The State of Theory

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Overview

- Motivations - background, and the problem of cosmic acceleration

- Why consider Modified gravity?

- What are the **theoretical issues** facing any such approach? Screening mechanisms - focusing on the Vainshtein mechanism.

- How to Construct Models - An example: Galileons - origins and novel features

An alternative title: "What Are Theorists Thinking/Worrying About Today?"
This is a story in progress - no complete answers yet.
Useful (hopefully) reference for a lot of what I'll say is

Beyond the Cosmological Standard Model
Bhuvnesh Jain, Austin Joyce, Justin Khoury and MT

arXiv:1407.0059; to appear in *Physics Reports (2015)*.

Modifying Gravity (Motivated by Acceleration)

Maybe cosmic acceleration is due to corrections to GR!

One thing to understand is: what degrees of freedom does the metric $g_{\mu\nu}$ contain in general?

$g_{\mu\nu}$

The graviton:
a spin 2 particle

Scalar fields:
spin 0 particles

A_μ

$h_{\mu\nu}$

A vector field:
a spin 1 particle

ϕ

We're familiar with this.

These are less familiar.

GR pins vector A_μ and scalar ϕ fields, making non-dynamical, and leaving only familiar graviton $h_{\mu\nu}$

Almost any other action will free some of them up

e.g., f(r) models
[Carroll, Duvvuri, M.T. & Turner, (2003)]

More interesting things also possible - massive gravity - see later

A common Language - EFT

How do theorists think about all this? In fact, whether dark energy or modified gravity, ultimately, around a background, a theory consists of a set of interacting fields in a Lagrangian. The Lagrangian contains 3 types of terms:

- Kinetic Terms: e.g.

$$\partial_\mu\phi\partial^\mu\phi \quad F_{\mu\nu}F^{\mu\nu} \quad i\bar{\psi}\gamma^\mu\partial_\mu\psi \quad h_{\mu\nu}\mathcal{E}^{\mu\nu;\alpha\beta}h_{\alpha\beta} \quad K(\partial_\mu\phi\partial^\mu\phi)$$

- Self Interactions (a potential)

$$V(\phi) \quad m^2\phi^2 \quad \lambda\phi^4 \quad m\bar{\psi}\psi \quad m^2 h_{\mu\nu}h^{\mu\nu} \quad m^2 h^\mu{}_\mu h^\nu{}_\nu$$

- Interactions with other fields (such as matter, baryonic or dark)

$$\Phi\bar{\psi}\psi \quad A^\mu A_\mu\Phi^\dagger\Phi \quad e^{-\beta\phi/M_p}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \quad (h^\mu{}_\mu)^2\phi^2 \quad \frac{1}{M_p}\pi T^\mu{}_\mu$$

Depending on the background, such terms might have functions in front of them that depend on time and/or space.

Many of the concerns of theorists can be expressed in this language

Consistency e.g. I: Weak Coupling

When we write down a classical theory, described by one of our Lagrangians, we are usually implicitly assuming that the effects of higher order operators are small, and therefore mostly ignorable. This needs us to work below the strong coupling scale of the theory, so that quantum corrections, computed in perturbation theory, are small. We therefore need.

- The dimensionless quantities determining how higher order operators, with dimensionful couplings (irrelevant operators) affect the lower order physics be $<<1$ (or at least <1)

$$\frac{E}{\Lambda} << 1 \qquad \text{(Energy << cutoff)}$$

But be careful - this is tricky! Remember that our kinetic terms, couplings and potentials all can have background-dependent functions in front of them, and even if the original parameters are small, these may make them large - the ***strong coupling problem***! You can no longer trust the theory!

