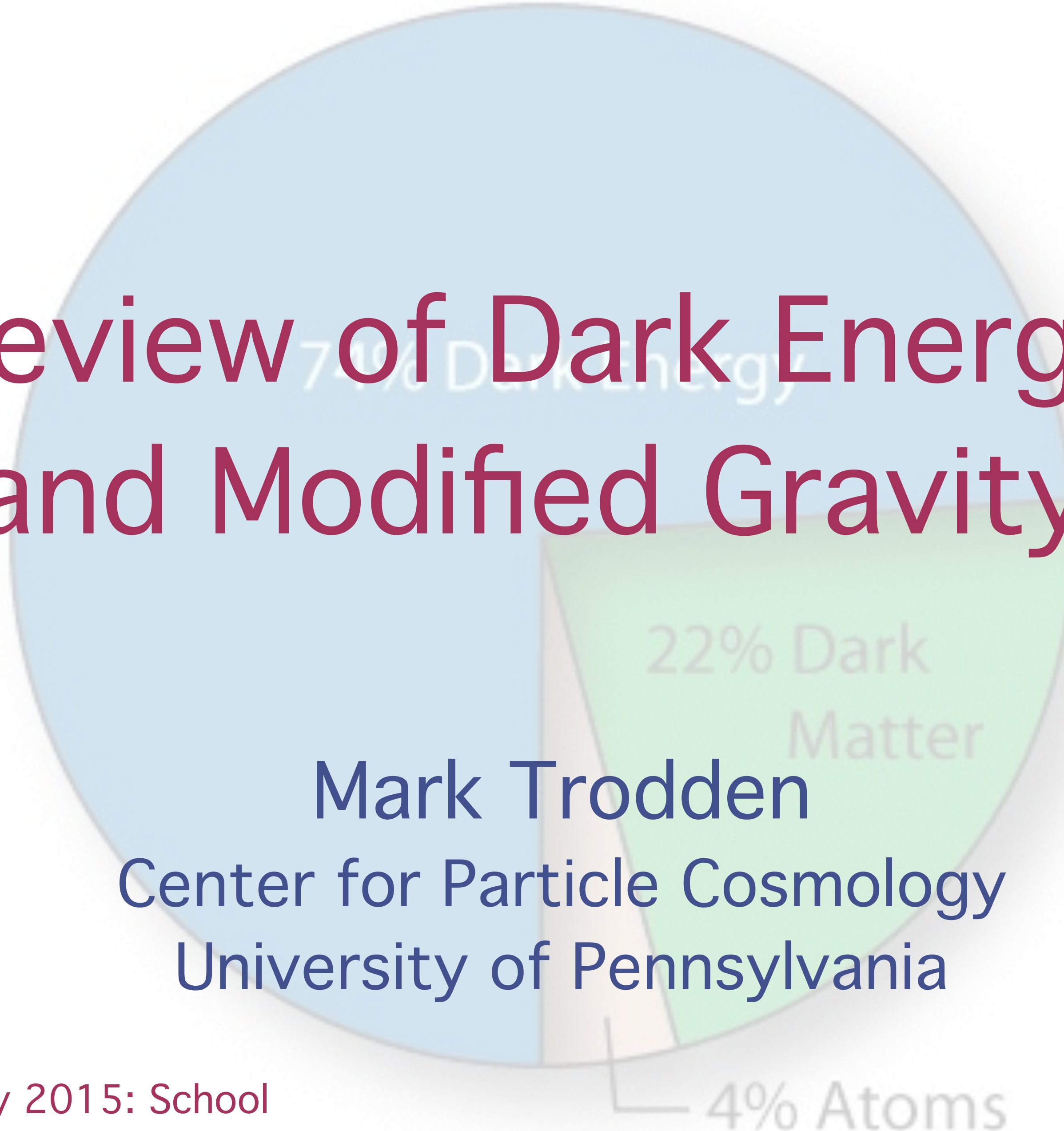
Review of Dark Energy and Modified Gravity



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Testing Gravity 2015: School
Simon Fraser University

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Overview I

Primary goals: (i) To provide familiarity with many concepts that will appear in the workshop; (ii) To argue for the primacy of theoretical consistency.

- Motivations, then a sketch of theoretical approaches
  - The cosmological constant
  - Dynamical dark energy
  - <u>Modified gravity</u>

- What does it mean to have a state-of-the-art model? Technical challenges: General discussion and EFT approach

- Screening mechanisms - part I, the chameleon mechanism

- Screening mechanisms - part II, the Vainshtein mechanism

Overview II

- More technical discussion of an interesting example.
  - Brane induced gravity, but mostly massive gravity
  - Their natural limit: galileons and their properties
  - General construction of interesting Galileon-like theories
  - Mathematical structure of Galileon-like theories.

- Prospects and tests.

Useful (hopefully) reference for a lot of what I'll say is an upcoming review,

Beyond the Cosmological Standard Model
Bhuvnesh Jain, Austin Joyce, Justin Khoury and MT

arXiv:1407.0059; to appear in Physics Reports (2015).

Simple Cosmology - a Reminder

Evolution of the universe governed by Einstein eqns

$$G_{\mu\nu}(g) = 8\pi G T_{\mu\nu}$$

Metric Matter

Use simple metric for cosmology and model matter as a perfect fluid with energy density ρ and pressure p

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 \propto \rho$$ The Friedmann equation

$$\frac{\ddot{a}}{a} \propto -(\rho + 3p)$$ The "acceleration" equation

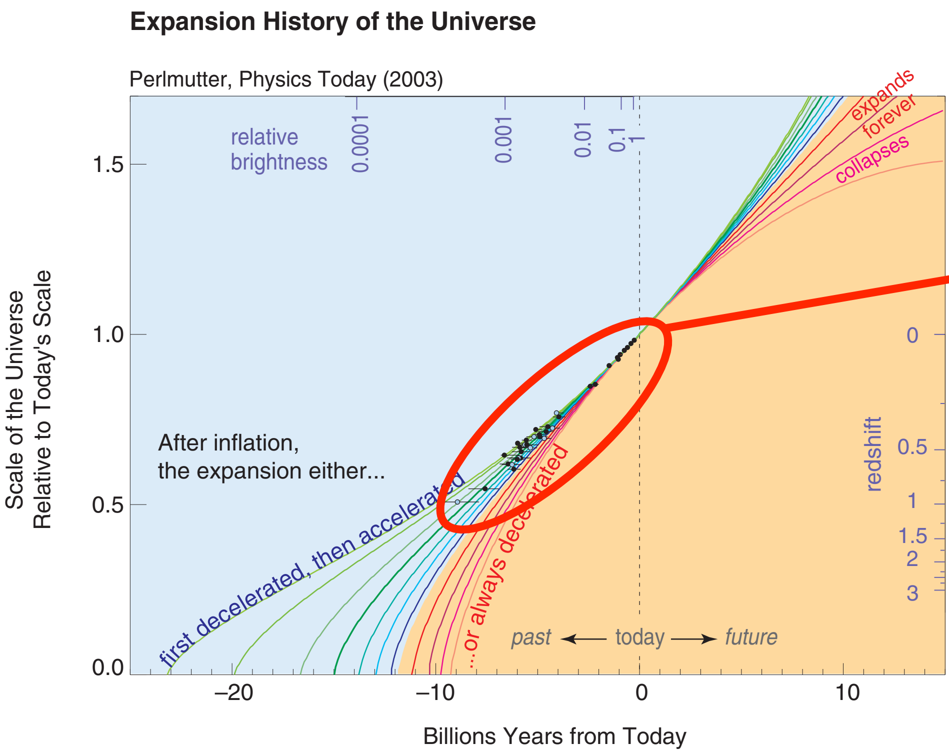
Parameterize different matter by equations of state: $p_i = w_i \rho_i$

When evolution dominated by type i, obtain

$$a(t) \propto t^{2/3(1+w_i)} \qquad \rho(a) \propto a^{-3(1+w_i)} \qquad (w_i \neq -1)$$

The Cosmic Expansion History

What does data tell us about the expansion rate?



