TESTING GRAVITY 2015
15-17 JANUARY 2015, SFU HARBOUR CENTER, VANCOUVER, BC, CANADA

Some recent advances on « massive gravity »
(a short and biased review)

1. Generic properties and problems of massive gravity.

2. Some recent progresses

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FP7/2007-2013
« NIRG » project no. 307934

1.1. Quadratic massive gravity: the Pauli-Fierz theory and the vDVZ discontinuity

Pauli-Fierz action: second order action for a massive spin two

$$\int d^4x \underbrace{\sqrt{g} R_g}_{\text{second order in } h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}} + m^2 \int d^4x h_{\mu\nu} h_{\alpha\beta} \left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

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second order in $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$

Only Ghost-free (quadratic) action for a massive spin two Pauli, Fierz 1939

(NB: $h_{\mu\nu}$ is TT: **5 degrees of freedom**)

1.1. Quadratic massive gravity: the Pauli-Fierz theory and the vDVZ discontinuity

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vDVZ discontinuity
(van Dam, Veltman; Zakharov; Iwasaki 1970)

The propagators read

propagator for $m=0$ $\quad D_0^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2p^2} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{2p^2} + \mathcal{O}(p)$

propagator for $m\neq 0$ $\quad D_m^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2(p^2-m^2)} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{3(p^2-m^2)} + \mathcal{O}(p)$

2.2. Non linear Pauli-Fierz theory and the « Vainshtein Mechanism »

Can be defined by an action of the form

Isham, Salam, Strathdee, 1971

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R_g + L_g \right) + S_{int}[f,g],$$

Einstein-Hilbert action for the *g* metric

Matter action (coupled to metric *g*)

Interaction term coupling the metric *g* and the <u>non dynamical</u> metric *f*

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The interaction term $S_{int}[f,g]$, is chosen such that

- It is invariant under diffeomorphisms
- It has flat space-time as a vacuum
- When expanded around a flat metric ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $f_{\mu\nu} = \eta_{\mu\nu}$)
  It gives the Pauli-Fierz mass term

- Some working examples

$$S_{int}^{(2)} = -\frac{1}{8} m^2 M_P^2 \int d^4x \, \sqrt{-f} \, H_{\mu\nu} H_{\sigma\tau} \left(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau}\right)$$

(Boulware Deser)

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(Arkani-Hamed, Georgi, Schwartz)

with $H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$

- Infinite number of models with similar properties
- Have been investigated in different contexts
  - « f-g, strong, gravity » Isham, Salam, Strathdee 1971
  - « bigravity » Damour, Kogan 2003
  - « Higgs for gravity » t'Hooft 2007, Chamseddine, Mukhanov 2010

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Generically: a 6th ghost-like degree of freedom propagates (Boulware-Deser 1972)

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  - de Rham, Gabadadze, Tolley 2010, 2011

Generically: a 6th ghost like degree of freedom propagates (Boulware-Deser 1972)

Look for static spherically symmetric solutions with the ansatz (not the most general one)

$$\begin{cases} g_{AB}dx^A dx^B &= -J(r)dt^2 + K(r)dr^2 + L(r)r^2 d\quad{}^2 \\ f_{AB}dx^A dx^B &= -dt^2 + dr^2 + r^2 d\quad{}^2 \end{cases}$$

Gauge transformation

$$\begin{cases} g_{\mu\nu}dx^\mu dx^\nu &= -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\quad{}^2 \\ f_{\mu\nu}dx^\mu dx^\nu &= -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\quad{}^2 \end{cases}$$

Which can easily be compared to Schwarzschild

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Then look for an expansion in

G_N (or in $R_S \propto G_N M$) of the would-be solution

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2$$

$$\nu(R) = -\frac{R_S}{R}(1 + O(1)\epsilon \quad \text{With} \quad \epsilon = \frac{R_S}{m^4 R^5}$$

$$\lambda(R) = +\frac{1}{2}\frac{R_S}{R}(1 + O(1)\epsilon$$

Vainshtein 1972
In « some kind »
[Damour et al. 2003]
of non linear PF

Wrong light bending!

This coefficient equals +1
in Schwarzschild solution

Introduces a new length scale R_v in the problem
below which the perturbation theory diverges!