Consistency e.g. II: Technical Naturalness

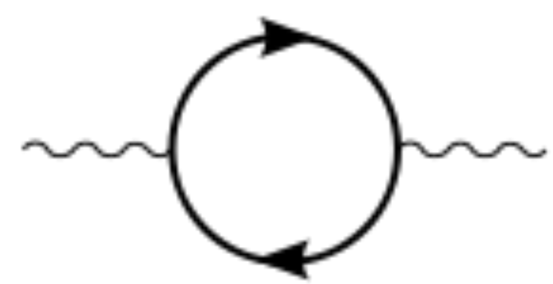
Even if your quantum mechanical corrections do not ruin your ability to trust your theory, any especially small couplings you need might be a problem.

- Suppose you need a very flat potential, or very small mass for some reason

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 \qquad m \sim H_0^{-1}$$

Then unless your theory has a special extra symmetry as you take m to zero, then quantum corrections will drive it up to the cutoff of your theory.

$$m_{\text{eff}}^2 \sim m^2 + \Lambda^2$$



- Without this, requires extreme fine tuning to keep the potential flat and mass scale ridiculously low - *challenge of technical naturalness.*

Consistency e.g. III: Ghost-Free

The Kinetic terms in the Lagrangian, around a given background, tell us, in a sense, whether the particles associated with the theory carry positive energy or not.

- Remember the Kinetic Terms: e.g.

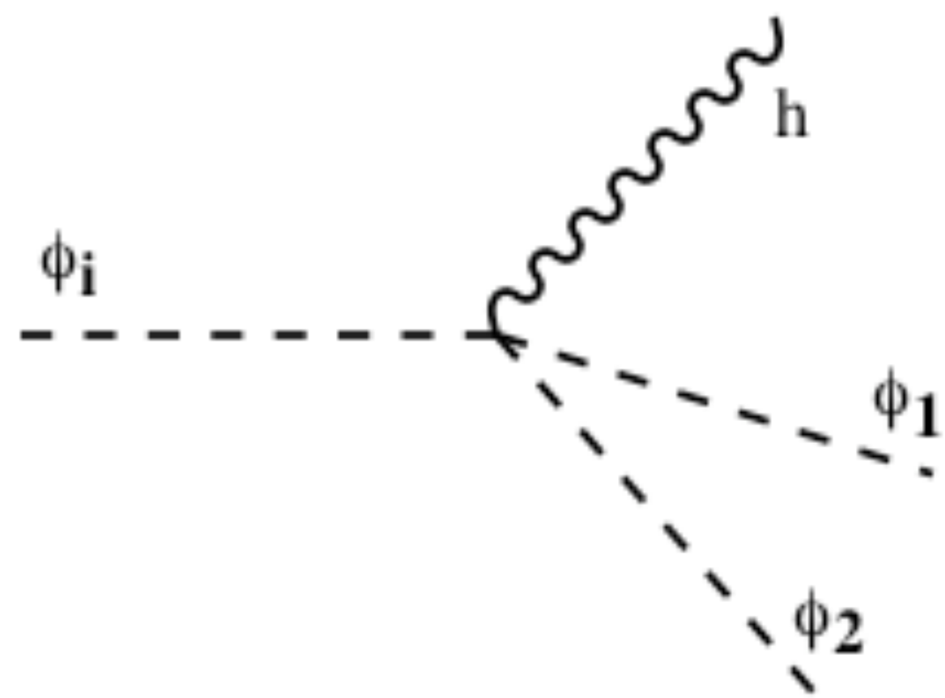
$$-\frac{f(\chi)}{2}K(\partial_\mu\partial^\mu\phi) \rightarrow F(t,x)\frac{1}{2}\dot{\phi}^2 - G(t,x)(\nabla\phi)^2$$

This sets the sign of the KE

- If the KE is negative then the theory has ***ghosts***! This can be catastrophic!

If we were to take these seriously, they'd have negative energy!!

- Ordinary particles could decay into heavier particles plus ghosts
- Vacuum could fragment



(Carroll, Hoffman & M.T.,(2003); Cline, Jeon & Moore. (2004))

Consistency IV, V, … - Superluminality, Analyticity, …

See me afterwards -
it's exhausting!