We now know, partly through this data, that the universe is not only expanding ...

$$\dot{a} > 0$$

... but is accelerating!!

$$\ddot{a} > 0$$

<u>If</u> we trust GR and recall that $$\frac{\ddot{a}}{a} \propto -(\rho + 3p)$$

<u>Then</u> we infer that the universe must be dominated by some strange stuff with $p<-\rho/3$. We call this **dark energy!**

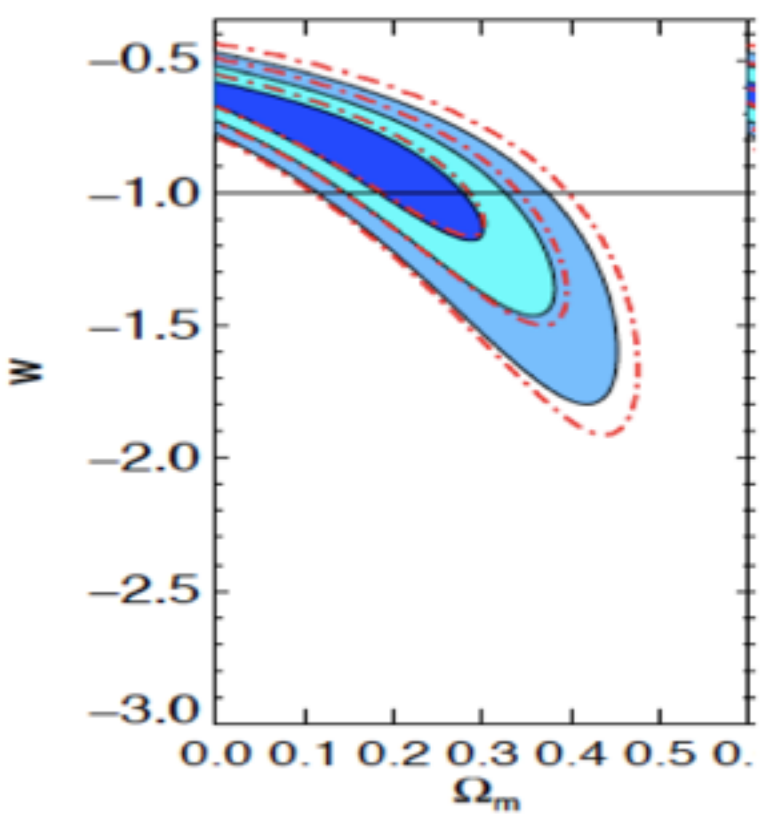
Cosmic Acceleration

$$\frac{\ddot{a}}{a} \propto -(\rho + 3p)$$

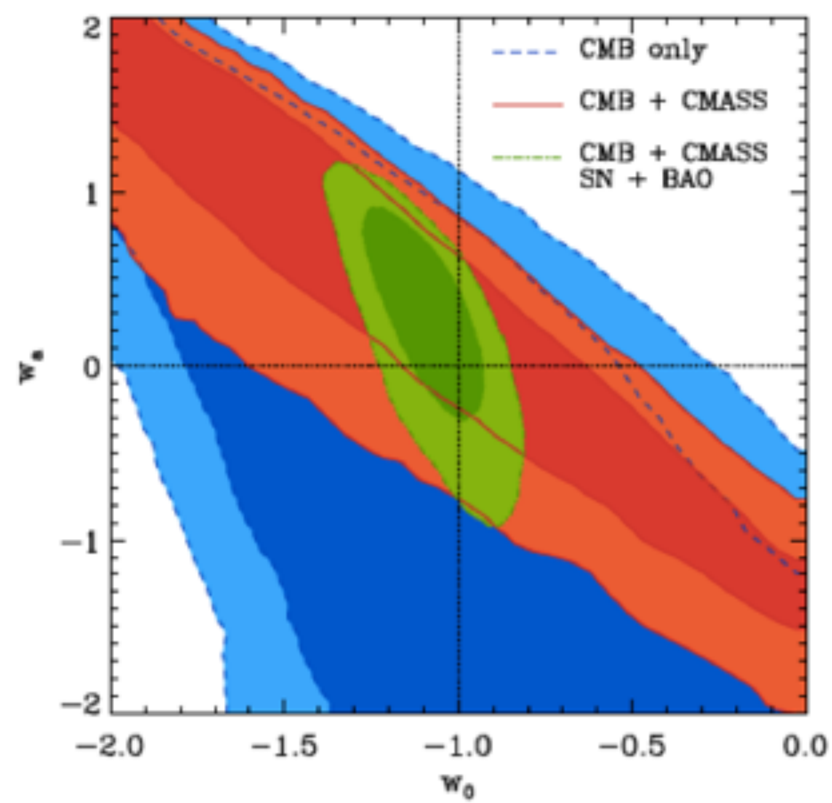
So, writing p=wρ, accelerating expansion means p<-ρ/3 or

$$\boxed{w<-1/3}$$

Conley et al. 2011



Sanchez et al. 2012



Cosmic Acceleration

$$\frac{\ddot{a}}{a} \propto -(\rho + 3p)$$

Three Broad Possibilities $a(t) \propto t^{2/3(1+w_i)}$ $\rho(a) \propto a^{-3(1+w_i)}$

	-1<w<-1/3	w=-1	w<-1
Evolution of Energy Density	Dilutes slower than any matter	Stays absolutely constant (Λ)	Increases with the expansion!!
Evolution of Scale Factor	Power-law quintessence	Exponential expansion	Infinite value in a finite time!!

The Cosmological Constant

Vacuum is full of virtual particles carrying energy. Equivalence principle (Lorentz-Invariance) gives

$$\langle T_{\mu\nu}\rangle \sim -\langle\rho\rangle g_{\mu\nu}$$

A constant vacuum energy! How big? Quick & dirty estimate of size only by modeling SM fields as collection of independent harmonic oscillators and then summing over zero-point energies.

$$\langle\rho\rangle \sim \int_0^{\Lambda_{\rm UV}} \frac{d^3k}{(2\pi)^3}\frac{1}{2}\hbar E_k \sim \int_0^{\Lambda_{\rm UV}} dk\; k^2\sqrt{k^2+m^2} \sim \Lambda_{\rm UV}^4$$

Most conservative estimate of cutoff: ~ 1 TeV. Gives

$$\Lambda_{\rm theory} \sim ({\rm TeV})^4 \sim\; 10^{-60}\; M_{\rm Pl}^4 << \Lambda_{\rm obs.} \sim M_{\rm Pl}^2 H_0^2 \sim 10^{-60}({\rm TeV})^4 \sim 10^{-120}\; M_{\rm Pl}^4$$

An enormous, and entirely unsolved problem in fundamental physics, made more pressing by the discovery of acceleration!

The Landscape & the Multiverse

At this stage, fair to say we are almost completely stuck! - No known dynamical mechanism, and a no-go theorem (Weinberg) to be overcome.

Anthropics provide a logical possibility to explain this, and the string landscape, with eternal inflation, may provide a way to realize it.

