YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Testing Gravity
SFU Harbour Center
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Creation of Universe in Bigravity

－ Resurrectiong Hartle-Hawking's no boundary proposal －

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Zhang, Saito & MS, JCAP 1302, 029 (2013) [arXiv:1210.6224 [hep-th]]
MS, Yeom & Zhang, CQG 30, 232001 (2013) [arXiv:1307.5948 [gr-qc]]
Zhang, MS & Yeom, arXiv:1411.6769 [hep-th]

1. Introduction

modified gravity = popular scenario for the accelerated expansion

f(R), Chameleon, Galileon (Horndeski), massive gravity, DGP,…

Trend is to modify IR behavior of gravity

However, gravity deviates from GR most likely in the early universe.

cosmological implications? observational signatures?

How about quantum cosmology?

Problem(?) with GR quantum cosmology

Hartle-Hawking (HH) wave fcn with no boundary boundary condition

Hartle & Hawking (1983)

HH wave fcn ~ Euclidean vacuum state

most popular/natural wave fcn of the Universe

GR action with matter field (=φ)

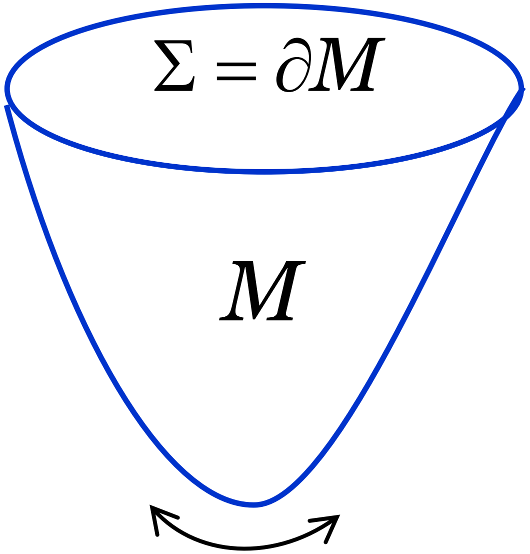
$$S[g,\varphi]=\int d^4x\left[\frac{M_P^2}{2}\sqrt{-g}R+L_m(\varphi)\right]$$

$$\Psi_{HH}[h,\phi]=\int_M^{\Sigma[h,\phi]} Dg D\varphi\, e^{-S_E[g,\varphi]}$$

path integral is over all compact

Euclidean manifolds M with Σ=∂M

$$(h,\phi)=(g,\varphi)\ \text{on}\ \Sigma=\partial M$$



regular, no boundary

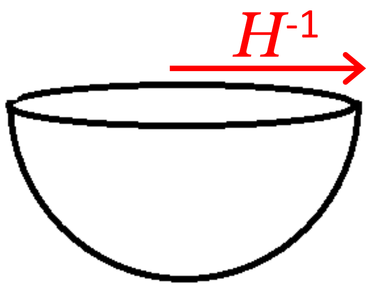
natural generalization of ground state in QFT

HH wave fcn fails to predict inflation:

$$L_m(\phi)=\int d^4x\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi-V(\phi)\right]$$

For $V<M_P^4$, $V'^2 \ll V^2/M_P^2$, $V'' \ll V/M_P^2$, path integral is dominated by

$M \approx$ a half of S^4 with $\partial M = S^3 = \Sigma[h,\phi]$; $\phi \approx const.$



$S^4 =$ Euclidean de Sitter /w radius H^{-1}; $H^2=\dfrac{V(\phi)}{3M_P^2}$

$$\Rightarrow \quad \Psi_{HH}[h,\phi]\approx \exp\left(-S_E^{dS}/2\right);\quad S_E^{dS}=-\frac{24\pi^2 M_P^4}{V(\phi)}$$

Ψ_{HH} is exponentially peaked at $V(\phi)=0$.

$$e.g.,\ \Psi_{HH}\propto \exp\left[\frac{24\pi^2 M_P^4}{m^2\phi^2}\right] \text{ for } V(\varphi)=\frac{1}{2}m^2\phi^2$$

inflation is exponentially unlikely!

2. Massive Gravity

Brief History

- Massive Gravity: can gravitons have mass? Yes! at linear level

  = massive spin 2 on flat bkgr: Fierz-Pauli (1939)

  GR: 2 tensor dof ⟺ Massive G: 5=2 tensor+2 vector +1 scalar

- But nonlinear extension seemed formidable…

  No diffeo inv means no gauge dof: 10-4=6 dof

  6= 2 tensor+2 vector + 2 scalar

  6-5=1 additional scalar dof ~ conformal dof = ghost!