The Need for Screening in the EFT

Look at the general EFT of a scalar field conformally coupled to matter

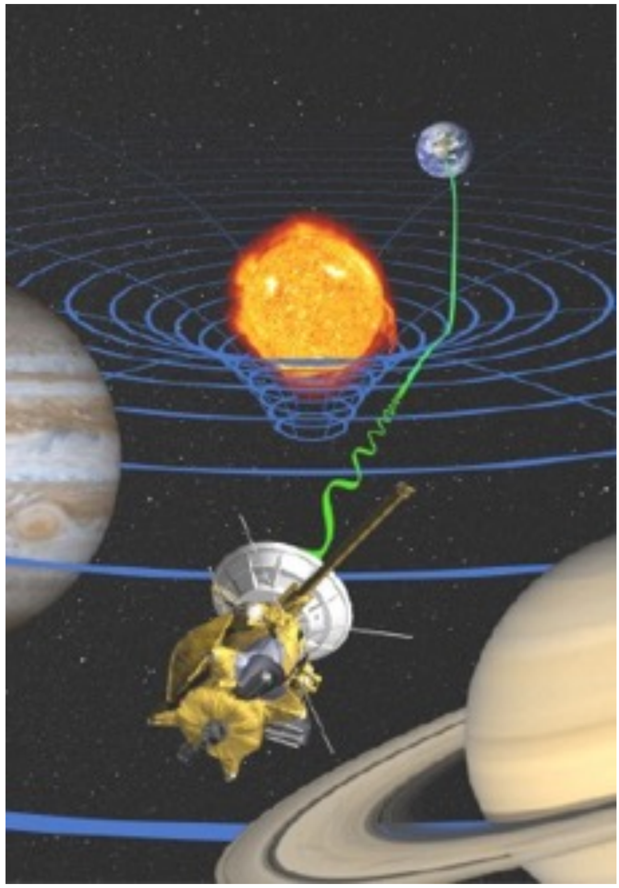
$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \ldots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T^\mu_\mu$$

Specialize to a point source $T^\mu_\mu \to -\mathcal{M}\delta^3(\vec{x})$ and expand $\phi = \bar{\phi} + \varphi$

$$Z(\bar{\phi})\left(\ddot{\varphi} - c_s^2(\bar{\phi})\nabla^2\varphi\right) + m^2(\bar{\phi})\varphi = g(\bar{\phi})\mathcal{M}\delta^3(\vec{x})$$

Expect background value set by other quantities; e.g. density or Newtonian potential. Neglecting spatial variation over scales of interest, static potential is

$$V(r) = -\frac{g^2(\bar{\phi})}{Z(\bar{\phi})c_s^2(\bar{\phi})}\frac{e^{-\frac{m(\bar{\phi})}{\sqrt{Z(\bar{\phi})}c_s(\bar{\phi})}r}}{4\pi r}\mathcal{M}$$



So, for light scalar, parameters O(1), have gravitational-strength long range force, ruled out by local tests of GR! If we want workable model need to make this sufficiently weak in local environment, while allowing for significant deviations from GR on cosmological scales!

Screening

So a general theme here, in both dark energy and modified gravity is the need for new degrees of freedom, coupled to matter with gravitational strength, and hence extremely dangerous in the light of local tests of gravity.

- Successful models exhibit "screening mechanisms". Dynamics of the new degrees of freedom are rendered irrelevant at short distances and only become free at large distances (or in regions of low density).
- There exist several versions, depending on parts of the Lagrangian used
  - Vainshtein: Uses the kinetic terms to make coupling to matter weaker than gravity around massive sources.
  - Chameleon: Uses coupling to matter to give scalar large mass in regions of high density
  - Symmetron: Uses coupling to give scalar small VEV in regions of low density, lowering coupling to matter
- In each case should "resum" theory about the relevant background, and EFT of excitations around a nontrivial background is not the naive one.
- Around the new background, theory is safe from local tests of gravity.

e.g. Massive gravity

Very recent concrete suggestion - consider massive gravity

- Fierz and Pauli showed how to write down a linearized version of this, but...

$$\propto m^2(h^2 - h_{\mu\nu}h^{\mu\nu})$$

- ... thought all nonlinear completions exhibited the "Boulware-Deser ghost".