[Bousso, Freivogel, Leichenauer, ...;Vilenkin, Guth, Linde, Salem, ...]

An important step is understanding how to compute probabilities in such a spacetime

No currently accepted answer, but quite a bit of serious work going on.

Too early to know if can make sense of this.

If a dynamical understanding of a small CC is found, it would be hard to accept this.

[Image: SLIM FILMS. Looking for Life in the Multiverse, A. Jenkins & G. Perez, Scientific American, December 2009]

If DE is time or space dependent, would be hard to explain this way.

A Comment on Model Building

- Now >15 years after the discovery of cosmic acceleration.
- It was the Wild West at first, but now the bar for interesting ideas is pretty high - questions of theoretical consistency and observational viability are key.
- Without a formulation in which such questions can be asked, models are intrinsically less interesting.
- Phenomenological approaches at the level of background cosmology can be good starting points, but without further fundamental development can't be more than reparametrizations of the expansion history.

Dynamical Dark Energy

Once we allow dark energy to be dynamical, we are imagining that is is some kind of honest-to-goodness mass-energy component of the universe.

It isn't enough for a theorist to model matter as a perfect fluid with energy density ρ and pressure p (at least it shouldn't be enough at this stage!)

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu}$$

Our only known way of describing such things, at a fundamental level is through quantum field theory, with a Lagrangian. e.g.

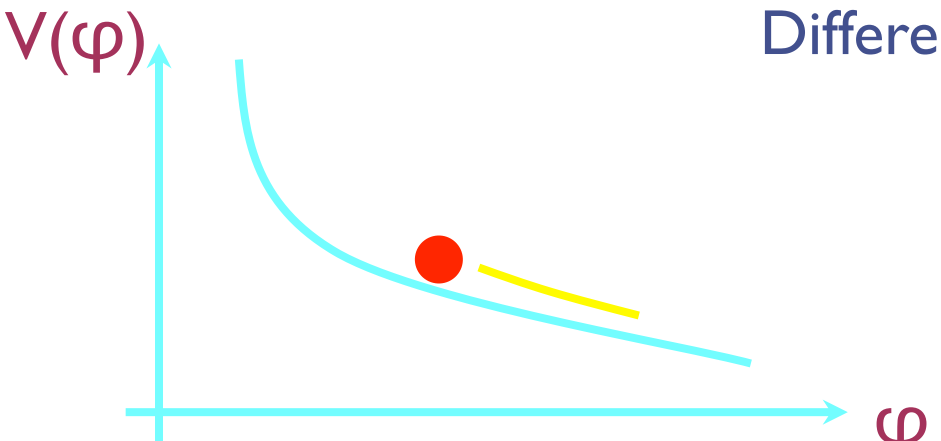
$$S_m = \int d^4x \, L_m[\phi, g_{\mu\nu}] \qquad L_m = \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi) \, \partial_\nu \phi - V(\phi)$$

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}} \qquad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Dynamical Dark Energy

Maybe there's some principle that sets vacuum energy to zero. Then dark energy might be like low-scale inflation today.

Use scalar fields to source Einstein's equation - *Quintessence*.



Difference: no minimum or reheating

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)\partial^\mu \phi - V(\phi)$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Small slope $\rho_\phi \approx V(\phi) \approx constant$

$$w = -\left[\frac{2V(\phi) - \dot{\phi}^2}{2V(\phi) + \dot{\phi}^2}\right]$$

Issues and Advantages

- Such an idea requires its own extreme fine tuning to keep the potential flat and mass scale ridiculously low - challenge of technical naturalness. Can be tackled if field respects an approximate global symmetry (e.g. a pseudo-Goldstone boson). Qw'll give more details soon.

  [Frieman, Hill, Stebbins, Waga]

- But then there are other fascinating constraints - e.g. such a field can have derivative couplings to the SM, and a slowly varying field leads to rotation of polarized radio light from distant galaxies

  [Carroll]

- On the other hand, some models, including those with exotic kinetic structure (k-essence), have the possibility of addressing the coincidence problem, and so there are advantages.

  [Armendariz-Picon, Mukhanov, Steinhardt; Caldwell. …]

- At present there are no compelling models.

Are we Being Fooled by Gravity?

(Carroll, De Felice & M.T., *Phys.Rev.* **D71:** 023525 (2005) [astro-ph/0408081])

We don't *really* measure w - we infer it from the Hubble plot via

$$w_{eff} = -\frac{1}{1-\Omega_m}\left(1+\frac{2}{3}\frac{\dot{H}}{H^2}\right)$$

Maybe, if gravity is modified, can infer value not directly related to energy sources (or perhaps without them!)

One example - Brans-Dicke theories

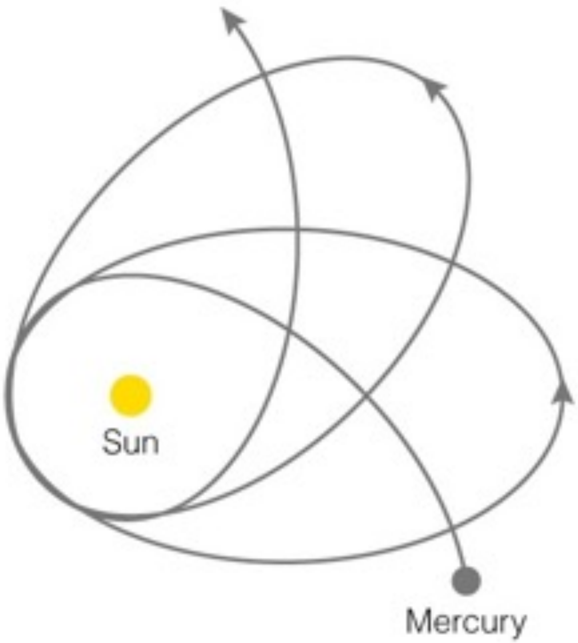
$$S_{BD} = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi}\left(\partial_\mu\phi\right)\partial^\mu\phi - 2V(\phi)\right] + \int d^4x\sqrt{-g}L_m(\psi_i, g)$$

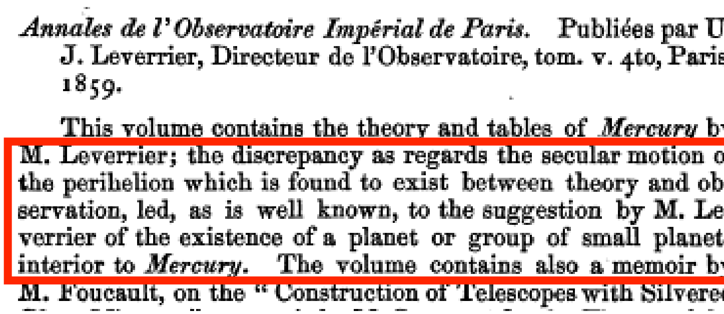
$\omega > 40000$ (Signal timing measurements from Cassini)

We showed that (with difficulty) can measure $w<-1$, even though no energy conditions are violated.

A Cautionary Tale

A related tale played out over 50 years over a century ago

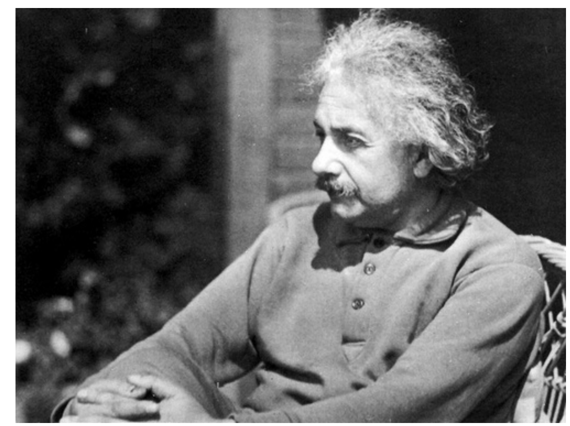




Annales de l'Observatoire Impérial de Paris. Publiées par U. J. Leverrier, Directeur de l'Observatoire, tom. v. 4to, Paris, 1859.

