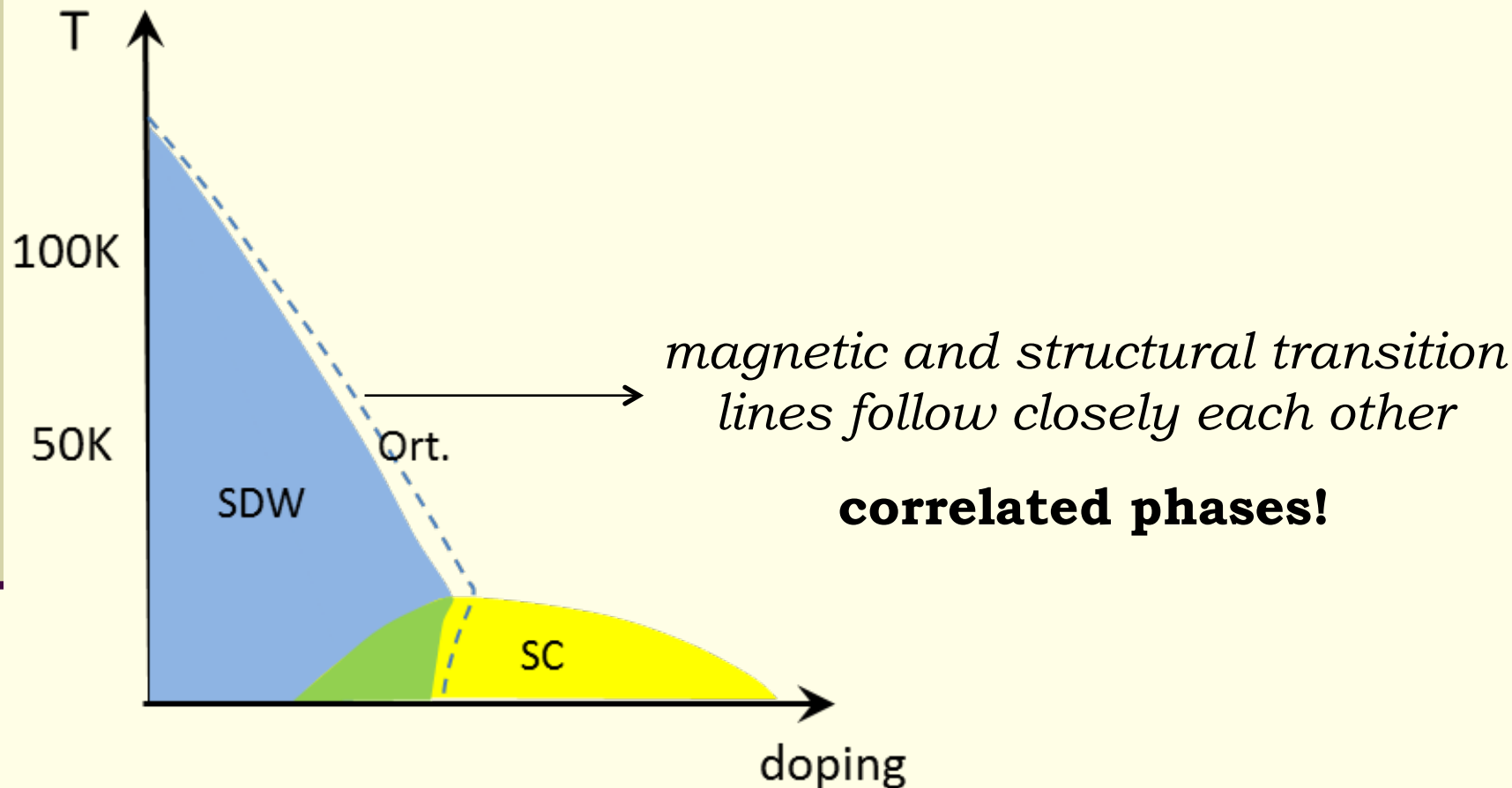


- structural transition
- orbital order

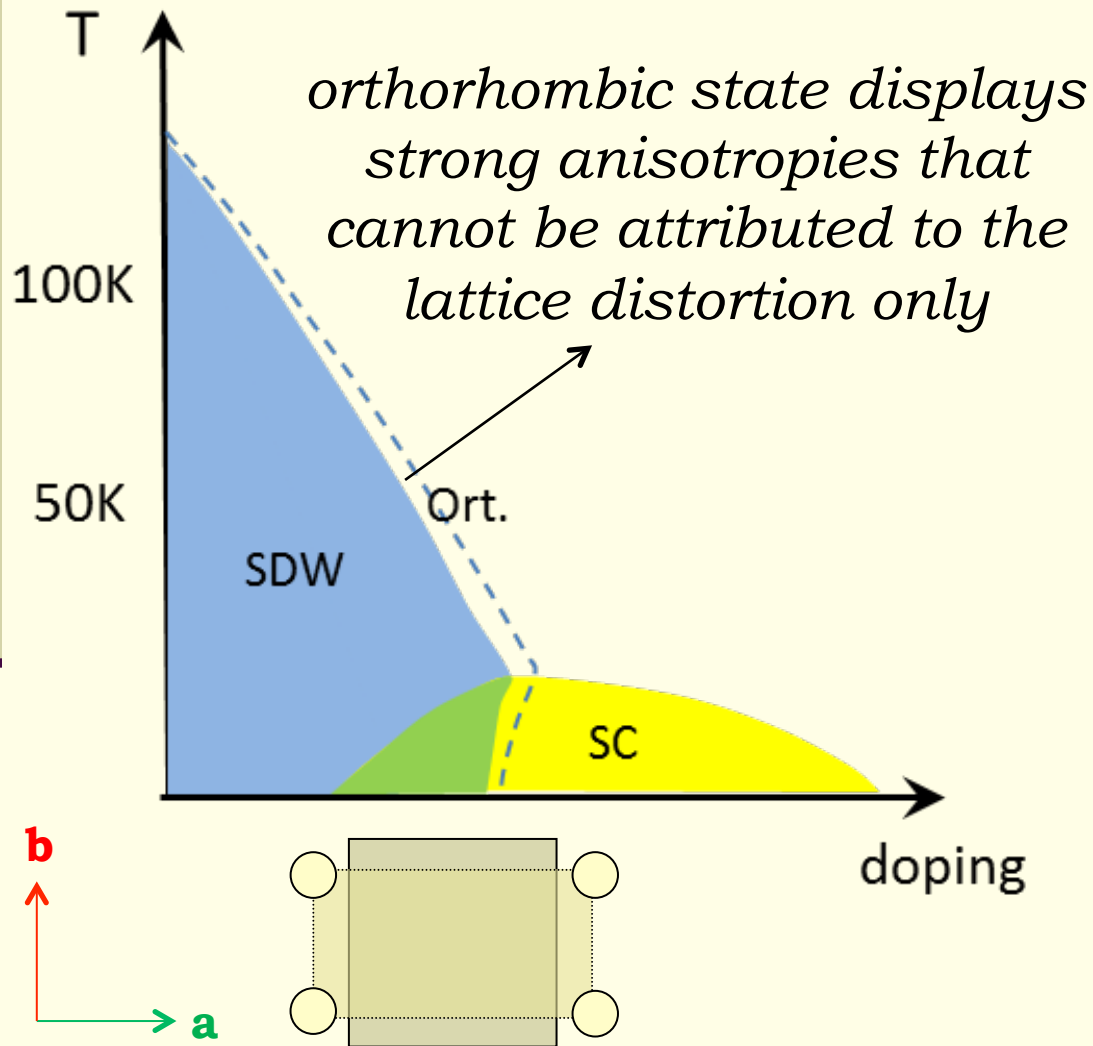
**nematic phase**

**Why nematics???**

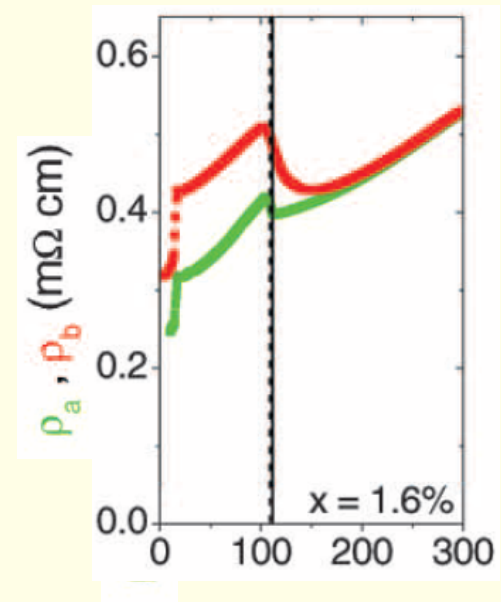
# Iron pnictides: normal state properties