  Boulware-Deser ghost (1972)

- discovery of ghost-free nonlinear (dRGT) massive gravity

  ghost removed by additional constraints

  de Rham, Gabadadze, Tolley (2011)

- extension of dRGT to bimetric (bi-)gravity Hassan & Rosen (2012)

dRGT massive gravity

de Rham, Gabadaze, Tolley (2011)

$$S = M_P^2 \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right]$$

$$\mathcal{L}_2 = \frac{1}{2} \left([\mathcal{K}]^2 - [\mathcal{K}^2] \right)$$

nonlinear FP term

$$\mathcal{L}_3 = \frac{1}{6} \left([\mathcal{K}]^3 - 3 [\mathcal{K}] [\mathcal{K}^2] + 2 [\mathcal{K}^3] \right),$$

$$\mathcal{L}_4 = \frac{1}{24} \left([\mathcal{K}]^4 - 6 [\mathcal{K}]^2 [\mathcal{K}^2] + 3 [\mathcal{K}^2]^2 + 8 [\mathcal{K}] [\mathcal{K}^3] - 6 [\mathcal{K}^4] \right)$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \left(\sqrt{g^{-1} f} \right)^\mu{}_\nu \qquad \left(\sqrt{g^{-1} f} \sqrt{g^{-1} f} \right)^\alpha_\beta = \left(g^{-1} f \right)^\alpha_\beta = g^{\alpha\nu} f_{\nu\beta}$$

$f_{\mu\nu}$: fixed metric (non-dynamical)

m_g : graviton mass on flat bkgr.

Hamiltonian dof: 6x2-2=5x2: 5=2 tensor + 2 vector + 1 scalar

↳ 2nd class constraints (remove BD ghost)

homogeneous instanton in dRGT gravity

Zhang, Saito & MS (2012)

$$S = M_P^2 \int \sqrt{-g}\, d^4x \left[\frac{R}{2} + \mathcal{L}_{\rm mg} - \frac{1}{2}(\nabla\phi)^2 - V(\phi) \right]$$

$$\mathcal{L}_{\rm mg} = \mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4$$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right),$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right),$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right)$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \left(\sqrt{g^{-1}f}\right)^\mu{}_\nu$$

$$f_{\mu\nu}dx^\mu dx^\nu = dT^2 + b^2(T)d\Omega_{(3)}^2 : \quad b(T) = F^{-1}\cos FT$$

$f_{\mu\nu}$ = fixed to (Euclidean) de Sitter

instanton solution

O(4) symmetry

$$g_{\mu\nu}dx^\mu dx^\nu = d\tau^2 + a^2(\tau)d\Omega_{(3)}^2$$

$$\Rightarrow \quad \dot{a}^2 - 1 - \frac{a^2}{3}\left(\frac{\dot{\phi}^2}{2} - V_{\rm eff}\right) = 0\ ; \qquad V_{\rm eff} = V + f(a,b)$$

slow-roll approx: ϕ = const.

$$f_{\mu\nu}dx^\mu dx^\nu = dT^2 + b^2(T)d\Omega_{(3)}^2 : \quad b(T) = F^{-1}\cos FT$$

$$\Rightarrow \quad f(a,b) = \text{fcn of } X = \frac{b}{a} = const.: f = f(X)$$

$$a = \frac{1}{H}\cos H\tau\ ; \quad H^2 = \frac{V_{\rm eff}}{3M_P^2}$$

$$T = F^{-1}\cos^{-1}(\alpha\cos H\tau);\quad \alpha \equiv \frac{F}{H}X,\quad 0<\alpha<1$$

$$X = X_\pm = \left(1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}\right)/(\alpha_3 + \alpha_4)$$

cond. for soln. to exist

HH wave function in dRGT gravity

MS, Yeom & Zhang (2013)

