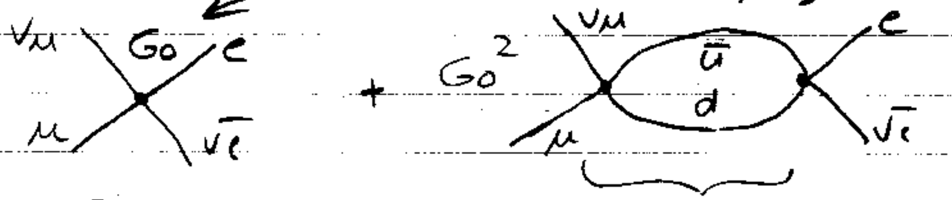
Universality of $G_F \Rightarrow$ Gauge Theory of Weak Interactions

VIII . 1

-  G_F = same for all quarks and leptons
  e.g. $\mu \to e \bar{\nu}_e \nu_\mu$, $\tau \to e \bar{\nu}_e \nu_\tau$, $K^- \to \pi^0 e \bar{\nu}_e$, $\pi^- \to \pi^0 e \bar{\nu}_e$ etc...

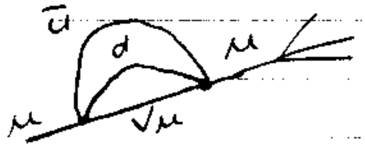
  ("Briefly discussed pgs. 72-73")

⊛ How do we explain this universality once we turn on loop corrections — Bare or Tree value coupling

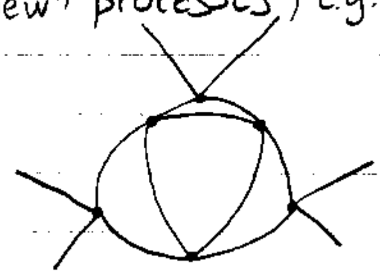
e.g. G_0  + G_0^2

$$= G_0 \left[Z_2 = 1 + G_0 \int d^4q \frac{1}{\not{q}+m} \frac{1}{\not{q}+m} \sim 1 + G_0 \Lambda^2 + G_0 m^2 \log \Lambda \right]$$

⊛ Also there is a divergent wave-function (and mass) renormalization:

-  $\Rightarrow Z_2 = 1 + G_0 \Lambda^2 + G_0 m^2 \log \Lambda$

⊛ The divergences get worse in higher loops, especially because they appear in "new" processes, e.g.:



$\sim G_0^3 (\bar{\psi}\psi)^3$ } A six fermion interaction

$\times\ G_0^3 (\underbrace{\Lambda^1}_{\text{probably gone by symmetric integration}} + m \log \Lambda + \text{finite})$

⊛ All of these divergences must be absorbed by introducing and then renormalizing new terms in the action

- e.g. $\mathcal{L}_{\substack{\text{six} \\ \text{fermion}}} = (G_0^{(6)}) (\bar{\psi}\psi)^3$ — dimension $(\text{mass})^{-5}$

  $G_{Ren}^{(6)} = G_0^{(6)} + G_0^3 m \log \Lambda + \dots$

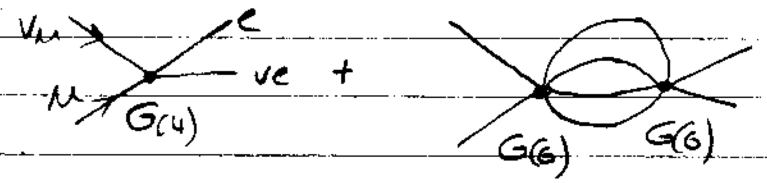
* An infinite number of new interactions (and associated couplings) must in fact be introduced

$\mathcal{L}_{\text{six fermions}} + \mathcal{L}_{\text{eight fermions}} + \ldots$

* In the "old days" ($\lesssim$ 1980's) this was thought to be nonsense: an infinite # of adjustable coupling constants would require an infinite amount of experimental data to determine them

∴ A non-renormalizable theory would have zero predictive power?

* Not true! Take $\mathcal{L}_{6\text{-fermion}}$. It would contribute to ordinary 4-fermion V−A scattering

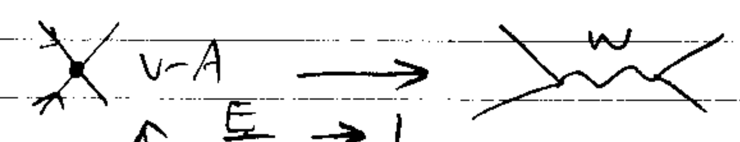


* But by dimensional analysis the second graph, after renormalization, can only contribute as
→ and fitting $G(6)$'s to some measured process

$$\sigma(\mu\bar{\nu}_\mu \to e\bar{\nu}_e) = \underbrace{G_{(4)}^2}_{\text{i.e. } G_F} E^2 \left(1 + \frac{G_{(6)}^2}{G_{(4)}} E^8 + \ldots\right)$$

* As long as $\frac{G_{(6)}^2}{G_{(4)}} E^8 \ll 1$ this expansion should still converge rapidly, and the effects of higher dimension operators will be negligible

* When we get to high energies, we will resolve the virtual massive force carriers that underlie effective theory



$\boxed{\text{All of physics is an effective theory!}}$

* But we'll simply replace "this" effective theory by another one!

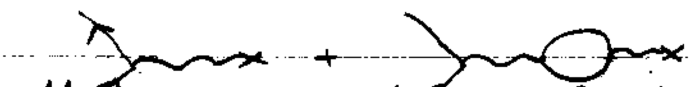
(*) The <u>real</u> reason why we turn to a gauge theory for the weak interactions is <u>not</u> renormalizability, but rather that this is the "only" natural explanation for <u>universality</u> of G_F

cf. QED: $e_{ren} = Z_1 Z_2 \sqrt{Z_3}\, e_{bare}$

(*) Also look at Eq. (10.38)
(*) Gauge Invariance demands $\delta_1 = -\delta_2$!!

(*) Depend on particle masses, but gauge symmetry forces $\boxed{Z_1^{-1} = Z_2}$ [Look at proof in P+S Ch 9.6]

(*) Z_3 = universal to <u>all</u> charged particles, coming from:

$Z_3^{(\mu)} =$ 

$= Z_3^{(e)}$

↑ <u>all</u> charged particles here, <u>whatever</u> the external particle.

∴ We "naturally" have:

$$|Q_p + Q_e| \lesssim 10^{-21}$$

(*) similarly, but not so precisely known, for $|Q_\mu - Q_e|$

<u>if</u> we assume that the as yet unknown fundamental theory of everything sets $|Q_p^{bare}| = |Q_e^{bare}|$

$\boxed{\text{Likewise, in the case of the weak interactions}}$

(*) We turn to a gauge theory in the search for a natural way to get $G_{F,ren}^{(\mu)} = G_{F,ren}^{(e)} = \ldots$

(*) Otherwise, we must assume that the fundamental theories <u>fine-tunes</u> the bare couplings for each particle, in order to end up with

$$G_{ren}^{(\mu)} \text{ " = " } G_0^{(\mu)} Z_1(m_\mu)\, Z_2(m_\mu) \sqrt{Z_3}$$
$$= G_0^{(e)} Z_1(m_e) \cdots$$

(*) Universality of G_F and short-range of interactions suggests a massive gauge theory ?