This volume contains the theory and tables of *Mercury* by M. Leverrier; the discrepancy as regards the secular motion of the perihelion which is found to exist between theory and observation, led, as is well known, to the suggestion by M. Leverrier of the existence of a planet or group of small planets interior to *Mercury*. The volume contains also a memoir by M. Foucault, on the "Construction of Telescopes with Silvered

"[General Relativity] explains ... quantitatively ... the secular rotation of the orbit of Mercury, discovered by Le Verrier, ... without the need of any special hypothesis.", **SPAW, Nov 18, 1915**



Could a similar story be unfolding today, with cosmic acceleration the canary in the mine, warning of the breakdown of gravity?*

*The EFT approach should make you wary of this line of argument!

Modifying Gravity

A crucial first question (for particle theorists) is: what degrees of freedom does the metric $g_{\mu\nu}$ contain in general?

(Decompose as irreducible repns. of the Poincaré group.)

$g_{\mu\nu}$

The graviton: a spin 2 particle

$h_{\mu\nu}$

We're familiar with this.

A_μ

A vector field: a spin 1 particle

Scalar fields: spin 0 particles

ϕ

These are less familiar.

Almost any other action will free some of them up

Propagating Degrees of Freedom

Which d.o.f.s propagate depends on the action.

In GR, the action is the Einstein-Hilbert action

$$S_{EH} = \frac{M_p^2}{2}\int d^4x\,\sqrt{-g}\,R$$

Its resulting equations of motion - the Einstein equations - contain constraints, similar to Gauss' law.

These pin the vector A_μ and scalar ϕ fields, making them non-dynamical, and leaving only the familiar graviton $h_{\mu\nu}$

Almost any other action will free up ϕ and/or A_μ, or more!

S_{EH}

Einstein-Hilbert
-gravitons

Modified gravity
-gravitons + new degrees of freedom

A common Language - EFT

In fact, whether dark energy or modified gravity, ultimately, around a background, it consists of a set of interacting fields in a Lagrangian. The Lagrangian contains 3 types of terms:

- Kinetic Terms: e.g.

$$\partial_\mu\phi\partial^\mu\phi \qquad F_{\mu\nu}F^{\mu\nu} \qquad i\bar{\psi}\gamma^\mu\partial_\mu\psi \qquad h_{\mu\nu}\mathcal{E}^{\mu\nu;\alpha\beta}h_{\alpha\beta} \qquad K(\partial_\mu\phi\partial^\mu\phi)$$

- Self Interactions (a potential)

$$V(\phi) \qquad m^2\phi^2 \qquad \lambda\phi^4 \qquad m\bar{\psi}\psi \qquad m^2h_{\mu\nu}h^{\mu\nu} \qquad m^2h^\mu{}_\mu h^\nu{}_\nu$$

- Interactions with other fields (such as matter, baryonic or dark)

$$\Phi\bar{\psi}\psi \qquad A^\mu A_\mu\Phi^\dagger\Phi \qquad e^{-\beta\phi/M_p}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \qquad (h^\mu{}_\mu)^2\phi^2 \qquad \frac{1}{M_p}\pi T^\mu{}_\mu$$

Depending on the background, such terms might have functions in front of them that depend on time and/or space.

Consistency I: Weak Coupling

When we write down a classical theory, described by one of our Lagrangians, we are usually implicitly assuming that the effects of higher order operators are small, and therefore mostly ignorable. This needs us to work below the strong coupling scale of the theory, so that quantum corrections, computed in perturbation theory, are small. We therefore need.

- The dimensionless quantities determining how higher order operators, with dimensionful couplings (irrelevant operators) affect the lower order physics be <<1 (or at least <1)

$$\frac{E}{\Lambda} << 1$$

(Energy << cutoff)

But be careful - this is tricky! Remember that our kinetic terms, couplings and potentials all can have background-dependent functions in front of them, and even if the original parameters are small, these may make them large - the ***strong coupling problem***! You can no longer trust the theory!

Consistency II: Technical Naturalness

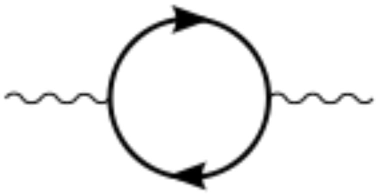
Even if your quantum mechanical corrections do not ruin your ability to trust your theory, any especially small couplings you need might be a problem.

- Suppose you need a very flat potential, or very small mass for some reason

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 \qquad m \sim H_0^{-1}$$

Then unless your theory has a special extra symmetry as you take m to zero, then quantum corrections will drive it up to the cutoff of your theory.

$$m_{\rm eff}^2 \sim m^2 + \Lambda^2$$



- Without this, requires extreme fine tuning to keep the potential flat and mass scale ridiculously low - ***challenge of technical naturalness.***

Consistency III: Ghost-Free

The Kinetic terms in the Lagrangian, around a given background, tell us, in a sense, whether the particles associated with the theory carry positive energy or not.

- Remember the Kinetic Terms: e.g.

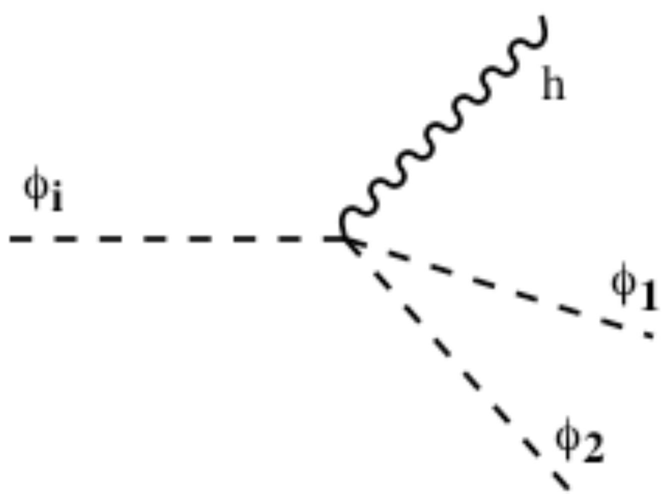
$$-\frac{f(\chi)}{2}K(\partial_\mu\partial^\mu\phi) \to F(t,x)\frac{1}{2}\dot{\phi}^2 - G(t,x)(\nabla\phi)^2$$

This sets the sign of the KE

- If the KE is negative then the theory has ***ghosts***! This can be catastrophic!

If we were to take these seriously, they'd have negative energy!!

- Ordinary particles could decay into heavier particles plus ghosts
- Vacuum could fragment



A Ghostly Example

The most obvious place this happens is when there are uncontrolled higher derivatives in the theory. A simple example illustrates this easily.

$$\mathcal{L} = -\frac{1}{2}(\partial\psi)^2 + \frac{1}{2\Lambda^2}(\Box\psi)^2 - V(\psi)$$

- Introduce an auxiliary field via

$$\mathcal{L} = -\frac{1}{2}(\partial\psi)^2 + \chi\Box\psi - \frac{\Lambda^2}{2}\chi^2 - V(\psi)$$ w/ EOM $$\chi = \frac{\Box\psi}{\Lambda^2}$$

(easy to check that substituting this back in yields original Lagrangian)

- Now make a field redefinition $\psi = \phi - \chi$ and integrate by parts in action

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \boxed{\frac{1}{2}(\partial\chi)^2} - \frac{\Lambda^2}{2}\chi^2 - V(\phi,\chi)$$

A ghost, with mass at the cutoff (so might be OK in full theory, but not always true)

This is why, within GR, almost all attempts to get a sensible model of w<-1 have failed. Many authors just ignore this fundamental problem.