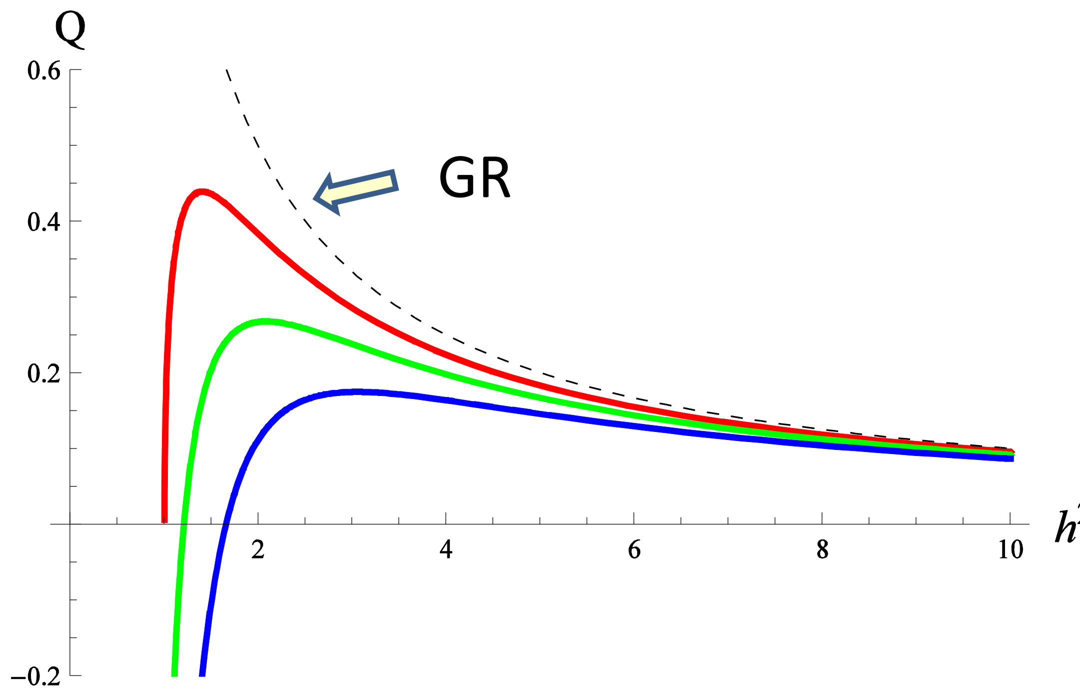
$$S_E = -\frac{4\pi^2 M_P^2}{H^2}\left[1 - \frac{m_g^2}{F^2} Z(X) D(\alpha^2)\right] = -\frac{4\pi^2 M_P^2}{X^2 F^2}\left[\alpha^2 - \frac{m_g^2}{F^2} Z(X)\alpha^2 D(\alpha^2)\right]$$

$$\equiv -\frac{4\pi^2 M_P^2}{X^2 F^2} Q(h^2)$$

$$h^2 \equiv \frac{H^2}{F^2} X^2 = \frac{1}{\alpha^2} = h^2(\phi)$$

$$\Psi_{\rm HH}(\phi) \approx \exp\left[-S_E(\phi)\right]$$

$$= \exp\left[\frac{4\pi^2 M_P^2}{X^2 F^2} Q\left(h^2(\phi)\right)\right]$$



$\Psi_{\rm HH}$ is peaked at $H^2 = O(m_g{}^2)$!

bimetric theory (bigravity)

- dRGT gravity may save HH wave fcn
- but fixed de Sitter $f_{\mu\nu}$ seems too ad hoc
- dRGT cosmology may suffer from instabilities

Hassan & Rosen (2012)

de Felice et al. (2012), ...

making *f* dynamical and theory stable ⇨ bimetric theory

$$S = \frac{1}{2}\int d^4x \left[\sqrt{-g}M_{\rm P}^2\left(R_{\rm g}-2\lambda_{\rm g}\right)+\sqrt{-f}M_{\rm f}^2\left(R_{\rm f}-2\lambda_{\rm f}\right)\right] + m_{\rm g}^2 M_{\rm e}^2\int d^4x\sqrt{-g}\sum_{n=1}^{4}\alpha_n\mathcal{U}_n(\mathcal{K}) + \int d^4x\left[\sqrt{-g}\mathcal{L}_{\rm mg}+\sqrt{-f}\mathcal{L}_{\rm mf}\right]$$

$$\mathcal{U}_1(\mathcal{K}) = [\mathcal{K}] \equiv \mathcal{K}^\mu_\mu,$$ ~ cosmological const.