For the sun: bigger than solar system! with $R_v = (R_S m^{-4})^{1/5}$

So, what is going on at smaller distances?

Vainshtein 1972

There exists an other perturbative expansion at smaller distances, defined around (ordinary) Schwarzschild and reading:

$$\left.\begin{aligned}\nu(R) &= -\frac{R_S}{R}\left\{1 + \mathcal{O}\left(R^{5/2}/R_v^{5/2}\right)\right\} \\ \lambda(R) &= +\frac{R_S}{R}\left\{1 + \mathcal{O}\left(R^{5/2}/R_v^{5/2}\right)\right\}\end{aligned}\right\} \quad \text{with} \quad R_v^{-5/2} = m^2 R_S^{-1/2}$$

- This goes smoothly toward Schwarzschild as *m* goes to zero

- This leads to corrections to Schwarzschild which are non analytic in the Newton constant

The **Vainshtein mechanism** is widely used in various attempts to modify gravity in the IR

- DGP model
- Massive gravity
- Degravitation
- Cascading DGP
- Galileons
- GR with an auxiliary dimension
- k-Mouflage

Good indications that it does work…

… However no definite proof (up to recently only one work of Damour et al. ‘03 concluding that it does not work) that this is indeed the case !

The **Vainshtein mechanism** is widely used in various attempts to modify gravity in the IR

- DG
- Ma
- De
- Ca
- Ga
- GI
- k-I

Goo

e.g. in DGP:

Various arguments in favour of a working Vainshtein mechanism,

Including
- some exact cosmological solutions

C.D., Dvali, Gabadadze, Vainshtein ‘02
- Sphericall symmetric solution on the brane

Gabadadze, Iglesias ‘04
- Approximate solutions

Gruzinov ‘01, Tanaka ‘04

… However no definite proof (up to recently only one work of Damour et al. ‘03 concluding that it does not work) that this is indeed the case !

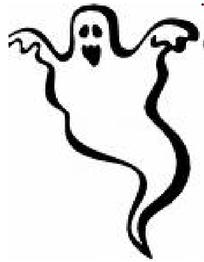
1.3.The crucial properties (and possible sickness) of massive gravity can all be seen taking its « decoupling limit »

Originally proposed in the analysis of Arkani-Hamed, Georgi and Schwartz (2003) using « Stückelberg » fields …

and leads (For a generic theory in the PF universality class) to the cubic action in the scalar sector (helicity 0) of the model

$$\frac{1}{2}\tilde{\phi}^{,\mu}\tilde{\phi}_{,\mu} - \frac{1}{M_P}\tilde{\phi}T - \frac{1}{\Lambda^5}\left\{\alpha\,(\Box\tilde{\phi})^3 + \beta\,(\Box\tilde{\phi}\;\tilde{\phi}_{,\mu\nu}\;\tilde{\phi}^{,\mu\nu})\right\}$$

« Strong coupling scale »
(hidden cutoff of the model ?)



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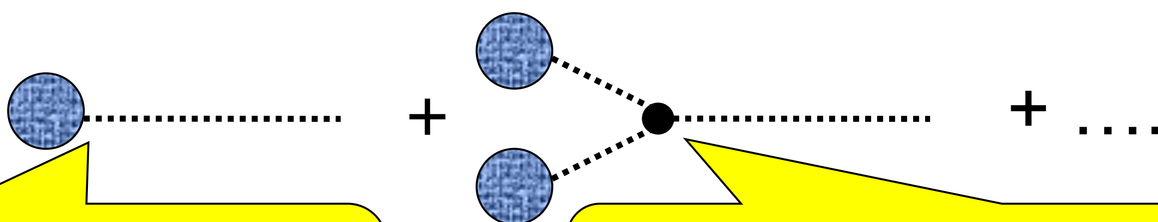
With $\Lambda = (m^4 M_P)^{1/5}$ and α and β model dependent coefficients

In the decoupling limit, the Vainshtein radius is kept fixed, and one can understand the Vainshtein mechanism as

Can one have a higher cutoff ?