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} M^2 A_\mu^2$$

Seems bad: destroys gauge invariance ??

$$= +\frac{1}{2} A_\mu \left[g^{\mu\nu}(\partial^2 + m^2) - \partial^\mu \partial^\nu \right] A_\nu$$

(*) Euler-Lagrange (either classically or as operator equation)

$$[\quad\quad] A_\nu = 0$$

(*) Contract with ∂_μ implies a constraint equation:

$$M^2 \boxed{\partial^\nu A_\nu = 0} \quad \text{i.e. } K \cdot \varepsilon = 0$$

or, in terms of polarization vectors, just 3 independent components

(*) Can quantize as usual – eliminate A^0 by equations of motion (non-covariant?) or introduce Gupta-Bleuler modifier

[see Itzykson + Zuber pgs. 134 – 137 and also D. Lurié]

(full propagator depends on how we treat A^0)

(*) Get (part of!) the propagator by:

$$\left[g^{\mu\nu}(\partial^2 + m^2) - \partial^\mu \partial^\nu \right] D^F_{\nu\alpha}(x-y) = \delta^\mu_\alpha \, \delta^4(x-y)$$

$$\text{or } D_F^{\mu\nu}(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \frac{\left(-g^{\mu\nu} + \boxed{p^\mu p^\nu / M^2}\right)}{p^2 - M^2}$$

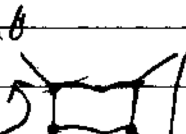
Higher loops now diverge and require new renormalization

(*) This causes severe ultra-violet divergence in loop diagrams violating our conclusions about renormalizability, which were based on $1/p^2$ behavior for gauge-field

∞ We lose universality

$$\boxed{\text{UNLESS}} \quad \text{e.g. } K^\mu J_\mu(k) = 0$$

$J^\mu(k)$



only where massless ghosts can restore $K_\mu M^\mu = 0$

(*) Actually ok in QED!

(*) But not for $J_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$

(*) And not for QCD

Spontaneous Symmetry Breaking — To the Rescue!

Case I: A Discrete Symmetry

* $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4$ $\quad \boxed{\phi = \text{Hermitian}}$

* $\mathcal{L}$ is symmetric under $\phi \to -\phi$

(There is no conserved current here: Noether's Thm applies to continuous symmetries).

* $H = \int d^3\vec{x} \left[\pi \dot{\phi} - \mathcal{L} \right]$ $\quad \dot{\phi} = \pi$ as usual

$$= \int d^3x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \underbrace{\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4}_{} \right]$$

∴ Minimum energy (classical) solution is:

$$\phi(x) = \text{constant} = 0$$

$\phi(x) = 0$ minimizes these "potential" terms

* In the QFT, the ground state is state of least energy

$$\langle \Omega | \phi_{op}(x) | \Omega \rangle = 0$$

Now do something (superficially) radical

* $\boxed{\text{Take } m^2 \equiv -\mu^2 < 0}$ $\quad E_p = \sqrt{\vec{p}^2 + m^2}$ = imaginary for small $|\vec{p}|$

* OK, don't get too excited: we must lose the interpretation of m^2 as a mass parameter

* Also, we don't expect "free fields" $(\Box + m^2)\phi_0(x) = 0$ to be stable solution

i.e. $\phi(x) = \phi_0(x) + \int d^4y\, D_F(x-y)\, J(y)\, \phi(y)$ ← small

where
$(\Box + m^2)\phi_0(x) = 0$ → * Could choose $\phi_0(x) = a(e^{ipx} + e^{-ipx})$ with $E_p = \sqrt{\vec{p}^2 + m^2}$ = stable for large enough $|\vec{p}|$

$(\Box + m^2) D_F(x-y) = \delta^4(x-y)$

$(\Box + m^2)\phi(x) = J(x)\,\phi(x)$ → * But runaway solution at small $\vec{p}$ gets excited by $J(y)$, and we can't solve by perturbation theory.

$\boxed{J(x) = \frac{\lambda}{6}\phi^2(x)}$

in perturbation theory we would try to use plane waves

* What we are doing here is to motivate a potential:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

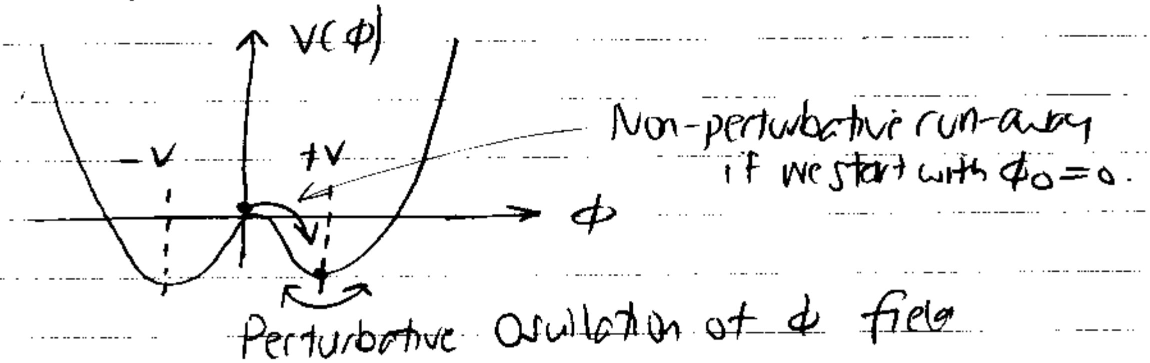
$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

* $H = \int d^3x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right]$

and now the ground state is determined by:

$$\phi(x) = \phi_0 \equiv v = \pm\sqrt{\frac{6}{\lambda}}\,\mu$$

where $\left.\frac{dV(\phi)}{d\phi}\right|_{\phi=v} = 0$



* The perturbative modes of the theory are identified by expanding about one of the minimum energy states

* Which one? Physics doesn't care!

[If you start the system at $\phi = 0$, then the "first" random quantum fluctuation in ϕ field will pick one]

$$\boxed{\phi(x) = v + \sigma(x)}$$

* Perturbative fluctuations

* We say that the vacuum breaks the symmetry

$$\langle \Omega | \phi_{op}(y) | \Omega \rangle = v$$

a coherent state of the ϕ field

But: $\langle \Omega | \sigma_{op}(y) | \Omega \rangle = 0$

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu(\underbrace{v}_{\text{constant}} + \sigma(x))\right]^2 - V(\phi(x))$$

$$V = \underbrace{-\frac{1}{2}\mu^2(v+\sigma)^2}_{\text{bad mass term}} = -\frac{1}{2}\mu^2 v^2 - \mu^2 v\sigma - \frac{1}{2}\mu^2\sigma^2$$

irrelevant constants, cancel

$$+\frac{\lambda}{24}(v+\sigma)^2 = +\frac{\lambda}{24}\left[v^4 + 4v^3\sigma + \underbrace{6v^2}_{36\frac{\mu^2}{\lambda}}\sigma^2 + 4v\sigma^3 + \sigma^4\right]$$

(*) The linear term in σ cancels precisely because: $\left.\dfrac{dV(v+\sigma)}{d\sigma}\right|_{\sigma=0} = 0!$

(*) The mass term for σ comes out with correct sign

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\frac{\lambda}{6}}\,\mu\,\sigma^3 - \frac{\lambda}{4!}\sigma^4}$$ + irrelevant constant

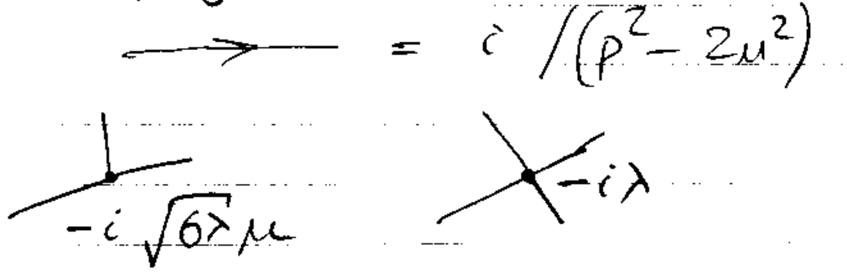
(*) Now we've identified a genuine mass $\sqrt{2}\,\mu$ for the σ fluctuations around the potential minimum

(*) The symmetry of the underlying Lagrangian,

$$\phi \to -\phi$$

is no longer evident, either in the ground state, or in the Lagrangian ($\sigma \not\to -\sigma$), unless we "move" all the degrees of freedom in the coherent state $|\Omega\rangle$ (leading to $v \to -v$ or $\mu \to -\mu$)

(*) However there is a deep connection between the three couplings of the theory, thanks to the underlying symmetry:



(*) 3 renormalization conditions, but only 2 independent parameters μ^2 and λ!