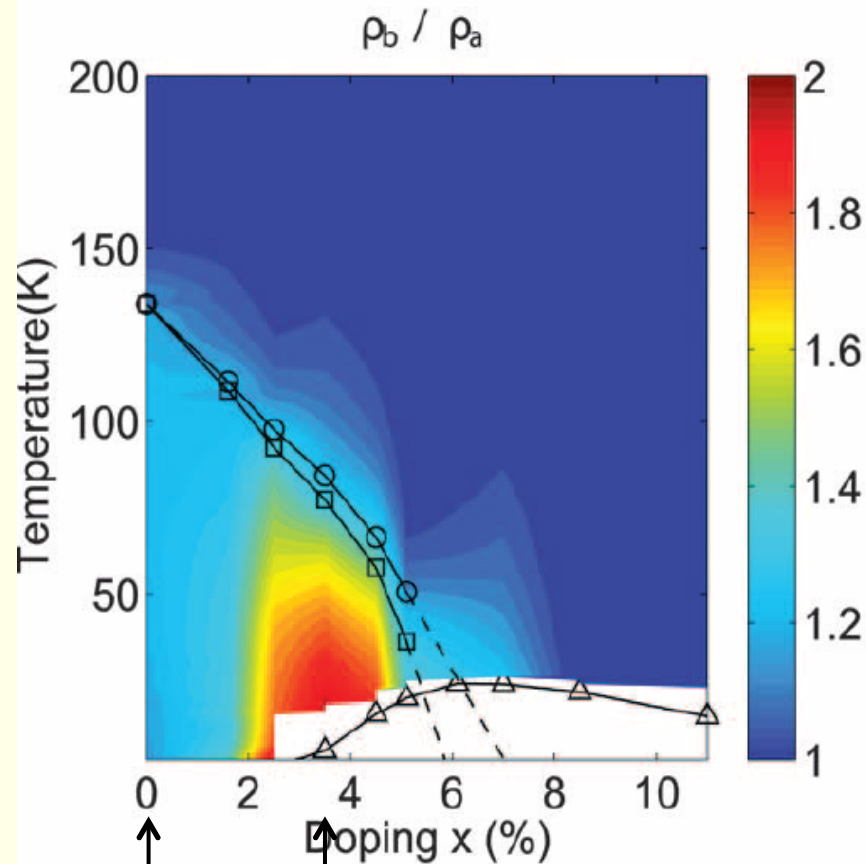
# Iron pnictides: normal state properties



- resistivity**



Chu et al, Science (2010)  
Tanatar et al, PRB (2010)



*Chu et al, Science (2010)*

$$\frac{a-b}{a+b} \approx 0.01$$

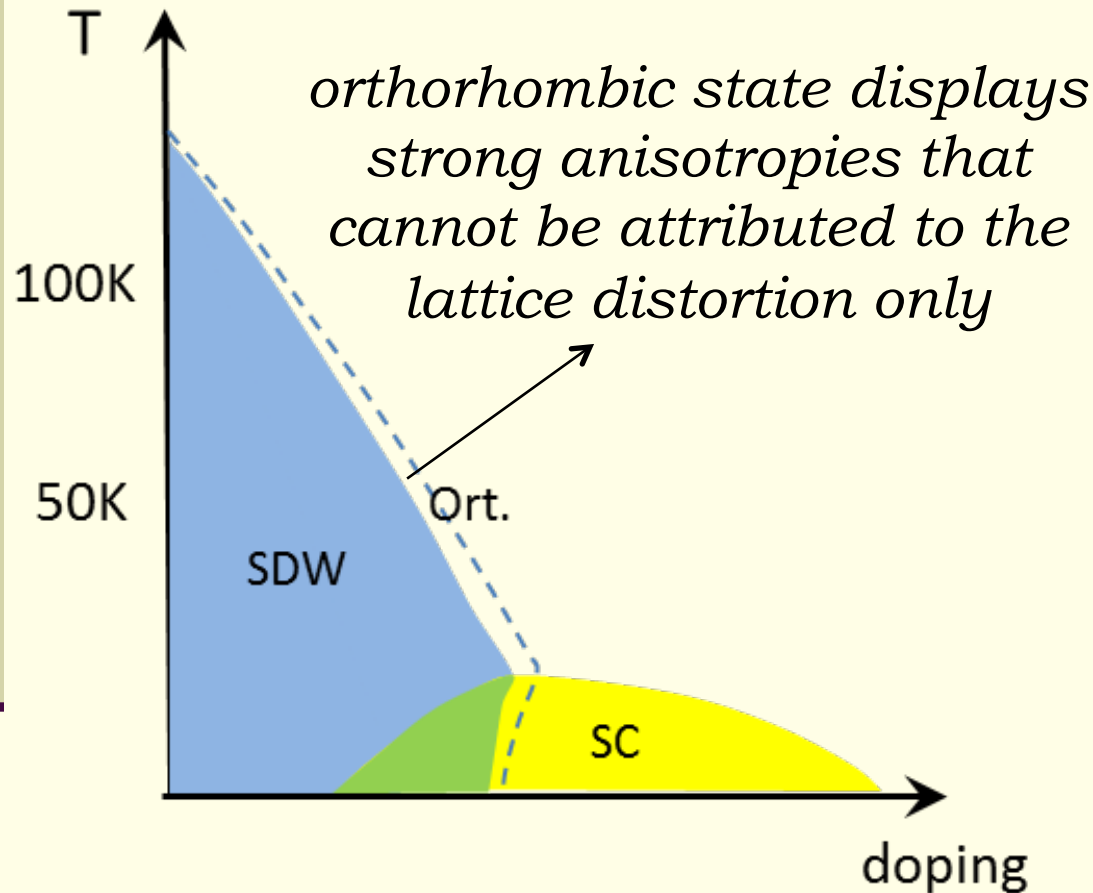
maximum  
orthorhombic  
distortion

maximum  
resistivity  
anisotropy

$$\frac{\rho_b}{\rho_a} \approx 2$$

**resistivity anisotropy cannot be attributed  
only to the orthorhombic distortion**

# Iron pnictides: normal state properties



- **resistivity**
- **optical spectrum**
- **orbital polarization**
- **density of states...**

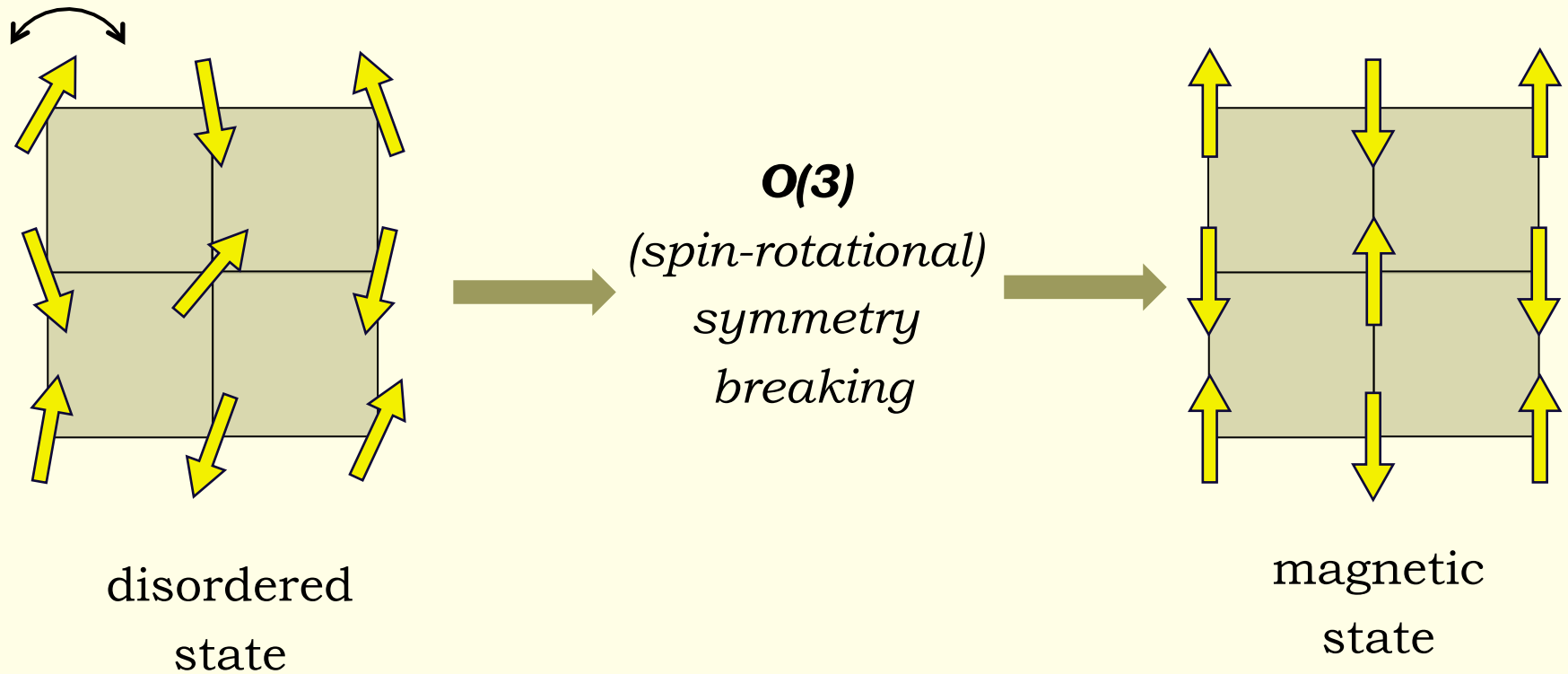
**underlying electronic order that spontaneously breaks tetragonal symmetry: nematic phase**

