



An introduction to gravitational wave interferometry



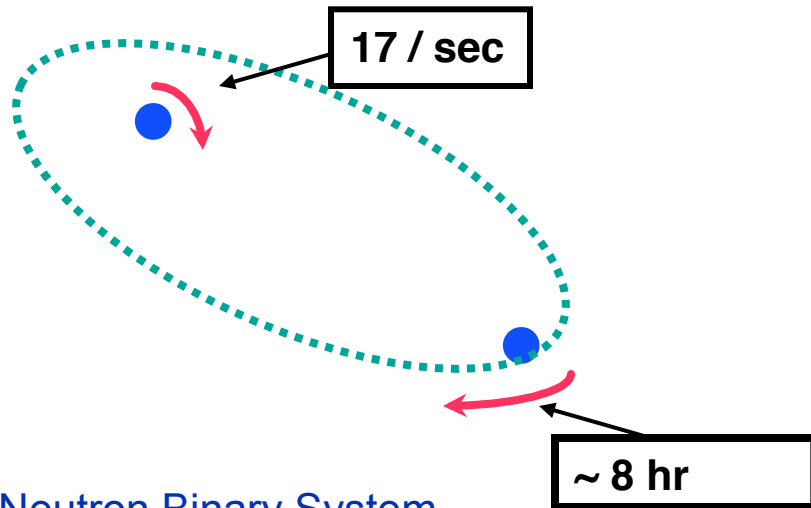
M. Landry
for the LIGO Scientific Collaboration
LIGO Hanford Observatory/Caltech
Testing Gravity Workshop – SFU Harbour Center
14 Jan 2015
LIGO-G1401332-v1



Hulse-Taylor binary pulsar

Neutron Binary System – Hulse & Taylor

PSR 1913 + 16 -- Timing of pulsars



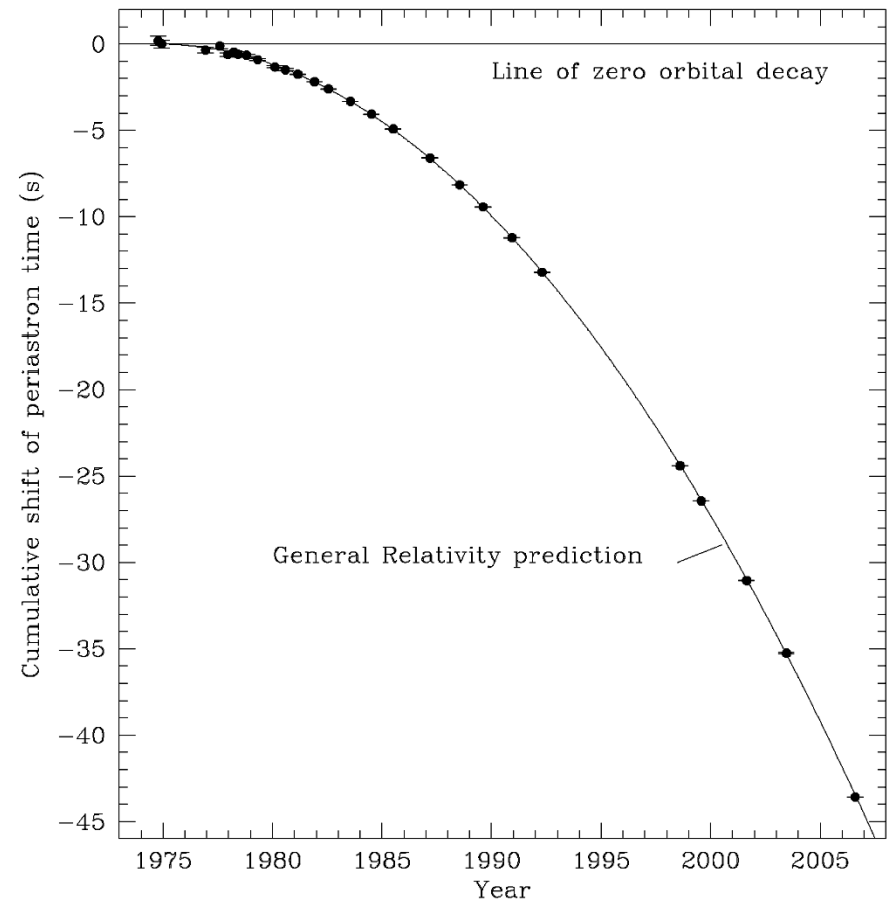
Neutron Binary System

- separated by 10^6 miles
- $m_1 = 1.4m_\odot$; $m_2 = 1.36m_\odot$; $\varepsilon = 0.617$

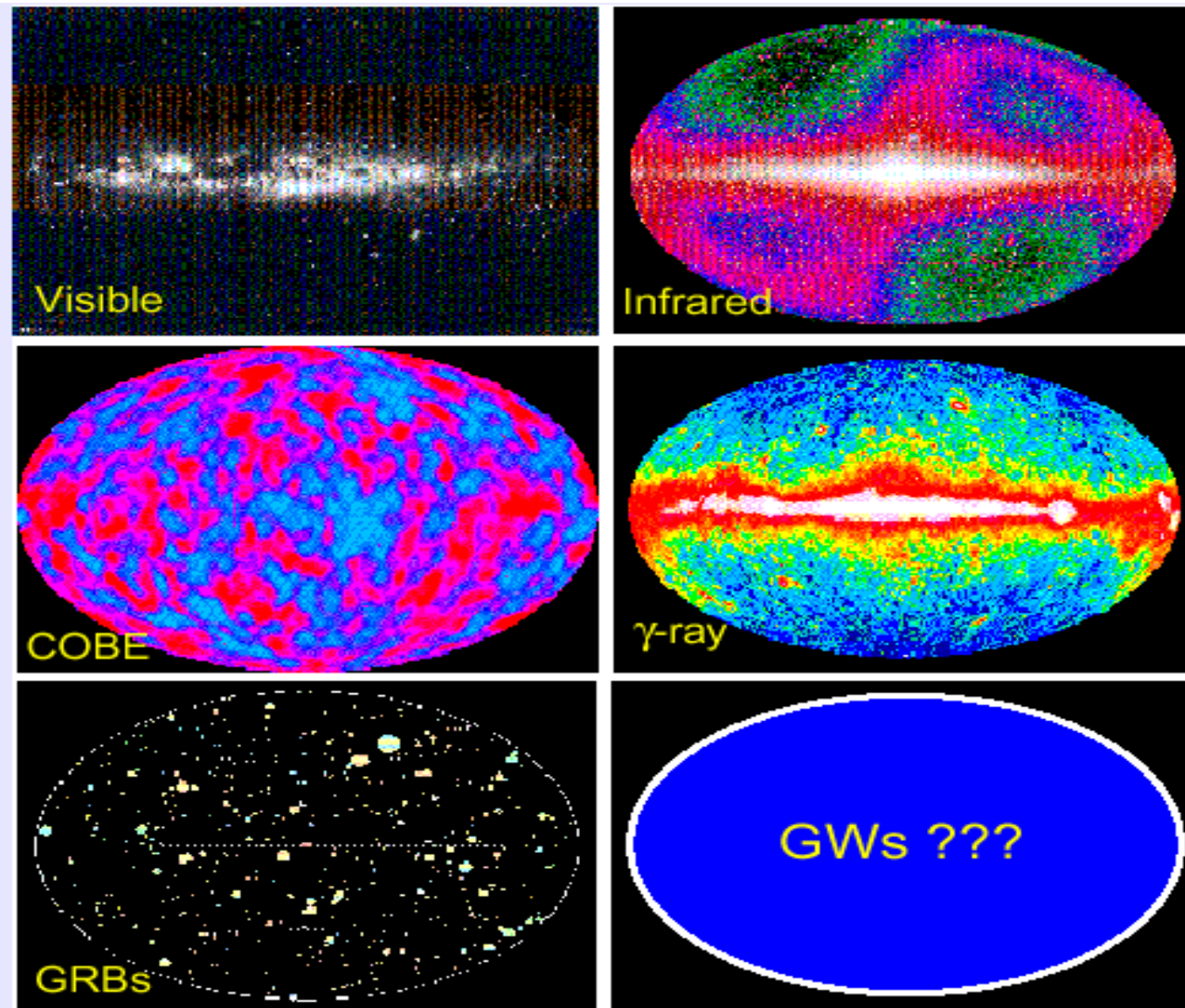
Prediction from general relativity

- spiral in by 3 mm/orbit
- rate of change orbital period

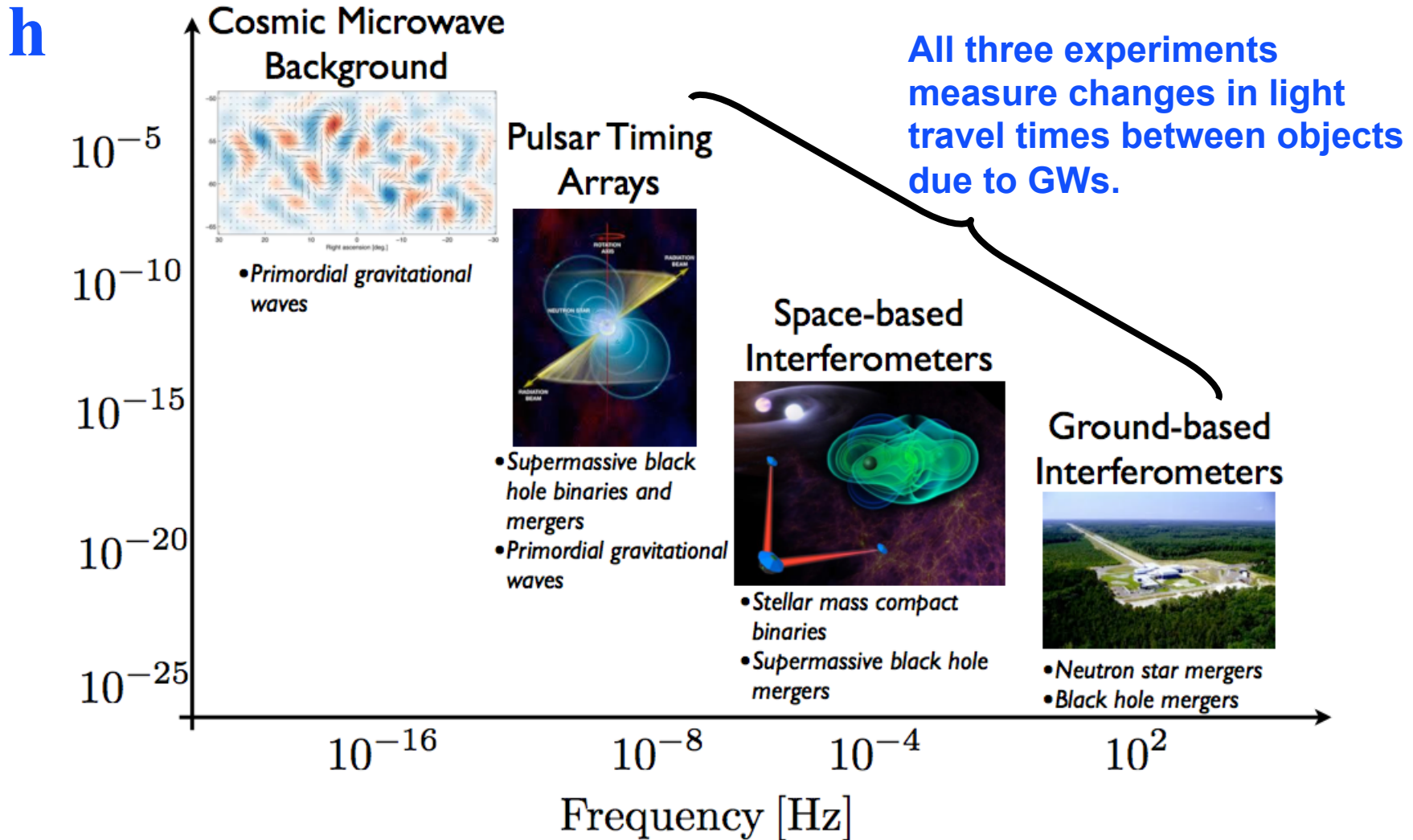
Emission of gravitational waves



Gravitational wave astronomy



Gravitational wave physics experiments



credit: X. Siemens and the NANOGrav Collaboration



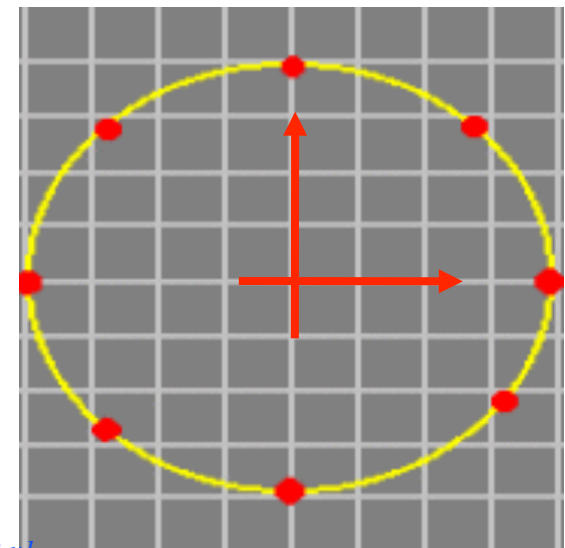
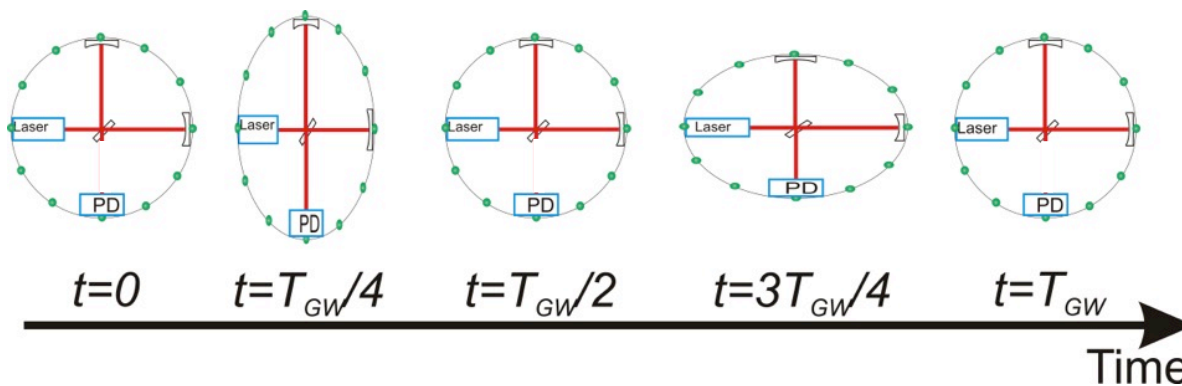
Outline

- Introduction to gravitational waves
 - » What are they? Expected signal strength?
 - » Astrophysical sources
 - » Interaction with interferometers
- Interferometry
 - » Optical configurations
 - » Controlling an interferometer
 - » Noise
 - » Calibration: how tiny a displacement can we record?
- The search for gravitational waves
 - » Advanced LIGO
 - » Plans for searches



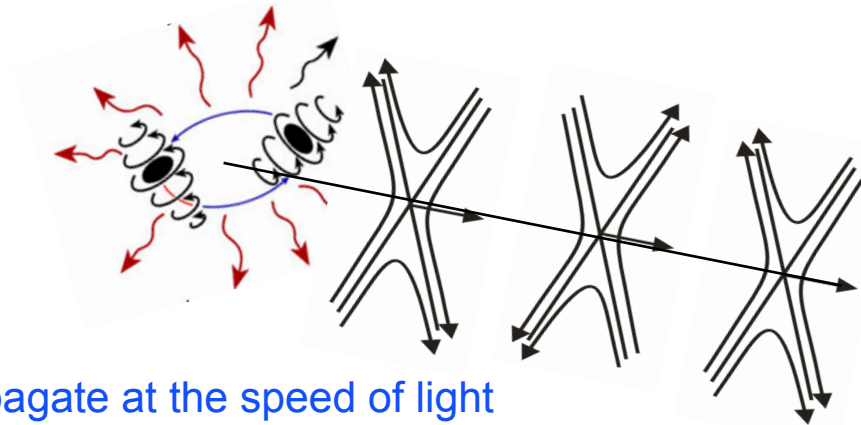
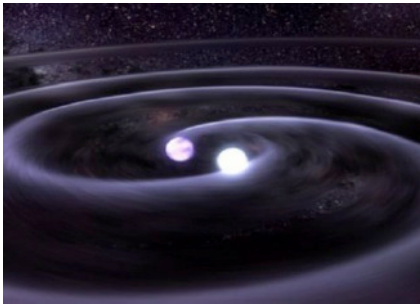
Gravitational waves

- Predicted by Einstein's theory of gravity, General Relativity, in 1916
- Generated by changing quadrupole moments such as in co-orbiting objects, spinning asymmetric objects
- Interact weakly with matter - even densest systems transparent to gravitational waves
- An entirely new spectrum in which to explore the universe



Gravitational waves

- Practically, need astrophysical objects moving near the speed of light



- » According to GR, GWs propagate at the speed of light
- » Quadrupolar radiation; two polarizations: h_+ and h_x

- Physically, gravitational waves are *strains*:

$$h = \frac{\Delta L(f)}{L}$$

- The above is an approximation good at low frequencies in the audio-band. More on this later.



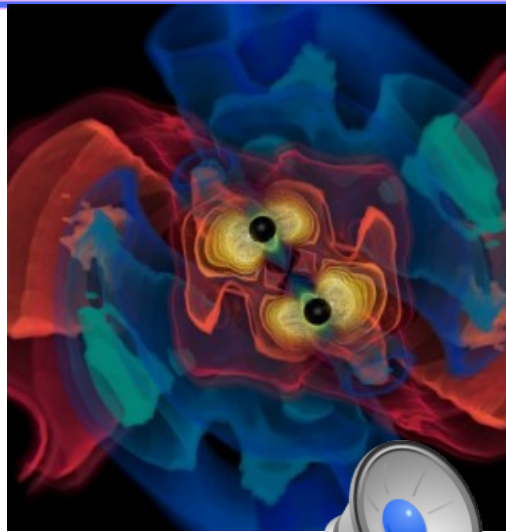
Expected strength

- Sense of scale: strain from a binary neutron star pair
 - » $M = 1.4 M_{\odot}$,
 - » $r = 10^{23}$ m (15 Mpc, Virgo),
 - » $R = 20$ km
 - » $f_{orb} = 400$ Hz

$$h \approx \frac{4\pi^2 G M R^2 f_{orb}^2}{c^4 r} \Rightarrow \boxed{h \sim 10^{-21}}$$



LIGO Astrophysical Sources of Gravitational Waves



Credit: AEI, CCT, LSU

Coalescing Compact Binary Systems: Neutron Star-NS, Black Hole-NS, BH-BH

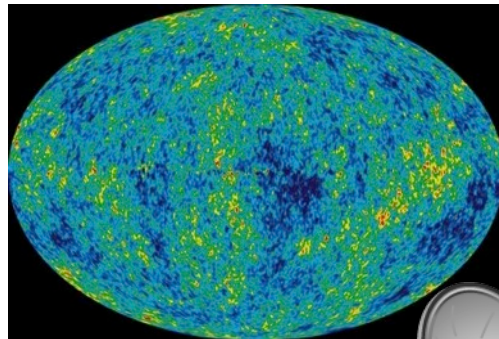
- Strong emitters, well-modeled,
- (effectively) transient



Credit: Chandra X-ray Observatory

Asymmetric Core Collapse Supernovae

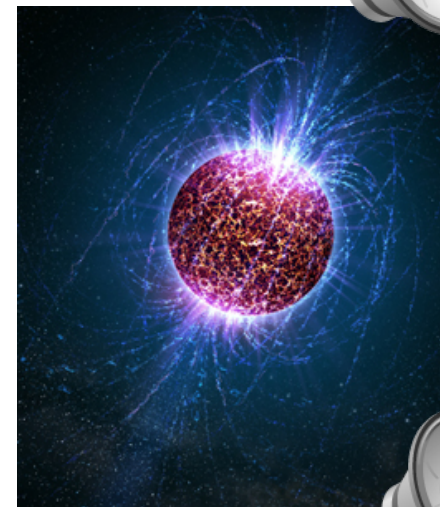
- Weak emitters, not well-modeled ('bursts'), transient
- Also: cosmic strings, SGRs, pulsar glitches