SSB Case II: Continuous Global Symmetry

$$\mathcal{L} = 1(\partial_\mu\phi)^\dagger(\partial_\mu\phi) - V(\phi^\dagger\phi)$$

where $V = -1\mu^2\phi^\dagger\phi - \frac{\lambda}{2}(\phi^\dagger\phi)^2$

"wrong" sign for mass term

N.B. "unconventional normalizations"

Now ϕ = complex : makes for Big changes!

$\mathcal{L}$ = invariant under continuous transformation:

$$\phi \rightarrow e^{i\alpha}\phi$$

Global for now.

Noether: $J^\mu = \frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi)}\underbrace{\delta\phi}_{i\alpha\phi} + \frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi^\dagger)}\underbrace{\delta\phi^\dagger}_{-i\alpha\phi^\dagger}$

$$\therefore \boxed{J^\mu = (\partial^\mu\phi^\dagger)\phi - \phi^\dagger(\partial^\mu\phi)}$$

Work through canonical quantization (see eg Mandl + Shaw)

$$\phi(x) = \sum_p \left(\frac{1}{2E_pV}\right)^{\frac{1}{2}}\left(a\,e^{-ipx} + b^\dagger e^{ipx}\right)$$

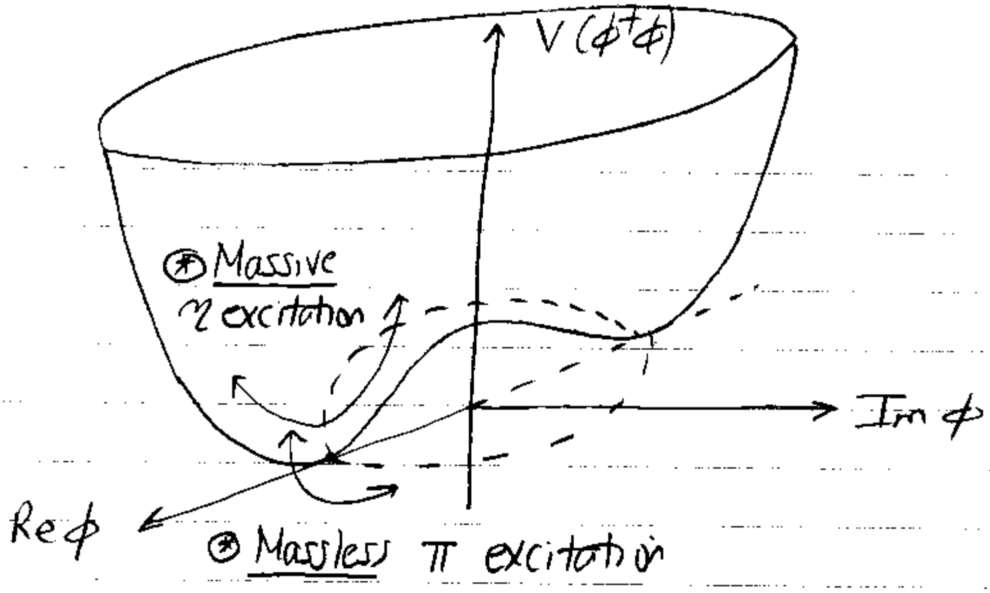
$$Q = \int_V d^3x \,:J^0(\vec{x},t): = \sum_p (a^\dagger a - b^\dagger b)$$

$$\boxed{H = \int d^3x\,\left[\pi^\dagger\pi + (\vec{\nabla}\phi)^\dagger\cdot(\vec{\nabla}\phi) + V(\phi^\dagger\phi)\right]}$$

$$V = -\mu^2\phi^\dagger\phi + \frac{\lambda}{2}(\phi^\dagger\phi)^2$$

or $\boxed{V(\phi^\dagger\phi) = \frac{\lambda}{2}\left(\phi^\dagger\phi - |\phi_0|\right)^2} \quad \boxed{|\phi_0|^2 = \frac{\mu^2}{\lambda}}$

$\left(-\frac{\lambda}{2}\phi_0^2\right)$ = irrelevant constant



Again, Nature randomly picks some point on the circle of degenerate minima of the potential

Let's $\boxed{\text{choose}}$ to expand about ϕ = real

$$\boxed{\phi(x) = \left[\phi_0 + \frac{1}{\sqrt{2}}\sigma(x)\right] e^{i\pi(x)/(\sqrt{2}\phi_0)}}$$

$$\boxed{\sigma, \pi = \text{Hermitian}}$$

ϕ_0 sets the scale for dimension-one field $\pi(x)$

$$\therefore \quad (\partial_\mu \phi)^\dagger (\partial_\mu \phi)$$

Phase factor cancels

$$= \left[\frac{1}{\sqrt{2}}\partial_\mu\sigma + \left(\phi_0 + \frac{1}{\sqrt{2}}\sigma\right)\left(-i\frac{\partial_\nu\pi}{\sqrt{2}\phi_0}\right)\right]\left[\frac{1}{\sqrt{2}}(\partial_\nu\sigma) + \left(\phi_0 + \frac{1}{\sqrt{2}}\sigma\right)\left(+i\frac{\partial_\nu\pi}{\sqrt{2}\phi_0}\right)\right]$$

$$= \frac{1}{2}(\partial_\mu\sigma)^2 + \left(1 + \frac{1}{\sqrt{2}}\frac{\sigma(x)}{\phi_0}\right)^2 \frac{1}{2}(\partial_\nu\pi)^2$$

$\boxed{\text{The } \pi \text{ kinetic term !!}}$

$$V(\phi^\dagger\phi) = \frac{\lambda}{2}\left[\left(\phi_0 + \frac{1}{\sqrt{2}}\sigma(x)\right)^2 - \phi_0^2\right]^2$$

$$= \frac{\lambda}{2}\left[\sqrt{2}\phi_0\sigma + \frac{1}{2}\sigma^2\right]^2$$

$\boxed{\text{The } \sigma \text{ mass term}}$

$$= \frac{\lambda}{2}\left[\frac{1}{4}\sigma^4 + \sqrt{2}\phi_0\sigma^3 + 2\phi_0^2\sigma^2\right]$$

$$\therefore \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$$

⊛ $$\mathcal{L}_0 = \tfrac{1}{2}(\partial_\mu \sigma)^2 - \tfrac{1}{2}(2\mu^2)\sigma^2 + \tfrac{1}{2}(\partial_\mu \pi)^2$$

⊛ $$\mathcal{L}_I = \tfrac{1}{2}(\partial_\mu \pi)^2 \left[\sqrt{2\lambda}\,\frac{\sigma}{\mu} + \frac{1}{2}\lambda^2 \frac{\sigma^2}{\mu^2} \right] - \sqrt{\frac{\lambda}{2}}\,\mu\,\sigma^3 - \frac{\lambda}{8}\sigma^4$$

⊛ So we end up with a massive σ as before

⊛ But we also have an exactly massless particle as well

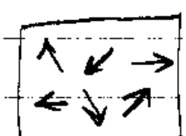
$\boxed{\text{Goldstone's Theorem}}$

"For every spontaneously broken continuous global symmetry, the theory must contain a massless particle"

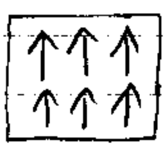
⊛ The details of $\mathcal{L}_I$ are not important here, but now 4 interaction terms with only 2 coupling constants.