E.g. around a heavy source: of mass M



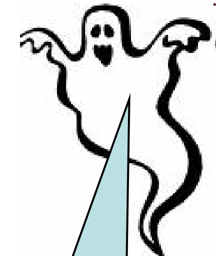
Interaction M/M_P of the external source with $\tilde{\phi}$

The cubic interaction above generates O(1) coorrection at $R=R_v= (R_S\, m^{-4})1/5$

« Strong coupling scale »
(hidden cutoff o

• 4. Superluminality !

Problems with causality ?



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With Λ = $(m^4 M_P)^{1/5}$ and α and β model de... ent coefficients

In the de... limit, the Vainshtein radius is kept ... and
o... is

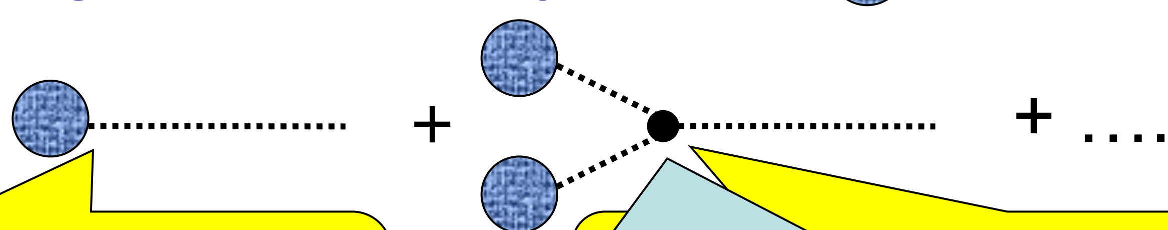
• 3. Low Strong Coupling scale

Can one have a higher cutoff ?

• 2. Boulware Deser ghost

Can one get rid of it ?

E.g. around a heavy source: of mass M



Interaction M/M$_P$ of
t
w

...interaction above generates
...t R=R_V= (R_S m^{-4})1/5

• 1. vDVZ discontinuity

Cured by the Vainshtein mechanism ?