NASA/WMAP Science Team

Cosmic Gravitational-wave Background

- Residue of the Big Bang
- Long duration, stochastic background



Casey Reed, Penn State

Spinning neutron stars

- (nearly) monotonic waveform
- Long duration

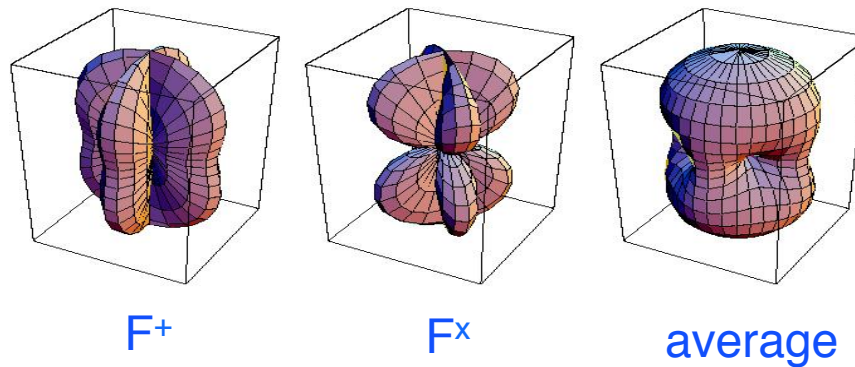
Audio credit: E. Thrane, CIT

Detector beam patterns

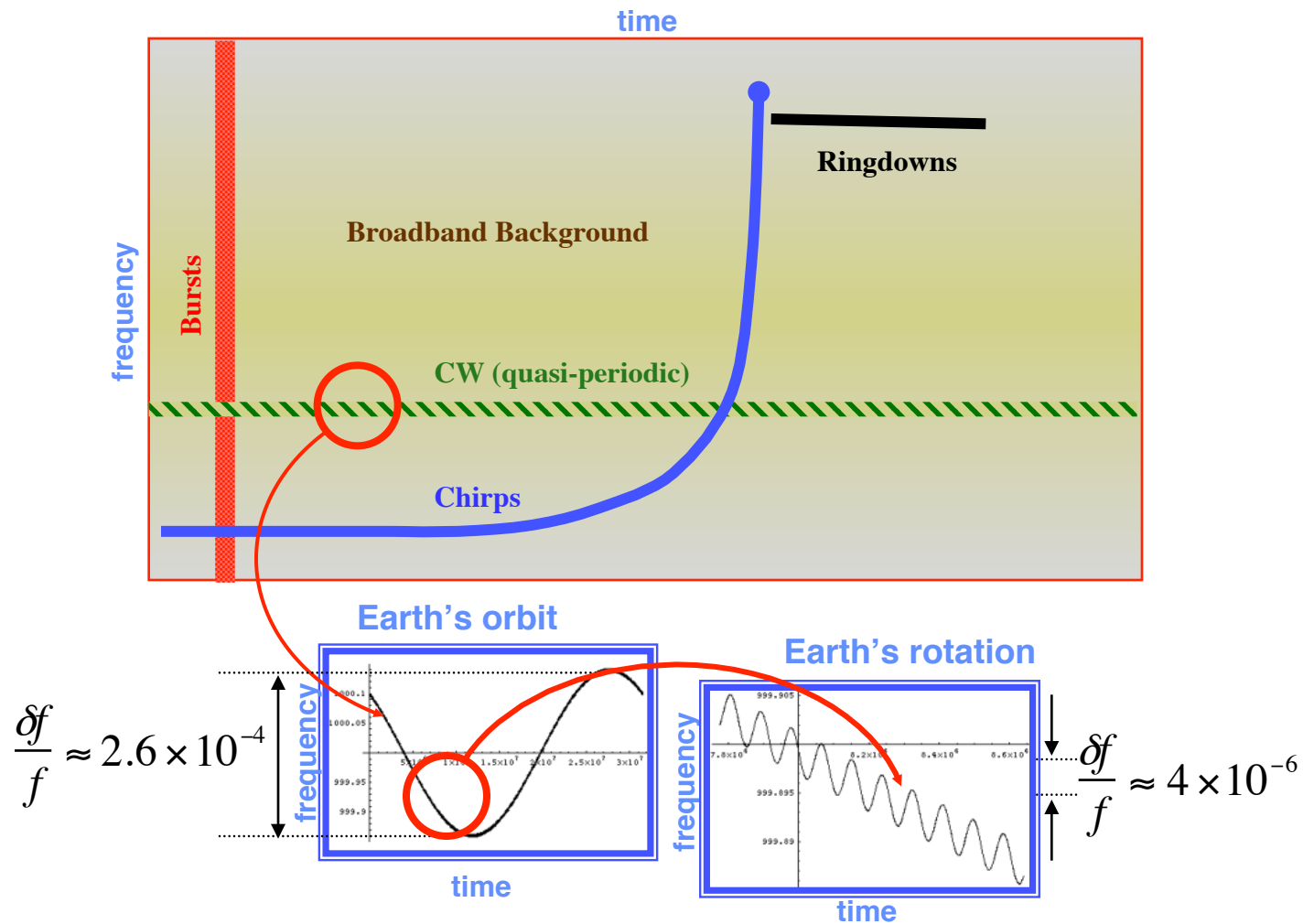
$$\frac{\delta L(t)}{L} = h(t) = F^+ h_+(t) + F^\times h_\times(t)$$

- $F^+, F^\times : [-1, 1]$
- $F = F(t; \alpha, \delta)$

LIGO example:

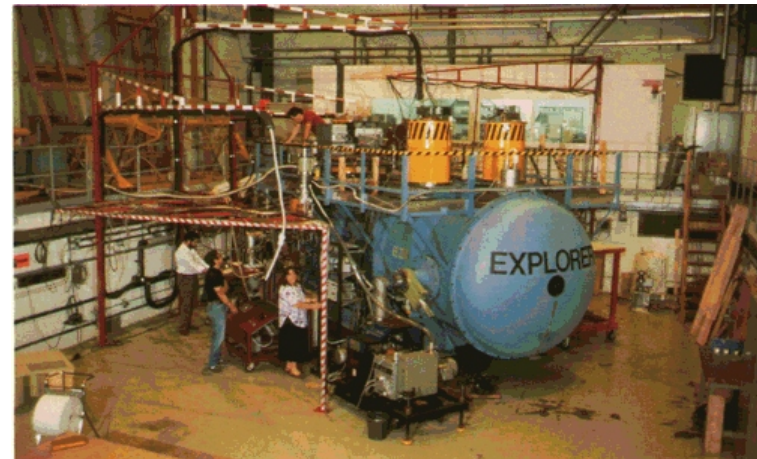


Frequency-Time Characteristics of GW Sources



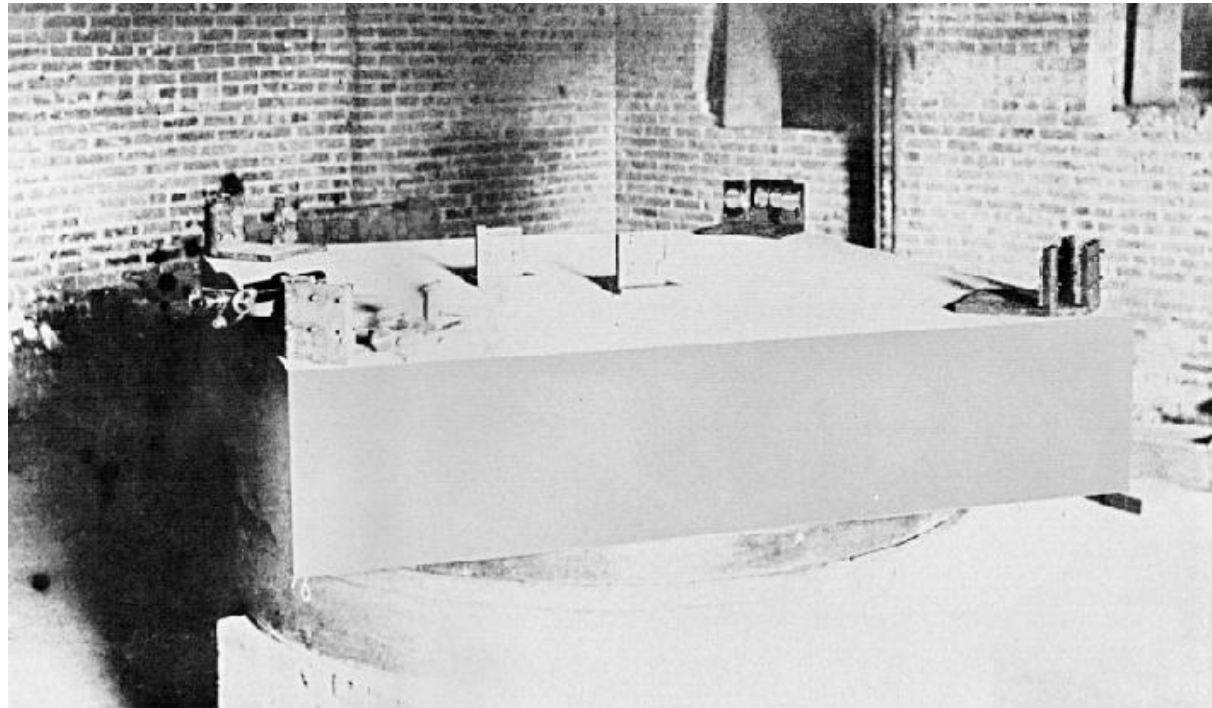
Resonant mass detectors

- Pioneered by Joseph Weber in the early 1960s, room temperature in-vacuum resonant mass detectors
- Employed piezoelectric strain gauges at center of bar; narrow band instruments with sensitivity near 1kHz
- Controversy in detection claims that have not been verified in follow up searches
- Modern resonant mass detectors employ
 - » Cryogenic bars with end transducers
 - » Use of SQUID low-noise amplifiers
 - » Vibration isolation
 - » Coincident running of detectors (IGEC)

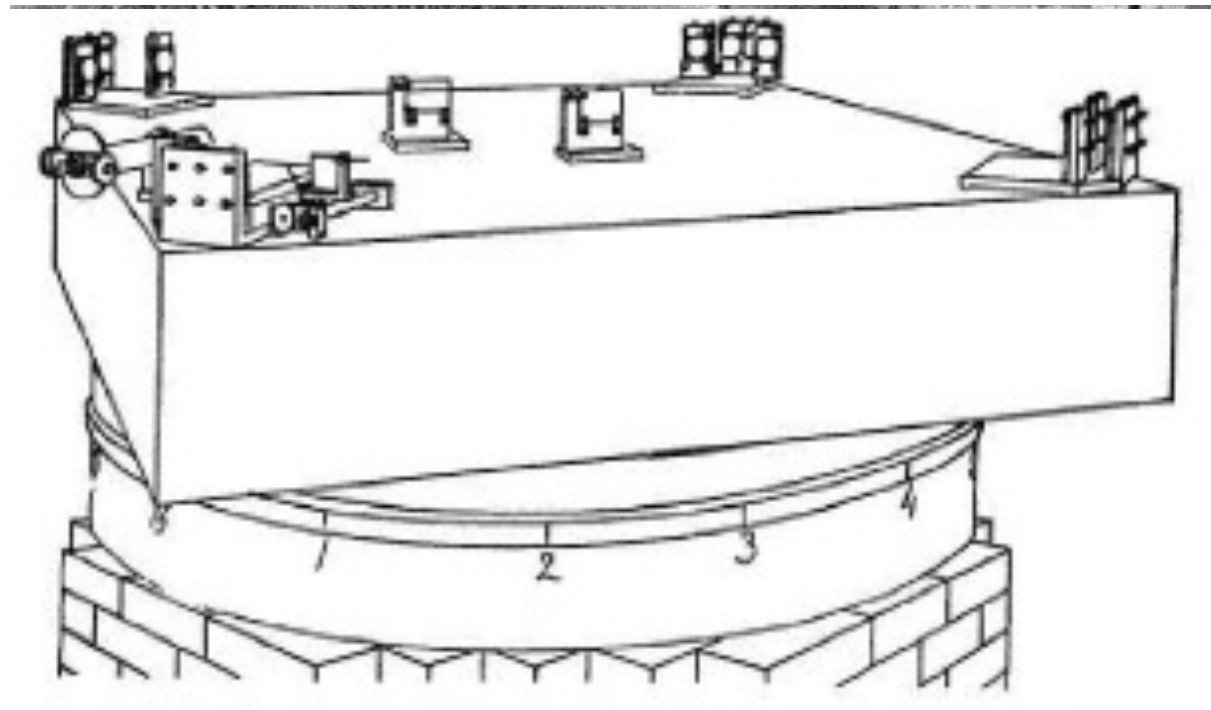


Explorer @CERN

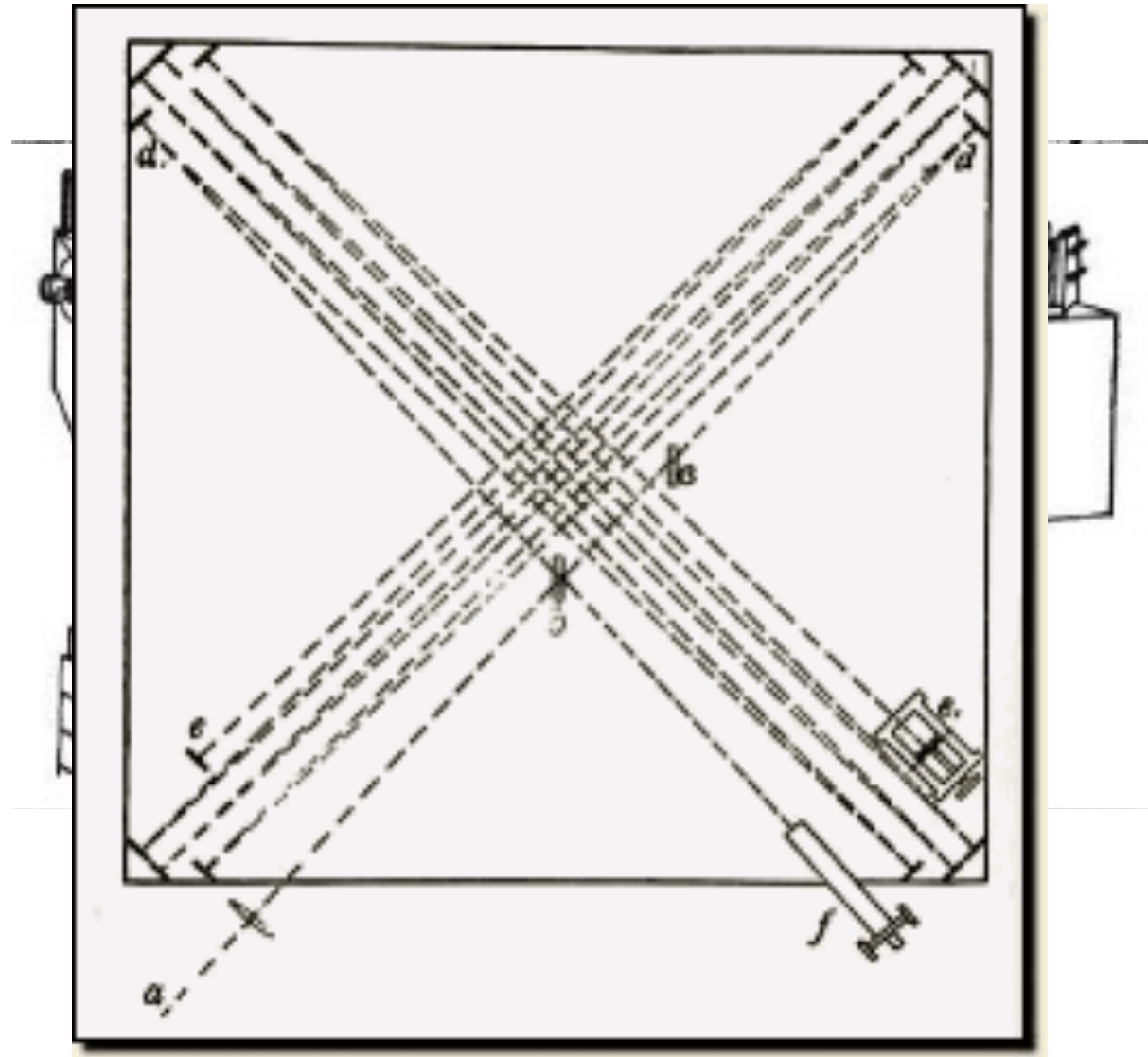
Michelson and Morley's apparatus



Michelson and Morley's apparatus

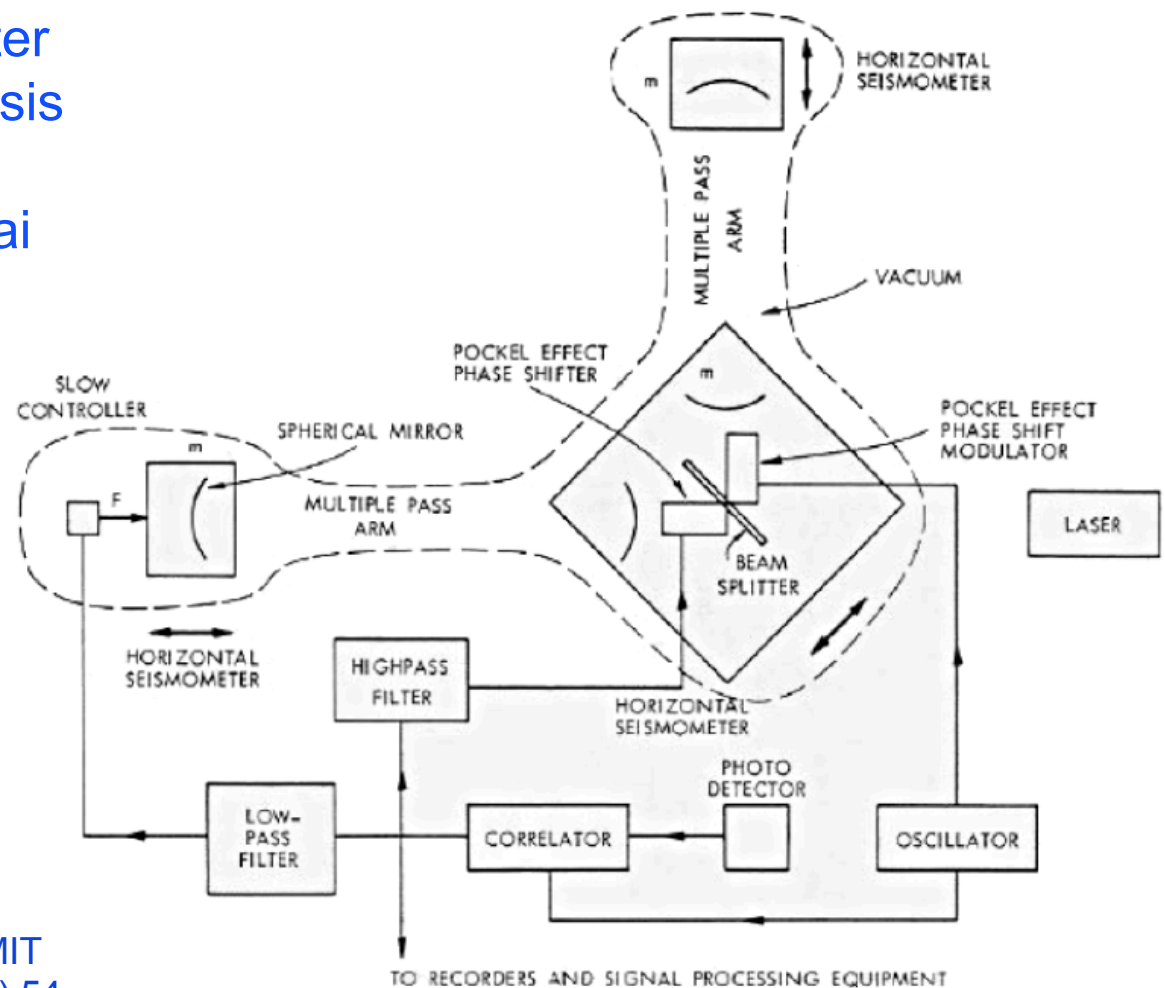


Michelson and Morley's apparatus



Interferometry envisioned

- Large-scale interferometer concept and noise analysis set down in 1972 in unpublished report by Rai Weiss



R. Weiss, Quarterly Progress Report, MIT
Research Lab of Electronics 105 (1972) 54



The *space-time interval* in special relativity

Special relativity says that the interval

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

between two events is *invariant*.

In shorthand, we write it as $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$
with the Minkowski *metric* given as

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

P. Saulson, Fundamentals of
Interferometric Gravitational
Wave Detectors (1994)
and
LIGO doc G100558



Generalizing

General relativity says almost the same thing, except the metric can be different.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

The trick is to find a metric $g_{\mu\nu}$ that describes a particular physical situation.

The metric carries the information on the space-time curvature that, in GR, embodies gravitational effects.



Gravitational waves

Gravitational waves propagating through flat space are described by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

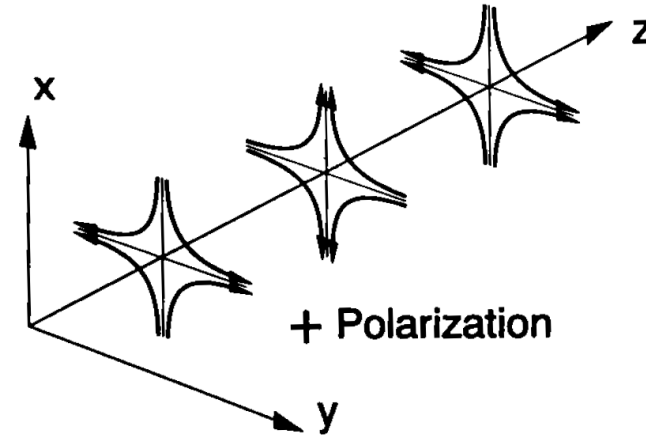
A wave propagating in the z-direction is described by

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

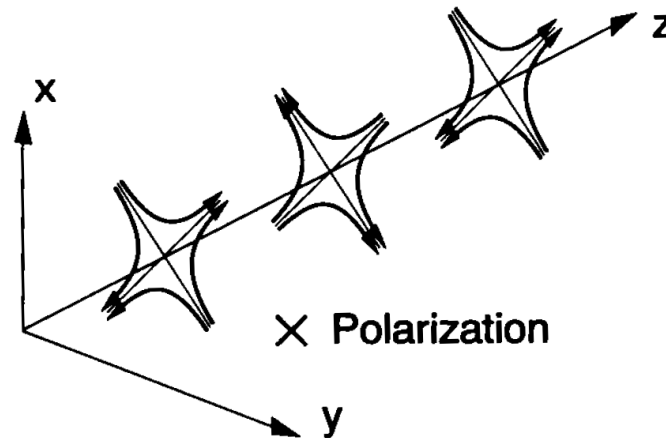
Two free parameters implies two polarizations

Two polarizations

$$\hat{h}_+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



$$\hat{h}_\times = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$





Light travel time difference between x and y arms

Start with x arm.