*cf. Kivelson, Fradkin & Emery, Nature (1998)*



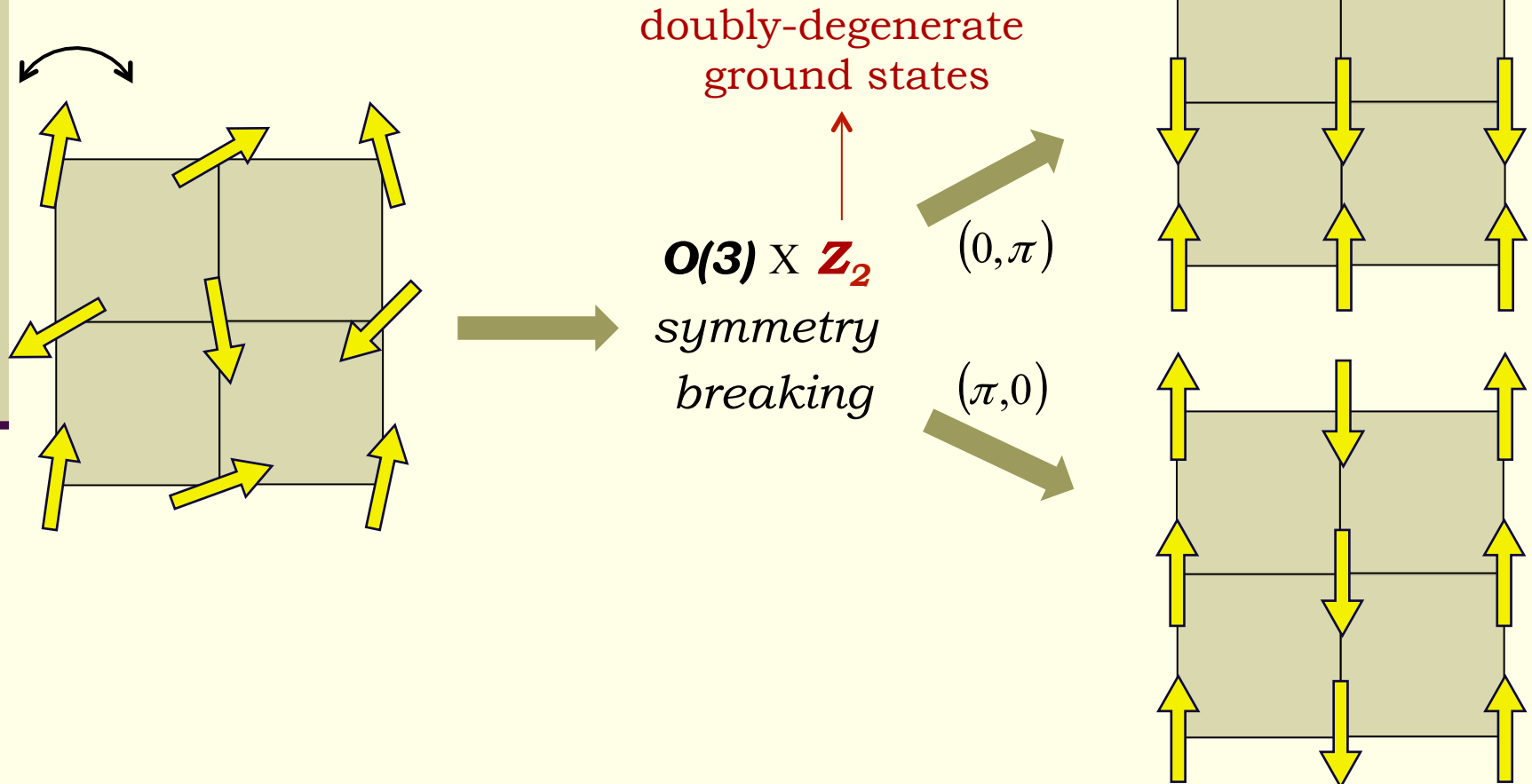
# Iron pnictides: nematic order

- Symmetry breaking in a regular antiferromagnet:



# Iron pnictides: nematic order

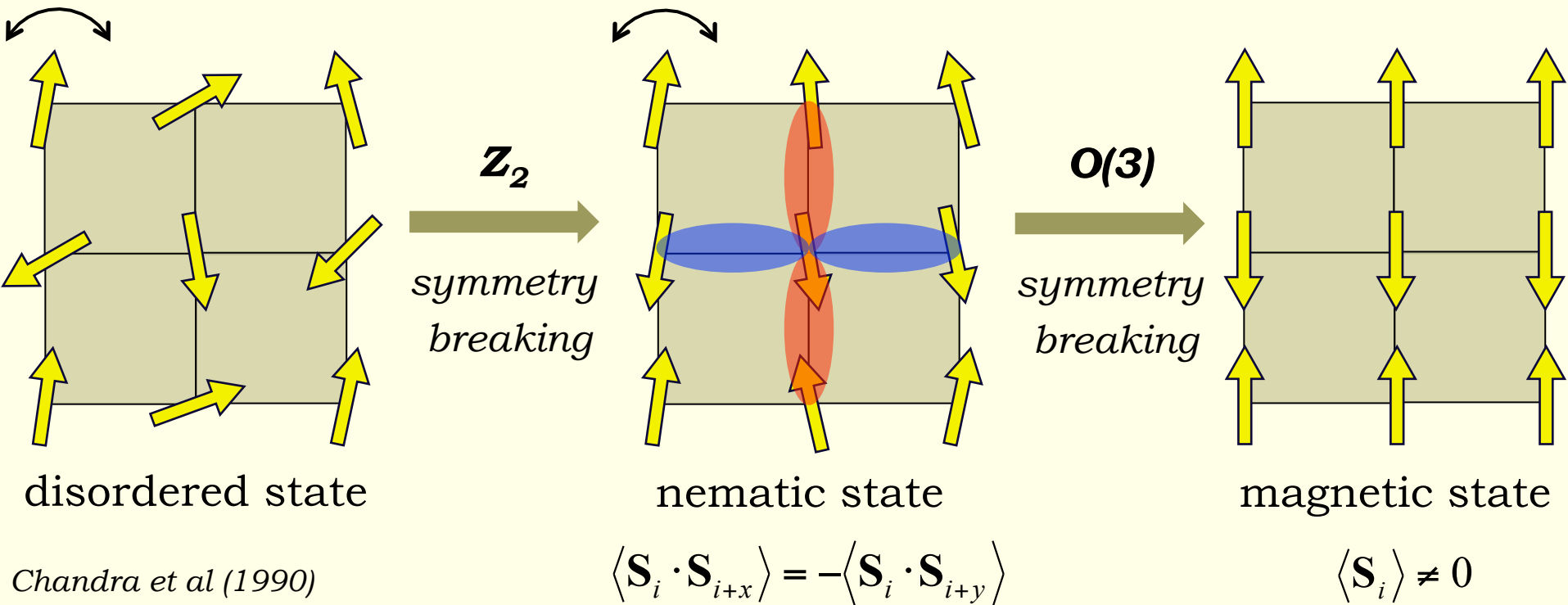
- Symmetry breaking in the striped magnetic state of the iron pnictides:



# Iron pnictides: nematic order

- A state that breaks  $Z_2$  symmetry but remains paramagnetic

**spontaneous tetragonal symmetry breaking**



Chandra et al (1990)

Fang et al (2008)

Xu et al (2008)

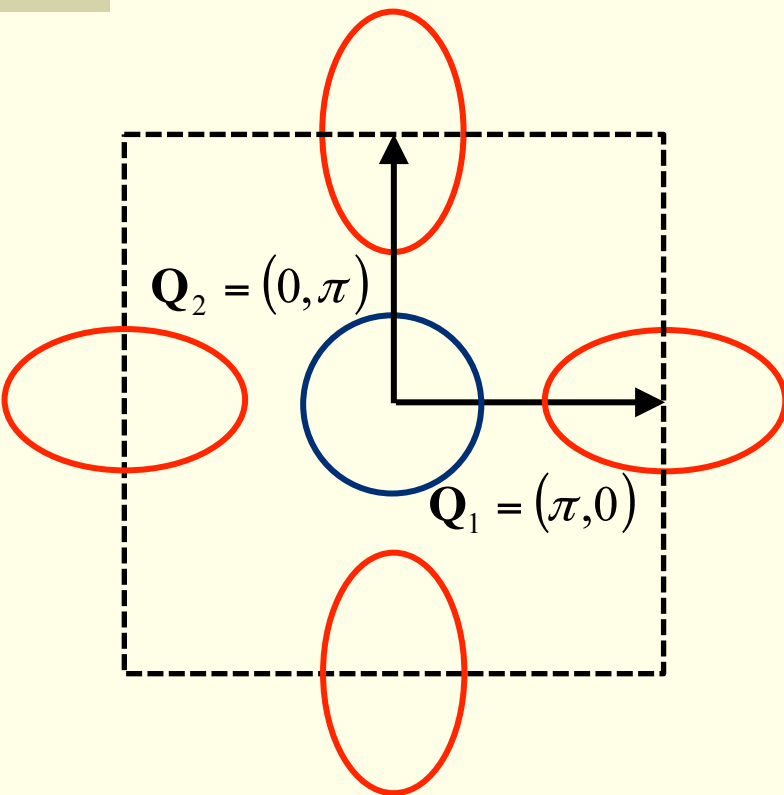
# Itinerant approach to the nematic state