$$\mathcal{U}_2(\mathcal{K}) = \frac{1}{2!}\left([\mathcal{K}]^2-[\mathcal{K}^2]\right),$$

$$\mathcal{U}_3(\mathcal{K}) = \frac{1}{3!}\left([\mathcal{K}]^3-3[\mathcal{K}][\mathcal{K}^2]+2[\mathcal{K}^3]\right),$$

$$\mathcal{U}_4(\mathcal{K}) = \frac{1}{4!}\left([\mathcal{K}]^4-6[\mathcal{K}^2][\mathcal{K}]^2+8[\mathcal{K}^3][\mathcal{K}]+3[\mathcal{K}^2]^2-6[\mathcal{K}^4]\right)$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_\nu \qquad M_{\rm e} = (M_{\rm P}^{-2}+M_{\rm f}^{-2})^{-1/2}$$

Hamiltonian degrees of freedom in bigravity

$g_{\mu\nu}$: $g^{(3)}_{ab} + N^{\mu}$

$f_{\mu\nu}$: $f^{(3)}_{ab} + F^{\mu}$

lapse & shift = Lagrange multipliers

4 + (3 + 1)

gauge dof

3-metrics: 6 + 6 = 12, conjugate mom: 6 + 6 = 12

12 + 12 = 24

4 gauge (1st class) constraints + 4 gauge conditions

24 – 4 – 4 = 16 = 4 massless + 12 massive

3 + 1 (+1) remove 3 Lag multp. + 2 massive dof

consistency (2ndary constr.)

BD ghost dof

massless: 4 = (2 tensor) x 2

massive: 12 – 2 = 10 = (2 tensor + 2 vector + 1 scalar) x 2

tensor mass eigenstates are mixture of g & f ⇨ massive GW osc.

De Felice, Tanaka, Nakamura (2013)

Euclidean cosmological solution

Zhang, MS & Yeom (2014)

O(4) symmetry

$$ds_g^2 = N^2(\tau)d\tau^2 + a^2(\tau)d\Omega_{(3)}^2$$

$$ds_f^2 = N_f^2(\tau)d\tau^2 + b^2(\tau)d\Omega_{(3)}^2$$

2x2=4 dof

one 1st class constraint (~Friedmann eq for a) + one time reparm

two 2nd class constraints (~Friedmann eq for b + consistency)

remove BD ghost

$$\begin{aligned} S_{\rm E} = 2\pi^2\Big\{ & -3M_{\rm P}^2\int d\tau a\left(\frac{\dot{a}^2}{N}+N\right) - 3M_{\rm f}^2\int d\tau b\left(\frac{\dot{b}^2}{N_{\rm f}}+N_{\rm f}\right) \\ & + \int d\tau a^3 N\left[M_{\rm P}^2\lambda_{\rm g} + V_{\rm g} + \frac{\dot{\phi}_{\rm g}^2}{2N^2} + m_g^2 M_e^2\sum_{n=0}^{3} A_n\left(\frac{b}{a}\right)^n\right] \\ & + \int d\tau b^3 N_{\rm f}\left[M_{\rm f}^2\lambda_{\rm f} + V_{\rm f} + \frac{\dot{\phi}_{\rm f}^2}{2N_{\rm f}^2} + m_g^2 M_e^2\sum_{n=0}^{3} B_n\left(\frac{b}{a}\right)^{n-3}\right]\Big\} \end{aligned}$$

HH wave function in bigravity

Euclidean solution : O(4)-sym + slow-roll ⇨ O(5)-sym

$$a = H^{-1}\sin H\tau$$

$$b = H_f^{-1}\sin H_f f(\tau)$$

$$X \equiv \frac{b}{a} = const. = \text{fcn of model prm's}$$

⇨ $$f(\tau) = \frac{1}{H_f}\sin^{-1}(\alpha \sin H\tau); \quad \alpha \equiv \frac{H_f}{H} X$$

$0 < \alpha < 1$

$$H = H(\phi_g, \phi_f)$$

$$H_f = H_f(\phi_g, \phi_f)$$

$$\Psi_{HH}(a_{\max}, b_{\max}, \phi_g, \phi_f) \approx \exp[-S_E(\phi_g, \phi_f)]$$