For light moving along the x axis, we are interested in the interval between points with non-zero dx and dt , but with $dy = dz = 0$:

$$ds^2 = -c^2 dt^2 + (1 + h_{11})dx^2 = 0$$



Solving for variation in light travel time

$$ds^2 = -c^2 dt^2 + (1 + h_{11})dx^2 = 0$$

$h(t)$ can have any time dependence, but for now assume that $h(t)$ is constant during light's travel through arm.

Rearrange, take square root, and replace square root with 1st two terms of binomial expansion

$$\int dt = \frac{1}{c} \int \left(1 + \frac{1}{2} h_{11} \right) dx$$

then integrate from $x = 0$ to $x = L$:

$$\Delta t = h_{11} L / 2c$$



LIGO Solving for variation in light travel time (II)

In doing this calculation, we choose coordinates that are marked by free masses.

“Transverse-traceless (TT) gauge”

Thus, end mirror is always at $x = L$.

Round trip back to beam-splitter:

$$\Delta t = h_{11} L / c$$

Now, calculating the same for y-arm ($h_{22} = -h_{11} = -h$):

$$\Delta t_y = -h L / c$$

Hence the difference between x and y round-trip times:

$$\Delta \tau = 2h L / c$$



Multipass, phase diff

To make the signal larger, we can arrange for N round trips through the arm instead of 1.

$$\Delta\tau = h \frac{2NL}{c} \equiv h\tau_{stor}$$

It is useful to express this as a phase difference between the light beams from the two arms when they recombine at the beamsplitter:

$$\Delta\phi = h\tau_{stor} \frac{2\pi c}{\lambda}$$



Interferometer response to $h(t)$

Free masses are free to track time-varying h .

As long as τ_{stor} is short compared to time scale of $h(t)$, then output tracks $h(t)$ faithfully.

If not, then put time-dependent h into integral of slide 24 before carrying out the integral.

Response “rolls off” for fast signals.

This is what is meant by interferometers being *broad-band* detectors.

But, noise is stronger at some frequencies than others. (More on this later.) This means some frequency bands have good sensitivity, others not.



Interpretation

A gravitational wave's effect on one-way travel time:

$$\Delta t = \frac{h}{2} \frac{L}{c}$$

Just as if the arm length is changed by a fraction

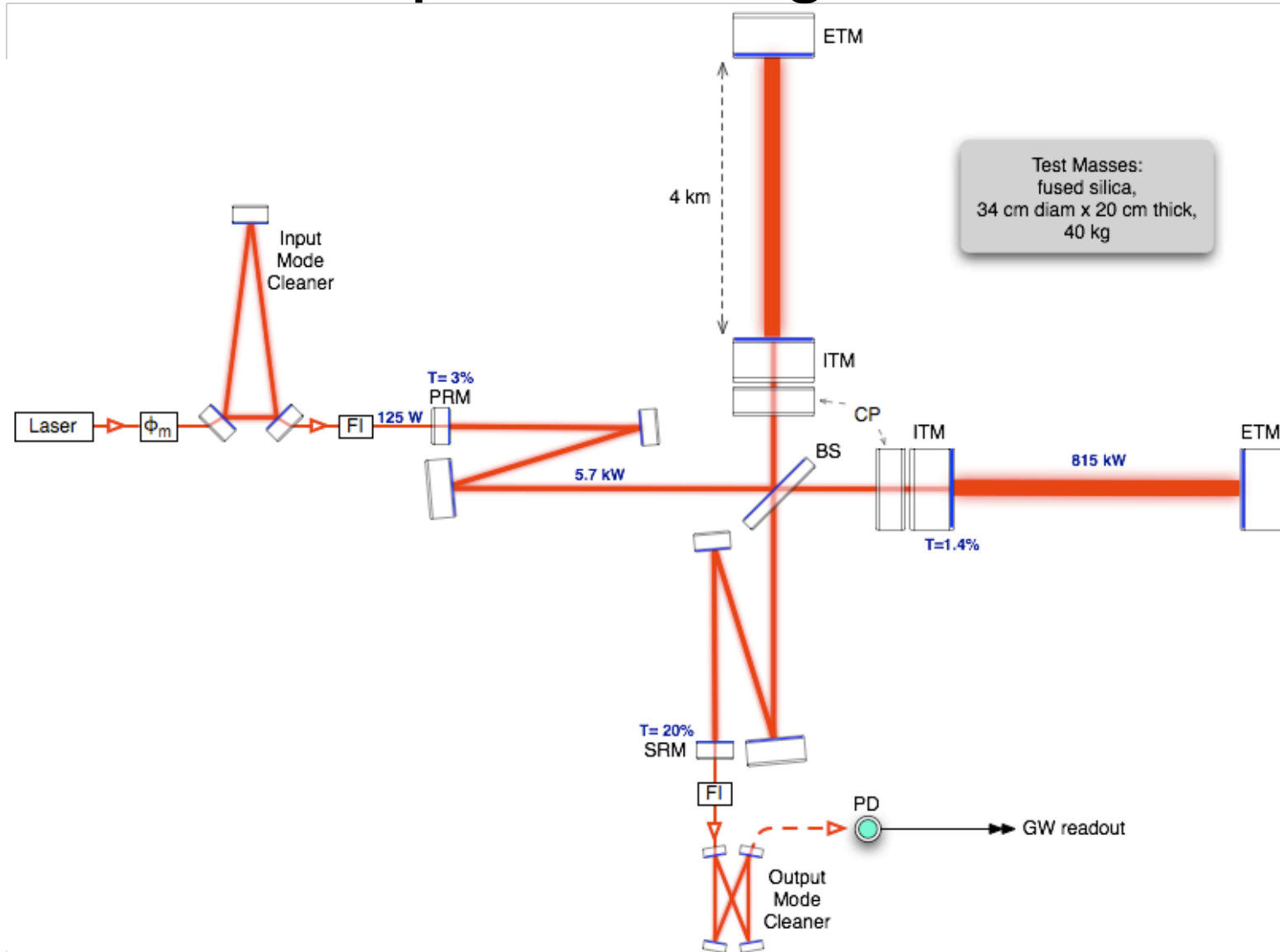
$$\frac{\Delta L}{L} = \frac{h}{2}$$

However, in the TT gauge, we say that the masses didn't move (they mark coordinates), but that the separation between them changed.

The metric of the space between them changed.

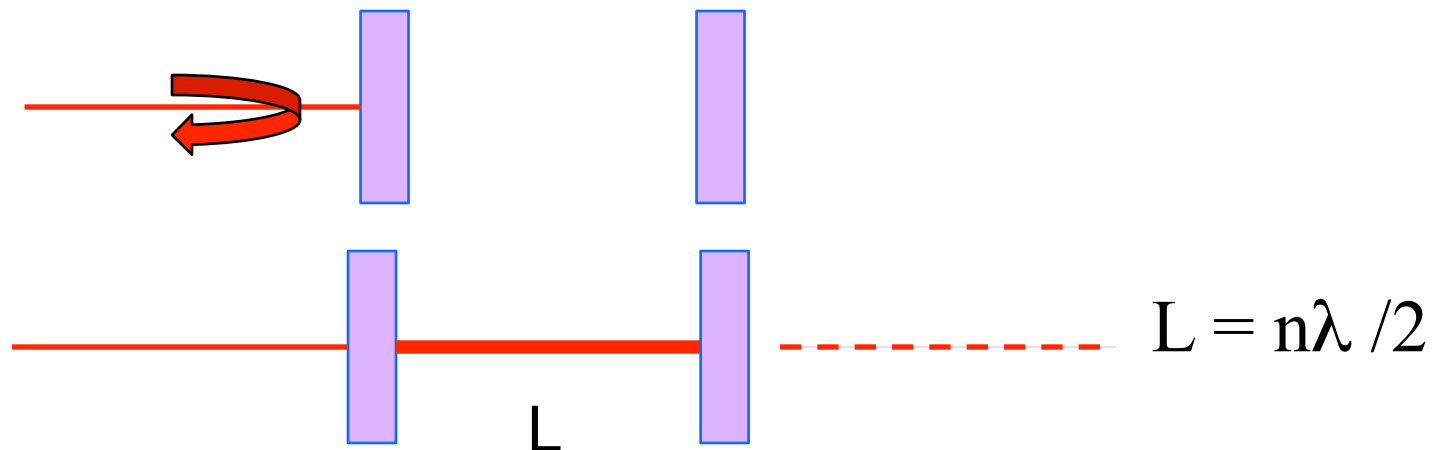


Optical configuration

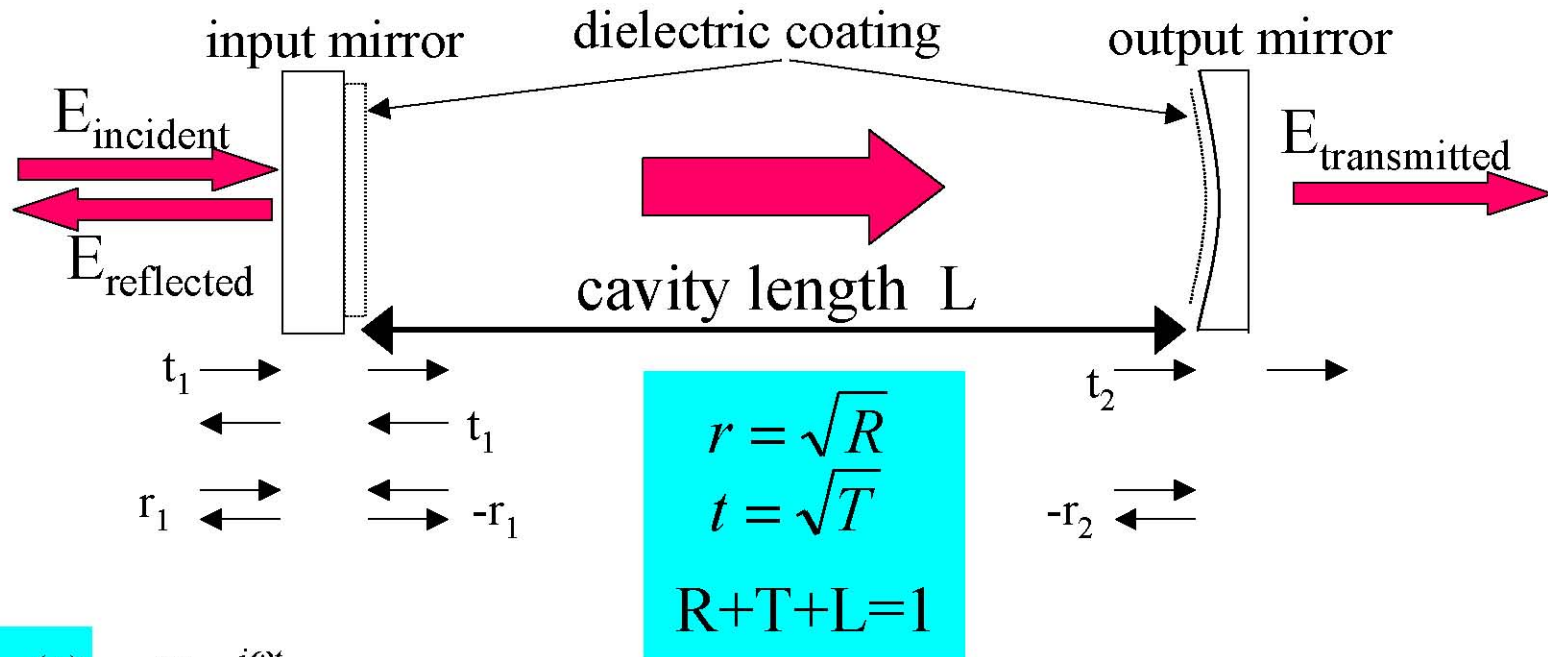


Pound-Drever-Hall locking

- At the heart of a working interferometer is a series of digital and analog control loops
- Loops control optical (Fabry-Perot) cavities
- *Pound-Drever-Hall* locking is used routinely in GW detectors
- Consider a Fabry-Perot cavity: lock via length, or frequency



Cavity Description



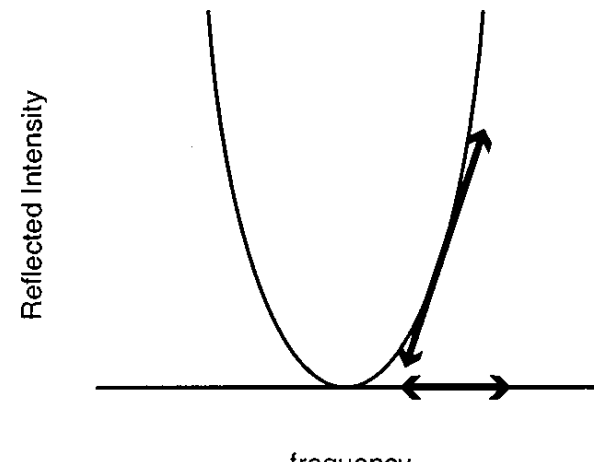
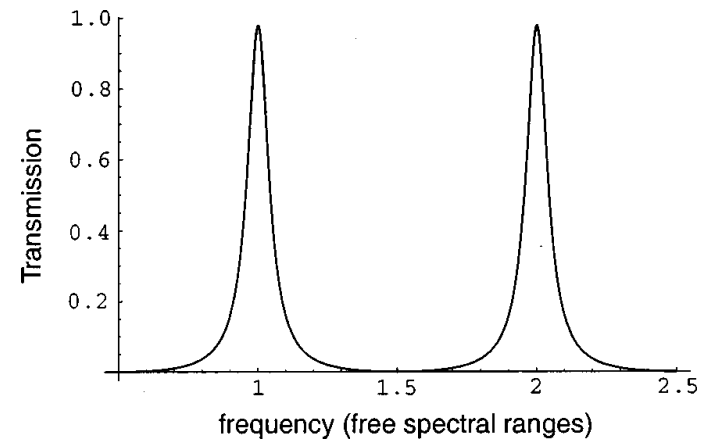
$$E_{\text{incident}}(t) = E_0 e^{i\omega t}$$

$$E_{\text{transmitted}}(t) = E_0 t_1 t_2 e^{i\omega(t - \frac{L}{c})} + E_0 t_1 t_2 r_1 r_2 e^{i\omega(t - \frac{3L}{c})} + E_0 t_1 t_2 r_1^2 r_2^2 e^{i\omega(t - \frac{5L}{c})} + \dots$$

$$E_{\text{reflected}}(t) = E_0 r_1 e^{i\omega t} - E_0 r_2 t_1^2 e^{i\omega(t - \frac{2L}{c})} - E_0 r_1 r_2^2 t_1^2 e^{i\omega(t - \frac{4L}{c})} - E_0 r_1^2 r_2^3 t_1^2 e^{i\omega(t - \frac{6L}{c})} - \dots$$

Pound-Drever-Hall locking

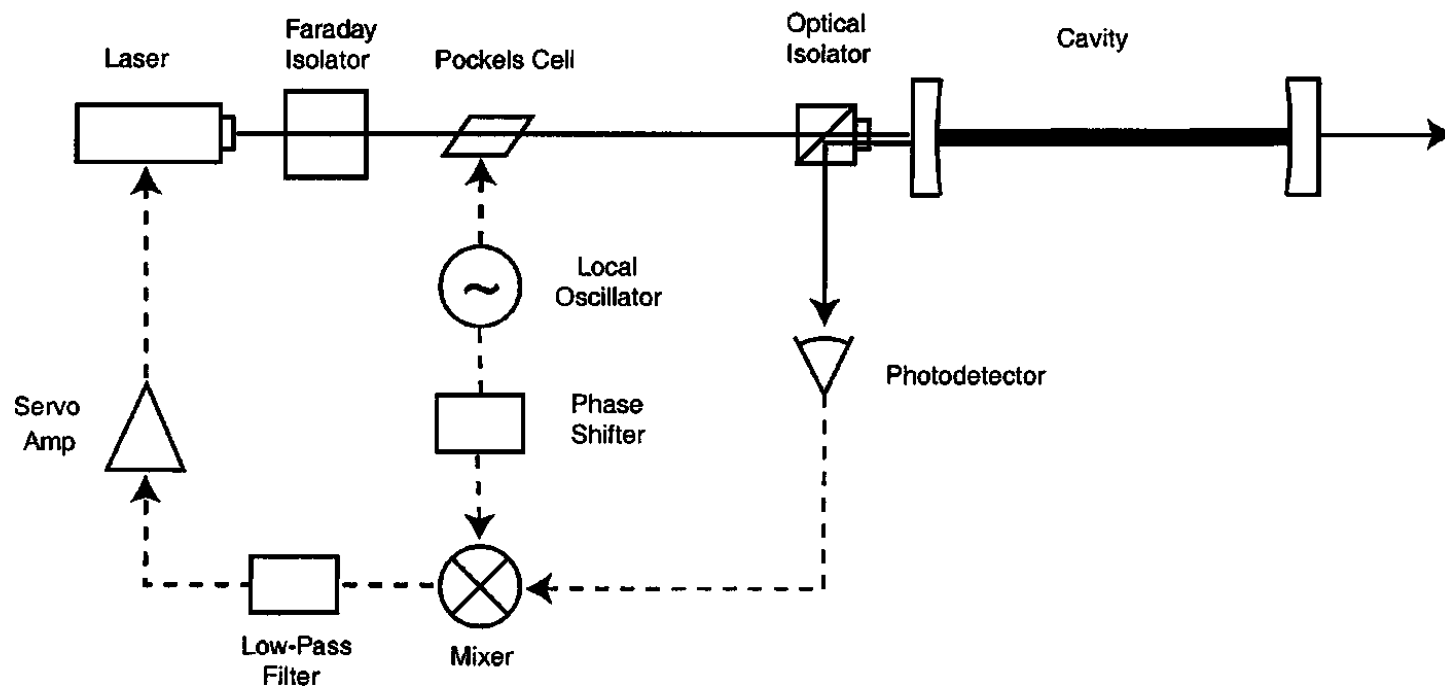
- $L = n\lambda / 2$, or, ν must be $n \cdot \Delta_{\text{fsr}} = n \cdot c / 2L$. Here, Δ_{fsr} = 'free spectral range'
- Could servo the cavity on transmission, but the laser intensity fluctuates too
- Instead: *reflection locking*
- Amplitude on reflection doesn't tell you which way to servo; need derivative



E. Black, Am. J. Phys., 69 (1) 2001

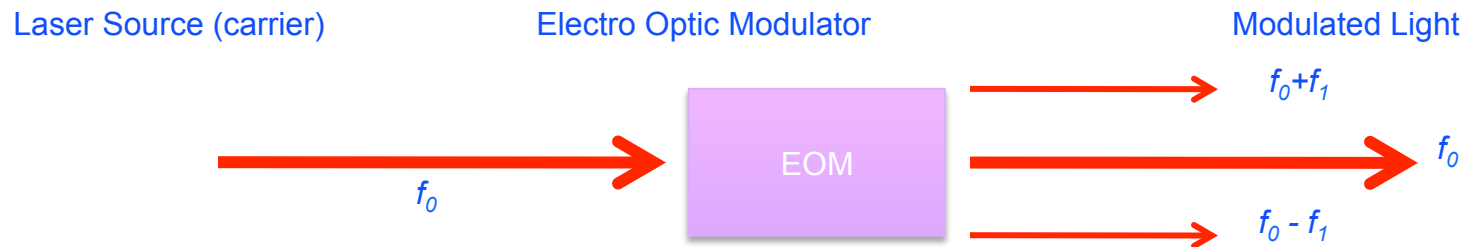
Pound-Drever-Hall locking

- A Pockels cell modulates the light phase, producing sidebands
- Mixer yields a signal at low f (near DC) and at $2*f_{\text{mod}}$
- Derivative information in the former; low pass and send to laser control – Null the error point

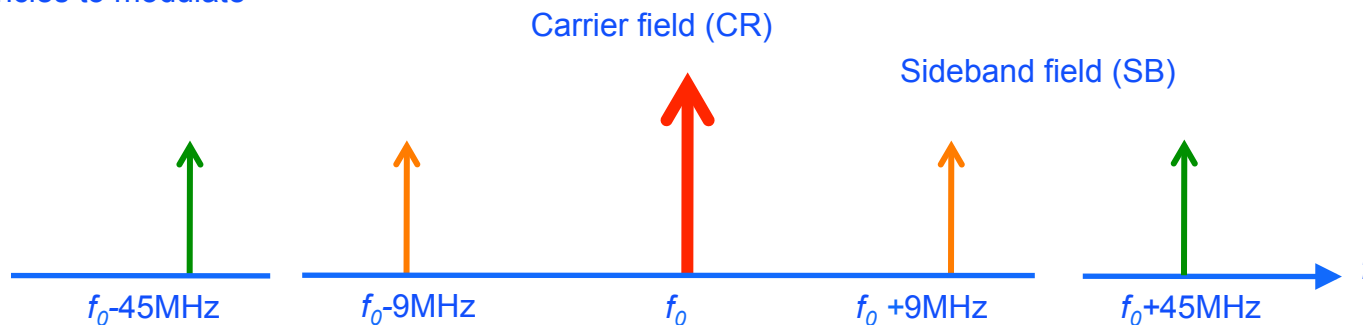




Adding frequency sidebands



EOM has an index of refraction that is linear to an electric field. Local oscillator provides E-field at specific frequencies to modulate phase/frequency.



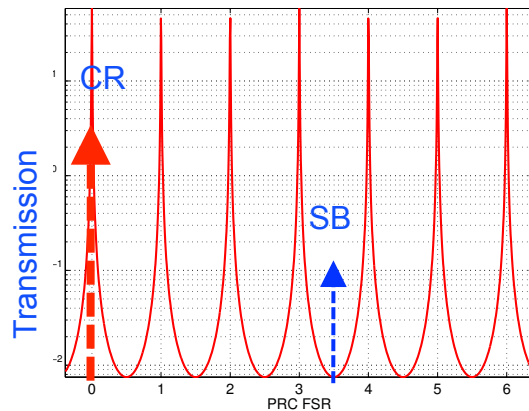
aLIGO has two sidebands (SBs), 9MHz and 45 MHz

TJ Massinger, J Kissel, K Kokeyama
LIGO doc G1400673



How does that help?