$\boxed{\text{Spin Waves are Goldstone Bosons}}$

Ferromagnet above TC



$\langle M \rangle = 0$



$\langle M \rangle \neq 0$

Below Tc

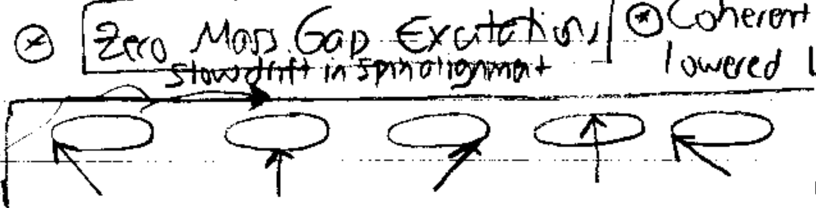
Rotational Symmetry spontaneously Broken by random impurity

⊛ $\boxed{\text{Zero Mass Gap Excitations}}$

⊛ Coherent collection of spins lowered by one unit along magnetization axis

"All" spins in this region alligned

slow drift in spin alignment



$\longleftarrow$ wavelength λ $\longrightarrow$

⊛ Energy $\rightarrow 0$ as $\lambda \rightarrow \infty$

SSB III : Continuous Local Symmetry

$$\circledast\ \mathcal{L}_{Abelian} = 1\,(D_\mu\phi)^\dagger(D^\mu\phi) - \frac{1}{4}(F_{\mu\nu})^2 - V(\phi^\dagger\phi)$$

$$D_\mu\phi = (\partial_\mu + ieA_\mu)\,\phi$$

$\circledast$ $\mathcal{L}$ is invariant under local $U(1)$ gauge transformations:

$$\phi(x) \rightarrow e^{i\alpha(\underline{x})}\,\phi(x) \qquad \phi = \underline{\underline{\text{complex}}}$$

$$A_\mu \rightarrow A_\mu - \frac{1}{e}\partial_\mu\alpha(x) \qquad \text{scalar field}$$

$\circledast$ So far looks like ordinary QED with massless A^μ:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} \qquad \underline{\underline{\ }}\ (\partial^\mu\phi^\dagger)\phi - \phi^\dagger(\partial^\mu\phi)$$

$$\mathcal{L}_{int} = -ie\,A_\mu J^\mu - V(\phi^\dagger\phi) \qquad = \text{particles} + \text{anti-particles of opposite charge}$$

$\circledast$ But now choose $\boxed{V = \frac{\lambda}{2}(\phi^\dagger\phi - v^2)^2}$ ← change of notation

as before

and let's choose to expand the action in fluctuations around $\phi(x)$ = real

$$\circledast\quad \phi(x) = \left(v + \frac{1}{\sqrt{2}}\rho(x)\right)e^{i\pi(x)/\sqrt{2}\phi_0}$$

$\circledast$ But thanks to local gauge symmetry,

$$\phi \rightarrow e^{i\alpha(x)}\phi\,,\quad \alpha(x) = -\pi(x)/\sqrt{2}\phi_0$$

we can always do the expansion in a gauge where

$$\boxed{\phi(x) = v + \frac{\rho(x)}{\sqrt{2}}} \quad \boxed{\text{“Unitary Gauge”}} = \text{purely real at all } x.$$

$\circledast$ What about gauge-invariance? Well, cf T III ...

⊛ N.B. We couldn't do this in the theory with only a global symmetry. ⊛ Where did the $\pi(x)$-field go?

$$\therefore \mathcal{L} = \left[(\partial_\mu - ieA_\mu)\left(v + \tfrac{1}{\sqrt{2}}\rho(x)\right)\right]\left[(\partial_\mu + ieA_\mu)\left(v + \tfrac{\rho}{\sqrt{2}}\right)\right]$$

$$- \tfrac{1}{4} F_{\mu\nu}^2 - \tfrac{\lambda}{2}\left[\left(v + \tfrac{\rho}{\sqrt{2}}\right)^2 - v^2\right]^2$$

$$= \left[\tfrac{1}{\sqrt{2}}\partial_\mu\rho - ieA_\mu\left(v + \tfrac{1}{\sqrt{2}}\rho\right)\right]\left[\qquad\qquad\right]^*$$

$$- \dots$$

$$= \tfrac{1}{2}(\partial_\mu\rho)^2 + e^2 A_\mu^2\left(v^2 + \sqrt{2}\,v\rho + \tfrac{1}{2}\rho^2\right)$$

$$- \tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{\lambda}{2}\left[\sqrt{2}\,v\rho + \tfrac{1}{2}\rho^2\right]^2$$

$$\therefore \boxed{\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}}$$

$$\mathcal{L}_0 = -\tfrac{1}{4}(F_{\mu\nu})^2 + \tfrac{1}{2} m_A^2 A_\mu^2 + \tfrac{1}{2}(\partial_\mu\rho)^2 - \tfrac{1}{2}(2\mu^2)\rho^2$$

$$m_A^2 = 2e^2v^2 \quad \text{and} \quad \mu^2 = \lambda v^2$$

→ Photon has acquired a mass!!

⊛ Can better understand this graphically – see P+S p.691 + esp. Aitchison + Hey

$$⊛\ \left.\begin{aligned}\mathcal{L}_{int} = {} & e^2 A_\mu^2\left[\sqrt{2}\,v\rho + \tfrac{1}{2}\rho^2\right] \\ & - \tfrac{\lambda}{\sqrt{2}} v\rho^3 - \tfrac{\lambda}{8}\rho^8\end{aligned}\right\} \text{the "usual suspects"}$$

⊛ The photon has a mass and $\therefore$ $\underline{3}$ polarization d.o.f

⊛ We originally started with a massless A_μ (2 d.o.f) and a massless Goldstone Boson $\pi(x)$ ← it was "eaten" by A_μ!

SSB of non Abelian Gauge Theory

⊛ $\mathcal{L} = 1\,(D_\mu\phi)^\dagger(D^\mu\phi) - \frac{1}{4}\sum_a (F_{\mu\nu}^a)^2 - V(\phi^\dagger\phi)$

⊛ $V = -\mu^2\,\phi^\dagger\phi + \underline{1}\,\lambda\,(\phi^\dagger\phi)^2$

⊛ Last goofy normalization change

$= \lambda\left[\phi^\dagger\phi - \phi_0^2\right]^2 + \text{constant}$

⊛ $\phi_0^{\,2} = \frac{1}{2}\frac{\mu^2}{\lambda} \equiv \frac{1}{2}v^2$

⊛ Take : $\phi = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$ = fundamental representation

$\boxed{\text{N.B}}$ As we will see, the pattern of the SSB depends both on the gauge group and on the representation of ϕ.

⊛ $\boxed{D_\mu\phi = \left(\partial_\mu - i g A^{\mu a} T^a\right)\phi \qquad T^a \equiv \frac{1}{2}\sigma^a}$, σ^a = Pauli Matrices

⊛ As usual, we must spontaneously break the symmetry :

$$\phi(x) \rightarrow e^{i w^a(x) T^a}\phi(x)$$

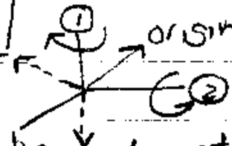
by choosing to expand the theory in fluctuations about one particular copy of the degenerate minima

⊛ By local gauge invariance, we can make $\phi(x)$ have isospin down at all x :

[To do this in non-unitary gauge, write $\phi = \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix}$ = all real elements, which shows that this model is equivalent to O(4)?]

$$\boxed{\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\rho(x) \end{pmatrix}}$$

i.e. two iso-rotations will do it, where independent (i.e. local) iso-rotations can be made at each x.



⊛ We already know that $\rho(x)$ – The Higgs Boson – will get a mass from:

$$V = \lambda\left[\frac{1}{2}(v+\rho)^2 - \frac{1}{2}v^2\right]^2$$

$$= \lambda\left[v\rho + \rho^2\right]^2 = \underbrace{\boxed{\lambda v^2}}_{\mu^2}\rho^2 + O(\rho^3) + O(\rho^4)$$

as well as self-interactions (ρ^3, ρ^4) and interactions with the gauge fields, from $|D_\mu\phi|^2$.