**Two spin-density wave instabilities driven by nesting:**

$$\mathbf{S} = \mathbf{M}_1 e^{i\mathbf{Q}_1 \cdot \mathbf{r}} + \mathbf{M}_2 e^{i\mathbf{Q}_2 \cdot \mathbf{r}}$$

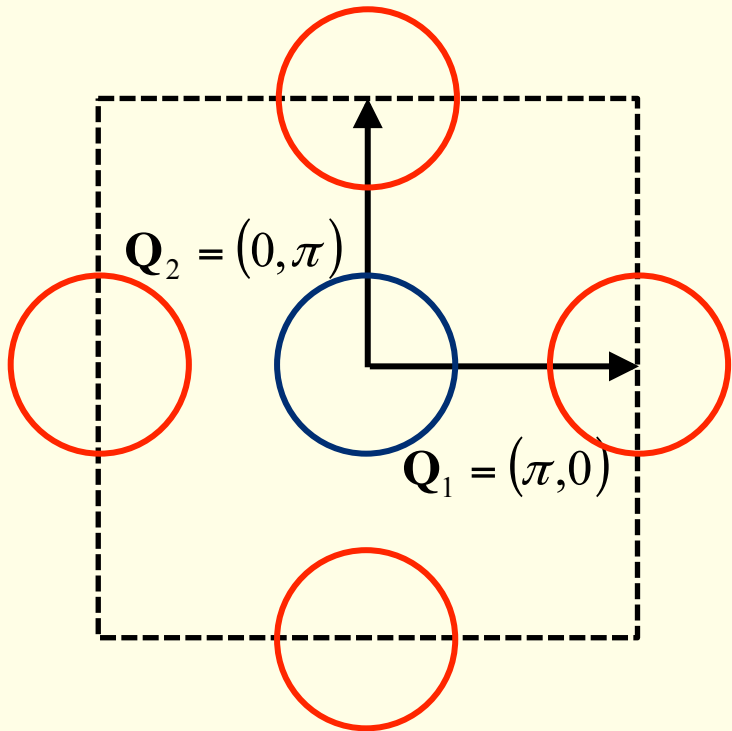
*Microscopic calculation of the free energy (Hertz-Millis approach)*

$$H = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}}^{(h)} d_{\mathbf{k}\alpha}^+ d_{\mathbf{k}\alpha} + \sum_{\mathbf{j}, \mathbf{k}} \varepsilon_{\mathbf{j}, \mathbf{k}}^{(e)} c_{\mathbf{j}, \mathbf{k}\alpha}^+ c_{\mathbf{j}, \mathbf{k}\alpha} \\ + I \sum_{\mathbf{j}, \mathbf{k}, \mathbf{k}', \mathbf{q}} \left( c_{\mathbf{j}, \mathbf{k}\alpha}^+ \sigma_{\alpha\beta} d_{\mathbf{k}+\mathbf{q}\beta} \right) \left( d_{\mathbf{k}'\gamma}^+ \sigma_{\gamma\delta} c_{\mathbf{j}, \mathbf{k}'-\mathbf{q}\delta} \right)$$



# Itinerant approach to the nematic state

For perfect nesting:  $F_{\text{mag}} = \frac{a}{2} (M_1^2 + M_2^2) + \frac{u}{4} (M_1^2 + M_2^2)^2$

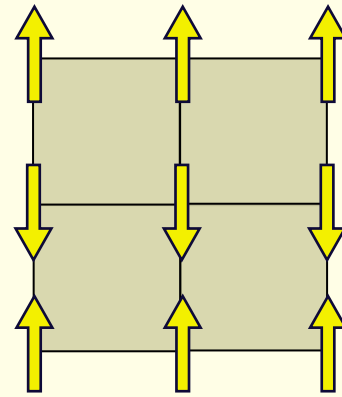
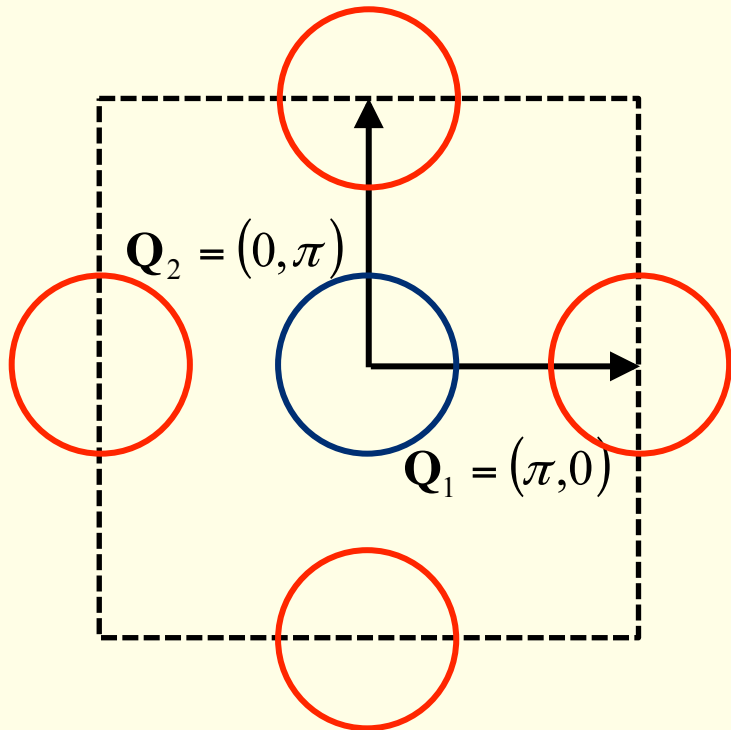


**mean-field solution:**  $\frac{\partial F_{\text{mag}}}{\partial M_i} = 0$

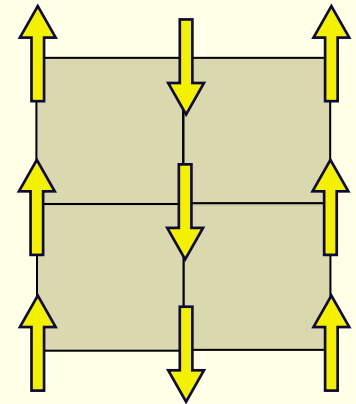
$$M_1^2 + M_2^2 = \text{constant}$$

# Itinerant approach to the nematic state

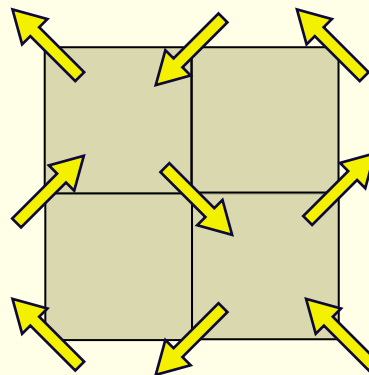
For perfect nesting:



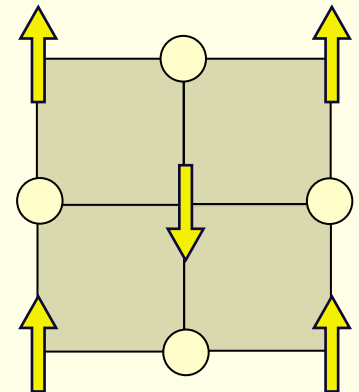
$$\mathbf{M}_1 = 0 \text{ \& } \mathbf{M}_2 \neq 0$$