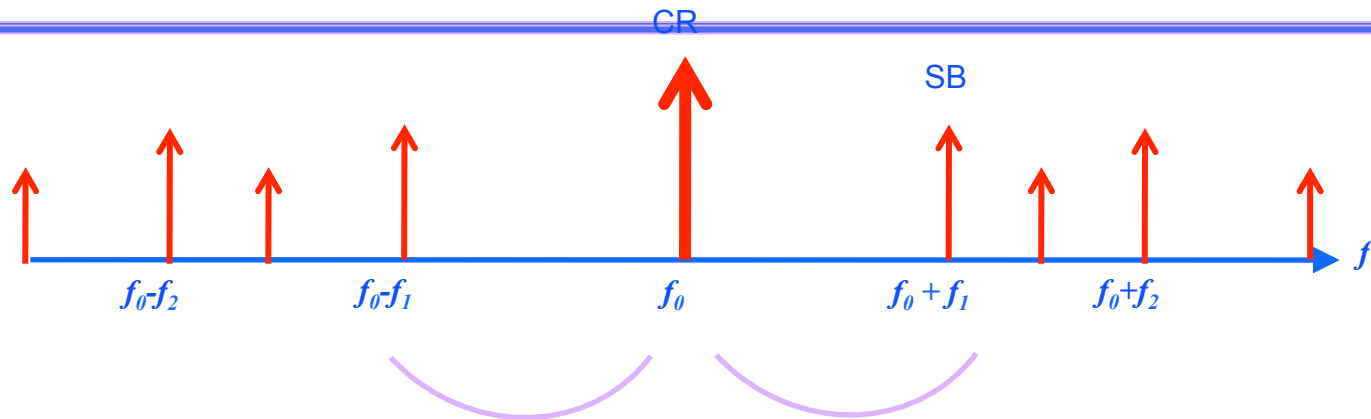
- Carrier and sideband frequencies are tuned such that the carrier is resonant (and thus fully transmitted) while the sidebands are anti-resonant and are fully reflected



- By comparing the relative amplitudes of these reflected signals, we can figure out how close we are to resonance
- Problem: we have two sideband frequencies and each has harmonics, photodiode picks up beat notes for all of them, we have a lot of signals to sift through



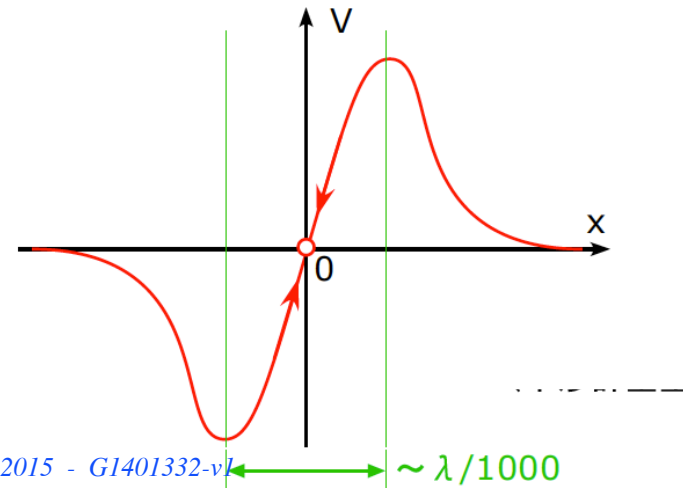
Demodulation



When we include harmonics, this suddenly looks like a mess. Luckily, we have the oscillator created these sidebands.

The resulting demodulated signal is DC and linear to cavity length for small displacements.

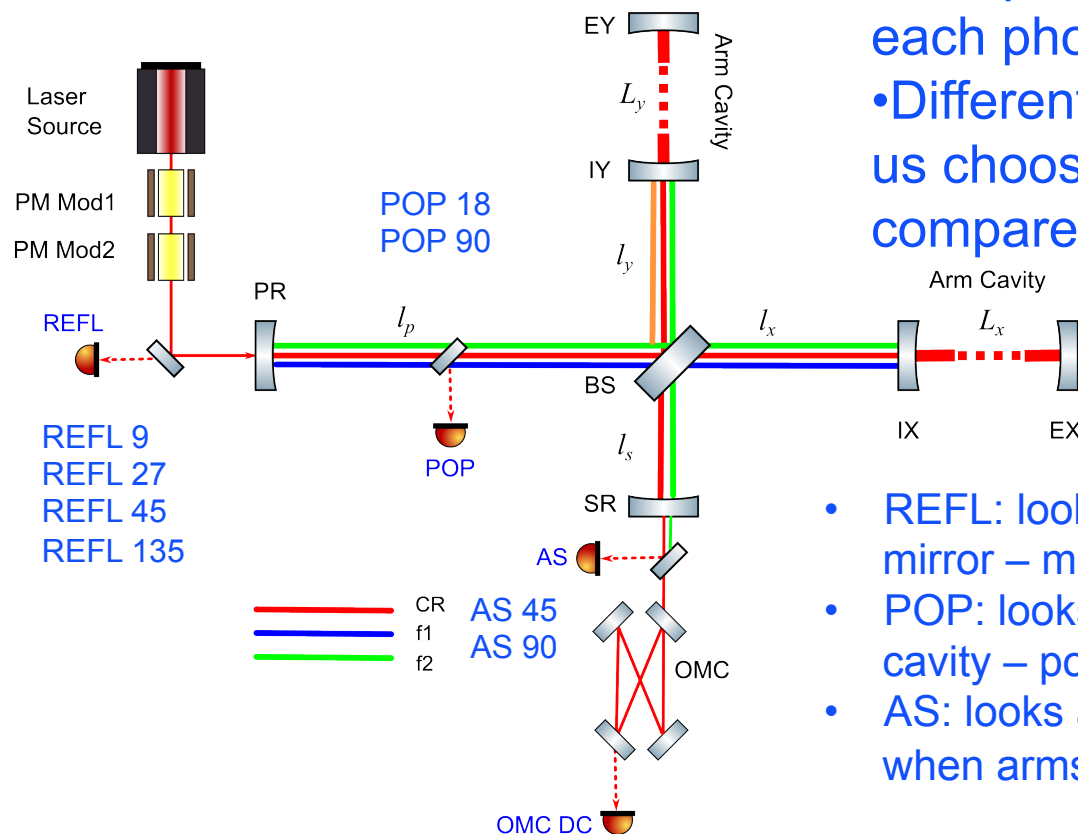
Since we know the frequency spacing between the signals, we can use our local oscillator to pick off the signal we want using a mixer.





Length control in aLIGO

- 3 RF photodiodes: REFL, POP, AS
- Multiple demodulation frequencies at each photodiode
- Different demodulation frequencies let us choose which parts of optical fields to compare to determine lock



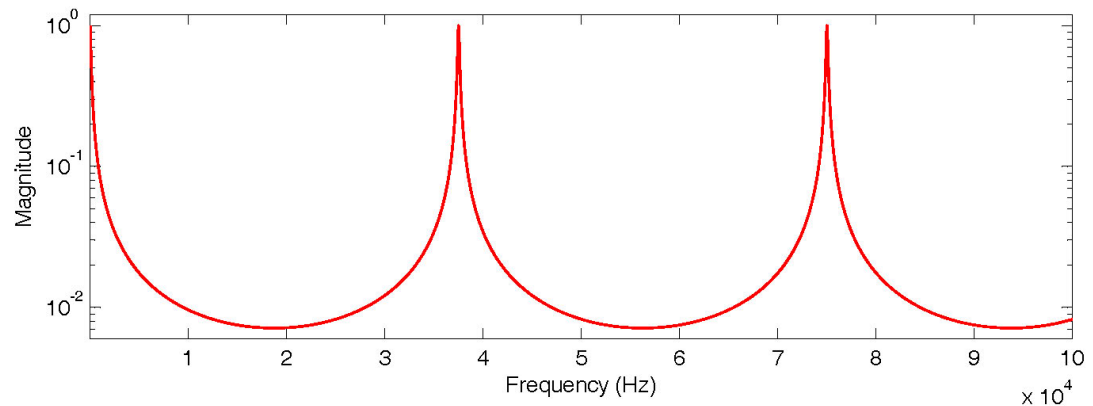
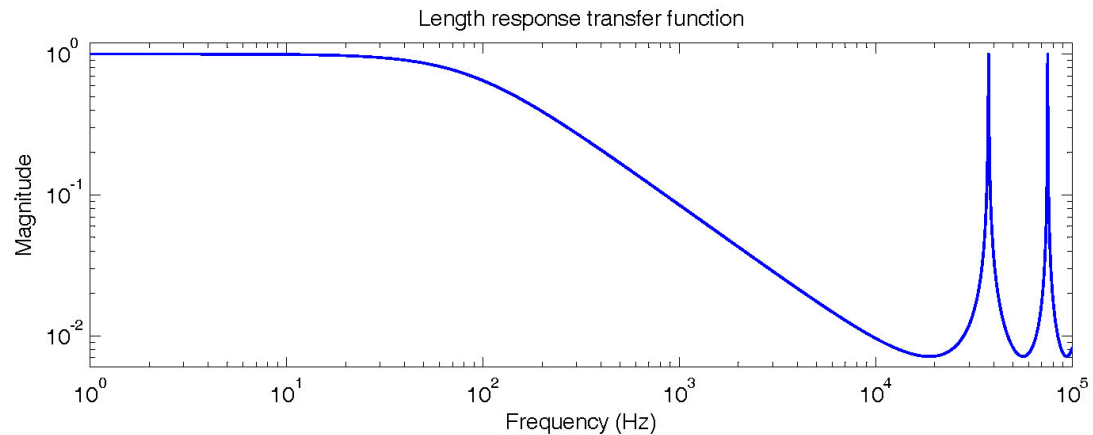
- REFL: looks at reflected light at power recycling mirror – minimized when IFO locked
- POP: looks at light circulating in power recycling cavity – power builds up when IFO locked
- AS: looks at reflected light from arm cavities - dark when arms are aligned, can be offset



Aside I : High-f GW detection

$$H_L(s) = \frac{1-r_a r_b}{1-r_a r_b e^{-2sT}}$$

r_a, r_b : mirror reflectivities
 T : cavity transit time



1FSR

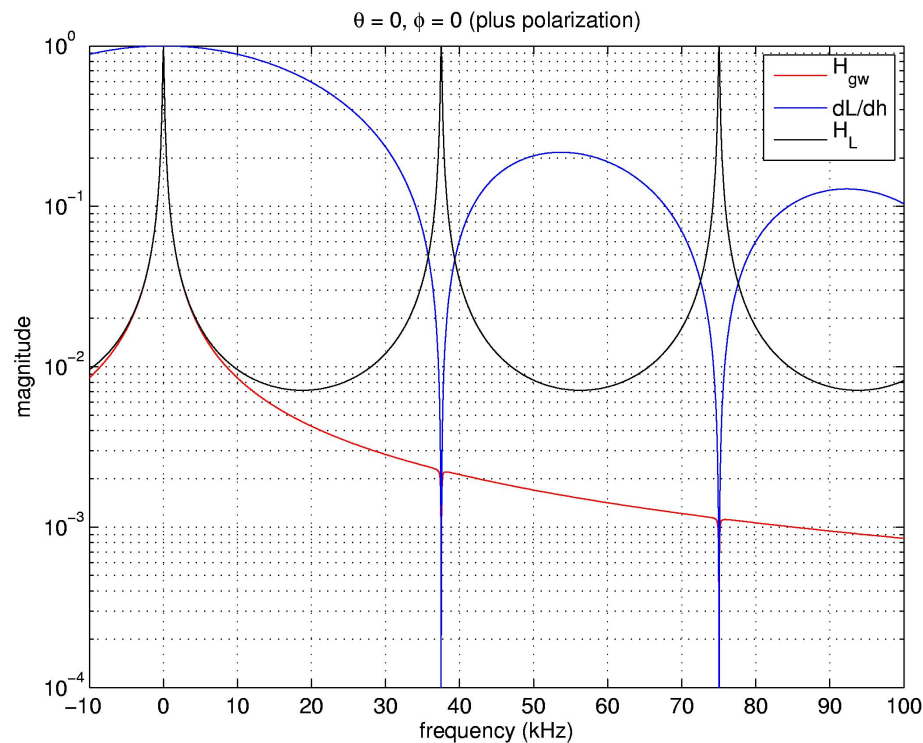
2FSR

R. Savage, M. Rakhmanov, H. Elliott: LIGO-G060667
See also Class. Quant. Grav., 25 (2008)184017

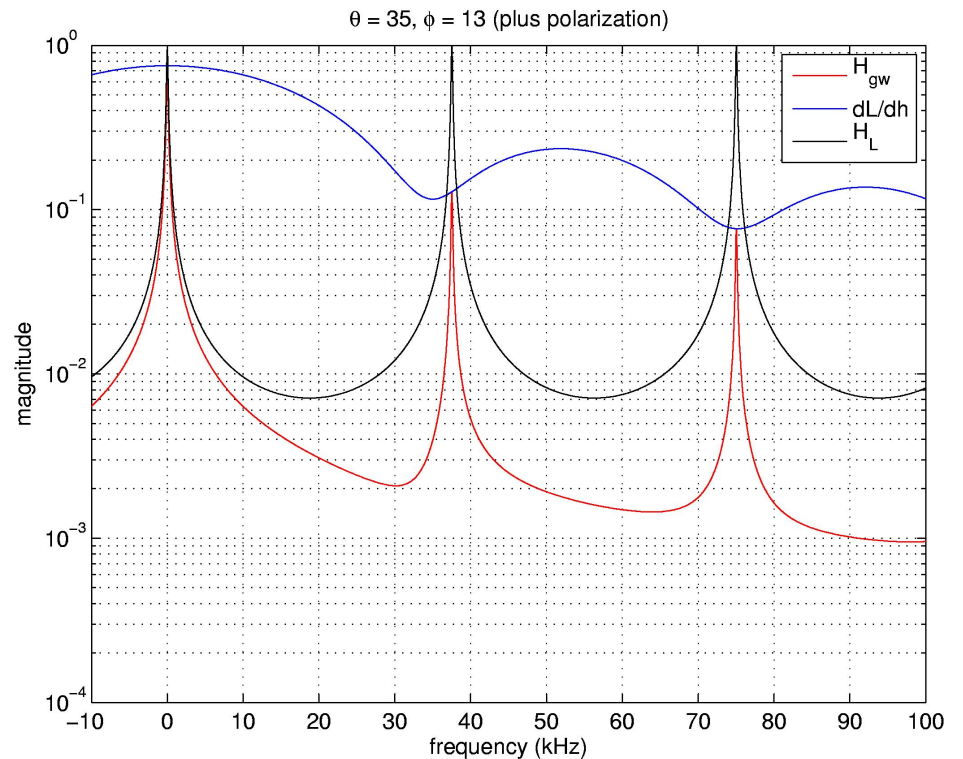


Interferometer response to GW

On zenith

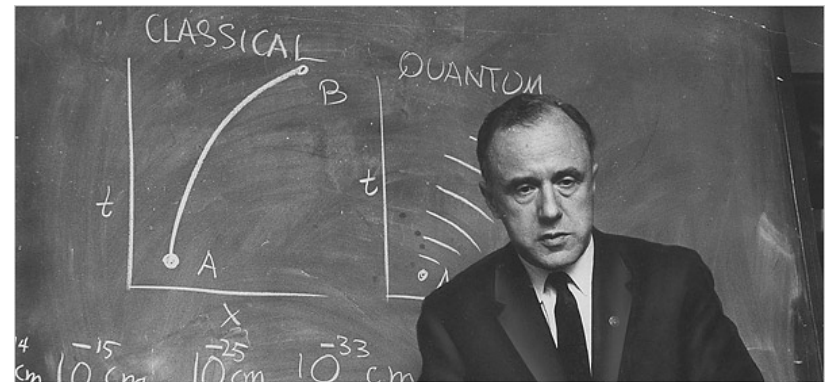


At some angle



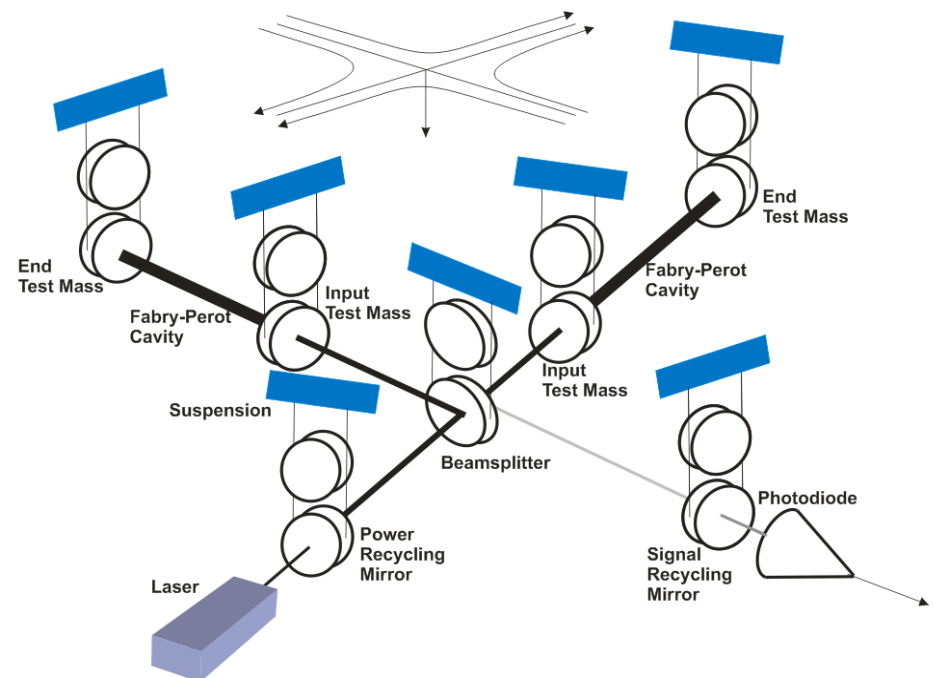
Aside II : John Wheeler

1. Wheeler's impact in the field of nuclear physics, and ties to Richland where he lived in 1943-44 and directed the scientific effort of the construction and commissioning of B-Reactor.
2. Wheeler's singular impact in the development of General Relativity and Gravity as a quantitative, testable field, leading to the development in Richland of LIGO
3. His dedication to education.
Wheeler's work with graduate and undergraduate students is legendary in the physics community. He directly supervised 51 PhD dissertations from 1935-1986



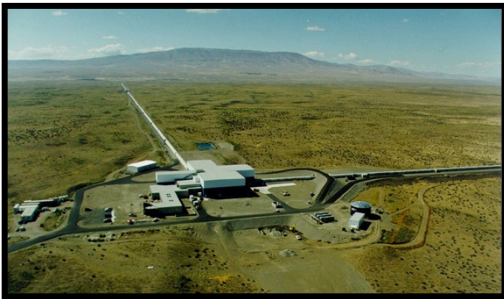
Advanced LIGO

- Power recycled Fabry-Perot Michelson with Signal recycling (increase sensitivity, add tunability)
- Active seismic isolation, quadruple pendulum suspensions (seismic noise wall moves from 40Hz to 10Hz)
- DC readout, Output Mode Cleaner (better use of photons)
- ~20x higher input power (lower shot noise)
- 40 kg test masses (smaller motion due to photon pressure fluctuations)
- Larger test mass surfaces, low-mechanical -loss optical coatings (decreased mid-band thermal noise)
- Fused Silica Suspension (decreased low-frequency thermal noise)

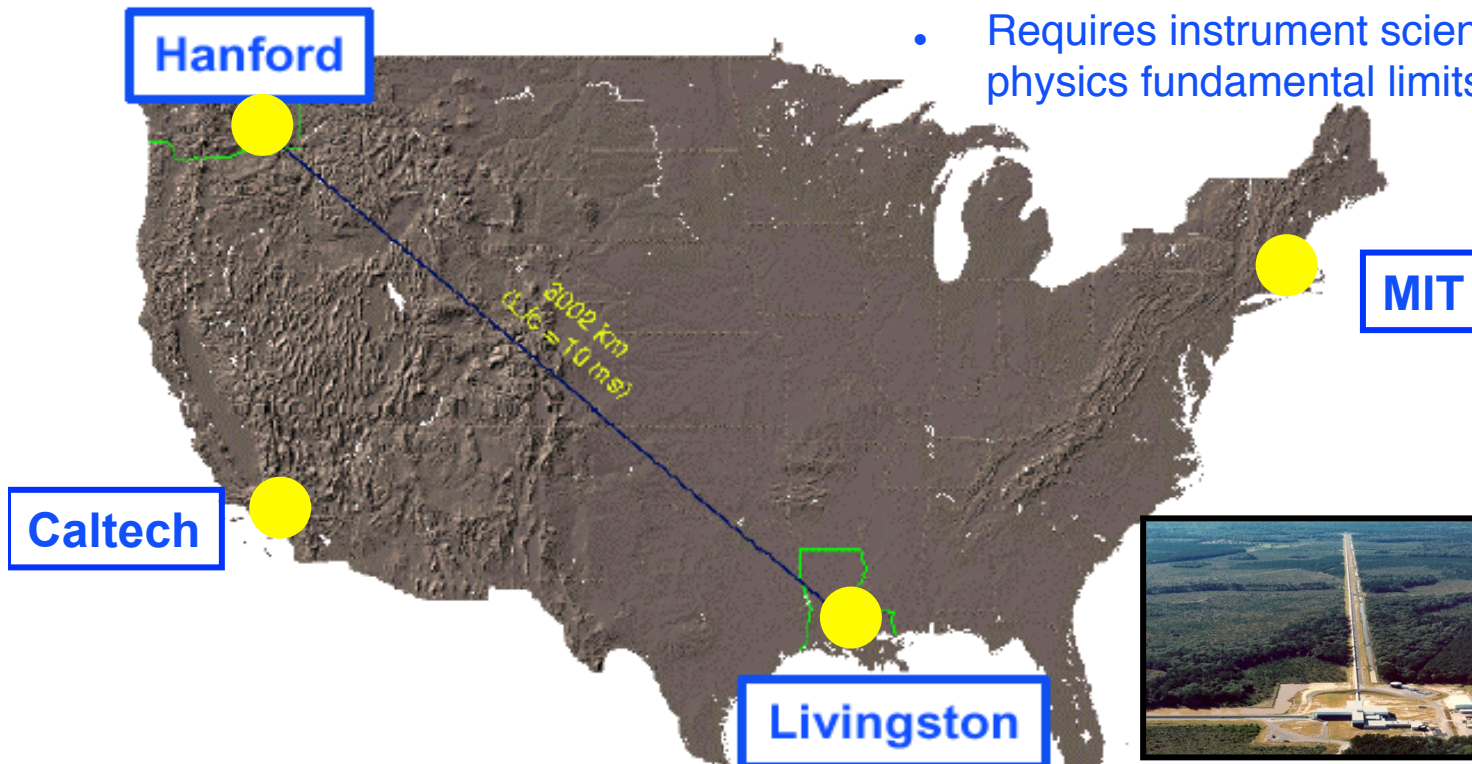




LIGO Laboratory: two Observatories, Caltech and MIT campuses



- Mission: to develop gravitational-wave detectors, and to operate them as astrophysical observatories
- Jointly managed by Caltech and MIT; responsible for operating LIGO Hanford and Livingston Observatories
- Requires instrument science at the frontiers of physics fundamental limits