⊛ Let's instead just concentrate on the mass generation for the gauge-fields: $\boxed{\text{So set } \rho(x) = 0 \text{ for now}}$

$$\therefore\ |D_\mu\phi|^2 \underset{\rho(x)=0}{=} \frac{1}{2}g^2(0\ \ v)\,T^a T^b \begin{pmatrix} 0 \\ v \end{pmatrix} \underbrace{A^a_\mu A^{b\mu}}$$

⊛ $a \leftrightarrow b$ symmetric

$\therefore$ only $\frac{1}{2}(T^aT^b + T^bT^a)$ contributes

$$= \frac{1}{2}\left(\frac{1}{4}\right)(\underbrace{\sigma^a\sigma^b + \sigma^b\sigma^a}_{2\delta^{ab}}) = \frac{1}{4}\delta^{ab}$$

$\therefore$ $|D_\mu\phi|^2$ contributes to the Lagrangian

$$\boxed{\Delta\mathcal{L} = \frac{1}{8}g^2v^2A^a_\mu A^{a\mu} \equiv \frac{1}{2}m_A^2 A^a_\mu A^{a\mu} \qquad m_A = \frac{1}{2}gv}$$

Count degrees of freedom

Well, we have yet to discuss this ...

* Original free-field Lagrangian should have only only 2 polarizations for each A^a, $a=1,\dots,N^2-1$
* Globally symmetric theory would have N^2-1 Goldstone bosons π^a, from equipotential lines at the chosen minimum $\phi_0 \to e^{i\pi^a T^a}\phi_0$
* Conclusion: the Goldstone Bosons were "eaten" by the gauge fields, to get their extra (massive) polarization

How to preserve a massless photon?

* We want mass for the W_μ^+ and W_μ^- weak force carriers, but not for the γ. Our first SSB scheme has given us 3 degenerate vector bosons.
* There are many (an infinite #!) of solutions, and only experiment can tell us which one is realized at "low" energies.

Pattern of Symmetry Breaking

* What happens if we change the representation of ϕ?

$$\phi_i \to \left(e^{i\alpha^a T^a}\right)_{ij}\phi_j$$

$$\approx 1 + i\alpha^a T^a_{ij}\phi_j$$

means $(D_\mu\phi)_i = \partial_\mu\phi_i - igA^a_\mu T^a_{ij}\phi_j$ for the covariant derivative

* To find what masses are generated for the gauge fields, let's again forget about $\partial_\mu \phi$, and write:

$$(\phi_0)_i = \frac{v}{\sqrt{2}} \begin{pmatrix} \text{some} \\ \text{nonzero} \\ \text{entries} \end{pmatrix}_i$$ ← There may be several distinct possibilities

$$\therefore \Delta \mathcal{L} = \frac{g^2}{2} A^a_\mu A^{b\mu} \underbrace{(T^a \phi_0)_i (T^b \phi_0)_j}$$

$$\equiv \frac{1}{2} \underbrace{m^2_{ab}} A^a_\mu A^{b\mu}$$

* m^2_{ab} is a positive semi-definite matrix since $m^2_{aa} = (T^a \phi_0)^2 \geq 0$, which means that m^2_{ab} has positive eigenvalues

[ ]

* But if there are generators for which

$$\boxed{T^a \phi_0 = 0}$$

then the corresponding gauge fields stay massless.

$\boxed{\text{Ex: } SU(\underline{3}) \text{ with fundamental representation } \phi.}$

* Choose $\phi_0 = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ and then:

$$T^{1,2,3} \phi_0 = 0$$ Recall e.g. $T^3 = \begin{bmatrix} 1 & & \\ & -1 & \\ & & 0 \end{bmatrix}$

* Hence $SU(3)$ gets "broken" to $SU(2)_{\text{isospin}}$, where the 3 gauge fields of $SU(2)_{\text{isospin}}$ stay massless, and the other 5 gauge fields of $SU(3)$ get (different) masses!

Right: ϕ_0 has 3 complex entries, with one constant $|\phi_0|^2 = v^2$. Thus 5 possible Goldstone rephasings to get eaten by 5 A's!

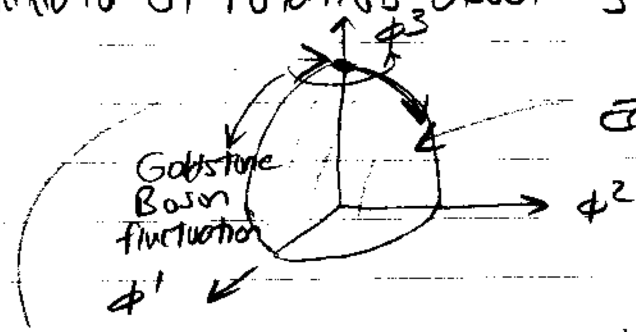
$\boxed{\text{Ex : } SU(2) \text{ with } \underline{\text{adjoint}} \text{ representation } \phi}$
Georgi-Glashow model

* There is a <u>real</u> representation with $N^2-1 = 3$ components
  i.e. $\phi_0{}^a$, $a = 1, 2, 3$ [or $J=1$ real representation of $O(3)$, the rotation group]

* Then the generators are $(T^a)_{bc} = i\,\varepsilon^{bac}$

[check : $\phi \to e^{\overbrace{i\omega^a T^a}^{}}_{\text{real!}}\phi$ stays real !]

* Then choosing
$$\phi_0{}^b = \phi_0\,\delta^{b3}, \quad \text{we find:}$$
$$(T^3\phi_0)^a = (T^3)^{ab}\phi_0{}^b = -i\varepsilon^{3a3}\phi_0 = 0$$
generator of rotations about $\hat{3}$ axis



Equipotential surface : $(\phi^a)^2 = v^2$

* We chose the vacuum state here, but this vacuum is <u>invariant</u> under rotations about the $\hat{3}$ axis

* So no Goldstone Boson for "rephasing" around the 3 axis

$e^{i\pi^3(x)T^3}\phi_0 = \phi_0$, independent of π^3!

or $\boxed{SU(2) \xrightarrow[\text{broken}]{\text{gets}} U(1)}$

with a real vector Higgs field [Good: Count 3 components, with 1 constraint $\phi^2 = \phi_0^2$ leaving 2 Goldstone Bosons to render two gauge fields massive]

* $\boxed{\text{Not realized in Nature}}$ There are in fact <u>two</u> "neutral currents" $e^- e^- \gamma\, e^- e^-$ <u>and</u> $e^- e^- Z^0\, \nu_\mu \nu_\mu$

So we want to <u>fully</u> break $SU(2)$, with a <u>further</u> unbroken $U(1)$

⊛ To double check the counting of the number of Goldstone Bosons, in the Georgi-Glashow model, go back to the theory with Global Symmetry only

$$\mathcal{L} = (\partial_\mu \phi)^T (\partial^\mu \phi)^T - V(\phi^T \phi)$$

transpose instead of $\dagger$ since we have a real representation

$$\phi = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \\ v + \eta(x) \end{pmatrix} = \text{real triplet representation of } SU(2), \text{ or fundamental representation of } O(3)\,!$$

⊛ $V = \lambda \left[\phi^T \phi - v^2 \right]^2 = \lambda \left[\phi_1^2 + \phi_2^2 + \underbrace{(v+\eta)^2 - v^2}_{2v\eta + \eta^2} \right]^2$

∴ V generates a mass for η, but not for ϕ_1 and ϕ_2

∴ Only 2 Goldstone Bosons for Global SU(2) with real triplet ϕ field, or:

$$\text{Global } O(3) \xrightarrow[\text{broken}]{\text{gets}} O(2) \equiv \underbrace{U(1)}$$

← i.e. $\mathcal{L}_{\text{Global}}$ is invariant under rotations of ϕ_1 and ϕ_2

⊛ So this globally unbroken symmetry gets gauged to give us a massless photon in the SSB of the Local Theory