$$\mathbf{M}_1 \neq 0 \text{ \& } \mathbf{M}_2 = 0$$



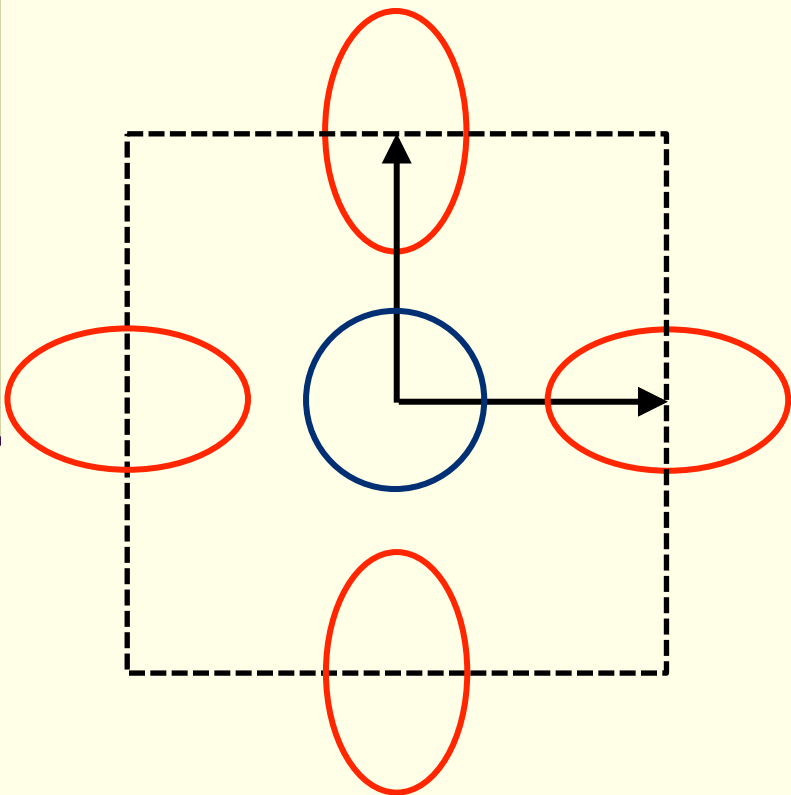
$$\mathbf{M}_1 \perp \mathbf{M}_2$$



$$\mathbf{M}_1 \parallel \mathbf{M}_2$$

# Itinerant approach to the nematic state

Away from perfect nesting:  $F_{\text{mag}} = \frac{a}{2} (M_1^2 + M_2^2) + \frac{u}{4} (M_1^2 + M_2^2)^2 - \frac{g}{4} (M_1^2 - M_2^2)^2$



$$\left\{ \begin{array}{l} a = \frac{2}{U_{\text{spin}}} + 2 \int_k G_{\Gamma,k} G_{X,k} \\ u = \frac{1}{2} \int_k G_{\Gamma,k}^2 (G_{X,k} + G_{Y,k})^2 \\ g = -\frac{1}{2} \int_k G_{\Gamma,k}^2 (G_{X,k} - G_{Y,k})^2 > 0 \end{array} \right.$$

$$G_{i,k}^{-1} = i\omega_n - \varepsilon_k$$

# Itinerant approach to the nematic state

**mean-field solution:**  $\frac{\partial F_{\text{mag}}}{\partial M_i} = 0$   $+ g M_1^2 M_2^2$

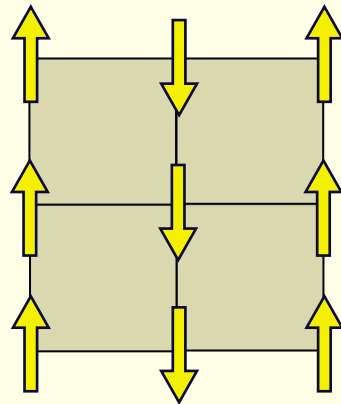
$$F_{\text{mag}} = \frac{a}{2} (M_1^2 + M_2^2) + \frac{u}{4} (M_1^2 + M_2^2)^2 - \overbrace{\frac{g}{4} (M_1^2 - M_2^2)^2}$$



# Itinerant approach to the nematic state

**mean-field solution:**  $\frac{\partial F_{\text{mag}}}{\partial M_i} = 0$   $+ g M_1^2 M_2^2$

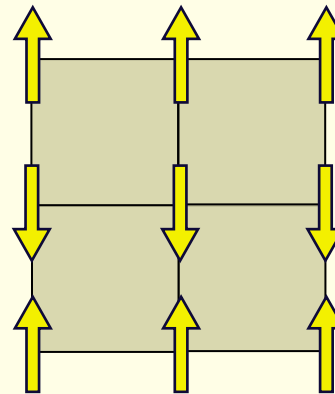
$$F_{\text{mag}} = \frac{a}{2} (M_1^2 + M_2^2) + \frac{u}{4} (M_1^2 + M_2^2)^2 - \overbrace{\frac{g}{4} (M_1^2 - M_2^2)^2}$$



$$M_1 \neq 0$$

$$M_2 = 0$$

OR



$$M_1 = 0$$

$$M_2 \neq 0$$

# Itinerant approach to the nematic state

**To consider the possibility of a nematic state, we need to include fluctuations**

$$F_{\text{mag}} = \chi_{\text{mag}}^{-1}(\mathbf{q})(M_1^2 + M_2^2) + \underbrace{\frac{u}{4}(M_1^2 + M_2^2)^2}_{\psi \propto M_1^2 + M_2^2} - \underbrace{\frac{g}{4}(M_1^2 - M_2^2)^2}_{\varphi \propto M_1^2 - M_2^2}$$

*nematic  
order parameter*

# Itinerant approach to the nematic state

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$$F_{\text{mag}} = \chi_{\text{mag}}^{-1}(\mathbf{q})(M_1^2 + M_2^2) + \underbrace{\frac{u}{4}(M_1^2 + M_2^2)^2}_{\psi \propto M_1^2 + M_2^2} - \underbrace{\frac{g}{4}(M_1^2 - M_2^2)^2}_{\varphi \propto M_1^2 - M_2^2}$$

*nematic  
order parameter*

- By integrating out the magnetic fluctuations, we obtain the free energy for the nematic degrees of freedom

$$F_{\text{nem}} = \frac{a_n}{2} \varphi^2 + \frac{u_n}{4} \varphi^4$$

# Itinerant approach to the nematic state

---

Equation of state for the nematic order parameter:

$$\varphi^3 = \varphi \left[ g \int \chi_{\text{mag}}^2(\mathbf{q}) - 1 \right]$$

**$\varphi \neq 0$  solution already in the paramagnetic phase, when the magnetic susceptibility is large enough**

$$\langle M_1^2 \rangle \neq \langle M_2^2 \rangle$$

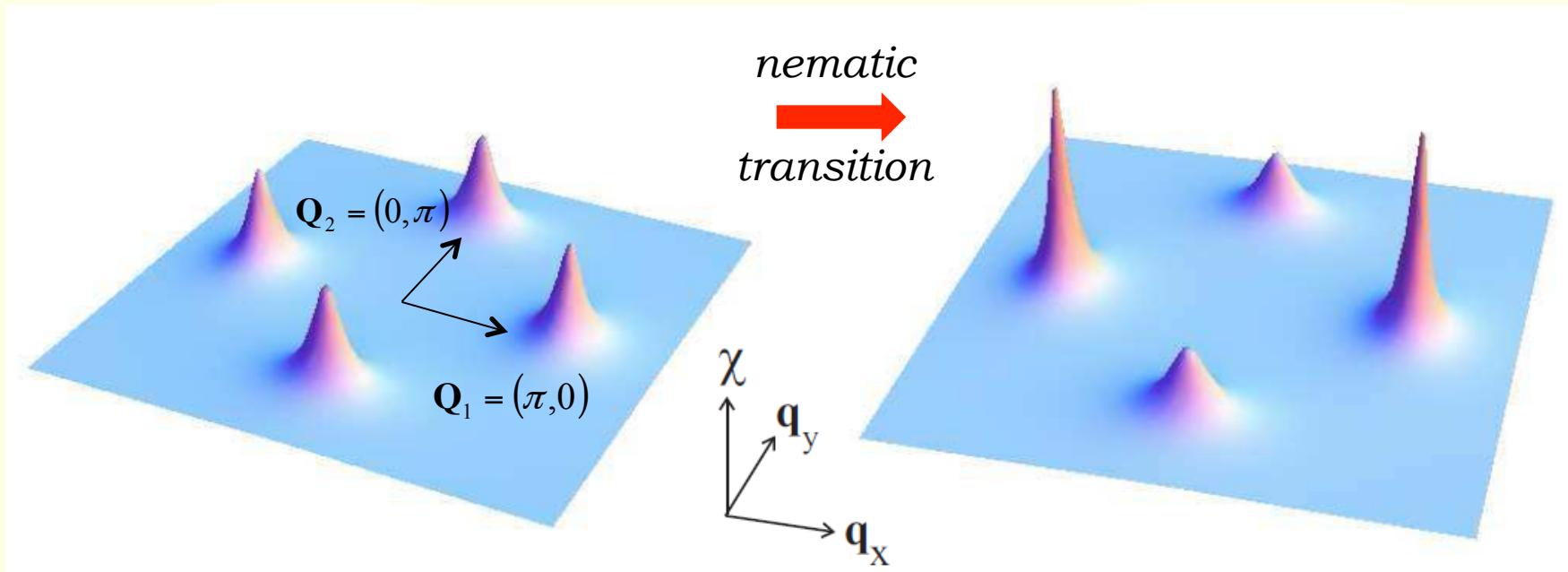
**magnetic  
fluctuations**



**nematic order**

# Itinerant approach to the nematic state

- Magnetic fluctuations become stronger around one of the ordering vectors in the paramagnetic phase