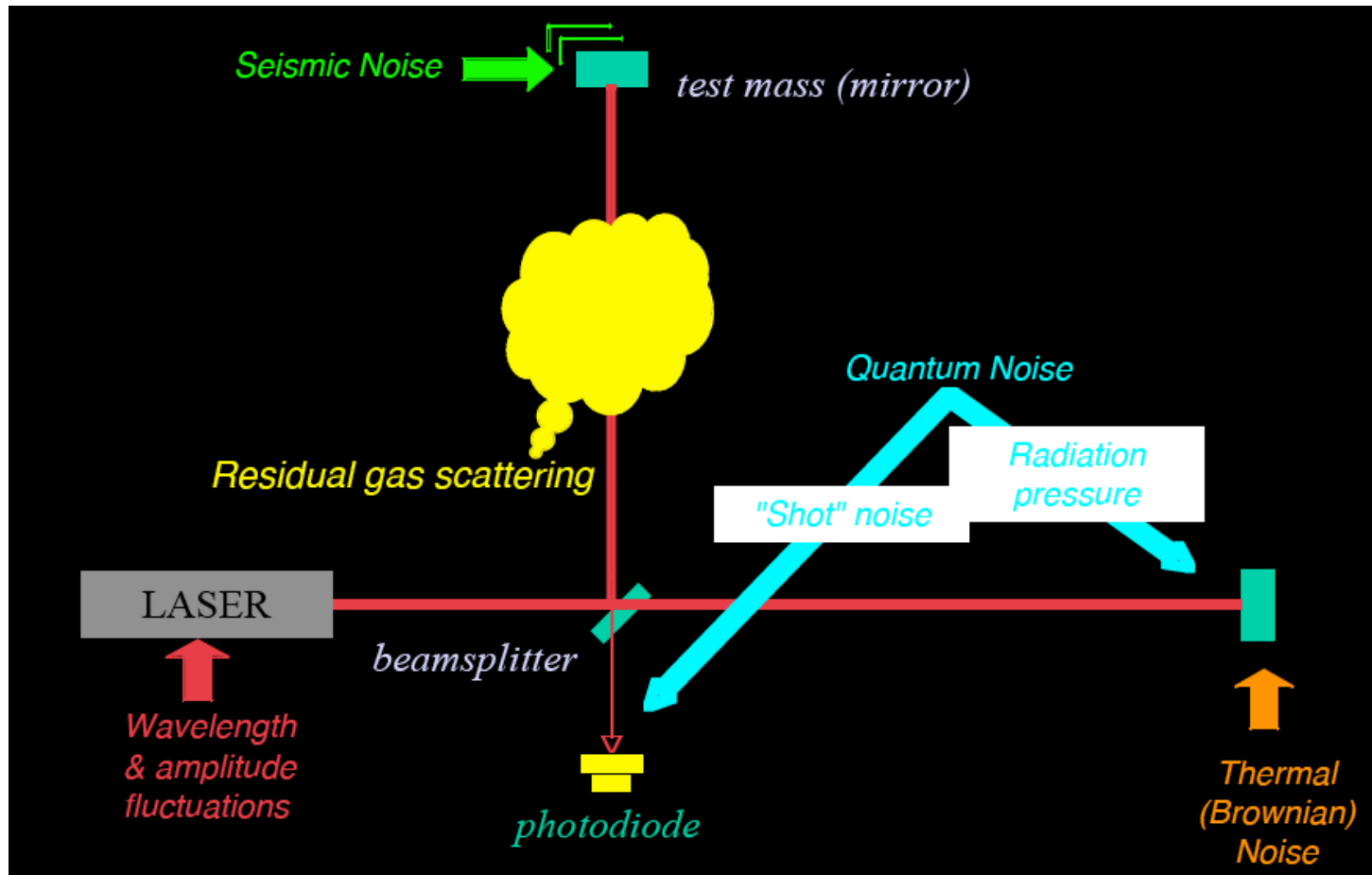


LIGO Scientific Collaboration

- 900+ members, 80+ institutions, 17 countries



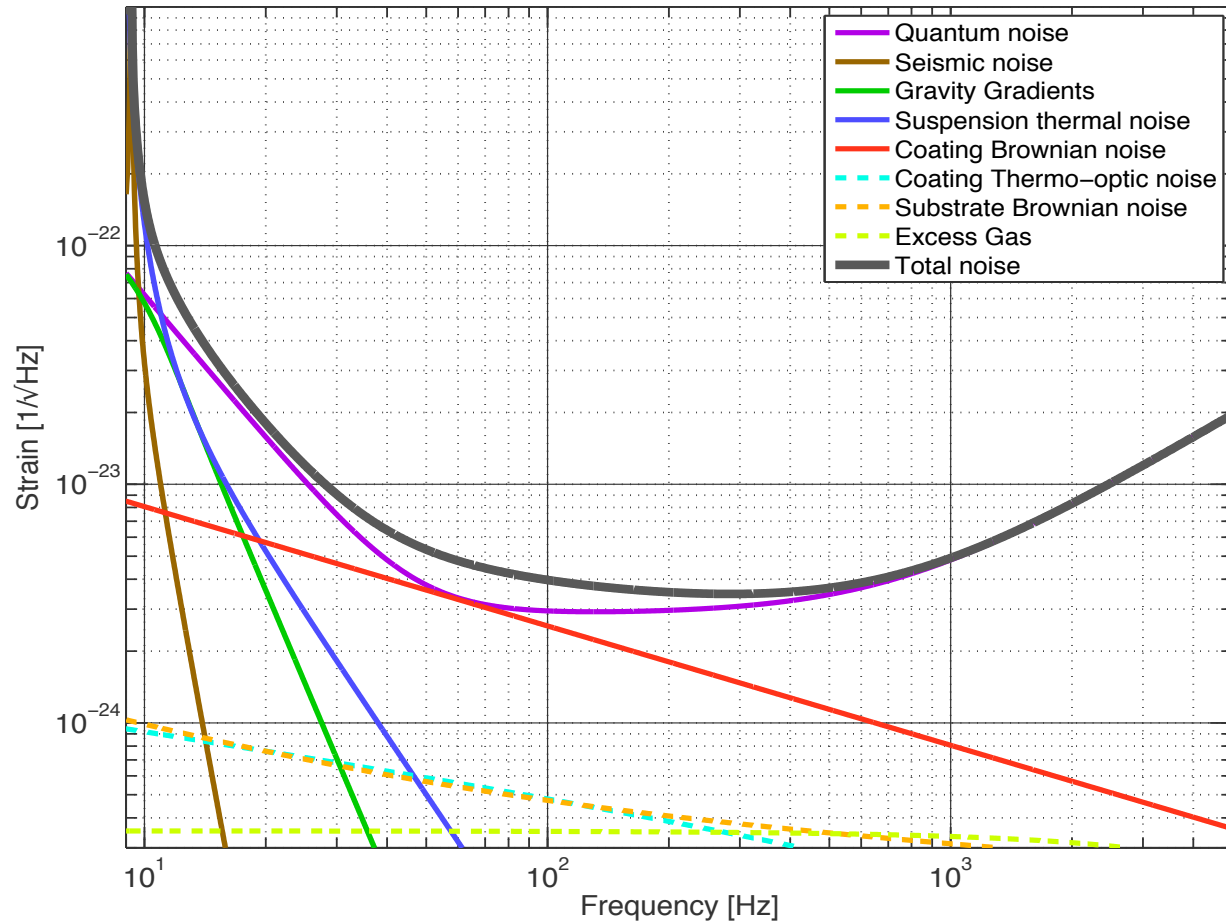
Noise cartoon

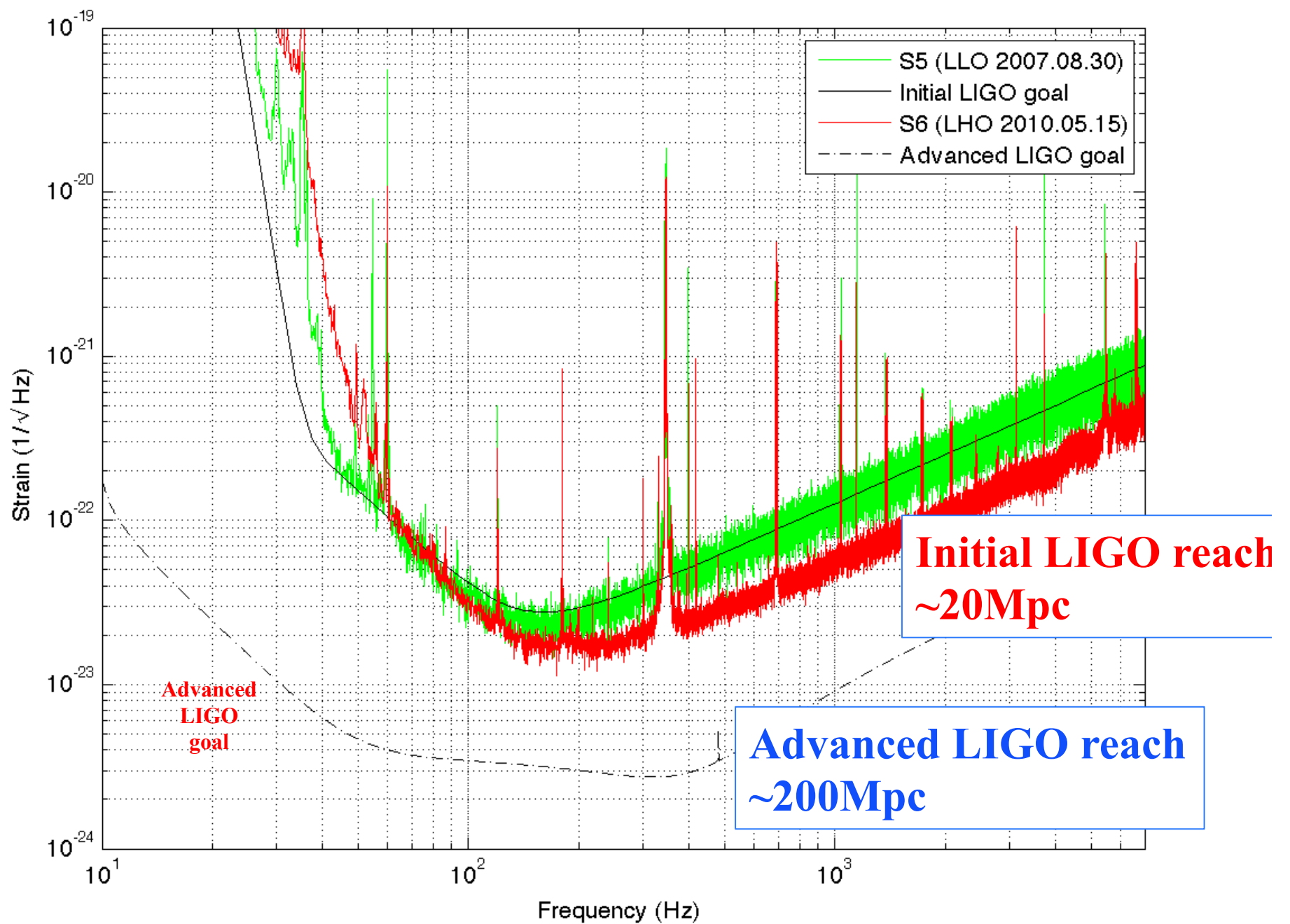


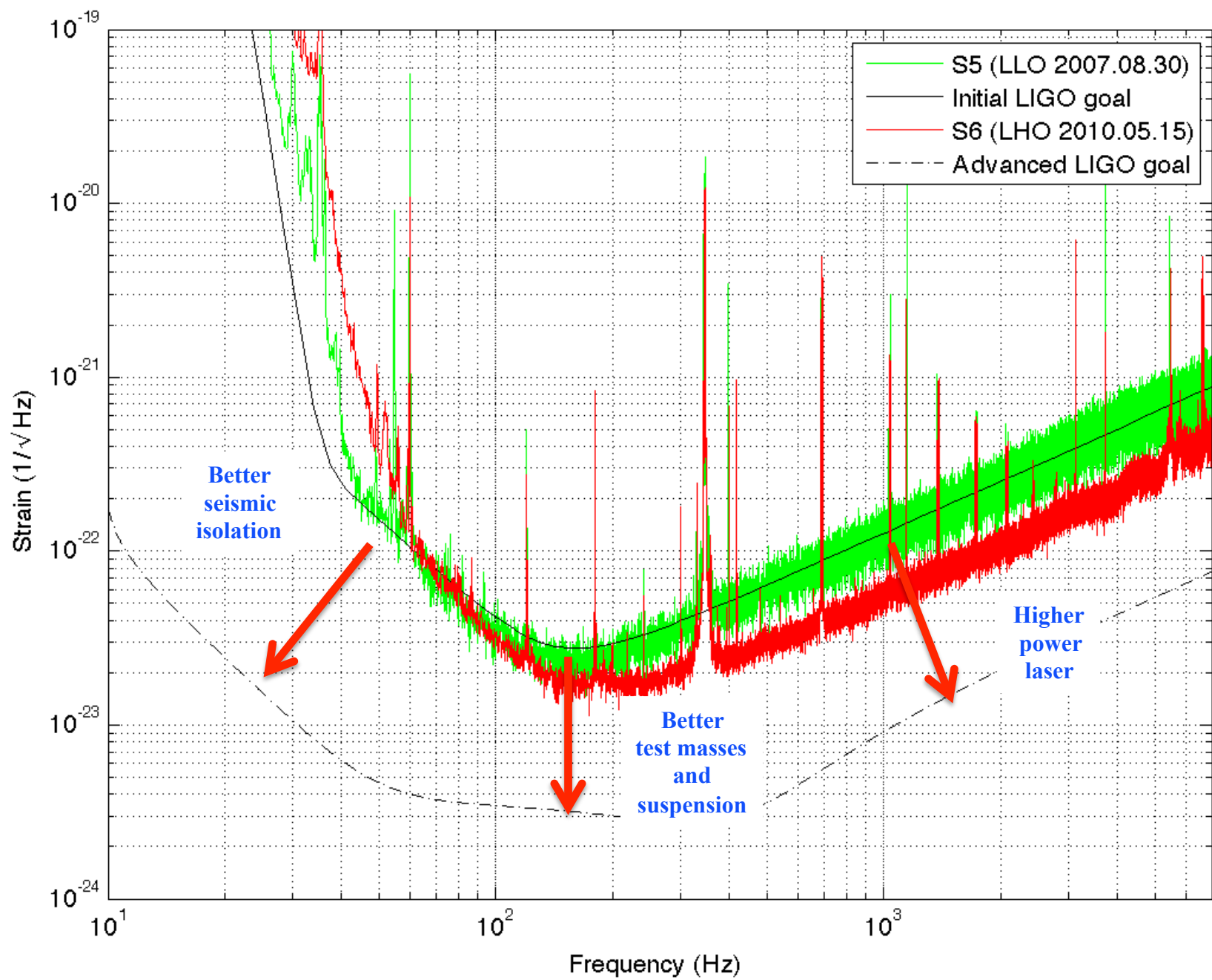
R. Adhikari

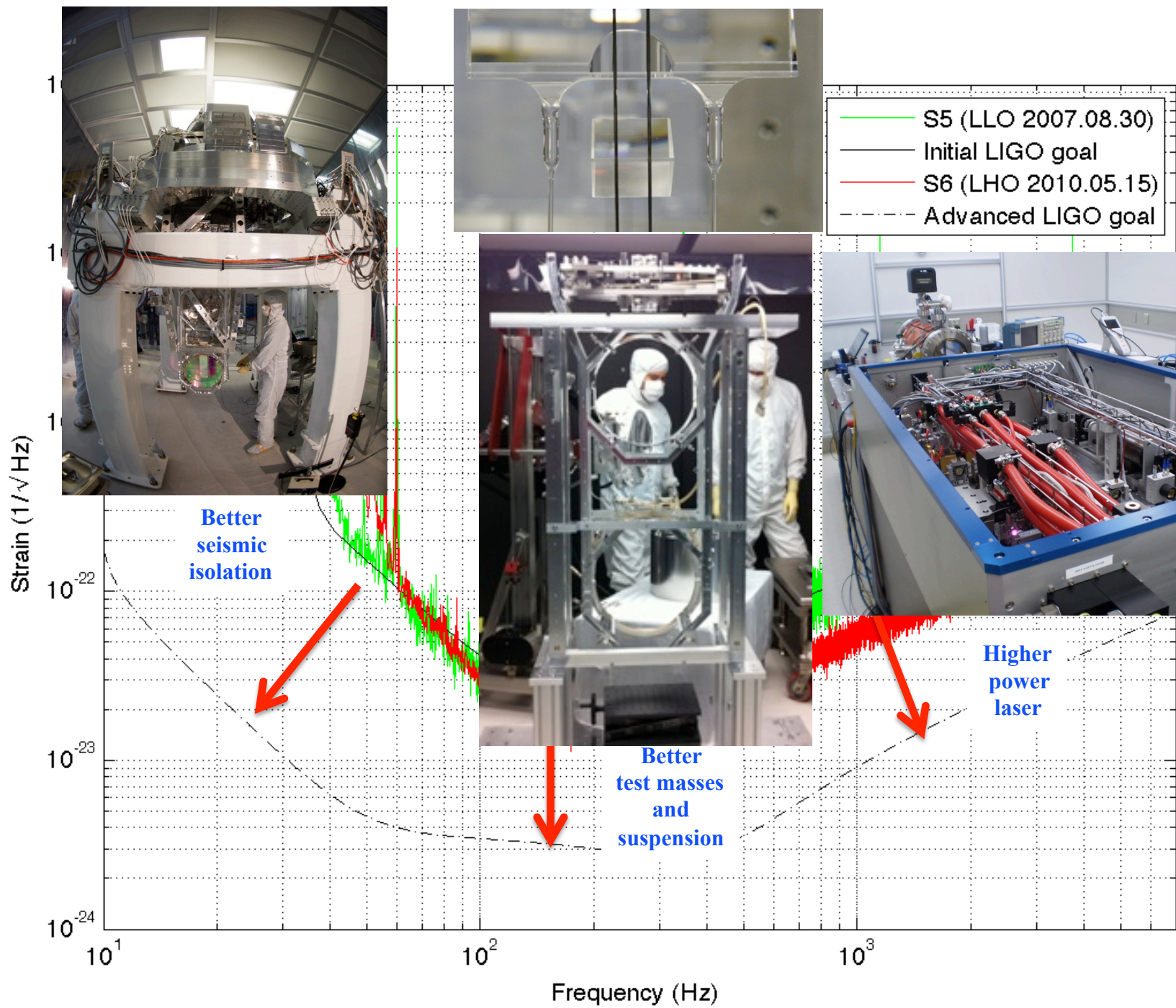


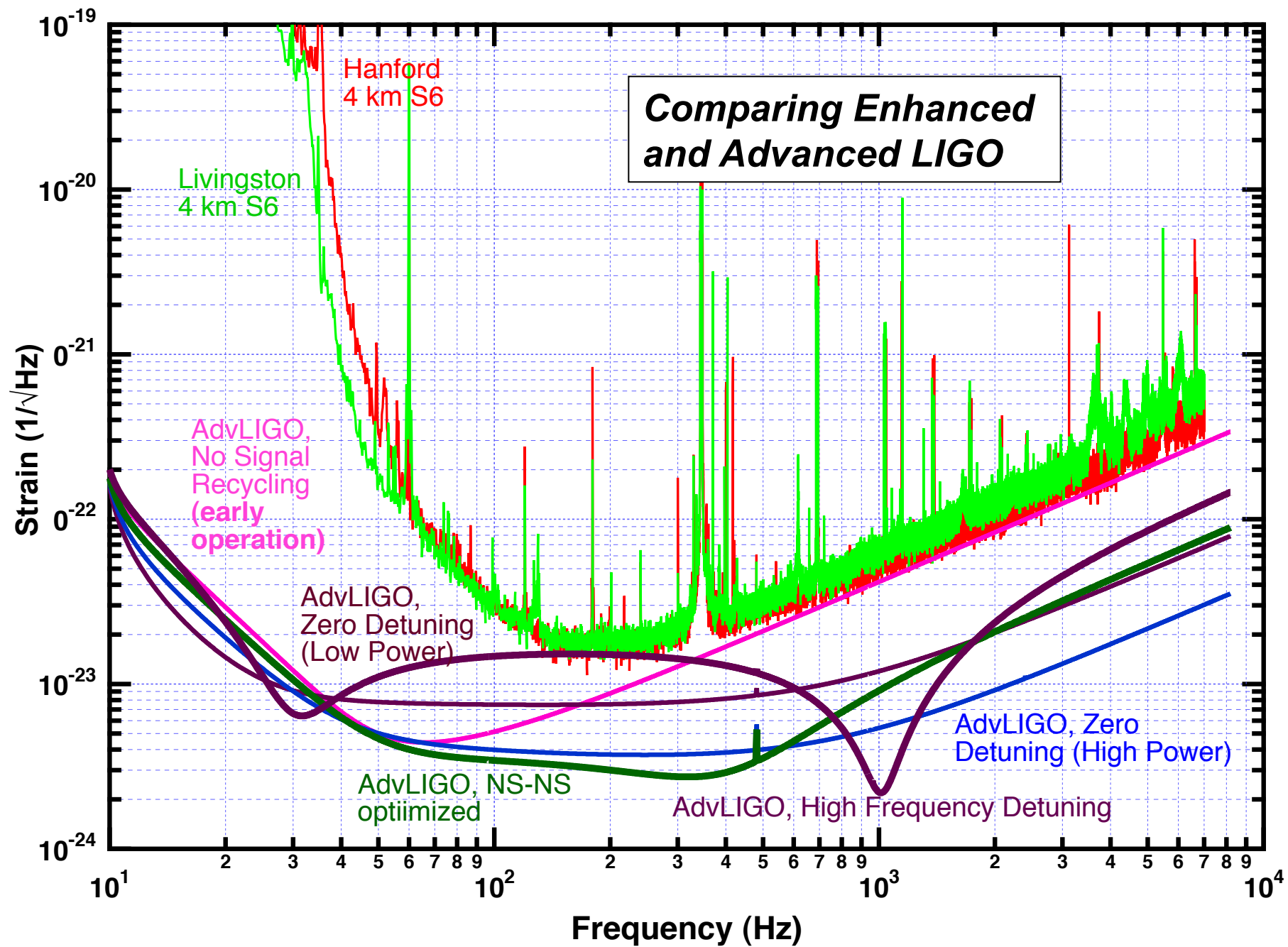
Principal noise terms







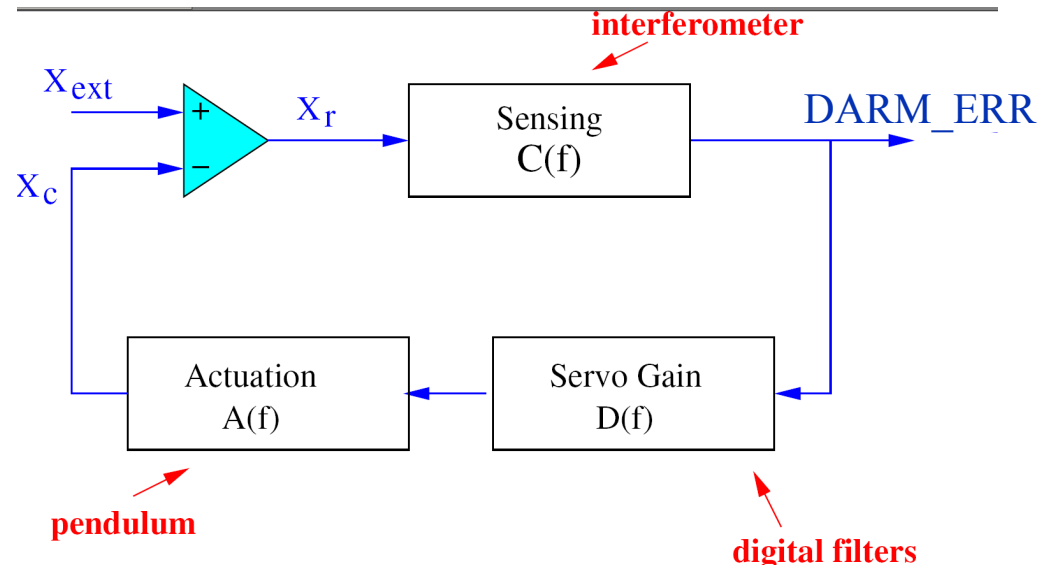






Length calibration

- Measure open loop gain G , input unity gain to model
- Extract gain of sensing function $C=G/AD$ from model
- Produce response function at time of the calibration, $R=(1+G)/C$
- Now, to extrapolate for future times, monitor calibration lines in $DARM_ERR$ error signal, plus any changes in gains.
- Can then produce R at any later time t .
- A photon calibrator, using radiation pressure, gives results consistent with the standard calibration.



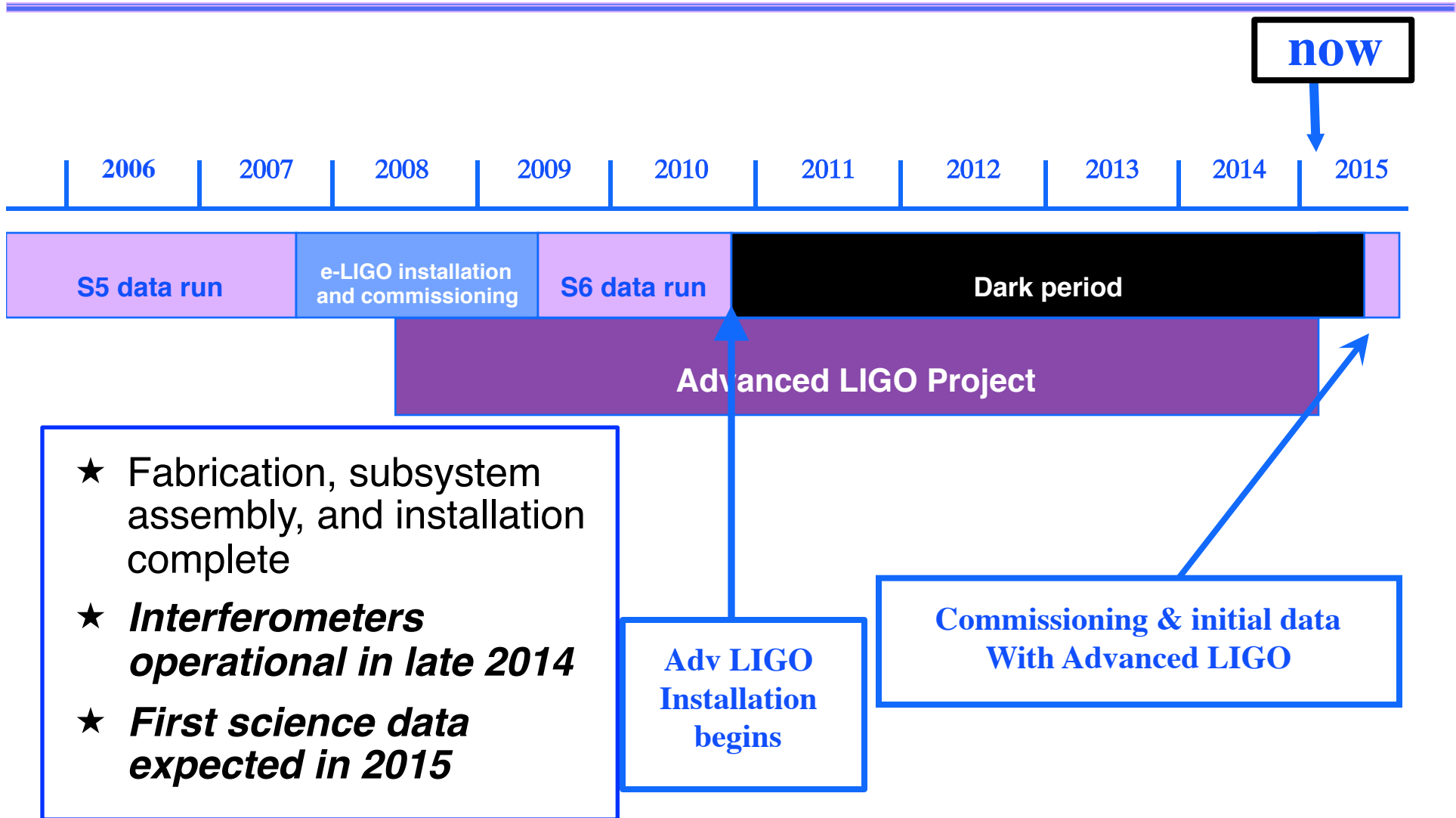
$$G(f) = C(f)D(f)A(f)$$

$$R(f) = \frac{1 + G(f)}{C(f)}$$

$$x_{ext}(f) = R(f)DARM_ERR(f)$$



LIGO time line



Phases in installation



deinstall



modify vacuum



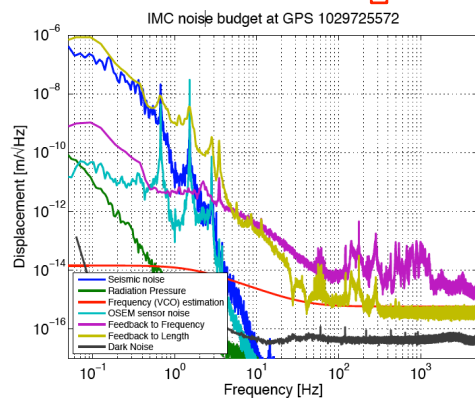
in-chamber
clean



install



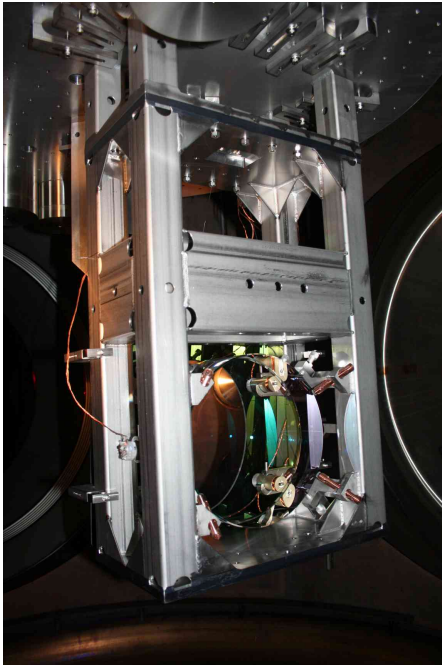
commission



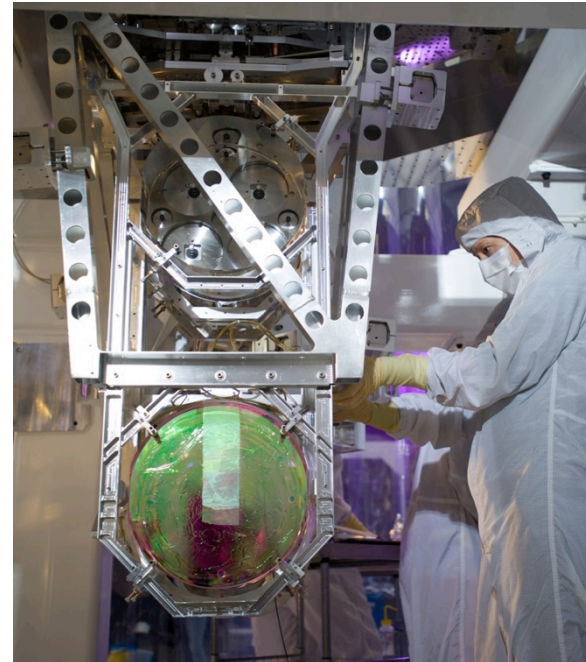


10X more sensitive, >10X harder...

- 14 unique fabricated parts