Within last few years a counterexample has been found. This is a very new, and potentially exciting development!

[de Rham, Gabadadze, Tolley (2011]

$$\mathcal{L} = M_P^2\sqrt{-g}(R + 2m^2\mathcal{U}(g,f)) + \mathcal{L}_m$$

Proven to be ghost free, and investigations of the resulting cosmology - acceleration, degravitation, ... are underway, both in the full theory and in its decoupling limit - galileons!

[Hassan & Rosen(2011)]

Focus on Galileons

In a limit yields novel and fascinating 4d EFT that many of us have been studying. Symmetry: $\pi(x) \to \pi(x) + c + b_\mu x^\mu$

Relevant field referred to as the *Galileon*

(Nicolis, Rattazzi, & Trincherini 2009)

$$\mathcal{L}_1 = \pi \qquad \mathcal{L}_2 = (\partial\pi)^2 \qquad \mathcal{L}_3 = (\partial\pi)^2\Box\pi$$

$$\mathcal{L}_{n+1} = n\eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n}\left(\partial_{\mu_1}\pi\partial_{\nu_1}\pi\partial_{\mu_2}\partial_{\nu_2}\pi\cdots\partial_{\mu_n}\partial_{\nu_n}\pi\right)$$

There is a separation of scales

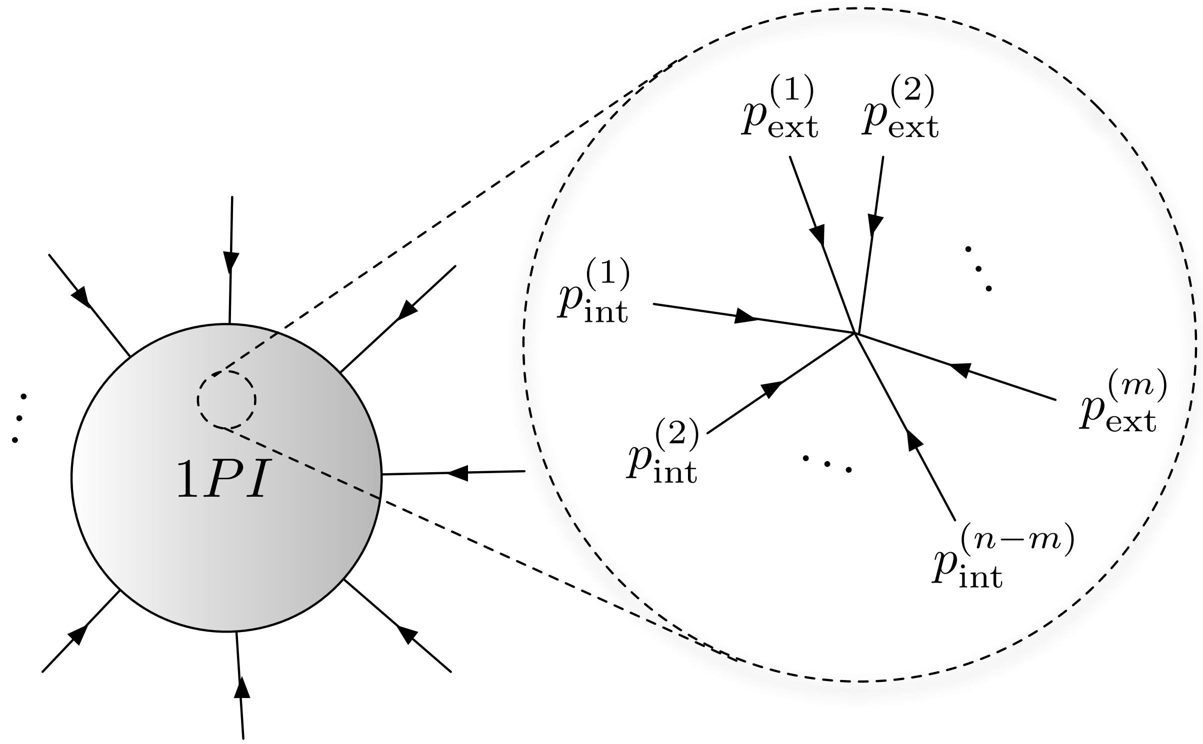
- Allows for classical field configurations with order one nonlinearities, but quantum effects under control.
- So can study non-linear classical solutions.
- Some of these very important (Vainshtein screening)