Consistency IV - Superluminality …

Crucial ingredient of Lorentz-invariant QFT: *microcausality*. Commutator of 2 local operators vanishes for spacelike separated points as operator statement

$$[\mathcal{O}_1(x), \mathcal{O}_2(y)] = 0 \; ; \qquad \text{when} \qquad (x-y)^2 > 0$$

Turns out, even if have superluminality, under right circumstances can still have a well-behaved theory, as far as causality is concerned. e.g.

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda^3}\partial^2\phi(\partial\phi)^2 + \frac{1}{\Lambda^4}(\partial\phi)^4$$

- Expand about a background: $\phi = \bar{\phi} + \varphi$
- Causal structure set by effective metric

$$\mathcal{L} = -\frac{1}{2}G^{\mu\nu}(x, \bar{\phi}, \partial\bar{\phi}, \partial^2\bar{\phi}, \ldots)\partial_\mu\varphi\partial_\nu\varphi + \cdots$$

- If G globally hyperbolic, theory is perfectly causal, but *may* have directions in which perturbations propagate outside lightcone used to define theory.

But: there is still a worry here! …

… & Analyticity

Theory may not have a Lorentz-Invariant UV completion! Sometimes can see from 2 to 2 scattering amplitude - related to superluminality: can think of propagation in G as sequence of scattering processes with background field

- Focus on 4-point amplitude $\mathcal{A}(s,t)$ expressed as fn of Mandelstam variables.
- Won't provide details here, but can use analyticity properties of this, with a little complex analysis gymnastics, plus the optical theorem to show

$$\frac{\partial^2}{\partial s^2}\mathcal{A}(s,0)\Big|_{s=0} = \frac{4}{\pi}\int_{s_*}^{\infty} ds \frac{\mathrm{Im}\mathcal{A}(s,0)}{s^3} \geq 0$$

So, in forward limit, amplitude must have +ve s^2 part. True for *any* L-I theory described by an S-matrix. Violation implies violation of L-I in the theory.

- There exist other consistency relations. In general can conclude

$$\begin{tikzcd}
\cancel{S\text{-matrix analyticity}} \arrow[r] \arrow[dr, dashed, leftrightarrow] & \cancel{\text{Lorentz invariance}} \\
 & \text{Superluminality} \arrow[u]
\end{tikzcd}$$

May have to have a non-Wilsonian, non-LI UV completion of the theory. Might be very hard!!

A Toy Example (for Aficionados)

Consider a simple and benign-looking model, that is clearly LI

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{\alpha}{4\Lambda^4}(\partial\phi)^4$$

Can compute 2 to 2 scattering amplitude in field theory

$$\mathcal{A}_{2\to 2}(s,t) = \frac{\alpha}{2\Lambda^4}(s^2+t^2+u^2) = \frac{\alpha}{\Lambda^4}(s^2+t^2-st)$$

Take the forward limit t = 0:

$$\mathcal{A}_{2\to 2}(s,0) = \frac{\alpha}{\Lambda^4}s^2$$

So are not free to choose alpha<0 in a Lorentz-invariant theory with an analytic S-matrix. Note also that, in this theory alpha<0 is naively interesting because it exhibits screening. It *also* exhibits superluminality for that choice: Circumstantial evidence for connection between superluminality and analyticity - but not a proof.

The Need for Screening in the EFT

Consider general EFT of a scalar field conformally coupled to matter

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, \ldots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T^\mu_\mu$$

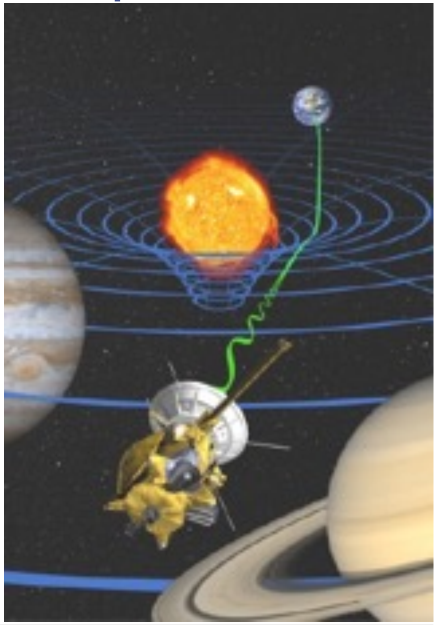
Specialize to a point source $T^\mu_\mu \to -\mathcal{M}\delta^3(\vec{x})$ and expand $\phi = \bar{\phi} + \varphi$

$$Z(\bar{\phi})\left(\ddot{\varphi} - c_s^2(\bar{\phi})\nabla^2\varphi\right) + m^2(\bar{\phi})\varphi = g(\bar{\phi})\mathcal{M}\delta^3(\vec{x})$$

Expect background value set by other quantities; e.g. density or Newtonian potential. Neglecting spatial variation over scales of interest, static potential is

$$V(r) = -\frac{g^2(\bar{\phi})}{Z(\bar{\phi})c_s^2(\bar{\phi})} \frac{e^{-\frac{m(\bar{\phi})}{\sqrt{Z(\bar{\phi})}c_s(\bar{\phi})}r}}{4\pi r}\mathcal{M}$$

So, for light scalar, parameters O(1), have gravitational-strength long range force, ruled out by local tests of GR! If we want workable model need to make this sufficiently weak in local environment, while allowing for significant deviations from GR on cosmological scales!



Screening

So a general theme here, in both quintessence and modified gravity is the need for new degrees of freedom, coupled to matter with gravitational strength, and hence extremely dangerous in the light of local tests of gravity.

- Successful models exhibit "screening mechanisms". Dynamics of the new degrees of freedom are rendered irrelevant at short distances and only become free at large distances (or in regions of low density).
- There exist several versions, depending on parts of the Lagrangian used
  - Vainshtein: Uses the kinetic terms to make coupling to matter weaker than gravity around massive sources.
  - Chameleon: Uses coupling to matter to give scalar large mass in regions of high density
  - Symmetron: Uses coupling to give scalar small VEV in regions of low density, lowering coupling to matter
- In each case should "resum" theory about the relevant background, and EFT of excitations around a nontrivial background is not the naive one.
- Around the new background, theory is safe from local tests of gravity.