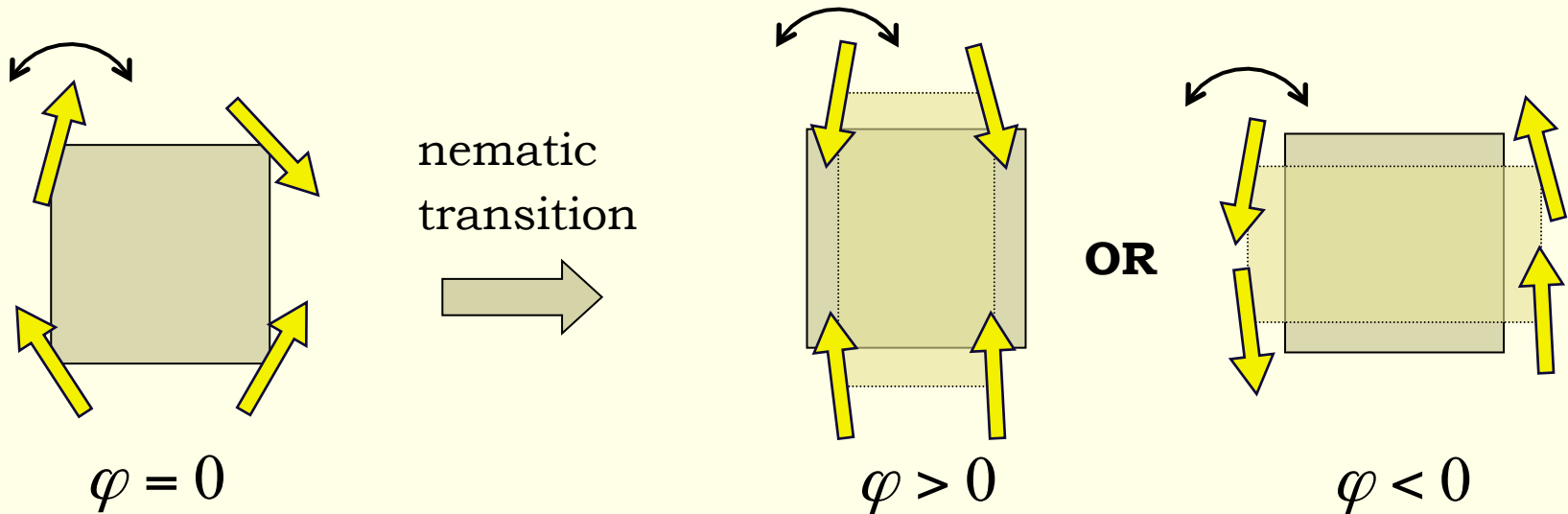
$x$  and  $y$  directions become inequivalent:  
tetragonal symmetry breaking

# Nematic transition triggers structural transition

**magneto-elastic coupling:**  $H_{\text{mag-el}} = \lambda \sum_{\mathbf{k}} \delta \left( c_{X,\mathbf{k}\alpha}^+ c_{X,\mathbf{k}\alpha} - c_{Y,\mathbf{k}\alpha}^+ c_{Y,\mathbf{k}\alpha} \right)$

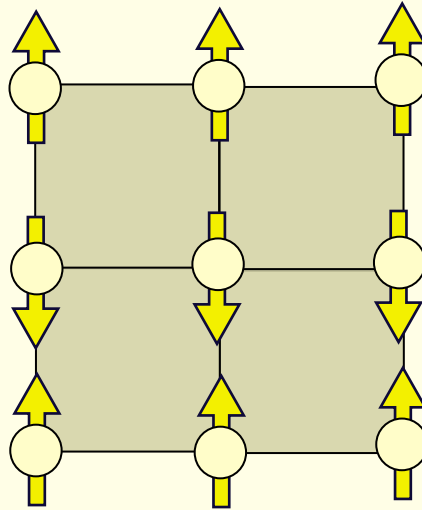
$$\delta = \frac{a-b}{a+b}$$

$$\Rightarrow \langle \delta \rangle \propto \langle \varphi \rangle$$



*Structural transition driven by magnetic fluctuations*

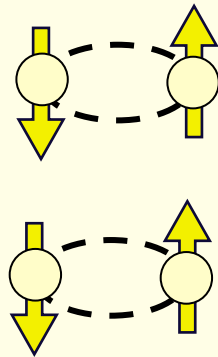
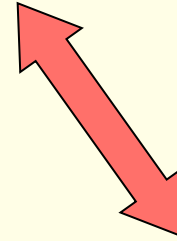
**magnetic phase**



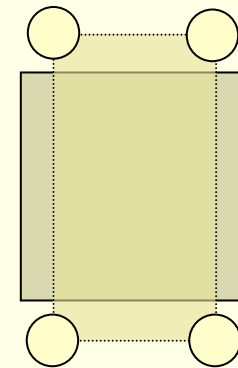
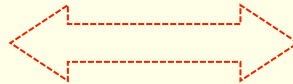
*competing  
order*



*emergent  
order*



*indirect  
competition*



- structural transition
- orbital order

**superconducting phase**

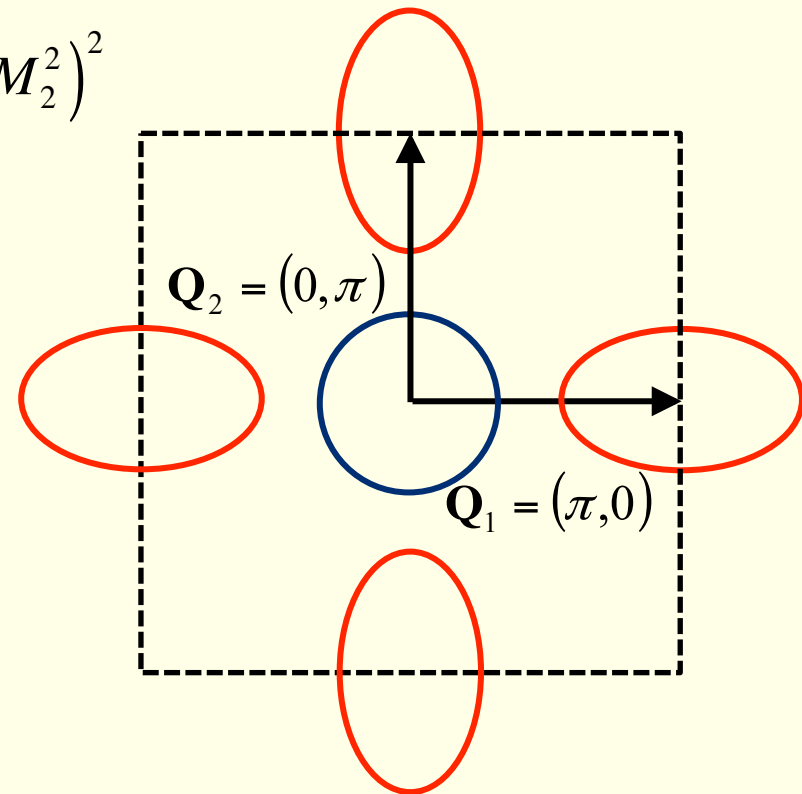
**nematic phase**

# Competition between superconductivity and nematicity

- Nematicity is a consequence of magnetic fluctuations

**Free energy (without SC)**

$$F = \frac{a_m}{2} (M_1^2 + M_2^2) + \frac{u_m}{4} (M_1^2 + M_2^2)^2 - \frac{g_m}{4} (M_1^2 - M_2^2)^2$$





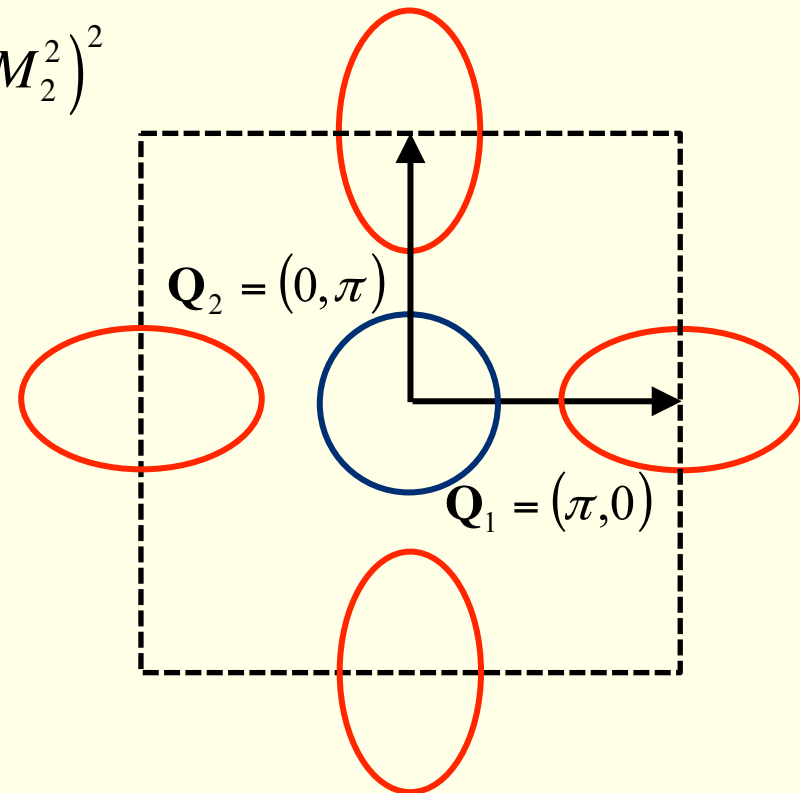
# Competition between superconductivity and nematicity