We now understand that there are many variations on this (Horndeski), some that share its attractive properties (more about this soon)

Nonrenormalization!

Amazingly terms of galilean form are nonrenormalized (c.f SUSY theories).
Possibly useful for particle physics & cosmology. We'll see.

Expand quantum effective action for the classical field about expectation value



The n-point contribution contains at least 2n powers of external momenta: cannot renormalize Galilean term with only 2n-2 derivatives.
Can show, just by computing Feynman diagrams, that at all loops in perturbation theory, for any number of fields, terms of the galilean form cannot receive new contributions.

[Luty, Porrati, Ratazzi (2003); Nicolis, Rattazzi (2004); Hinterbichler, M.T., Wesley, (2010)]

Can even add a mass term and remains technically natural

The Vainshtein Effect

Consider, for example, the "DGP cubic term", coupled to matter

$$\mathcal{L} = -3(\partial\pi)^2 - \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{M_{Pl}}\pi T$$

Now look at spherical solutions around a point mass

$$\pi(r) = \begin{cases} \sim \Lambda^3 R_V^{3/2}\sqrt{r} + const. & r \ll R_V \\ \sim \Lambda^3 R_V^3 \frac{1}{r} & r \gg R_V \end{cases} \qquad R_V \equiv \frac{1}{\Lambda}\left(\frac{M}{M_{Pl}}\right)^{1/3}$$

Looking at a test particle, strength of this force, compared to gravity, is then

$$\frac{F_\pi}{F_{\rm Newton}} = \frac{\pi'(r)/M_{Pl}}{M/(M_{Pl}^2 r^2)} = \begin{cases} \sim \left(\frac{r}{R_V}\right)^{3/2} & R \ll R_V \\ \sim 1 & R \gg R_V \end{cases}$$

So forces much smaller than gravitational strength within the Vainshtein radius - hence safe from 5th force tests.

The Vainshtein Effect

Suppose we want to know the the field that a source generates within the Vainshtein radius of some large body (like the sun, or earth)

Perturbing the field and the source

$$\pi = \pi_0 + \varphi, \quad T = T_0 + \delta T,$$

yields

$$\mathcal{L} = -3(\partial\varphi)^2 + \frac{2}{\Lambda^3}\underbrace{\left(\partial_\mu\partial_\nu\pi_0 - \eta_{\mu\nu}\Box\pi_0\right)}_{\sim\left(\frac{R_v}{r}\right)^{3/2}}\partial^\mu\varphi\partial^\nu\varphi - \frac{1}{\Lambda^3}(\partial\varphi)^2\Box\varphi + \frac{1}{M_4}\varphi\delta T$$

Thus, if we canonically normalize the kinetic term of the perturbations, we raise the effective strong coupling scale, and, more importantly, heavily suppress the coupling to matter!

Regimes of Validity

The usual quantum regime of a theory

$$r \ll \frac{1}{\Lambda}$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3/2} \gg 1$$

$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \gg 1$$

A new classical regime, with order one nonlinearities

$$\frac{1}{\Lambda} \ll r \ll R_V$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3/2} \gg 1$$

$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \ll 1$$

The usual linear, classical regime of a theory

$$r \gg R_V$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3} \ll 1$$

$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \ll 1$$

$$r \sim \frac{1}{\Lambda}$$

$$r \sim R_V$$

The Vainshtein Effect is Very Effective!