 - 68 fabricated parts total
 - 165 total including machined parts and hardware
- 188 unique fabricated parts
 - 1569 fabricated parts total
 - 3575 total including machined parts and hardware

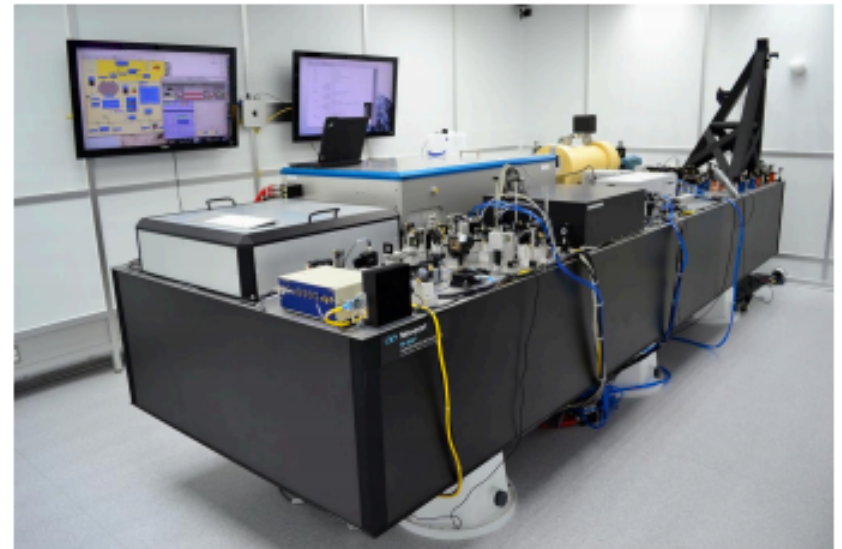
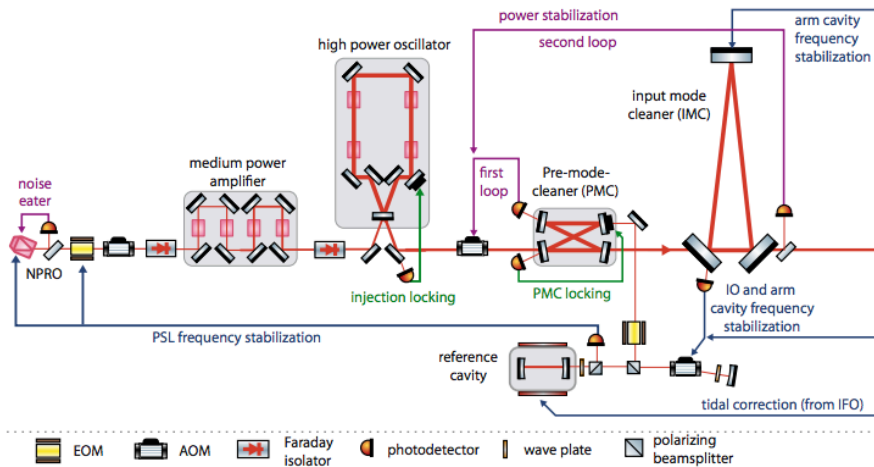


Test mass suspension
From **Initial LIGO**

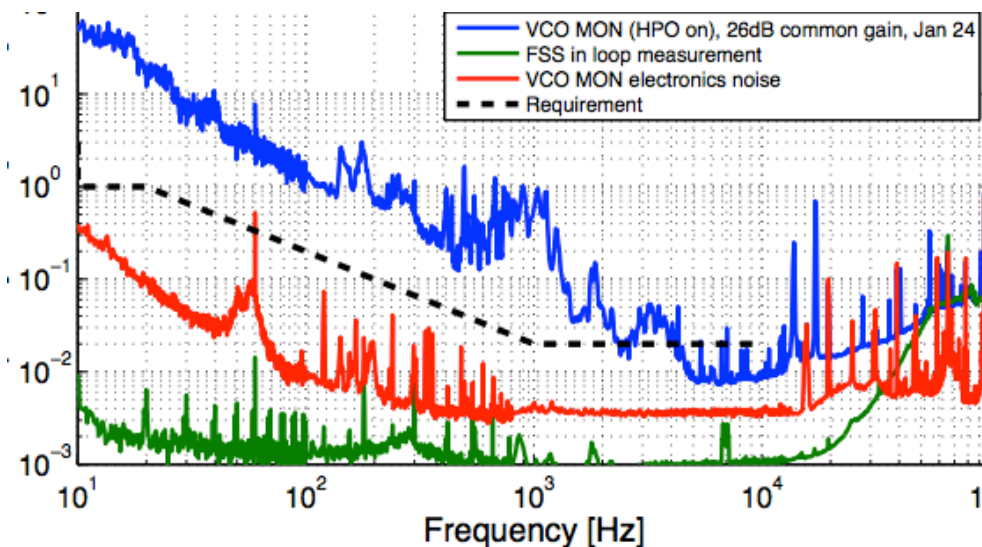


Test mass suspension
From **Advanced LIGO**

Pre-stabilized laser

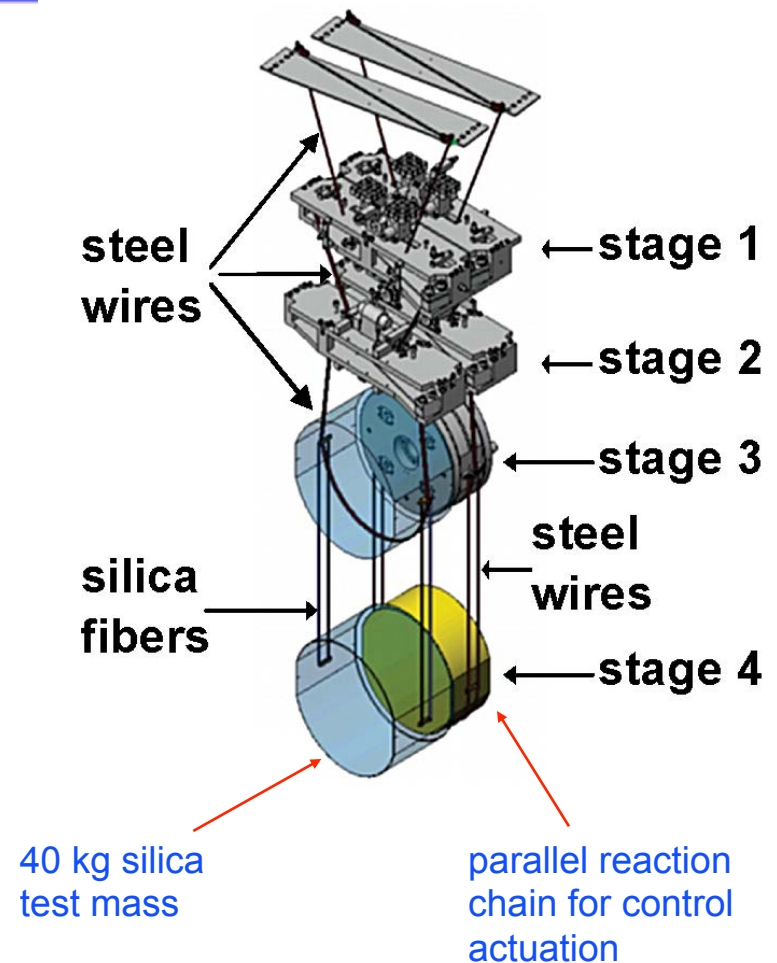


- Frequency noise measured at Livingston
- 3 W input to IMC
- noise between 10 and 100 Hz is already better; expect to meet spec without difficulty



Test mass suspensions

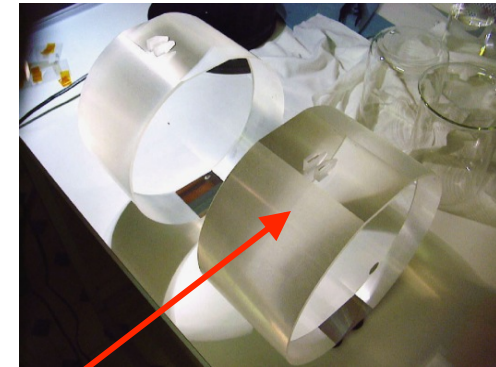
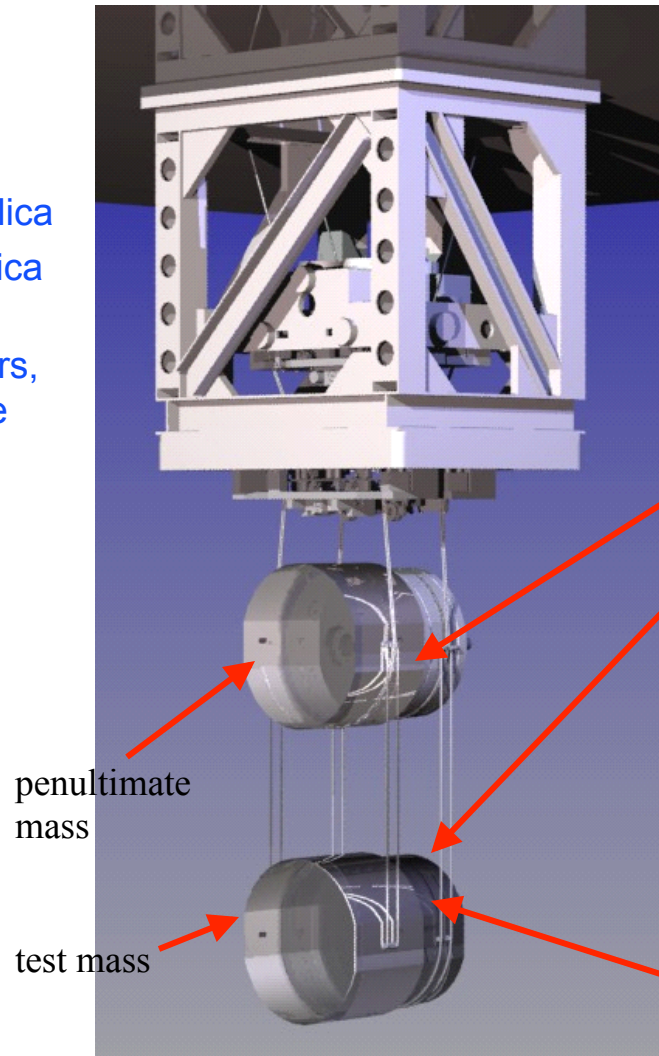
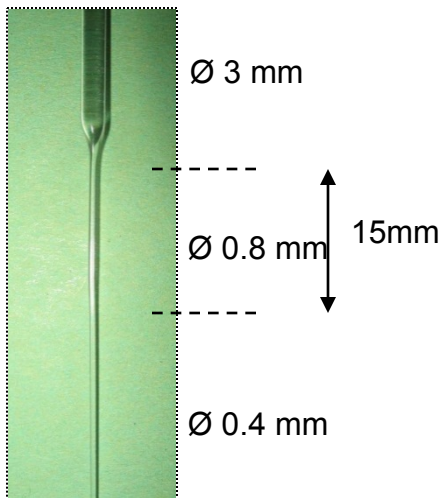
- Thermal noise reduction: monolithic fused silica suspension as final stage - low pendulum thermal noise and preservation of high mirror quality factor
- Seismic isolation: use quadruple pendulum with 3 stages of maraging steel blades for enhanced vertical isolation
 - » *isolation @ 10Hz: quad $\sim 3e-7$, c.f. single stage $\sim 5e-3$*
- Control noise minimisation: apply damping at top mass (for 6 degrees of freedom) and use quiet reaction pendulum for global control actuation in a hierarchical way
 - » Coil/magnet actuation at top 3 stages
 - » electrostatic drive at test mass