2. Some recent progresses and open issues

2.1. The Vainshtein mechanism.

2.2. Getting rid of the Boulware-Deser ghost: the dRGT model

2.1. The Vainshtein mechanism

To summarize: 2 regimes

with $\epsilon = \frac{R_S}{m^4 R^5}$

$$\nu(R) = -\frac{R_S}{R}\left(1 + \mathcal{O}(1)\epsilon + \cdots\right)$$

Valid for $R \gg R_V$

with $R_V = \left(R_S m^{-4}\right)^{1/5}$

Standard perturbation theory around flat space

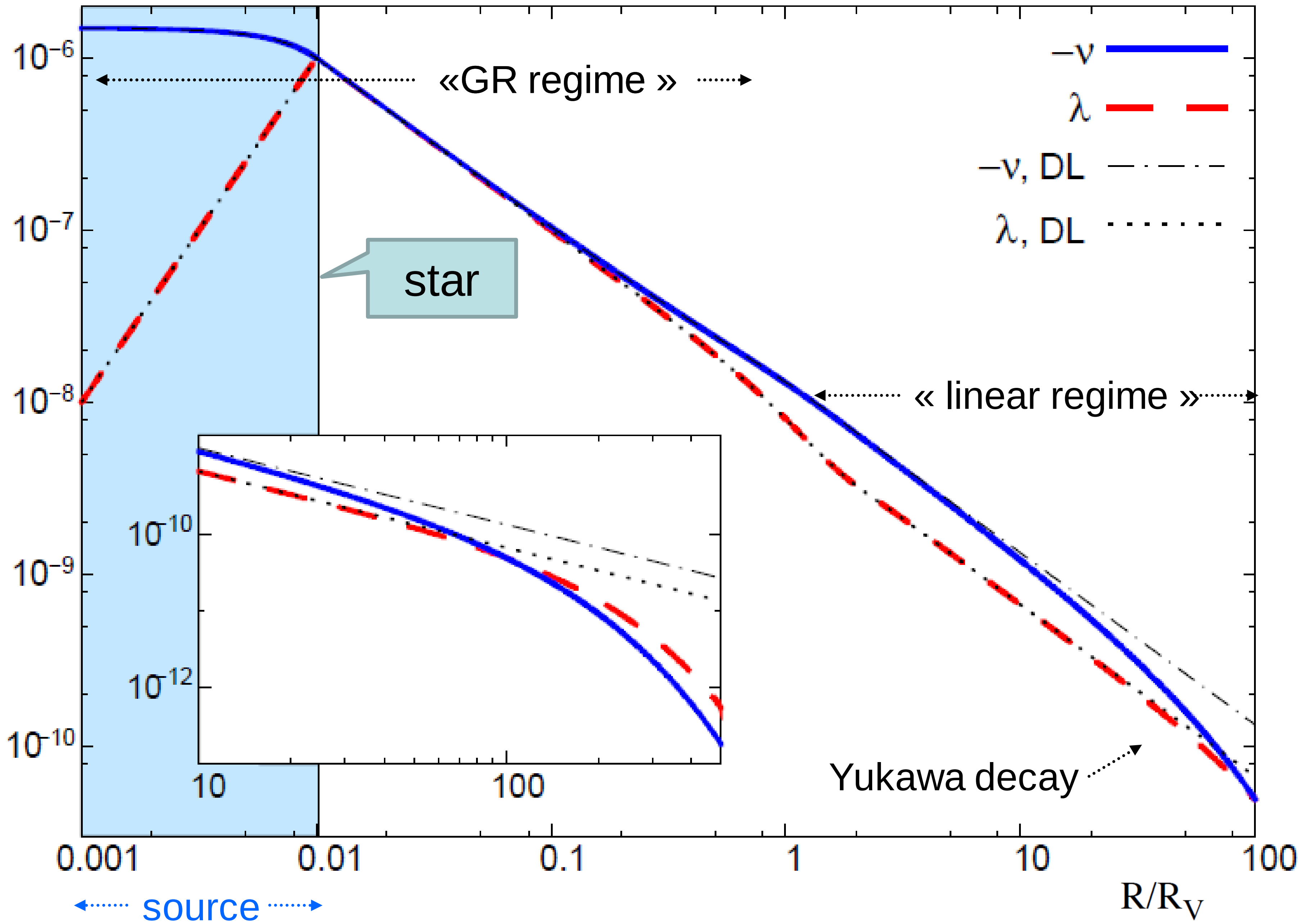
Crucial question: can one join the two regimes in a single existing non singular (asymptotically flat) solution? (Boulware Deser 72)

Expansion around Schwarzschild solution

$$\nu(R) = -\frac{R_S}{R}\left(1 + \mathcal{O}\left(R^{5/2}/R_V^{5/2}\right)\right)$$