Fix r_c to make solutions cosmologically interesting - 4000 Mpc =10^{10} ly

$$r^* = \left(\frac{2GM}{c^2} r_c^2\right)^{1/3}$$

$\rightarrow$ ~0.1 kpc = 10^7 AU

$\rightarrow$ ~Mpc ~ 30 galactic radii

$\rightarrow$ ~10 Mpc ~ 10 virial radii

Constructing Theories I

[Goon, Hinterbichler, M.T., *Phys. Rev.Lett.* 106, 231102 (2011).
Goon, Hinterbichler, M.T., *JCAP* 1107, 017 (2011).]

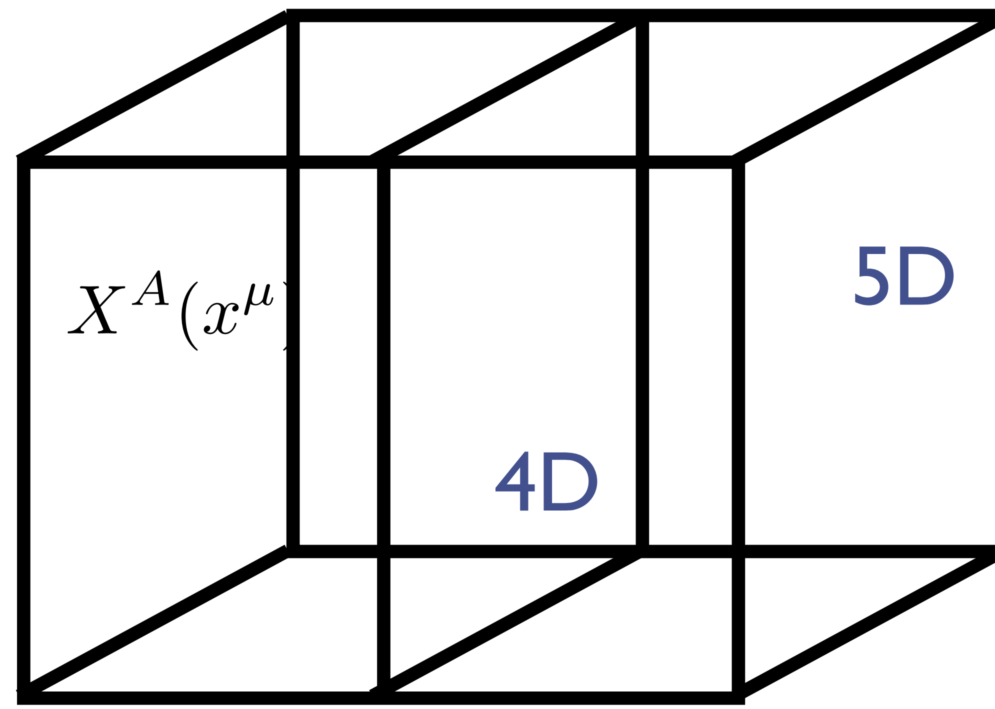
Main point:

- Can extend probe brane construction to more general geometries. e.g. other maximally-symmetric examples

Bulk $$ds^2 = d\rho^2 + f(\rho)^2 g_{\mu\nu}(x)dx^\mu dx^\nu$$

Induced on Brane $$\bar{g}_{\mu\nu} = f(\pi)^2 g_{\mu\nu} + \nabla_\mu\pi\nabla_\nu\pi$$

Bulk Killing Vectors $$\delta_K X^A = a^i K_i^A(X) + a^I K_I^A(X)$$



Galileons with symmetry

$$(\delta_K + \delta_{g,\mathrm{comp}})\pi = -a^i k_i^\mu(x)\partial_\mu\pi + a^I K_I^5(x,\pi) - a^I K_I^\mu(x,\pi)\partial_\mu\pi$$