Eg. The Chameleon Mechanism

Consider the following action:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_{\rm matter}\left[A^2(\phi)g_{\mu\nu},\psi\right]$$

Acceleration of test particle influenced by scalar field via

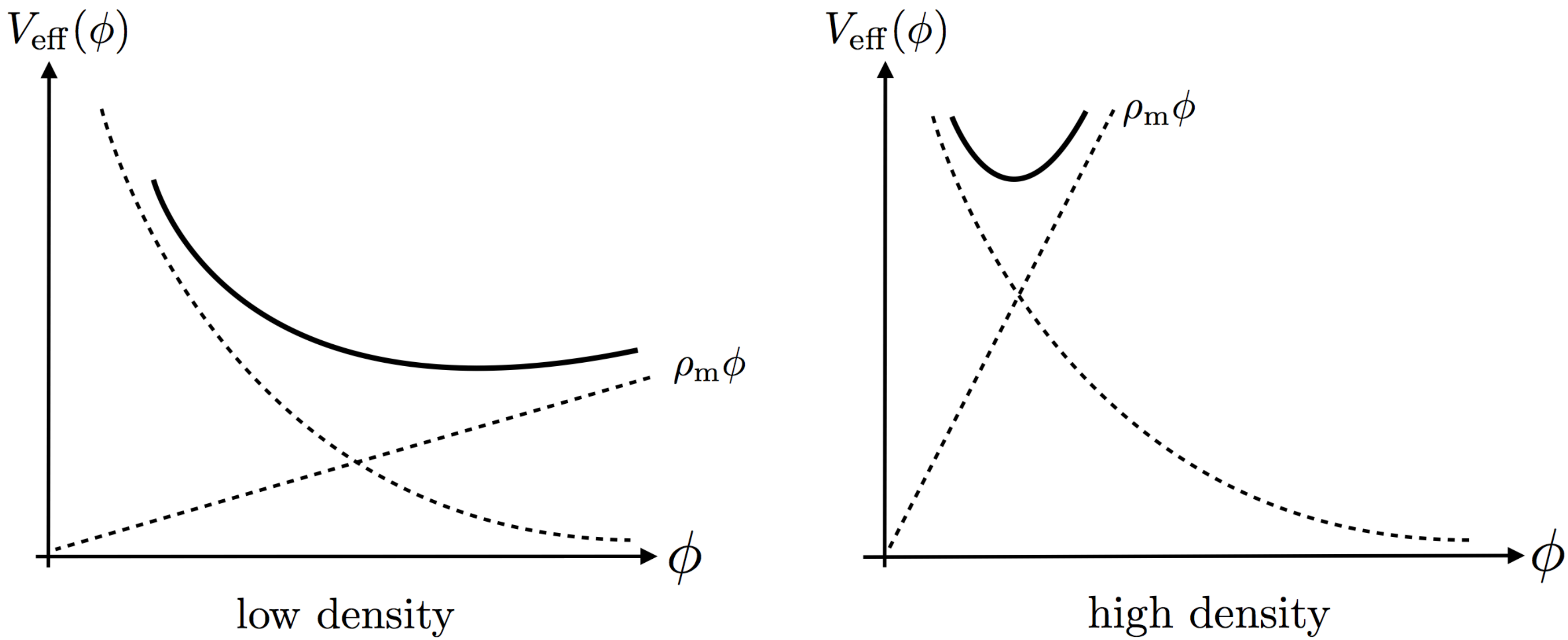
$$\vec{a} = -\vec{\nabla}\Phi - \frac{{\rm d}\ln A(\phi)}{{\rm d}\phi}\vec{\nabla}\phi = -\vec{\nabla}\Big(\Phi + \ln A(\phi)\Big)$$

Can choose V and A so that scalar propagates freely and mediates fifth force in regions of low Newtonian potential, but force shuts off in high density regions.

Equation of motion

$$\Box\phi = V_{\rm eff\,,\phi}(\phi) \qquad \text{where} \qquad V_{\rm eff}(\phi) = V(\phi) + A(\phi)\rho$$

Matter density appears in effective potential allows suppression of force due to the scalar field is hidden in regions of high density.



General limitation of chameleon (& symmetron) - and any mechanism with screening condition set by local Newtonian potential: range of scalar-mediated force on cosmological scales is bounded. So have negligible effect on linear scales today, and so deviation from LCDM is negligible. Remain very interesting as way to hide light scalars suggested by e.g. string theory. But won't discuss too much more here, except for an example later.

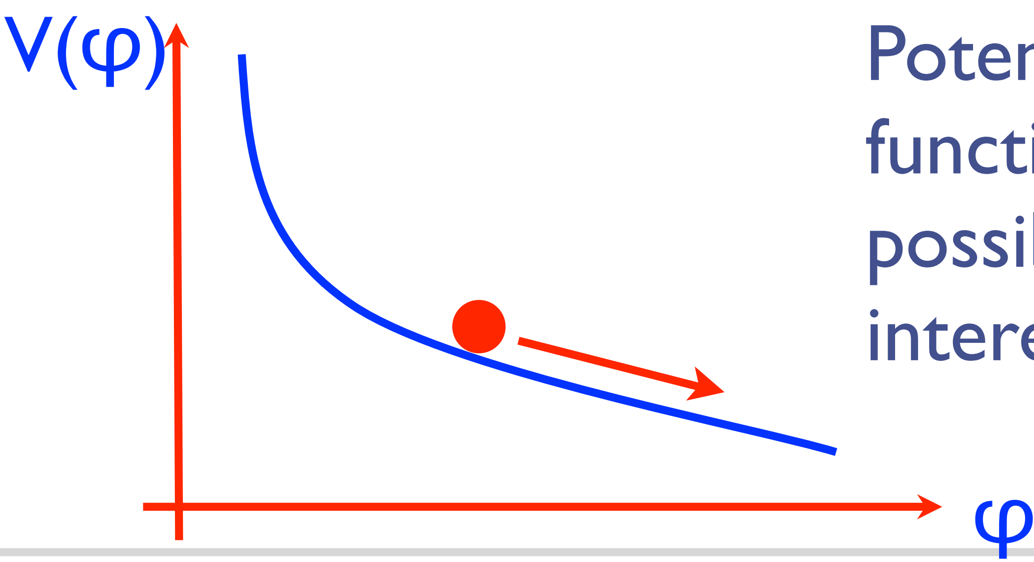
Example Model: f(R) Gravity

(Carroll, Duvvuri, M.T. & Turner, *Phys.Rev.* **D70:** 043528 (2004) [astro-ph/0306438])

Can modify the Einstein-Hilbert action

$$S = \frac{M_P^2}{2}\int d^4x\,\sqrt{-g}\,f(R) + \int d^4x\,\sqrt{-g}L_m$$

This frees up precisely one of those new degrees of freedom we talked about ϕ



Potential determined by the function f(R). Opens up the possibility of cosmologically interesting evolution.

A Fascinating Possibility

There exists an intriguing class of actions that yield late-time cosmic acceleration!

In fact: 4th order nature provides enough freedom to reproduce any cosmological evolution by appropriate choice of function f(R)

We fix the expansion history

$$\frac{H^2}{H_0^2} = \Omega_m a^{-3} + \Omega_r a^{-4} + \frac{\rho_{eff}}{\rho_c}$$

and solve the Friedmann eq. as a second order differential equation for f(R)

Thus, we can find a family of f(R) for each expansion history!

BUT: Disastrous disagreement with solar system constraints unless chameleon mechanism works - strongly restricts models!

Facing the (Solar System) Data

Easy to see model has problems agreeing with GR on scales smaller than cosmology. Can map theory to

$$S_{BD} = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi}\left(\partial_\mu\phi\right)\partial^\mu\phi - 2V\left(\phi\right)\right] + \int d^4x\sqrt{-g}L_m\left(\psi_i, g\right)$$

i.e., a Brans-Dicke theory, with a potential that we may ignore, with $\omega=0$

But, solar system measurements constrain $\omega>40000$

More complicated versions barely survive, as we'll see, constraints are strict.