- But magnetism competes with SC

## Free energy (with SC)

$$F = \frac{a_m}{2} (M_1^2 + M_2^2) + \frac{u_m}{4} (M_1^2 + M_2^2)^2 - \frac{g_m}{4} (M_1^2 - M_2^2)^2 \\ + \frac{a_s}{2} |\Delta|^2 + \frac{u_s}{4} |\Delta|^4 + \frac{\gamma_{ms}}{2} (M_1^2 + M_2^2) |\Delta|^2$$

*competition between  
magnetism and SC*



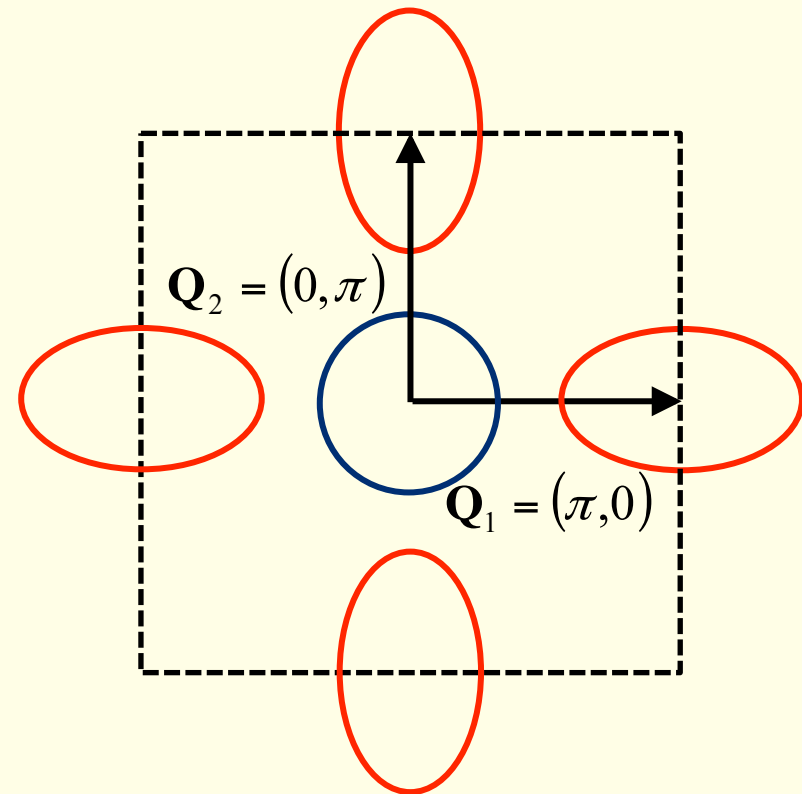
# Competition between superconductivity and nematicity

- By integrating out the magnetic fluctuations, we obtain an indirect competition between nematicity and SC

$$F = \frac{a_n}{2} \varphi^2 + \frac{u_n}{4} \varphi^4 + \frac{a_s}{2} |\Delta|^2 + \frac{u_s}{4} |\Delta|^4 + \frac{\gamma_{ns}}{2} \varphi^2 |\Delta|^2$$

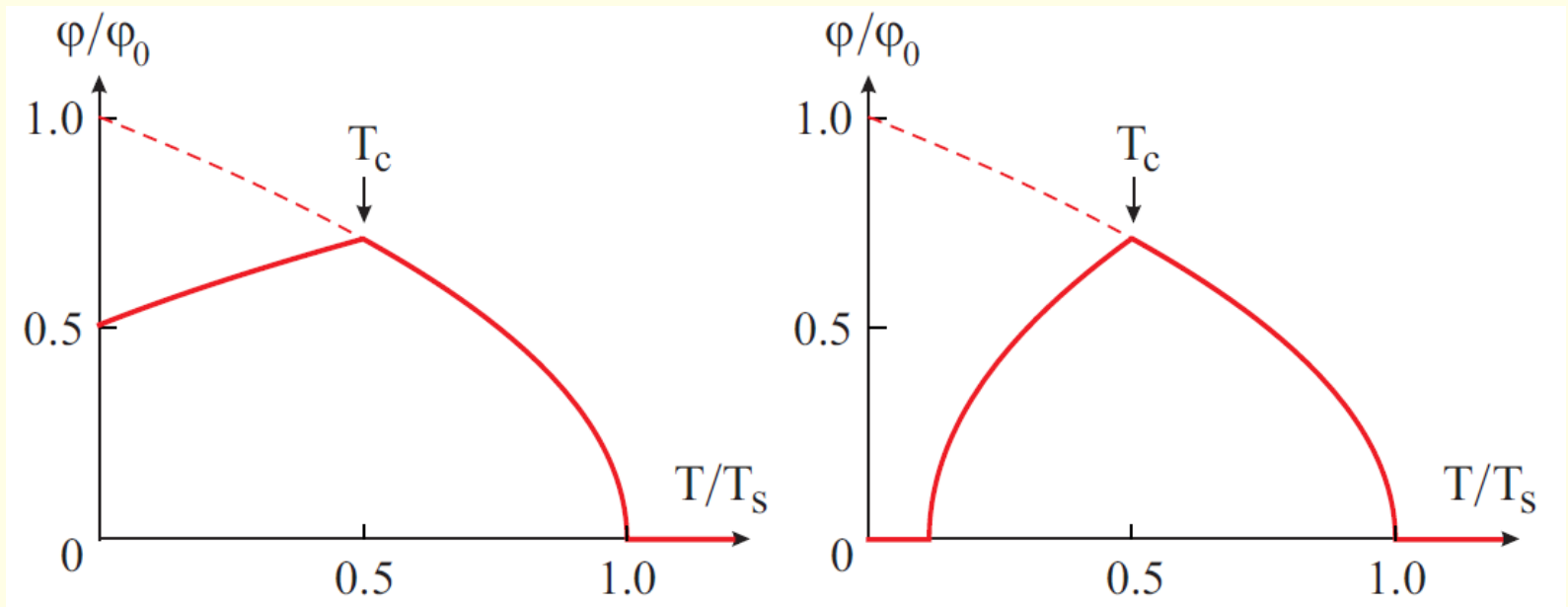
*competition between  
nematicity and SC*

$$\gamma_{ns} \propto \gamma_{ms}$$



# Competition between superconductivity and nematicity

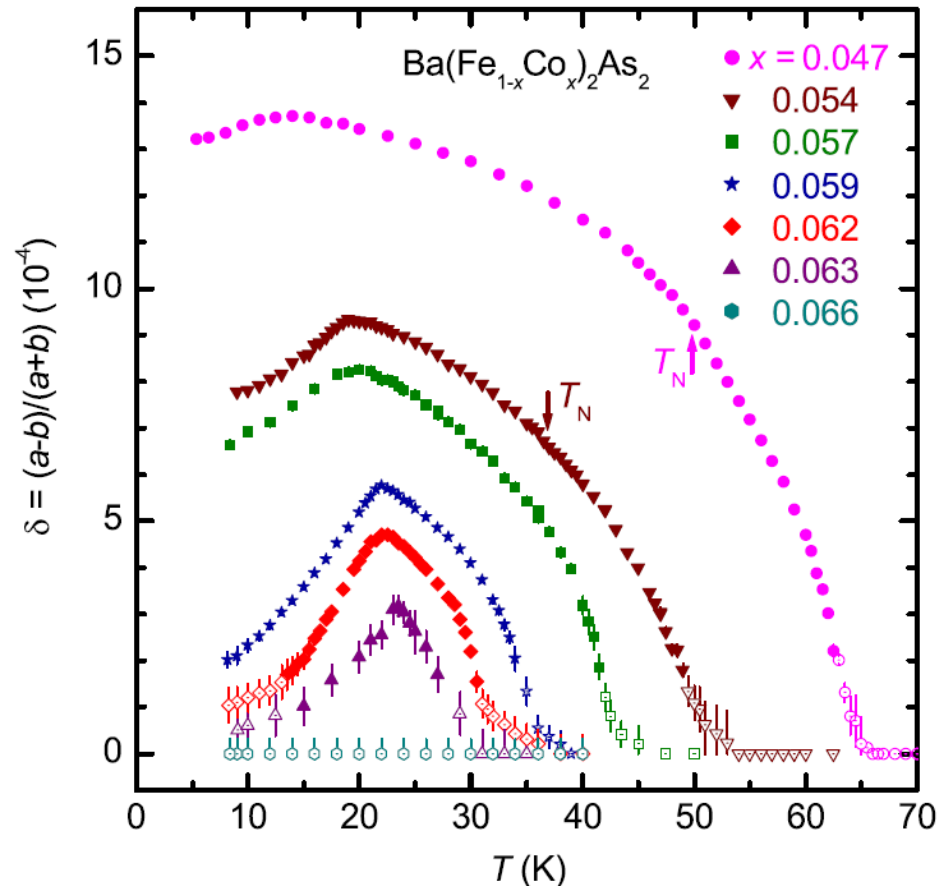
- Due to its magnetic origin, nematicity also competes with superconductivity
  - *even in the absence of long-range magnetic order*