∴ Local O(3) aka SU(2) with triplet Higgs $\xrightarrow[\text{broken}]{\text{gets}}$ U(1)

Glashow – Weinberg – Salam : $SU(2) \times U(1)$

⊛ With fundamental representation Higgs field

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \underset{\text{gauge}}{\overset{\text{unitary}}{=}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

⊛ The Higgs field

⊛ Seems most natural, because the V–A currents

$$J^\mu_{weak} = \bar{\nu}_e \gamma^\mu (1-\gamma_5) e \underset{\text{written as}}{\overset{\text{can be suggestively}}{=}} \bar{E} \gamma^\mu (1-\gamma^5) T_+ E$$

where the electron-type weak isodoublet

$$E_L = \begin{pmatrix} \nu_e(x) \\ e(x) \end{pmatrix}_L$$

a reminder : only left-chirality

and $T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ = isospin raising operator

⊛ Now, to accommodate a massless photon, allow fields to interact with a "B_μ" gauge boson

$$\phi \xrightarrow[SU(2)\times U(1)]{\text{Local}} \underbrace{e^{i\alpha^a(x) T^a}}_{SU(2)} \underbrace{e^{i\frac{1}{2}\beta(x)}}_{} \phi$$

⊛ overall $U(1)$ phase not contained in $SU(2)$

⊛ So the covariant derivative reads:

$$\boxed{D_\mu \phi = \left(\partial_\mu - i g A^a_\mu T^a - i \tfrac{1}{2} g' B_\mu\right)\phi}$$

and e.g. $\boxed{B_\mu \xrightarrow[\text{Abelian}]{} B_\mu + \frac{i}{g'} \partial_\mu \beta(x)}$ plus nonAbelian transformation law for $A^a_\mu(x)$

⊛ To identify the masses, set $h(x) = 0$ as we did before:

$$|D_\mu \phi|^2 = \frac{1}{2}(0 \ \ v)\left(g A^a_\mu T^a + \frac{1}{2} g' B_\mu\right) \times \left(g A^{b\mu} T^b + \frac{1}{2} g' B^\mu\right)\begin{pmatrix}0\\ v\end{pmatrix}$$

$$= \frac{1}{2} v^2 (0 \ \ 1)\left[g^2 A^a_\mu A^{b\mu} \underbrace{\frac{1}{2}(T^a T^b + T^b T^a)}_{\frac{1}{2}\delta^{ab} \text{ since } T^a = \frac{1}{2}\sigma^a \text{ and } \sigma^a\sigma^b + \sigma^b\sigma^a = 2\delta^{ab}}\right]\begin{pmatrix}0\\ 1\end{pmatrix}$$

$a \leftrightarrow b$ symmetric — so symmetrize as we've done before

$$+ g g' A^a_\mu B^\mu (0 \ 1) T^a \begin{pmatrix}0\\ 1\end{pmatrix} + \frac{1}{4} g'^2 B_\mu B^\mu (0 \ 1)\begin{pmatrix}0\\ 1\end{pmatrix}$$

⊛ only $T^3 = \frac{1}{2}\sigma^3$ survives

$$= \frac{v^2}{8}\left[g^2 (A^1_\mu)^2 + g^2 (A^2_\mu)^2 + g^2 (A^3_\mu)^2 - 2 g g' A^3_\mu B^\mu + g'^2 B_\mu B^\mu\right]$$

complete square!

$$= \frac{1}{2}\frac{v^2}{4}\left[g^2 (A^1_\mu)^2 + g^2 (A^2_\mu)^2 + (g A^3_\mu - g' B_\mu)^2\right]$$

⊛ When we include our fermion weak isodoublets, it will be convenient to use iso-raising/lowering combinations:

$$\boxed{W_\mu^{\pm} = \frac{1}{\sqrt{2}}\left(A^1_\mu \mp i A^2_\mu\right) \quad \Big| \quad \text{Mass } m_W = \frac{gv}{2}}$$

⊛ We must also normalize the linear combo in the 3rd basis

$$\boxed{Z_\mu^0 = \frac{1}{\sqrt{g^2 + g'^2}}\left(g A^3_\mu - g' B_\mu\right) \quad \Big| \quad \text{Mass } m_Z = \sqrt{g^2 + g'^2}\,\frac{v}{2}}$$

⊛ The normalization ensures that $\int [DA^3_\mu][DB_\mu] \underset{\text{Jacobian}}{\overset{\text{unit}}{=}} \int [dZ^0_\mu][DA^0_\mu]$

or $[Z_\mu^0(\vec{x}), \Pi_Z^\nu(\vec{y})] = \delta_\mu^{\ \nu}\delta^3(\vec{x}-\vec{y})$

or $[Z_\mu^0(x), Z^0_\nu(y)] = g_{\mu\nu}\Delta(x-y)$

} so we have properly normalized momentum space creation/annihilation, if we had it for A^3_μ and B_μ

⊛ The fourth linear combination of (A^a, B^μ), orthogonal to Z_ν^0, remains massless

$$\boxed{A_\mu^0 = \frac{1}{\sqrt{g^2+g'^2}}\left(g' A_\mu^3 + g B_\mu\right) \quad \Big| \quad \text{Mass } m_A = 0!}$$

⊛ This ensures e.g. $[A_\mu^0(x), Z_\nu^0(y)] = 0$, so that the two fields create / destroy distinct particles

⊛ So the unbroken $U_{EM}(1)$ is <u>not</u> the "original" $U(1)$ - not surprisingly, there is a <u>mixing</u> between the original charge-zero bosons

$$\begin{pmatrix} Z^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}, \quad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

⊛ The Weinberg, or weak mixing angle.

⊛ We also call the quantum number of the original B_μ field $\boxed{\text{"weak" Hypercharge } Y \quad SU(2) \times U_Y(1)}$

⊛ To confirm our charge assignments, let's consider a field with an arbitrary Hypercharge:

$$\boxed{\psi(x) \to e^{i\alpha^a(x) T^a} e^{i Y \beta(x)} \psi(x)}$$

Then the covariant derivative is written as:

$$D_\mu = \partial_\mu - i g A_\mu^a T^a - i g' Y B_\mu$$

$$= \partial_\mu - i \frac{g}{\sqrt{2}} \left(W_\mu^- T^+ + W_\mu^+ T^- \right) \quad \boxed{T^\pm = T_1 \pm i T_2}$$

$$- i \frac{1}{\sqrt{g^2+g'^2}} \underbrace{Z_\mu^0}_{= \frac{1}{\sqrt{g^2+g'^2}}(g A_\mu^3 - g' B_\mu)} \left(g^2 T^3 - g'^2 Y \right)$$

$$- i \frac{g g'}{\sqrt{g^2+g'^2}} \underbrace{A_\mu^0}_{= \frac{1}{\sqrt{g^2+g'^2}}(g' A_\mu^3 + g B_\mu)} \left(T^3 + Y \right)$$

so total $= \frac{1}{g^2+g'^2} g (g^2+g'^2)$ so total $= \frac{1}{g^2+g'^2}(\cancel{g'^2+g^2}) g'$

So we identify the electric charge as:

$$\boxed{Q = T^3 + Y} \quad \text{with:} \quad \boxed{e \equiv \frac{g g'}{\sqrt{g^2+g'^2}} = g \sin\theta_W}$$

$\boxed{\text{Ex:}}$ $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ $\leftarrow T^3 = +\frac{1}{2}$, $\leftarrow T^3 = -\frac{1}{2}$; since we set $Y = \frac{1}{2}$

∴ Higgs field $h(x)$ is electrically neutral, won't couple directly to A_μ^0
[the complex ϕ^+ is completely eaten ($m_{W^\pm}$) as is imaginary part of ϕ^0 (m_{Z^0}!)]

Also note that $\boxed{Q \begin{pmatrix} 0 \\ v \end{pmatrix} = 0}$, ∴ A_μ^0 remains massless, as per our general analysis on p. 85. yes!