Valid for $R \ll R_V$

Numerical investigations: Babichev, C.D., Ziour, 2009, 2010



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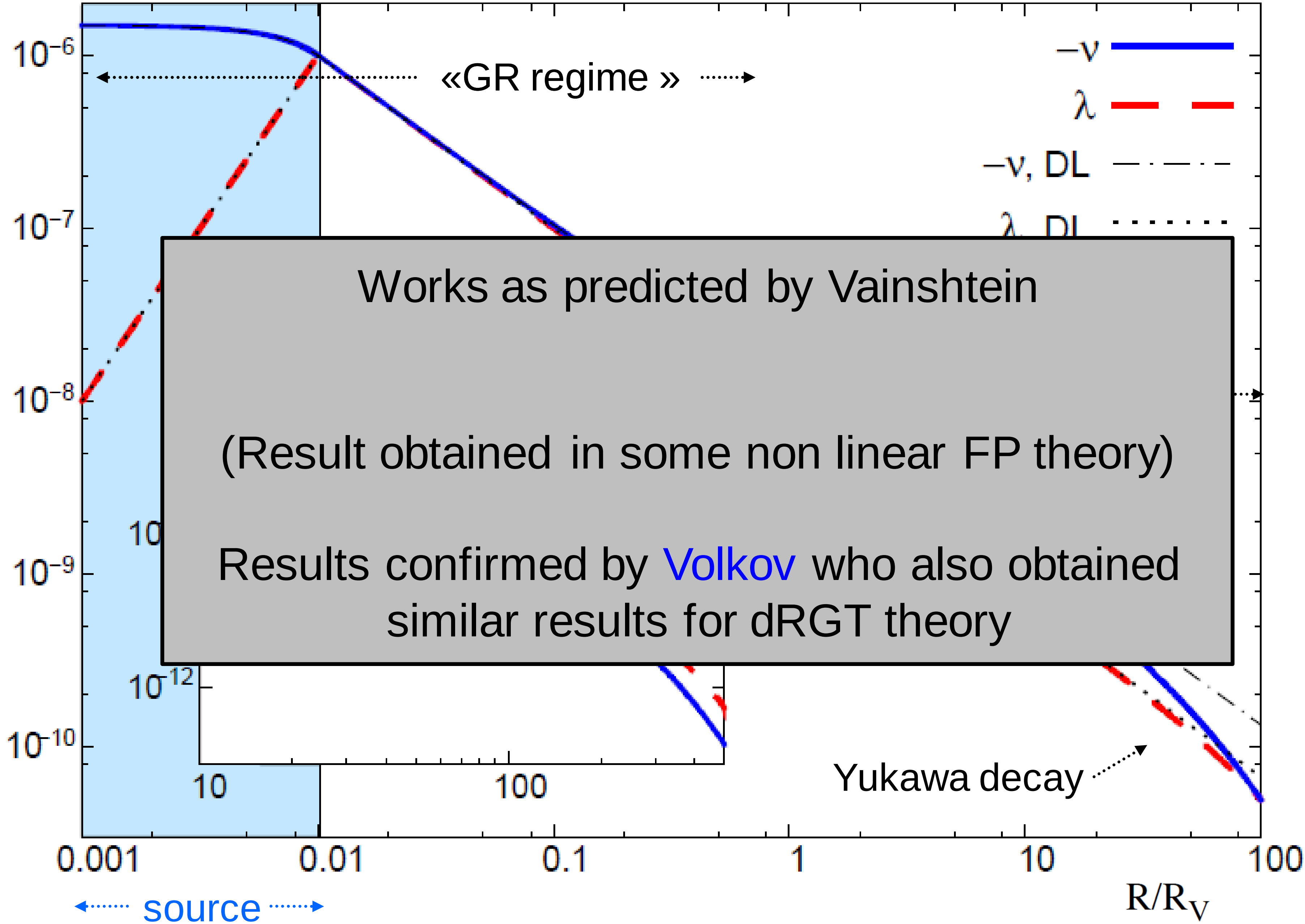

Works as predicted by Vainshtein

(Result obtained in some non linear FP theory)

Results confirmed by Volkov who also obtained similar results for dRGT theory

Solutions were obtained for very low density objects. We did (and still do) not know what is happening for dense objects (for BHs we now do know) or other more complicated solutions.

(Standard) Vainshtein mechanism does not work for black holes.

C.D.,T. Jacobson, 2012

∃ obstructions to have two metrics on the same manifold which do not share a common Killing horizon…

e.g. a the dynamical metric g and the non dynamical flat metric f of non linear Fierz-Pauli theory (applies to the case where metrics are commonly diagonal)

The (standard) Vainshtein mechanism does not work for Black Holes

End point of gravitational collapse ?

2.2. Getting rid of the Boulware Deser ghost : dRGT theory

de Rahm, Gabadadze; de Rham, Gababadze, Tolley 2010, 2011

Claim: the most general massive gravity (in the sense above) devoid of a Boulware Deser ghost is given by the 3 (4 counting Λ) parameters set of theories:

$$S = M_P^2 \int d^4x \sqrt{-g} \left\{ R + 2m^2 \sum_{n=1}^{3} \beta_n e_n(\mathbf{K}) \right\}$$

With

$$\begin{cases} K^{\mu}{}_{\nu} = \sqrt{g^{\mu\rho} f_{\rho\nu}} \\ e_1(\mathbf{K}) = tr\, \mathbf{K} \\ e_2(\mathbf{K}) = \frac{1}{2}\left((tr\, \mathbf{K})^2 - tr\, \mathbf{K}^2\right) \\ e_3(\mathbf{K}) = \frac{1}{6}\left((tr\, \mathbf{K})^3 - 3\, tr\, \mathbf{K}\, tr\, \mathbf{K}^2 + 2\, tr\, \mathbf{K}^3\right) \end{cases}$$