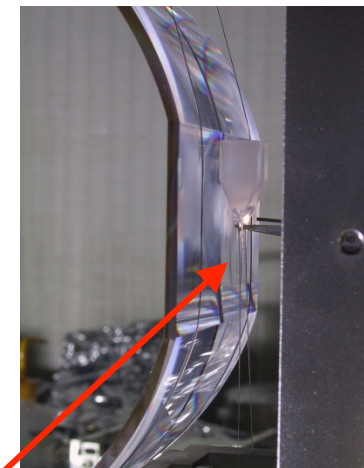
Monolithic stage

Monolithic stage

- 40 kg silica test mass suspended from 40 kg penultimate mass, also silica
- Four dumbbell shaped silica fibres (details below)
- Fibres welded to silica ears, bonded to the sides of the silica masses



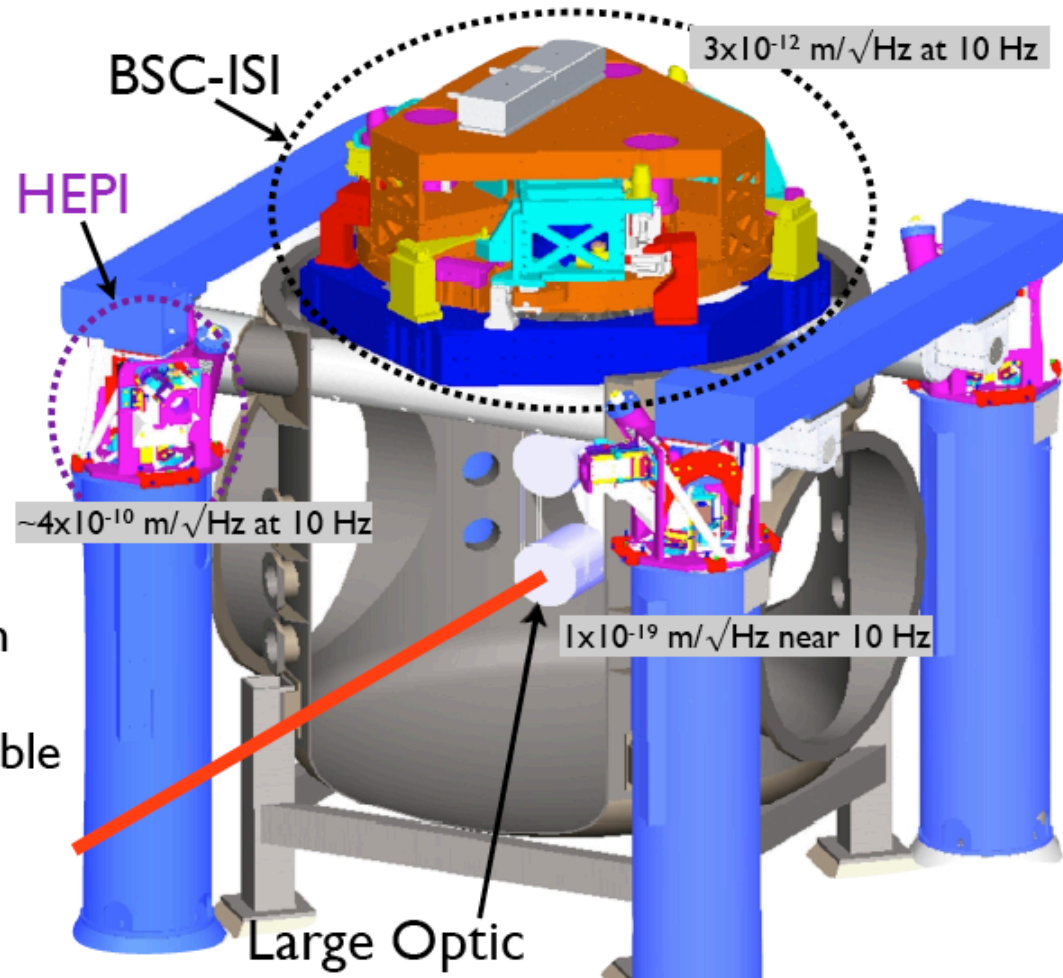
Ears silicate bonded to masses



Silica fibres, length 600 mm diameter 400 μ m, welded to silica ears

Seismic isolation

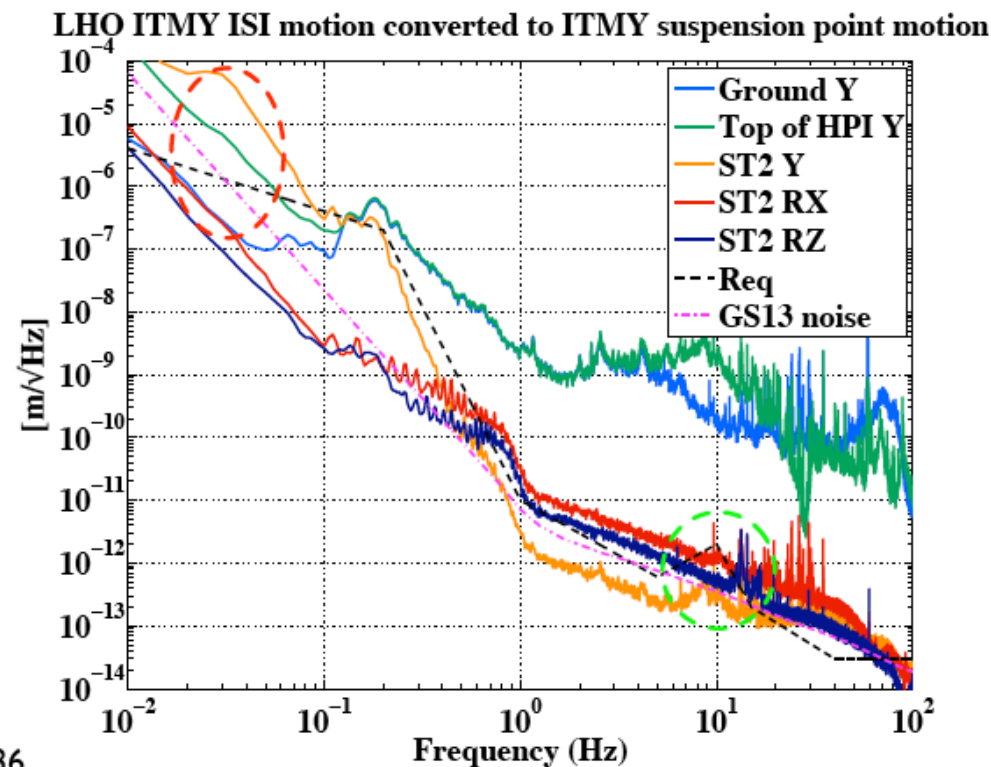
- 7 total layers
 - HEPI (1)
 - BSC-ISI (2)
 - Quad SUS (4)
- HEPI: Hydraulic External Pre-Isolator
large throw, isolation below ~ 5 Hz
- ISI
Internal Seismic Isolation
Isolates above ~ 0.2 Hz
Quiet, well controlled table
- Quad pendulum
superior performance at 10 Hz and above





Seismic isolation performance

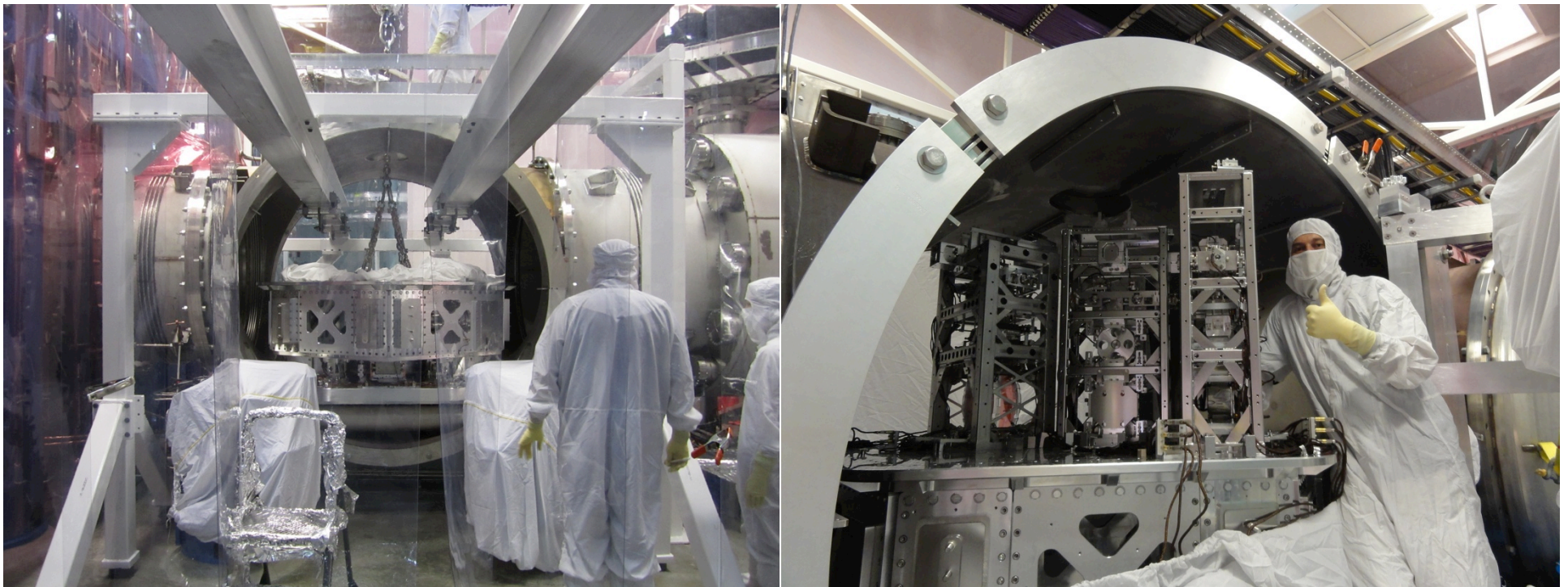
- At LHO the BSC ISI isolation is working at or better than the requirements from 0.1 Hz and up.
- Below 0.1 Hz there is some noise injection which may be removed with better tilt decoupling



R. deRosa, G1300936

HAM installations

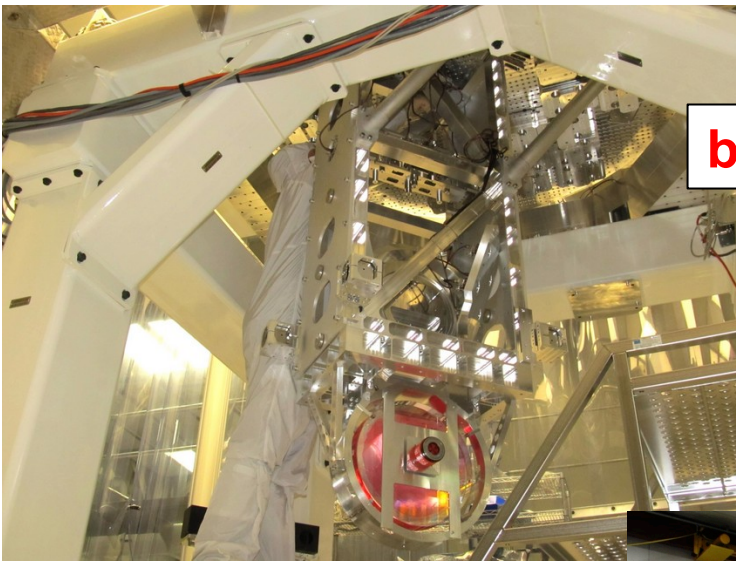
- For LIGO smaller chambers (“HAMs”), we install the seismic isolation platform into the chamber, and then populate it in situ



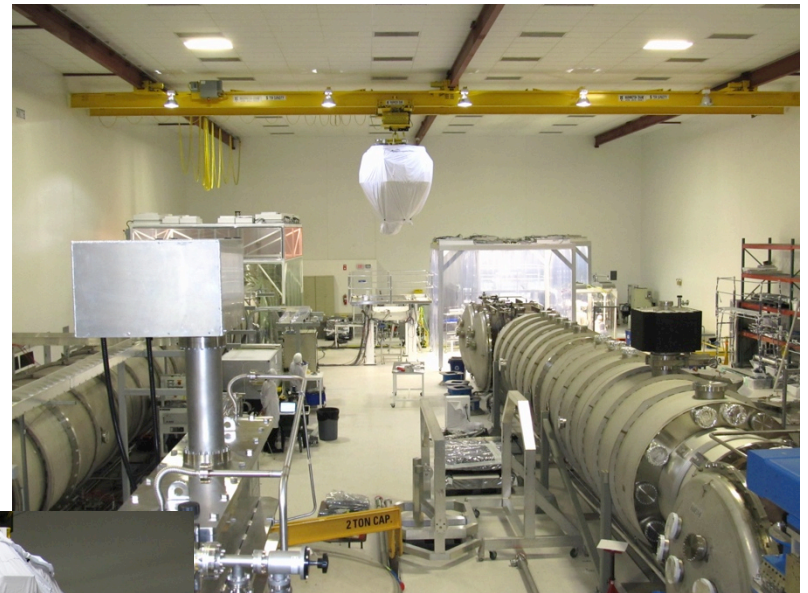
LLO HAM installation

BSC installations

- For LIGO large chambers ("BSCs"), we assemble a cartridge in a given hall, and then crane it into the vacuum envelope



beamsplitter



**cornerstation
Y mirror**



End Y mirror

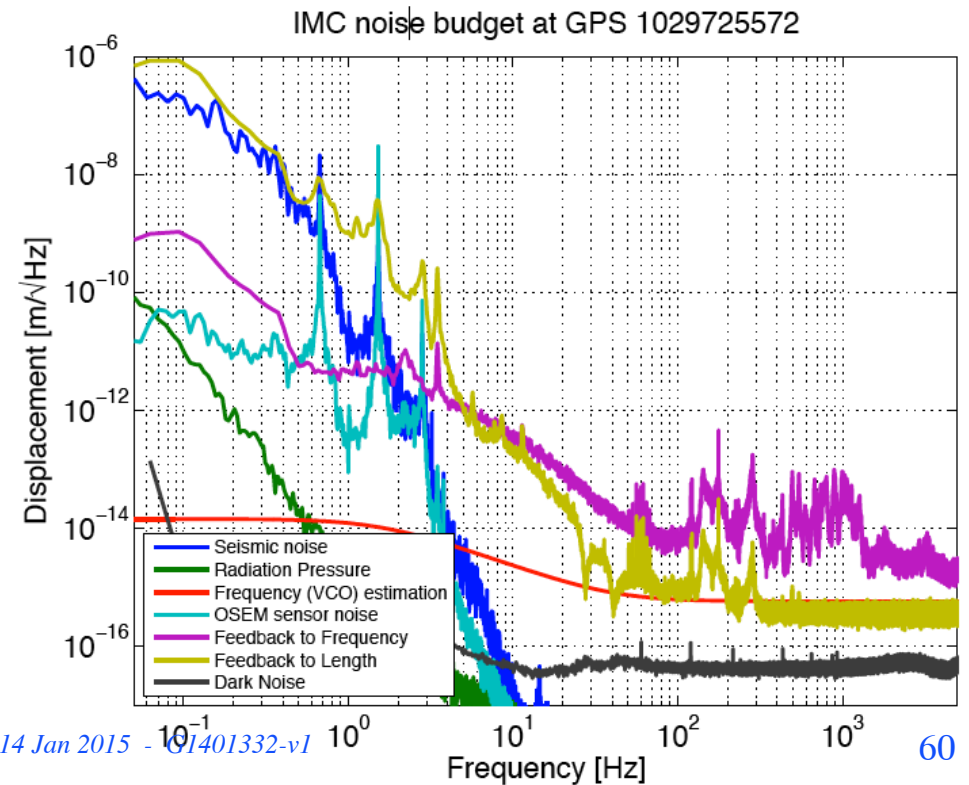
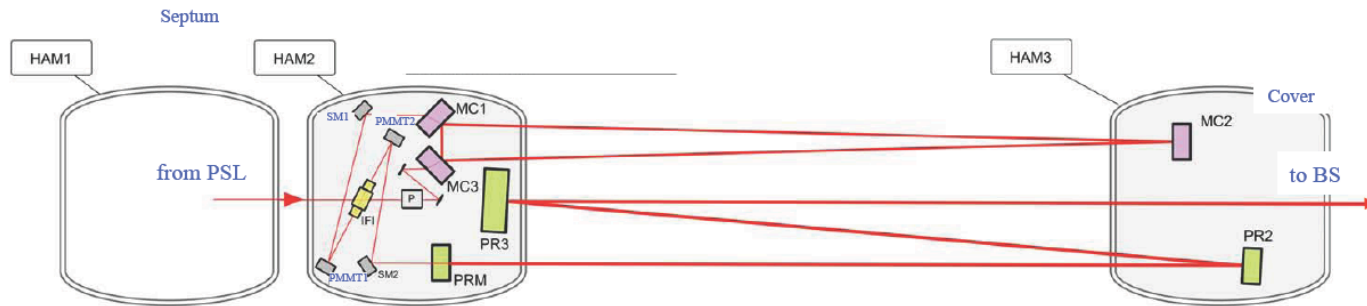


Short movie scene

- Installation scene from *LIGO: A Passion for Understanding*
- Full film at www.ligo.org



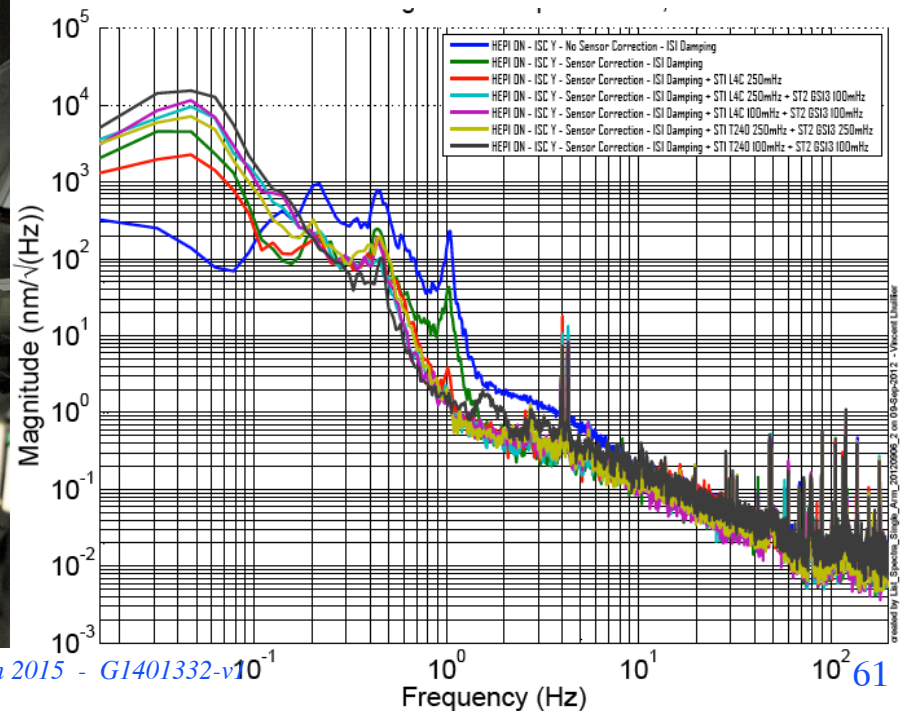
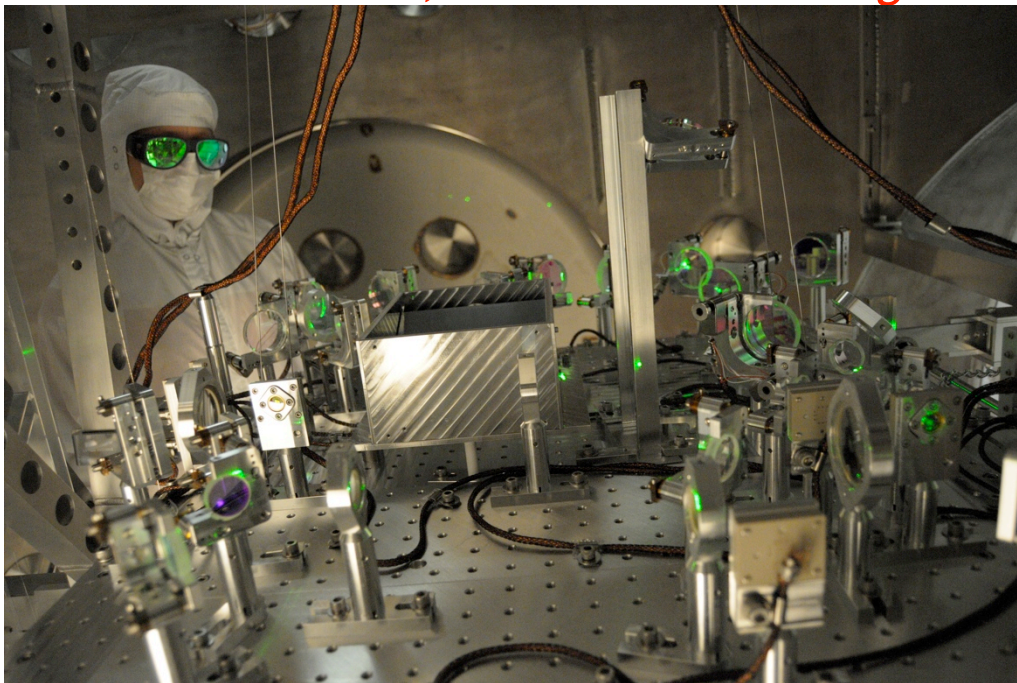
LIGO Livingston input mode cleaner