$\boxed{\text{Ex:}}$ $E_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ is assigned $Y = \ominus \frac{1}{2}$

so that only the e^- couples to A_μ^0, and not ν_e

$\boxed{\text{Ex:}}$ $Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$ is assigned $Y = \oplus \frac{1}{6}$ to get $Q_u = \frac{2}{3}$ and $Q_d = -\frac{1}{3}$.

$\boxed{\text{Ex:}}\ (T^3_{adj}\,W^\pm)^a = \underbrace{(T^3)_{ab}}_{= i\,\varepsilon^{a3b}} (W^\pm)_b \quad \{ = \frac{1}{\sqrt{2}}(\delta^{b1} \mp i\delta^{b2})$

$$\therefore = \frac{i}{\sqrt{2}}\left(\varepsilon^{a31} \mp i\varepsilon^{a32}\right) = \frac{i}{\sqrt{2}}\left(\delta^{a2} \pm i\,\delta^{a1}\right)$$

$$= \mp\frac{1}{\sqrt{2}}\left(\delta^{a1} \mp i\,\delta^{a2}\right) = \mp\, W^\pm$$

[Well W^+ destroys a W^+ particle and creates the opposite, the W^- particle: e.g. $\bar{E}\, T^+ E\, W^+$]

$\boxed{\text{Ex:}}\ (T^3_{adj}\ Z^0)^a = (T^3_{ab})\,\delta^{b3} = 0$ as expected

$\boxed{\text{Final, convenient form for the covariant derivative}}$

$$Z_\nu^0 \text{ coupling} = \frac{1}{\sqrt{g^2+g'^2}}\left(g^2\,T^3 - g'^2\,Y\right)$$

$$= \frac{1}{\sqrt{\quad}}\left[(g^2+g'^2)\,T^3 - g'^2\,\overset{= T^3+Y}{Q}\right]$$

$$= \sqrt{g^2+g'^2}\left[T^3 - \sin^2\theta_W\,Q\right] = \frac{g}{\cos\theta_W}\left[T^3 - \sin^2\theta_W\,Q\right]$$

$$\therefore\quad \boxed{\begin{aligned} D_\mu = \partial_\mu &- i\frac{g}{\sqrt{2}}\left(W^+_\mu T^+ + W^-_\mu T^-\right) \\ &- i\frac{g}{\cos\theta_W} Z^0_\mu\left(T^3 - \sin^2\theta_W\,Q\right) \\ &- i e A^0_\nu\, Q \end{aligned}}$$

$\boxed{\text{N.B. can eliminate } g = e/\sin\theta_W}$

The independent parameters of the standard model

* Can choose: $\boxed{e,\ \sin^2\theta_W,\ M_W} \leftrightarrow G_F$

* Plus the quark masses (later!)

* All other quantities can be expressed in terms of these:

e.g. $g = e/\sin\theta_W$

$$\boxed{M_Z = M_W/\cos\theta_W}$$

* Of particular interest is this. In a general theory, without an underlying symmetry, all elementary particle masses are independent parameters (and all must be separately, and "infinitely", renormalized).

* However, the original $SU(2)$ symmetry connects the A^3_μ component of Z^μ with the $A^{1,2}_\mu$ in $W^\pm_\mu$, so the squared mass matrix must look like

$$\underset{\substack{1,2,3,Y\\ \| \\ B}}{M^2_{ab}} = \frac{v^2}{4}\begin{bmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & g'^2 \end{bmatrix}$$

* If we turned off the coupling g', then all 3 A^a's would be degenerate, and the theory would have a global $SU(2)$ symmetry (i.e. interchange $A^{a=1} \leftrightarrow A^{a=3}$ in all space) } called "custodial" $SU(2)$ symmetry

* When we turn on g', we get mixing between B and A^3 (having broken custodial $SU(2)$), and we end up with:

$M_Z = M_W/\cos\theta_W$ at tree level

* A natural "zeroth-order" relation for any symmetry breaking mechanism having an underlying global $SU(2)$ symmetry [eg Technicolor]

Chiral Fermions

* We have yet to fully specify the V−A structure of the weak interactions in $SU(2) \times U_Y(1)$

* Recall that:

$$\bar{\psi} i \not{\partial} \psi = \bar{\psi}_L i \not{\partial} \psi_L + \bar{\psi}_R i \not{\partial} \psi_R$$

where $\boxed{\psi_{L,R} \equiv \frac{1}{2}(1 \mp \gamma^5)\, \psi_{L,R}}$

* Now we must end up with $\boxed{\text{only}}$ chiral-left fields coupled to the $W^\pm$, in accord with huge successes of the V-A theory.

* But, we must have $\boxed{\text{both}}$ L and R in the EM coupling.

* This is accomplished by giving SU(2) quantum numbers only to the chiral-left fermions, which also means giving a different Y to the chiral-right fields.

$$\boxed{SU(2)_L \times U(1)_Y}$$

$$\boxed{\mathcal{L} = \bar{E}_L i \not{\partial} E_L + \bar{e}_R i \not{\partial} e_R} \quad \left(\begin{array}{c} + \text{muon} + \text{tau} \\ + \text{quarks} \end{array}\right)$$

* $E_L \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ has $T_3 = \begin{array}{c} +\frac{1}{2} \\ -\frac{1}{2} \end{array}$ and $\boxed{Y = -\frac{1}{2}}$

* $e_R^- = SU(2)$ singlet i.e. $T_3 = 0$ and so $\boxed{Y = -1}$ to get correct $Q = -1$.

* Since the neutrino is <u>neutral</u>, this implies that

$$\nu_R \text{ has } T_3 = 0 \text{ and } Y = 0$$

i.e. the right-handed neutrino completely de-couples from the standard model (it's "sterile"!)

* In a real sense, we treat L and R components as <u>different</u> physical particles!
* So for the interaction part of $\mathcal{L}$, we have: [See bottom pgs. 94+92]

$g = e/\sin\theta_W$ not an independent parameter

$$\mathcal{L}_{int} = g\left(W_\mu^+ J_W^{\mu+} + W_\mu^- J_W^{\mu-} + Z_\mu^0 J_Z^\mu\right)$$

$$+ e A_\mu J^\mu_{EM}$$

where $J_W^{\mu+} = \frac{1}{\sqrt{2}}\Big(\underbrace{\bar{\nu}_L \gamma^\mu e_L}_{\bar{E}_L \gamma^\mu T_+ E_L} + \text{muon} + \ldots\Big)$ * See P+S Eq. (20.80) for quark currents

$$J_W^{\mu-} = \frac{1}{\sqrt{2}}\Big(\underbrace{\bar{e}_L \gamma^\mu \nu_L}_{\bar{E}_L \gamma^\mu T_- E_L} + \text{muon} + \ldots\Big)$$

$$J_Z^\mu = \frac{1}{\cos\theta_W}\Big[\overbrace{\tfrac{1}{2}\bar{\nu}_L\gamma^\mu \nu_L + \left(-\tfrac{1}{2} + \sin^2\theta_W\right)\bar{e}_L\gamma^\mu e_L}^{\bar{E}_L (T^3 - \sin^2\theta_W Q) E_L}$$

$$+ \underbrace{\sin^2\theta_W\, \bar{e}_R \gamma^\mu e_R}_{\bar{e}_R (T^3 - \sin^2\theta_W Q) e_R,\ T^3 = 0}\Big]$$

$+$ muon $+ \ldots$

$$\text{*}\ J^\mu_{EM} = \bar{E}_L \gamma^\mu Q E_L + \bar{e}_R \gamma^\mu Q e_R$$

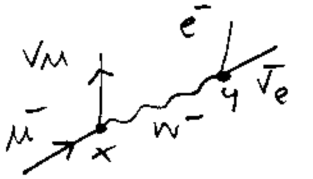
$$= -(\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R) = -\bar{e}\gamma^\mu e \;!!$$

* So although the <u>charged</u> weak current is purely <u>left-handed</u>, the <u>neutral</u> weak current also has right handed pieces, for all <u>charged</u> particles (since Z^μ is a mixture of $A^{3\mu}$ which is purely left, while B^μ is both right and left)

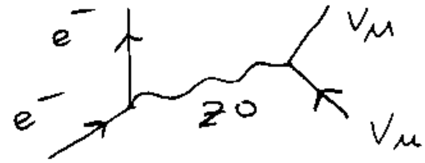
* $\boxed{\text{We're not done yet}}$ Where are the fermion masses?? Let's wait!