Challenges of f(R) Models

But there are other considerations. We need:

$f_{RR} > 0$ to have a stable high-curvature regime, to have a non-tachyonic scalar field

$1 + f_R > 0$ to have a positive effective Newton constant

$f_R < 0$ negative, monotonically increasing function of R that asymptotes to zero from below

$|f_R^0| \leq 10^{-6}$ must be small at recent epochs to pass LGC

(Hu and Sawicki Phys. Rev. D76 064004 (2007))

$$w_{eff} \simeq -1$$

More General Actions

(Carroll, De Felice, Duvvuri, Easson, M.T. & Turner, *Phys.Rev.* **D71:** 063513 (2005) [arXiv:astro-ph/0410031])

$$S = \frac{M_P^2}{2}\int d^4x\,\sqrt{-g}\,f(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, \ldots) + \int d^4x\,\sqrt{-g}L_m$$

Unlike f(R) models, it turn out to be relatively easy to satisfy solar system constraints in models like this - one can use the Riemann tensor, which doesn't essentially vanish in the solar system, whereas R does.

However

While f(R) models are ghost free, except for some very special examples, at least one d.o.f. freed up by these actions is a ghost!

(e.g. De Felice, Hindmarsh and M.T., JCAP 0608:005, (2006) [astro-ph/0604154])

This is bad!

Truly Modifying Gravity

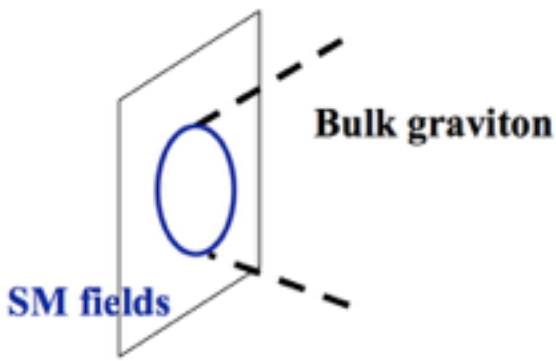
It would be very interesting to directly modify the dynamics of the graviton itself. This might help the cosmic acceleration question in two ways

- May exist new self-accelerating solution
- May be able to "degravitate" cosmological constant

$$8\pi G G_{\mu\nu} = T_{\mu\nu} \longrightarrow 8\pi G(\Box) G_{\mu\nu} = T_{\mu\nu}$$

Long-wavelength modes (CC?) do not gravitate.

[Dvali, Hofmann & Khoury]



Old example: DGP model. - Can get some degravitation, and some acceleration. But comes with some problems

[Dvali, Gabadadze & Porrati]

$$S = \frac{M_5^3}{2r_c}\int d^5x\sqrt{-G}\,R^{(5)} + \frac{M_4^2}{2}\int d^4x\sqrt{-g}\,R$$

Massive gravity

Very recent concrete suggestion - consider massive gravity

- Fierz and Pauli showed how to write down a linearized version of this, but...

$$\propto m^2(h^2 - h_{\mu\nu}h^{\mu\nu})$$

- ... thought all nonlinear completions exhibited the "Boulware-Deser ghost".

Within last two years a counterexample has been found. This is a very new, and potentially exciting development!

[de Rham, Gabadadze, Tolley (2011]

$$\mathcal{L} = M_P^2\sqrt{-g}(R + 2m^2\mathcal{U}(g,f)) + \mathcal{L}_m$$

Now proven to be ghost free, and investigations of the resulting cosmology - acceleration, degravitation, ... are underway, both as a gravity theory and as ...

[Hassan & Rosen(2011]

... Galileons

In a limit yields novel and fascinating 4d EFT that many of us have been studying. Symmetry: $\pi(x) \to \pi(x) + c + b_\mu x^\mu$

Relevant field referred to as the *Galileon*

(Nicolis, Rattazzi, & Trincherini 2009)

$$\mathcal{L}_1 = \pi \qquad \mathcal{L}_2 = (\partial\pi)^2 \qquad \mathcal{L}_3 = (\partial\pi)^2\Box\pi$$

$$\mathcal{L}_{n+1} = n\eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n}\left(\partial_{\mu_1}\pi\partial_{\nu_1}\pi\partial_{\mu_2}\partial_{\nu_2}\pi\cdots\partial_{\mu_n}\partial_{\nu_n}\pi\right)$$

There is a separation of scales

- Allows for classical field configurations with order one nonlinearities, but quantum effects under control.
- So can study non-linear classical solutions.
- Some of these very important (Vainshtein screening)

Amazingly terms of galilean form are nonrenormalized. *Possibly* useful for particle physics & cosmology. We'll see.

[Luty, Porrati, Ratazzi (2003); Nicolis, Rattazzi (2004); Hinterbichler, MT, Wesley (2012)]

More Specifics on Galileons

The Galilean terms take the form

$$\mathcal{L}_{n+1} = n\eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n}\left(\partial_{\mu_1}\pi\partial_{\nu_1}\pi\partial_{\mu_2}\partial_{\nu_2}\pi\cdots\partial_{\mu_n}\partial_{\nu_n}\pi\right)$$

$$\eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n} \equiv \frac{1}{n!}\sum_p (-1)^p\, \eta^{\mu_1 p(\nu_1)}\eta^{\mu_2 p(\nu_2)}\ldots\eta^{\mu_n p(\nu_n)}$$

- tensor is anti-symmetric in μ indices,
- anti-symmetric in ν indices, and
- symmetric under interchange of any μ, ν pair with any other

- Only first n of galileons terms non-trivial in n-dimensions.
- In addition, the tadpole term, π, is galilean invariant - include as the first-order galileon.

Interesting Mathematical Aside

The single field Galileon constitutes an example of what is known to mathematicians as an *Euler Hierarchy* [Thanks to David Fairlie]

Suppose have Lagrangian only depending on derivative:

$$L_1 = L_1(\dot{\phi}) \longrightarrow \mathcal{E}_1 = 0$$

$$L_2 = L_1 \mathcal{E}_1 \longrightarrow \mathcal{E}_2 = 0$$

$$L_3 = L_2 \mathcal{E}_2 \longrightarrow \mathcal{E}_3 = 0$$

$$\vdots$$

$$L_n = L_{n-1} \mathcal{E}_{n-1}$$ (total derivative)

Second order equations of motion, and series eventually terminates, as the Galileon one does

E.g. The Vainshtein Effect

Consider, for example, the DGP cubic term, coupled to matter

$$\mathcal{L} = -3(\partial\pi)^2 - \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{M_{Pl}}\pi T$$

Now look at spherical solutions around a point mass

$$\pi(r) = \begin{cases} \sim \Lambda^3 R_V^{3/2}\sqrt{r} + const. & r \ll R_V \\ \sim \Lambda^3 R_V^3 \frac{1}{r} & r \gg R_V \end{cases} \qquad R_V \equiv \frac{1}{\Lambda}\left(\frac{M}{M_{Pl}}\right)^{1/3}$$

Looking at a test particle, strength of this force, compared to gravity, is then

$$\frac{F_\pi}{F_{\rm Newton}} = \frac{\pi'(r)/M_{Pl}}{M/(M_{Pl}^2 r^2)} = \begin{cases} \sim \left(\frac{r}{R_V}\right)^{3/2} & R \ll R_V \\ \sim 1 & R \gg R_V \end{cases}$$

So forces much smaller than gravitational strength within the Vainshtein radius - hence safe from 5th force tests.

The Vainshtein Effect

Suppose we want to know the the field that a source generates within the Vainshtein radius of some large body (like the sun, or earth)

Perturbing the field and the source

$$\pi = \pi_0 + \varphi, \quad T = T_0 + \delta T,$$

yields

$$\mathcal{L} = -3(\partial\varphi)^2 + \frac{2}{\Lambda^3}\underbrace{\left(\partial_\mu\partial_\nu\pi_0 - \eta_{\mu\nu}\Box\pi_0\right)}_{\sim\left(\frac{R_v}{r}\right)^{3/2}}\partial^\mu\varphi\partial^\nu\varphi - \frac{1}{\Lambda^3}(\partial\varphi)^2\Box\varphi + \frac{1}{M_4}\varphi\delta T$$

Thus, if we canonically normalize the kinetic term of the perturbations, we raise the effective strong coupling scale, and, more importantly, heavily suppress the coupling to matter!