The Maximally-Symmetric Taxonomy

Potentially different Galileons corresponding to different ways to foliate a maximally symmetric 5-space by a maximally symmetric 4-d hypersurface

Brane metric

Ambient metric

	AdS_4	M_4	dS_4
AdS_5	AdS DBI galileons $so(4,2) \to so(3,2)$ $f(\pi) = \mathcal{R}\cosh^2(\rho/\mathcal{R})$	Conformal DBI galileons $so(4,2) \to p(3,1)$ $f(\pi) = e^{-\pi/\mathcal{R}}$	type III dS DBI galileons $so(4,2) \to so(4,1)$ $f(\pi) = \mathcal{R}\sinh^2(\rho/\mathcal{R})$
M_5	×	DBI galileons $p(4,1) \to p(3,1)$ $f(\pi) = 1$	type II dS DBI galileons $p(4,1) \to so(4,1)$ $f(\pi) = \pi$
dS_5	×	×	type I dS DBI galileons $so(5,1) \to so(4,1)$ $f(\pi) = \mathcal{R}\sin^2(\rho/\mathcal{R})$

Small field limit ↓

AdS galileons	normal galileons	dS galileons

These theories should fall into the Horndeski class, but here their symmetry properties are explicit.

Constructing Theories II

- For those of you who are more mathematically inclined, there is a nice story here that may have implications for, among other things, better understanding the nonrenormalization theorems.

- Since the galilean symmetry in nonlinearly realized, can use the coset construction to build the effective theory. (We've recently shown that one can do this for massive gravity also!)

- Galileons are Wess-Zumino terms! In d dimensions are d-form potentials for (d+1)-forms which are non-trivial co-cycles in Lie algebra cohomology of full symmetry group relative to unbroken one. Slightly different stories for DBI and conformal Galileons.

[Goon, Hinterbichler, Joyce & M.T., arxiv:1203.3191 [hep-th])

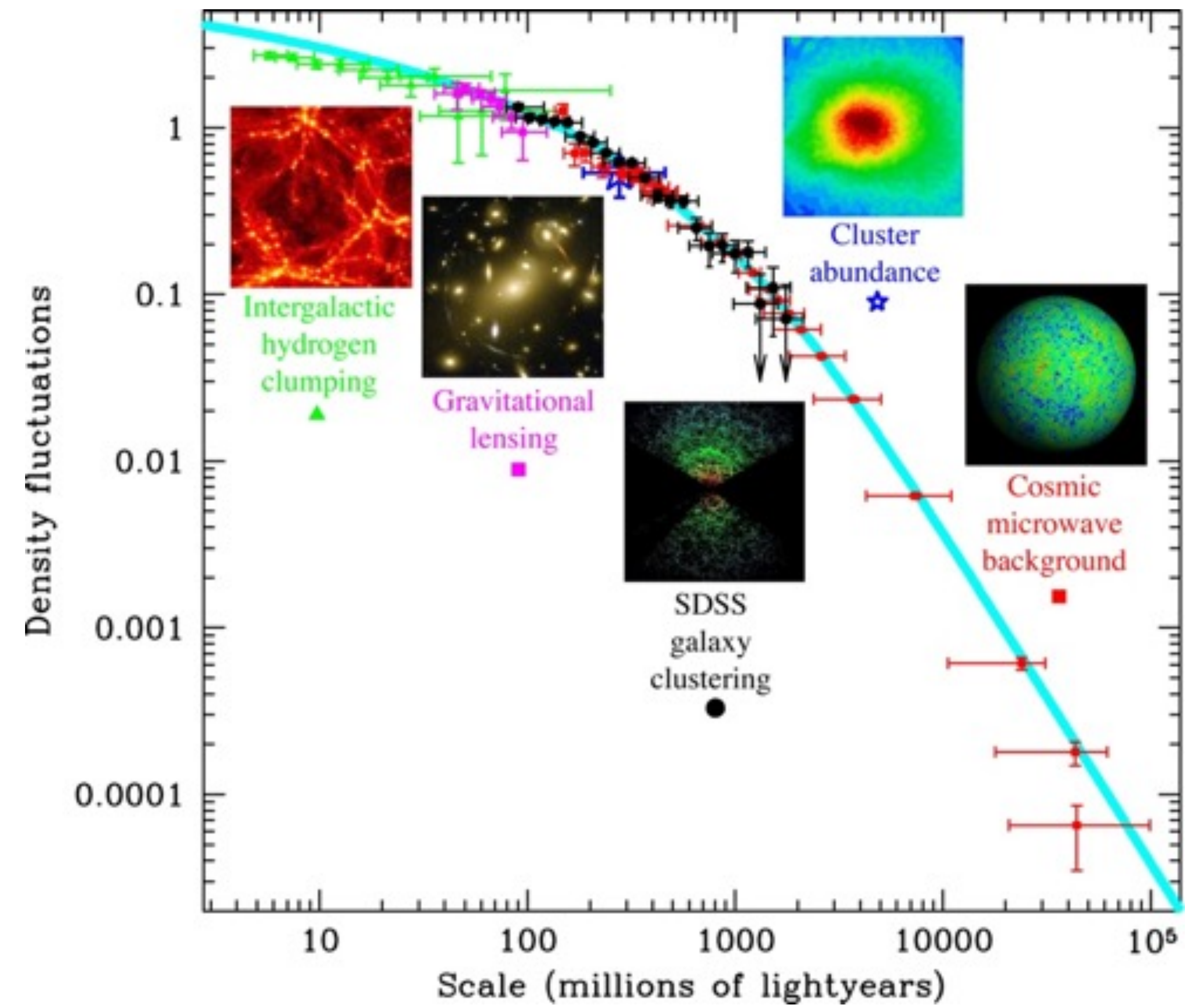
Can look for signals in, e.g., cosmology