Hanford single-arm integration

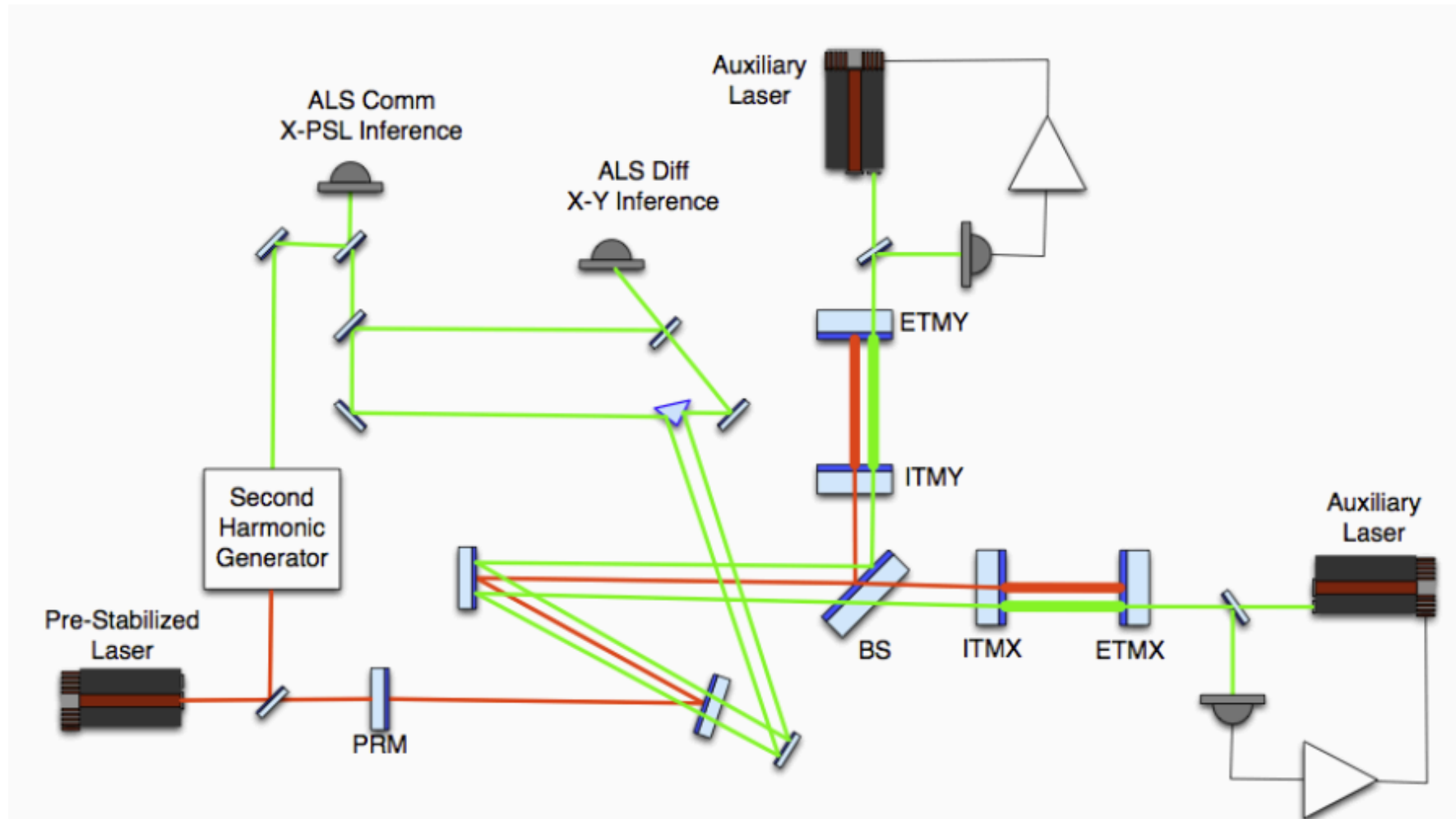
- New lock acquisition strategy developed for Advanced LIGO
 - Arm Length Stabilization system controls each arm cavity, putting them off-resonance
 - The 3 vertex lengths are controlled using robust RF signals
 - Arm cavities are brought into resonance in a controlled fashion
- *Therefore, commissioned single 4km arm*



Landry – Testing Gravity 14 Jan 2015 - G1401332-v10⁻¹

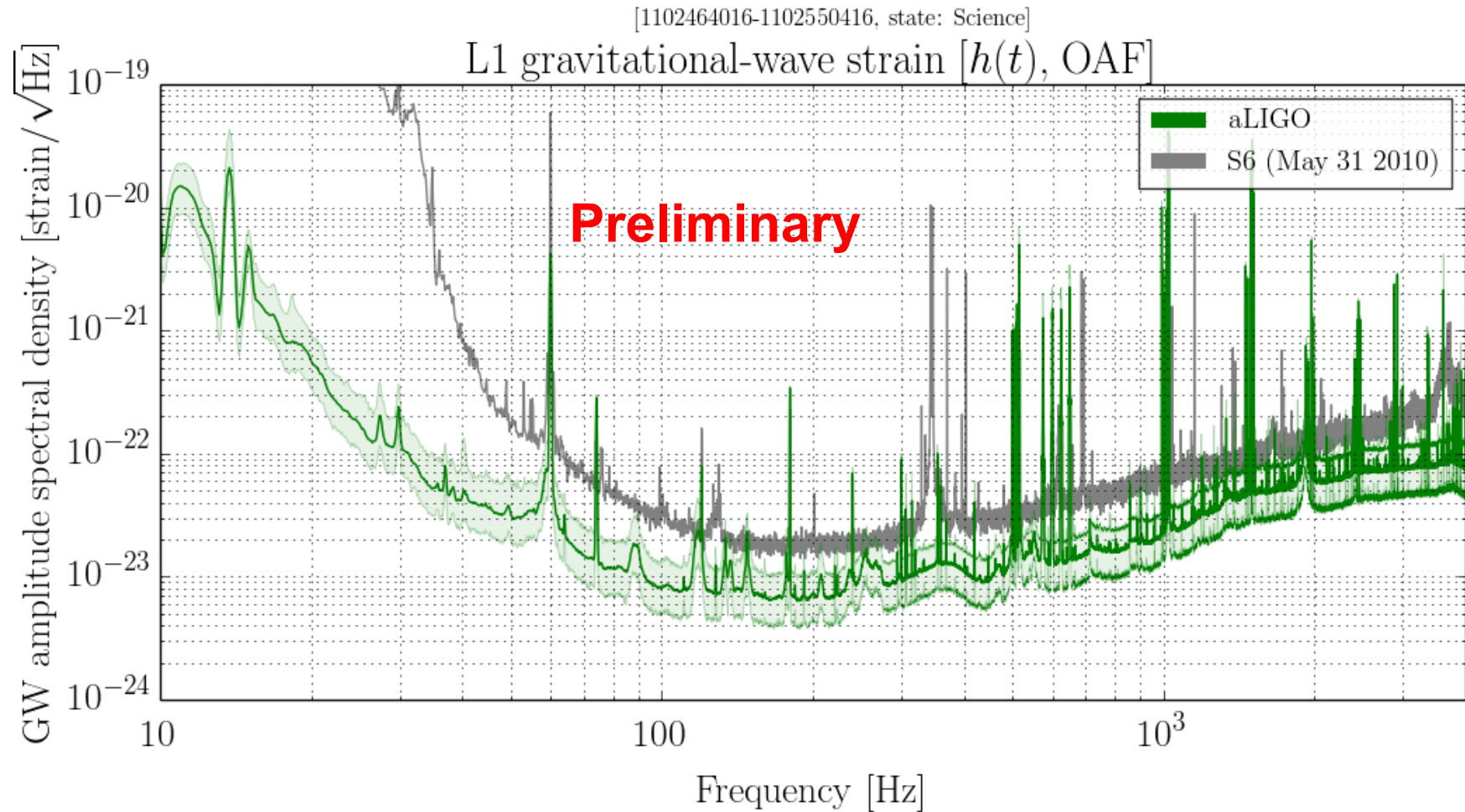
created by LAL_Spensing_Single_Arm_20150602_2 on 15-Sep-2015 - Vincent Leland

Arm length stabilization



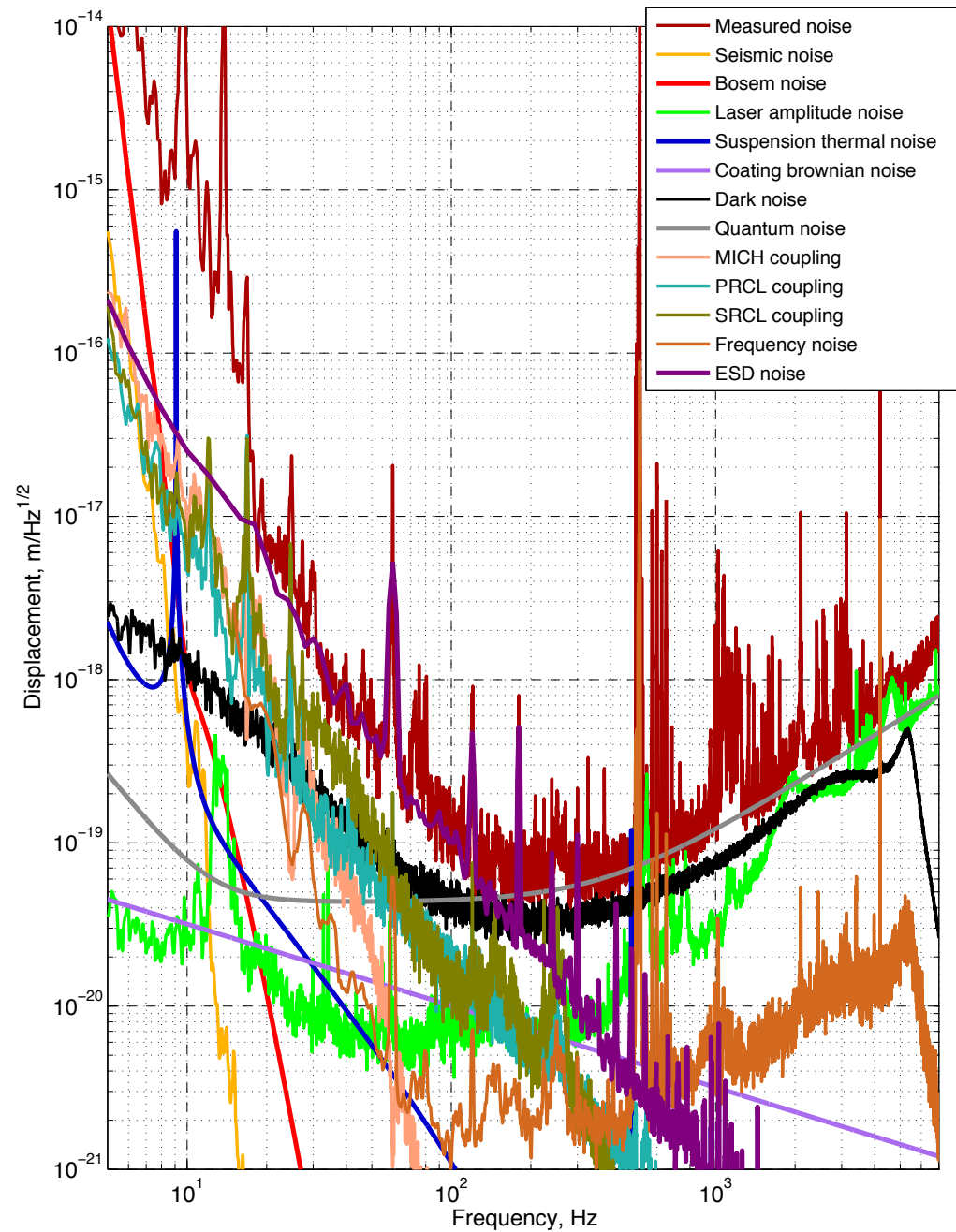


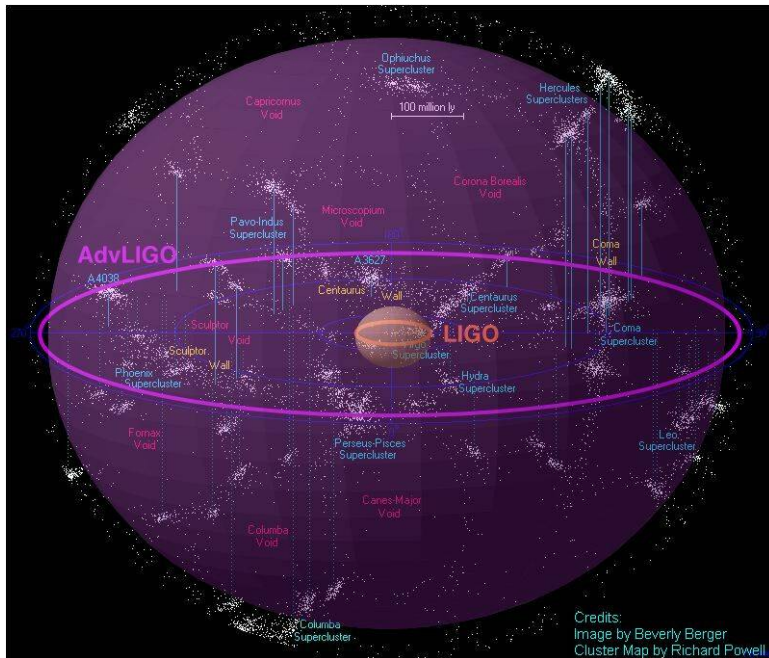
59Mpc lock during ER6





- Noise curve from lock at LLO
- ESD: electrostatic drive, the low noise actuator at the test mass
- Many technical noises still dominate these early spectra





Binary neutron stars

- Initial LIGO reach: 15Mpc; rate $\sim 1/50$ yrs
- Advanced LIGO ~ 200 Mpc
- ‘Realistic’ rate ~ 40 events/yr

Table 5. Detection rates for compact binary coalescence sources.

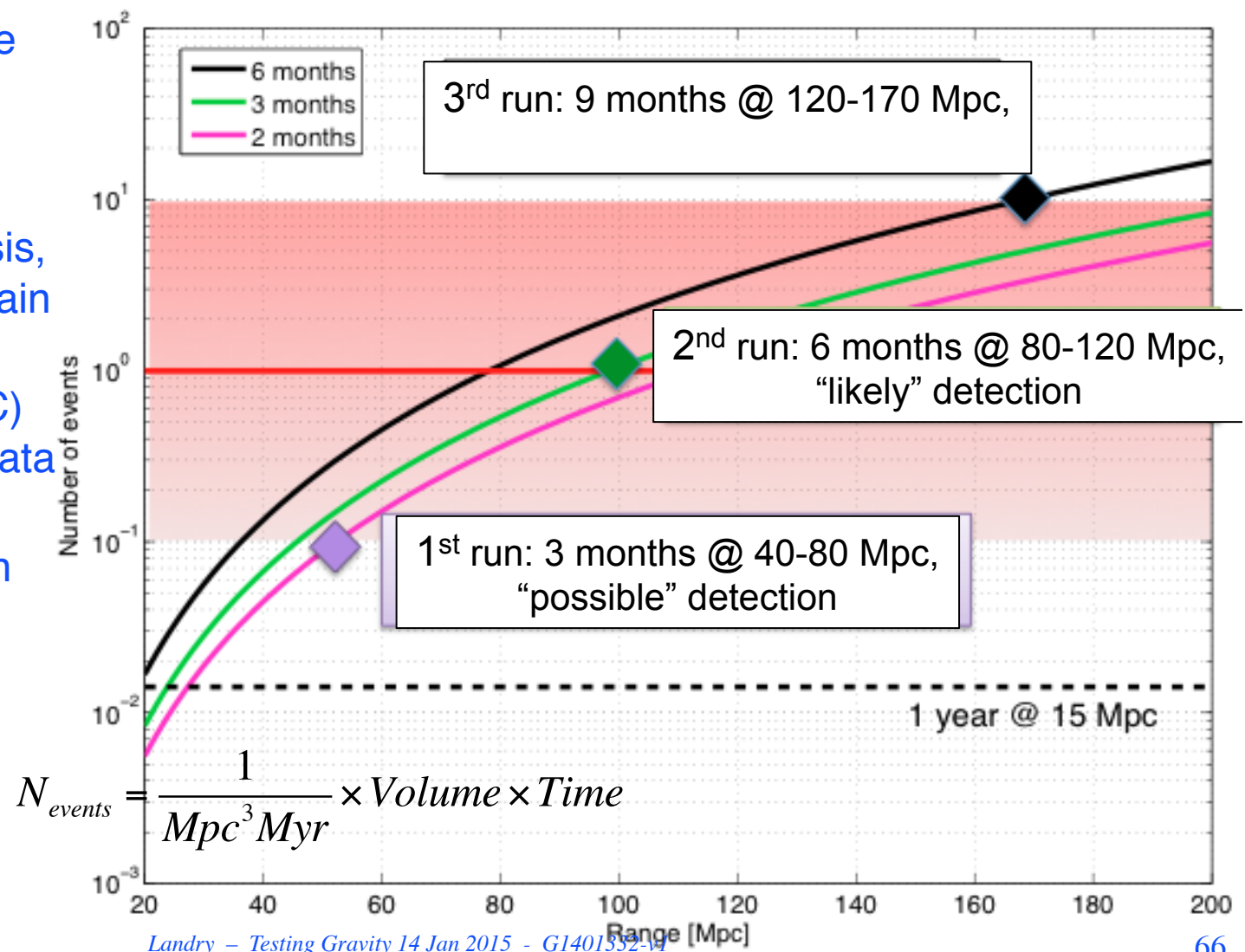
IFO	Source ^a	$\dot{N}_{\text{low}} \text{ yr}^{-1}$	$\dot{N}_{\text{re}} \text{ yr}^{-1}$	$\dot{N}_{\text{high}} \text{ yr}^{-1}$	$\dot{N}_{\text{max}} \text{ yr}^{-1}$
Initial	NS–NS	2×10^{-4}	0.02	0.2	0.6
	NS–BH	7×10^{-5}	0.004	0.1	
	BH–BH	2×10^{-4}	0.007	0.5	
	IMRI into IMBH			$< 0.001^b$	0.01^c
	IMBH–IMBH			10^{-4d}	10^{-3e}
Advanced	NS–NS	0.4	40	400	1000
	NS–BH	0.2	10	300	
	BH–BH	0.4	20	1000	
	IMRI into IMBH			10^b	300^c
	IMBH–IMBH			0.1^d	1^e

**Rates paper: Class. Quant. Grav,
27 (2010) 173001**



Current guess for sensitivity evolution, observation

- Vertical scale is the number of binary inspirals detected
- Rates based on population synthesis, realistic but uncertain
- LIGO Scientific Collaboration (LSC) preparing for the data analysis challenge
- Close collaboration with Virgo
- Early detection looks feasible
- [arXiv:1304.0670](https://arxiv.org/abs/1304.0670),
[arXiv:1003.2480](https://arxiv.org/abs/1003.2480)



Summary

- Advanced LIGO nearly complete as a project: L1 locking at 60Mpc, H1 fully locked but at the outset of noise hunting
- We expect to make first science run with the second generation detectors in 2015 and 2016, runs which may produce detections
- We will press onward with sensitivity improvements to design sensitivity
- We expect gravitational waves will be detected in the coming few years



Light at the end of a tunnel



LIGO Scientific Collaboration



•Australian Consortium
for Interferometric
Gravitational Astronomy

•The Univ. of Adelaide

•Andrews University

•The Australian National Univ.

•The University of Birmingham

•California Inst. of Technology

•Univ. of Cambridge

•Canadian Institute for Theoretical

Astrophysics and Perimeter Institute for
Theoretical Physics

•Cardiff University

•Carleton College

•Charles Sturt Univ.

•Columbia University

•CSU Fullerton

•Embry Riddle Aeronautical Univ.

•Eötvös Loránd Univ.

•University of Florida

•German/British Collaboration for
the Detection of Gravitational Waves

•University of Glasgow

•Goddard Space Flight Center

•Hanyang University

•Korea Institute of

Science and Tech Information

•Leibniz Universität Hannover

•Lund University

•Hobart & William Smith Colleges

•Inst. of Applied Physics of the

Russian Academy of Sciences

•Polish Academy of Sciences

•India Inter-University Centre for
Astronomy and Astrophysics

•Louisiana State University

•Louisiana Tech University

•Loyola Univ. of New Orleans

•University of Maryland

•Max Planck Institute for

Gravitational Physics