Regimes of Validity

The usual quantum regime of a theory

$$r \ll \frac{1}{\Lambda}$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3/2} \gg 1$$

$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \gg 1$$

$$\frac{1}{\Lambda} \ll r \ll R_V$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3/2} \gg 1$$

$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \ll 1$$

The usual linear, classical regime of a theory

$$r \gg R_V$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3} \ll 1$$

$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \ll 1$$

r

$$r \sim \frac{1}{\Lambda}$$

$$r \sim R_V$$

A new classical regime, with order one nonlinearities

The Meaning of the Vainshtein Radius

Fix parameters to make solutions cosmologically interesting

$$r^* = \left(\frac{2GM}{c^2} r_c^2\right)^{1/3}$$

~0.1 kpc = 10^7 AU

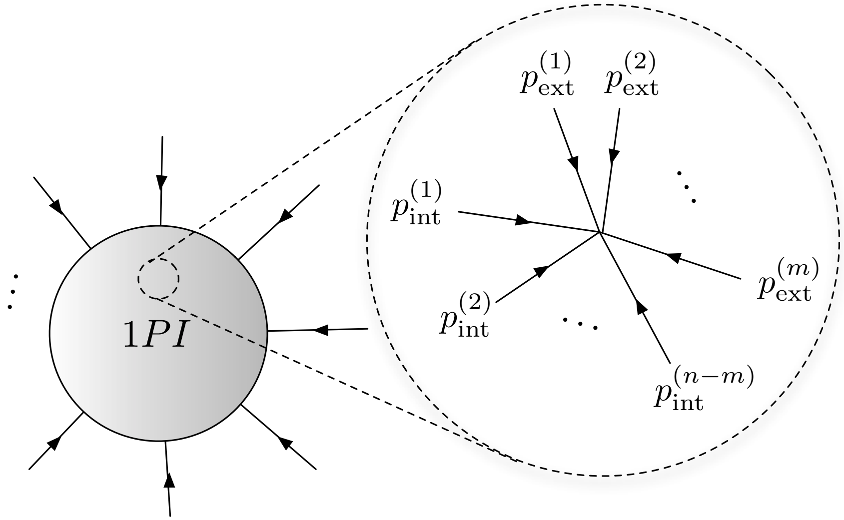
~Mpc ~ 30 galactic radii

~10 Mpc ~ 10 virial radii

Nonrenormalization!

Remarkable fact about these theories (c.f SUSY theories)

Expand quantum effective action for the classical field about expectation value



The n-point contribution contains at least 2n powers of external momenta: cannot renormalize Galilean term with only 2n-2 derivatives.
With or without the SO(N), can show, just by computing Feynman diagrams, that at all loops in perturbation theory, for any number of fields, terms of the galilean form cannot receive new contributions.

[K. Hinterbichler, M.T., D.Wesley, *Phys. Rev.* D82 (2010) 124018]

Can even add a mass term and remains technically natural

Constructing Galileons through SSB

- For those of you who are more mathematically inclined, there is a nice story here that may have implications for, among other things, better understanding the nonrenormalization theorems.

- Since the galilean symmetry in nonlinearly realized, can use the coset construction to build the effective theory. (We've recently shown that one can do this for massive gravity also!)

- Galileons are Wess-Zumino terms! In d dimensions are d-form potentials for (d+1)-forms which are non-trivial co-cycles in Lie algebra cohomology of full symmetry group relative to unbroken one. Slightly different stories for DBI and conformal Galileons.

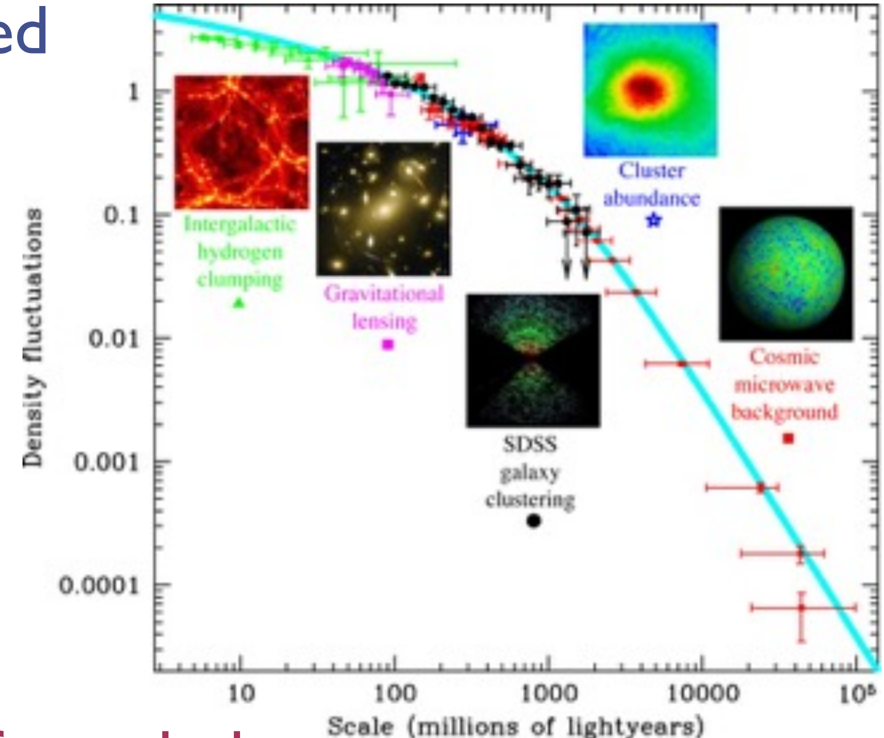
[Goon, Hinterbichler, Joyce & M.T., arxiv:1203.3191 [hep-th])

Prospects - it’s not all w!!

(Relatively) easy to get the observed expansion history from many different models - so how to test?

- Gravity is behind the expansion history of the universe

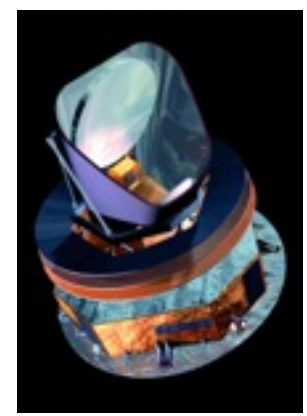
- But it is also behind how matter clumps up - potentially different.



- This could help distinguish a CC from dark energy from modified gravity

DARK ENERGY SURVEY

Large Synoptic Survey Telescope



You’re going to here a lot more detail about this in the coming days!

Summary

- Questions thrown up by the data need to find a home in fundamental physics, and theorists are hard at work on this. Requires particle physicists and cosmologists to work together.
- Many attractive ideas (as well as a lot of ugly ones) are being ruled out or tightly constrained by precision measurements in cosmology, in astrophysics, in particle physics and in the lab! This is extremely exciting!
- Serious models only need apply - theoretical consistency is a crucial question. We need (i) models in which right questions can be *asked* and (ii) A thorough investigation of the answers.
- Here mostly covered general approach to technical questions, and focused on specific example of Vainshtein screening.
- There are some terrific problems for students in this game!

Thank You!