- Weak gravitational lensing
- CMB lensing and the ISW effect
- Redshift space galaxy power spectra
- Combining lensing and dynamical cross-correlations
- The halos of galaxies and galaxy clusters

- Very broadly: Gravity is behind the expansion history of the universe

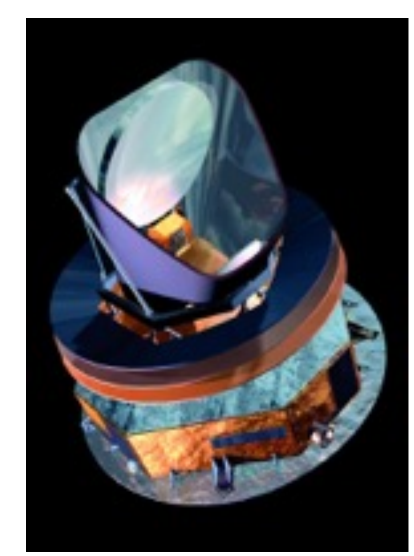
- But it is also behind how matter clumps up - potentially different.

- This could help distinguish a CC from dark energy from other possibilities



DARK ENERGY SURVEY

Large Synoptic Survey Telescope



These Theories are Difficult


Summary and Take-Away Points

- Cosmic acceleration: one of our deepest problems - observers should be general (don't test models yet) and vigilant.
- Questions thrown up by the data need to find a home in fundamental physics, and many theorists are hard at work on this. Requires particle physicists and cosmologists to work together.
- We still seem far from a solution in my opinion, but some very interesting ideas have been put forward in last few years.
- Many attractive ideas (as well as a lot of ugly ones) being ruled out or tightly constrained by these measurements. And fascinating new theoretical ideas are emerging *even without acceleration*
- Serious models only need apply - theoretical consistency is a crucial question. We need (i) models in which the right questions can be asked and (ii) A thorough investigation of the answers.
- I've mostly covered the general approach to the technical questions, and illustrated with a particular Vainshtein screened example; Galileons.

Thank You!