# Competition between superconductivity and nematicity

- X-ray diffraction: experimental observation of the suppression of the orthorhombic distortion below  $T_c$

**competition  
& coexistence**

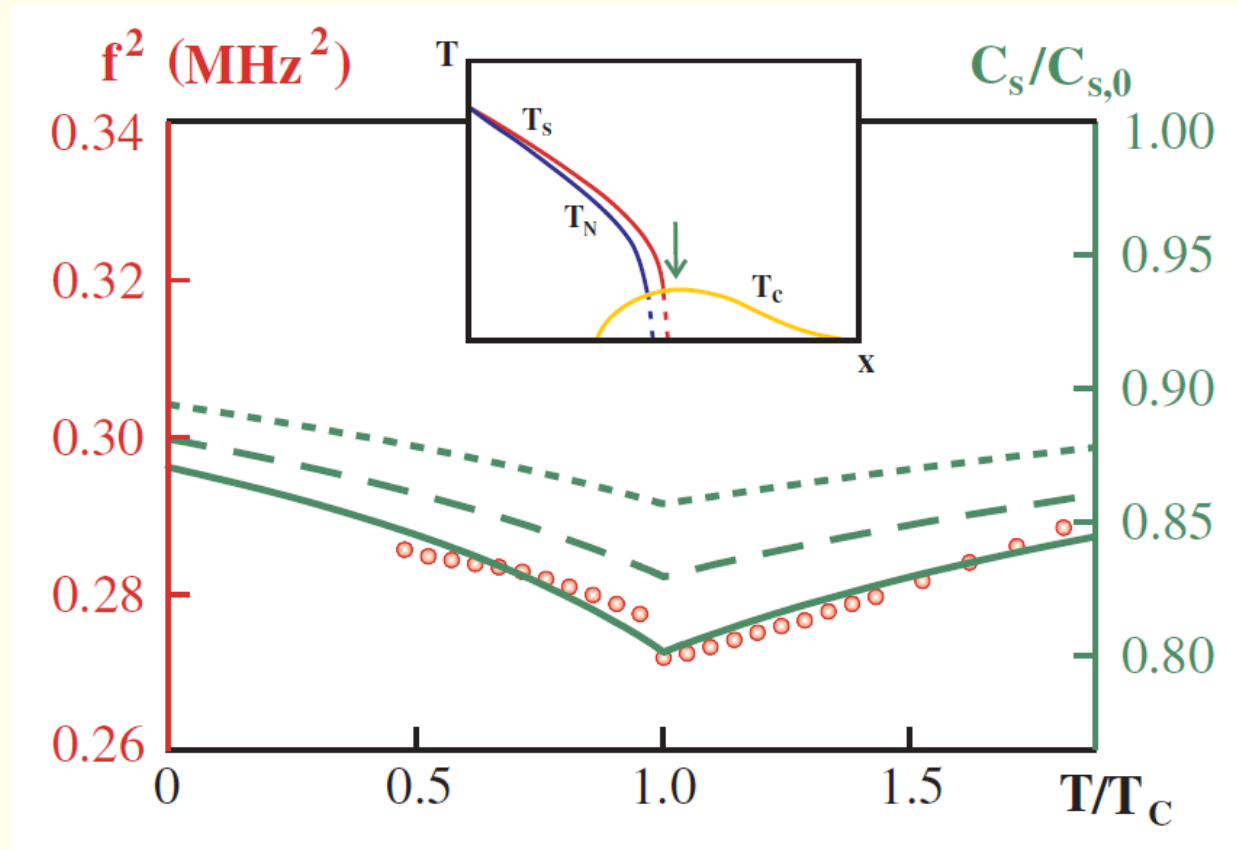


# Competition between superconductivity and nematicity

- The same model explains the hardening of the shear modulus below the SC transition temperature.

$$\left(\frac{C_s}{C_{s,0}}\right)^{-1} = 1 + \left(\frac{\lambda}{C_{s,0}}\right) \chi_{\text{nem}}$$

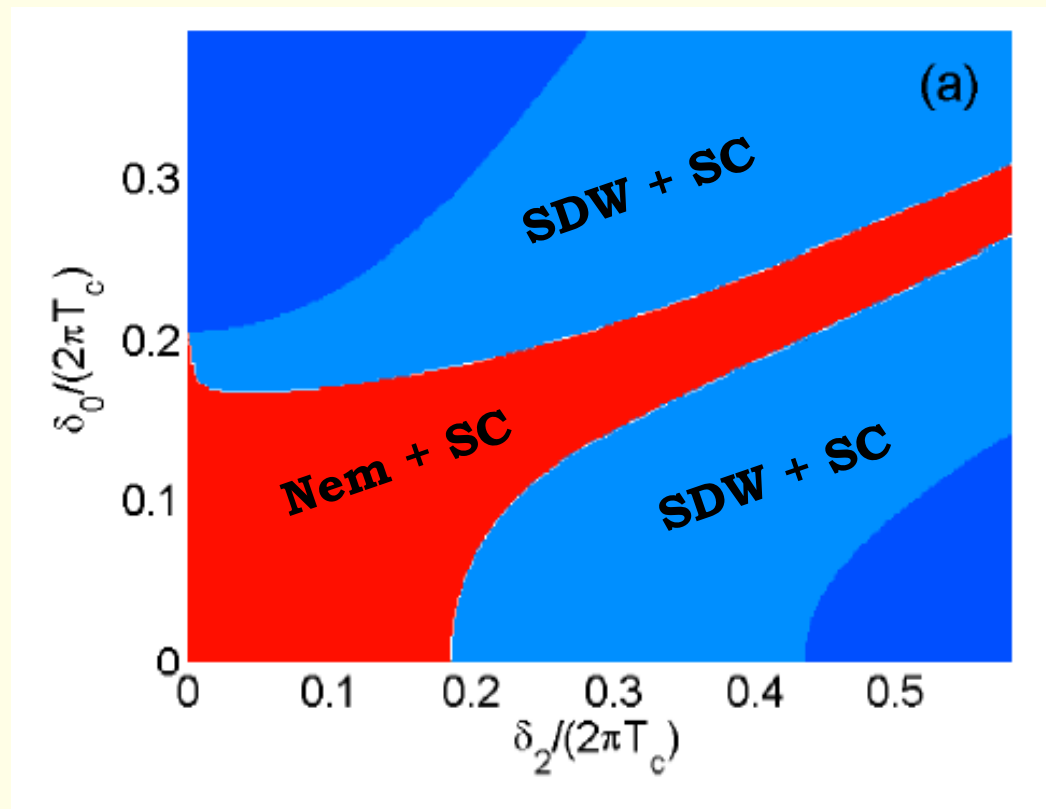
$$\chi_{\text{nem}} = \frac{\sum_{\mathbf{k}} \chi_{\text{mag}}^2(\mathbf{k})}{1 - g \sum_{\mathbf{k}} \chi_{\text{mag}}^2(\mathbf{k})}$$



# Competition between superconductivity and nematicity

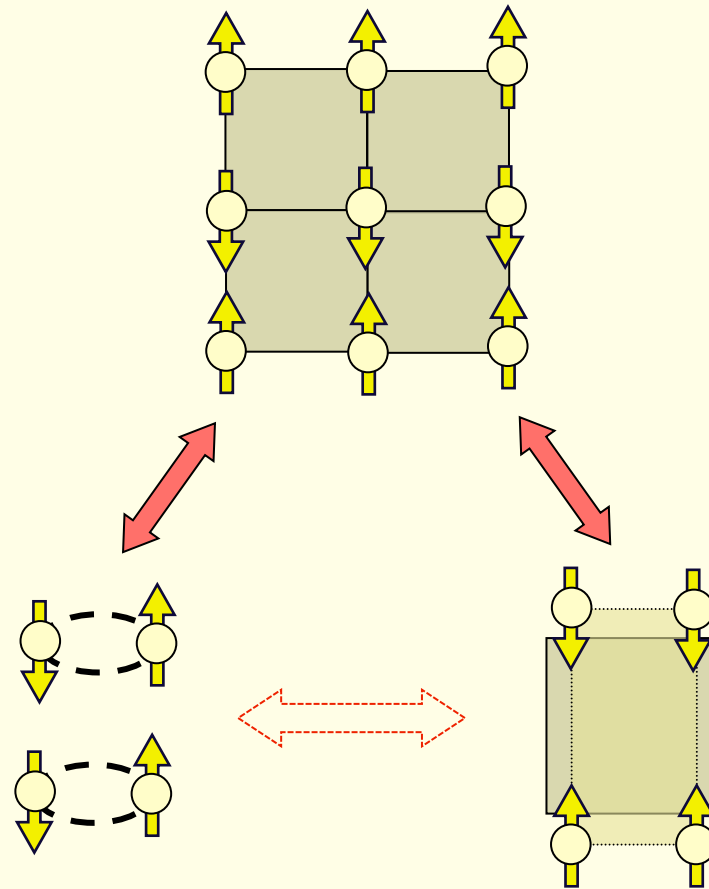
- Nematicity and superconductivity compete, but can coexist!

*band ellipticity  
(pressure)*



*carrier concentration (doping)*

# Conclusions



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