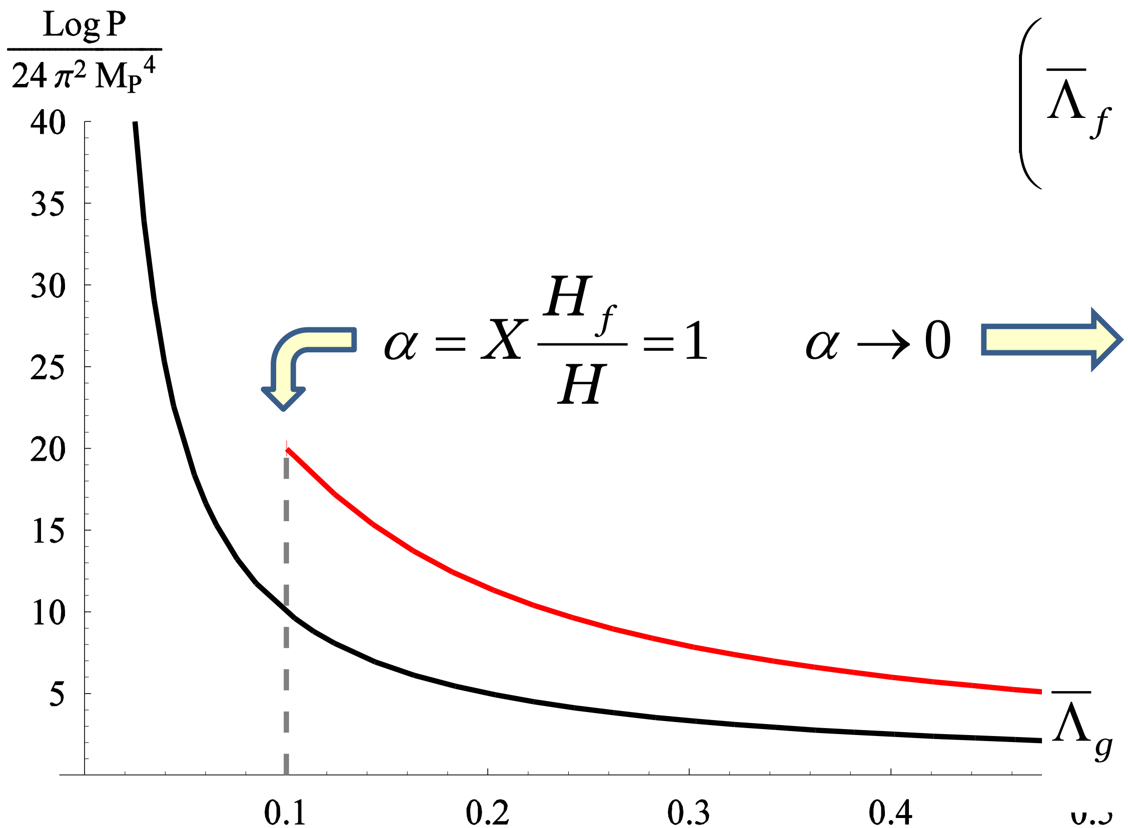
$$S_E = -12\pi^2\left[M_P^2 \int_0^{a_{\max}^2} da^2 \sqrt{1 - a^2 H^2} + M_f^2 \int_0^{b_{\max}^2} db^2 \sqrt{1 - b^2 H_f^2}\right]$$

$$= -8\pi^2 \left\{ \frac{M_P^2}{H^2} + \frac{M_f^2}{H_f^2}\left[1 - \left(1 - \alpha^2\right)^{3/2}\right] \right\}$$

Probability distribution

$$\text{Log}\, P(\phi) = \text{Log}\left|\Psi_{HH}(\phi)\right|^2 \approx -2S_E(\phi)$$