The absence of ghost is first seen in the decoupling limit
(using the observations of C.D., Rombouts 2005; Creminelli, Nicolis, Papucci, Trincherini 2005)

Which instead of the generic

$$\frac{1}{2}\tilde{\phi}\Box\tilde{\phi} + \frac{1}{\Lambda^5}\left\{\alpha\left(\Box\tilde{\phi}\right)^3 + \beta\left(\Box\tilde{\phi}\right)\tilde{\phi}_{,\mu\nu}\tilde{\phi}^{,\mu\nu}\right\}$$

With $\Lambda = (m^4 M_P)^{1/5}$

Looks like (de Rham, Gabadadze, 2010)

$$\begin{aligned}\frac{1}{2}\tilde{\phi}\Box\tilde{\phi} \quad &+ \quad \frac{1}{\Lambda_3^3}\tilde{\alpha}\left(\tilde{\phi}^{,\mu}\tilde{\phi}_{,\mu}\right)\Box\tilde{\phi} \\ &+ \quad \frac{1}{\Lambda_3^6}\tilde{\beta}\left(\tilde{\phi}^{,\mu}\tilde{\phi}_{,\mu}\right)\left(\tilde{\phi}_{,\mu\nu}\tilde{\phi}^{,\mu\nu} - \left(\Box\tilde{\phi}\right)^2\right) \\ &+ \quad \cdots\end{aligned}$$

With $\Lambda_3 = (m^2 M_P)^{1/3}$

The absence of ghost in the full theory has been heavily debated

Gabadadze, de Rham, Tolley;
Alberte, Chamseddine, Mukhanov;
Hassan, Rosen, Kluson, Alexandrov…

Easier to see using vierbeins

Hinterblicher, Rosen arXiv:1203.5783.
C.D., Mourad, Zahariade arXiv:1207.6338, 1208.4493

(Even though the metric and vierbein formulations are not totally equivalent
C.D., Mourad, Zahariade 2012; Banados, C.D., Pino, 2014)

The mass term

$$S = M_P^2 m^2 \int d^4x \sqrt{-g} \sum_{n=1}^{3} \beta_n e_n(\mathbf{K})$$

with $\begin{cases} g^{\mu\nu} &= \eta^{AB} e_A^\mu e_B^\nu \\ f_{\mu\nu} &= \eta_{AB} \omega_\mu^A \omega_\nu^B \end{cases}$

Can be written as Linear Combinations of

$$\left\{ \begin{array}{l} M_P^2 m^2 \beta_0 \int \epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} \wedge e^{A_4} \\ M_P^2 m^2 \beta_1 \int \epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} \wedge \omega^{A_4} \\ M_P^2 m^2 \beta_2 \int \epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge \omega^{A_3} \wedge \omega^{A_4} \\ M_P^2 m^2 \beta_3 \int \epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge \omega^{A_2} \wedge \omega^{A_3} \wedge \omega^{A_4} \end{array} \right.$$

Using the « symmetric vierbein condition »:
(or « Deser- van Nieuwenhuizen gauge condition »)

$$e_A^\mu \omega_{B\mu} = e_B^\mu \omega_{A\mu}$$

Rich phenomenology (self acceleration in particular) currently under investigation.

Extraction of a consistent theory (with always 5 – or less – dof) for a **massive graviton** living in a (single) **arbitrary metric** (hence extending FP theory)

L.Bernard, CD, M. von Strauss
1410.8302 + in preparation

$$S^{(2)} = -\frac{1}{2} M_g^2 \int \mathrm{d}^4 x \sqrt{|g|}\, h_{\mu\nu} \left(\tilde{\mathcal{E}}^{\mu\nu\rho\sigma} + m^2 \mathcal{M}^{\mu\nu\rho\sigma} \right) h_{\rho\sigma}$$

Einstein-Hilbert
kinetic operator

(curvature
dependent)
Mass term

Conclusions

- Massive gravity is a nice arena to explore large distance modifications of gravity.

- A first, possibly consistent (?), non linear theory has recently been proposed (after about 10 years of progresses following the DGP model)…

  … with many things still to be explored (in particular stability issues).