$$\left(\bar{\Lambda}_f \equiv \frac{3H_f^2}{M_f^2} = 1\text{: fixed}\right)$$



$$\alpha = X\frac{H_f}{H} = 1 \qquad \alpha \to 0$$

$$\bar{\Lambda}_g \equiv \frac{3H^2}{M_P^2}$$

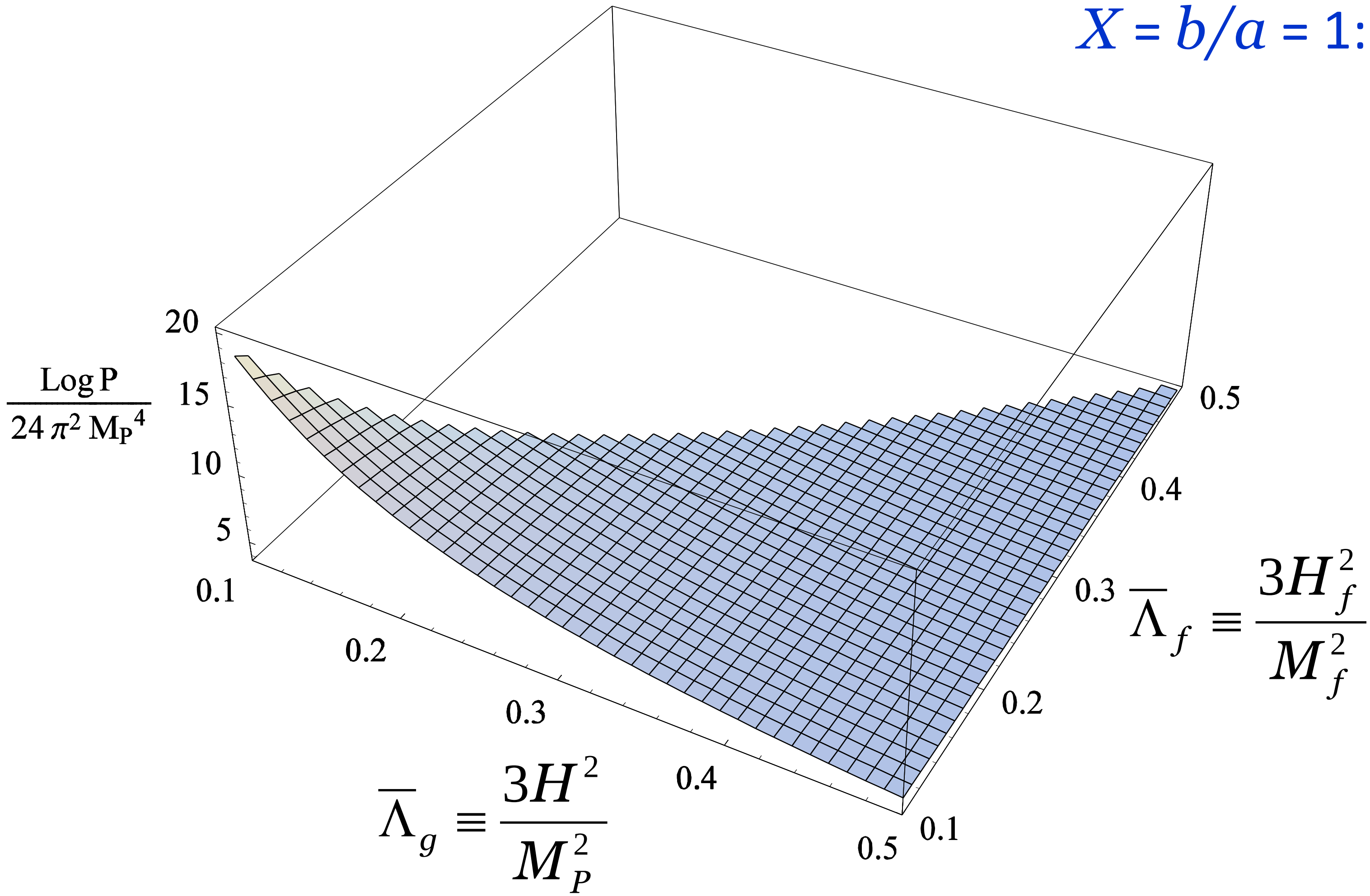
Real Euclidean soln ceases to exist at small H

Probability is peaked at around $\alpha = 1$

probability distribution (conti.)

$$\text{Log}\, P(\phi) = \text{Log}\left|\Psi_{HH}(\phi)\right|^2 \approx -2S_E(\phi)$$

$X = b/a = 1$: fixed



Conclusion

- In GR, HH no-boundary proposal fails to predict inflation
- dRGT gravity can successfully predict inflation after creation of the Universe
- Bigravity also predict successful inflation and is free from potential problems with dRGT gravity such as fixed, ad hoc fiducial metric & nonlinear ghosts in cosmological background
- Massless and massive GW dof ⇨ mixing (oscillation)!
- Other effects on the early universe?
- Any observational signatures?
- Can bigravity be embedded in higher dim gravity?

  eg, in brane world (Yamashita & Tanaka 2013)