Charged and Neutral Currents

- We already know e from low energy QED scattering
- We'll get M_W and $\sin^2\theta_W$ from low energy weak processes



charged current, because the lepton changes its charge



neutral current

- $$S_{fi}^{(2)} = \frac{(-i)^2}{\cancel{2!}} \int d^4x\, d^4y\, T\left(\mathcal{H}_I(x)\, \mathcal{H}_I(y)\right)$$

where the first-order S-matrix vanishes, because we need $\underline{\underline{4}}$ fermion fields to generate the external contractions.

$$-\mathcal{H}_I = \mathcal{L}_I = g\left(J_W^{+\mu} W_\mu^+ + J_W^{-\mu} W_\mu^- + J_Z^\mu Z^\mu\right)$$

- We now enlarge our $\mathcal{L}$ to include the E_L and M_L doublets:

$$M_L = \begin{pmatrix} \nu_\mu(x) \\ \mu^-(x) \end{pmatrix}_L \equiv \frac{1}{2}(1-\gamma^5)\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

- We get 2 cross-terms in all cases

e.g. $$T(\mathcal{H}\,\mathcal{H}) = \cancel{2} \times \left(\frac{g}{\sqrt{2}}\right)^2 N\left[\left(\bar{M}_L \gamma^\mu T_+ M_L\right)\cdot\left(\bar{E}_L \gamma^\nu T_- E_L\right)\right]$$

$$\times \quad \underbrace{W_\mu^+(x)\, W^{-\nu}(y)}$$

W^- creates the W^- at x
W^+ destroys the W^- at y

- We haven't really worked this out, but it ought to look like:

[in unitary gauge] $$\int \frac{d^4q}{(2\pi)^4}\, e^{-iq(x-y)}\, \frac{(-i)\left(g^{\mu\nu} - q^\nu q^\nu/M_W^2\right)}{q^2 - M_W^2} \approx \frac{+i g^{\mu\nu}}{M_W^2}\, \delta^4(x-y)$$

(see next page)

[when we convolute the propagator with lepton plane waves that yield $q^2 \ll M_W^2$, and where the $q^\mu q^\nu / M_W^2$ part will make a correction $\sim (M_{lepton}/M_W)^2 \ll 1$, when contracted into the lepton currents]

$$\therefore\ S_{fi}^{(2)}\Big|_{GWS}^{W^- \text{current}} = \underbrace{-}_{(-i)^2}\left[\frac{g}{\sqrt{2}}\right]^2 \times \underbrace{\frac{1}{4}}_{\text{From } (\frac{1}{2})(1-\gamma^5) \text{ projectors}} \times \frac{1}{M_W^2} \times \underbrace{(i)}_{\text{From W propagator}} = \left[ig^2/8M_W^2\right]$$

$$\therefore\ S_{fi}^{(2)}\Big| = -\frac{ig^2}{8M_W^2}\int d^4x\,\left[\bar{\nu}_\mu(x)\gamma^\mu(1-\gamma^5)\mu(x)\right]\left[\bar{e}(x)\gamma^\mu(1-\gamma^5)\nu_e(x)\right]$$

$$\equiv S_{fi}^{(1)}\Big|_{\text{theory}}^{V-A} = -i\int d^4x\,\underbrace{\mathcal{H}_I^{V-A}(x)}_{= -\mathcal{L}_I^{V-A}} = \frac{G_F}{\sqrt{2}}[\quad][\quad]$$

$1.166\times10^{-5}\ \text{GeV}^{-2}$ from measured $\mu \to e\nu\bar{\nu}$

$$\therefore\ \boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}}$$

⊛ Recall that $M_W = \frac{gV}{2}$ ← can swap M_W for the VEV of the Higgs Field

$$\therefore\ \boxed{V = \sqrt{\frac{1}{\sqrt{2}G_F}} \cong 250\ \text{GeV}}$$

⊛ What we really want to predict is M_W and M_Z

⊛ We need $g = e/\sin\theta_W$

$$\therefore\ \frac{G_F}{\sqrt{2}} = \frac{e^2}{8\sin^2\theta_W M_W^2} = \frac{4\pi\alpha_{EM}}{8\sin^2\theta_W M_W^2}$$

$$\therefore\ \boxed{M_W^2 = \frac{\pi\alpha_{EM}}{\sqrt{2}G_F\sin^2\theta_W}}$$

← We get this from neutral currents, the prediction that actually got GWS the Nobel in 1979

$$\boxed{M_Z = M_W/\cos\theta_W}$$

Remember: $\overbrace{T_3 - Q\sin^2\theta_W}$

$$\boxed{e^- \nu_\mu \longrightarrow e^- \nu_\mu}$$

⊛ $J_Z^\mu = \frac{1}{\cos\theta_W}\Big[\bar{\nu}_L\gamma^\mu\left(\tfrac{1}{2}\right)\nu_L + \bar{e}_L\gamma^\mu\left(\left(-\tfrac{1}{2}\right)+\sin^2\theta_W\right)e_L$
$+ \bar{e}_R\gamma^\mu(\sin^2\theta_W)e_R\Big]$

Now $\bar{e}_R\gamma^\mu e_R = \underbrace{\left[\tfrac{1}{2}(1+\gamma^5)e\right]^\dagger\gamma^0\gamma^\mu}_{\text{anticommutes twice}}\left[\tfrac{1}{2}(1+\gamma^5)e\right]$

$= \left(\tfrac{1}{2}\right)\bar{e}\gamma^\mu(1+\gamma^5)e$ since $(1+\gamma^5)^2 = 2(1+\gamma^5)$

$\therefore J_Z^\mu = \frac{1}{2}\frac{1}{\cos\theta_W}\Big[\tfrac{1}{2}\bar{\nu}\gamma^\mu(1-\gamma^5)\nu + \bar{e}\gamma^\mu(g_V - g_A\gamma^5)e\Big]$

i.e e_L part : $\gamma^\mu(1-\gamma^5)\left(-\tfrac{1}{2}+\sin^2\theta_W\right)$
e_R part : $\gamma^\mu(1+\gamma^5)\left(\sin^2\theta_W\right)$

$$\therefore \boxed{g_V = -\tfrac{1}{2} + 2\sin^2\theta_W\ ,\quad g_A = -\tfrac{1}{2}}$$

⊛ So exactly same analysis

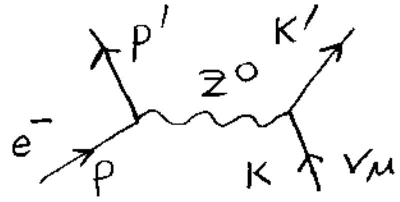
$\therefore S^{(2)}_{e\nu\to e\nu}\Big|_{GWS} \xrightarrow{s \ll M_Z^2} \frac{g^2}{\cos^2\theta_W}\times\left(\frac{1}{2}\right)^2\times\frac{1}{M_Z^2}\times i$ ($\frac{1}{2}$ to replace eg $\bar{e}_L\cdot e_L \to \bar{e}\cdot e$, and $\bar{\nu}_L\cdot\nu \to \bar{\nu}\cdot\nu$)

$\times\int d^4x\ \tfrac{1}{2}\left[\bar{\nu}\gamma^\mu(1-\gamma^5)\nu\right]\left[\bar{e}\gamma^\mu(g_V - g_A\gamma^5)e\right]$

⊛ Recall that $M_W = M_Z\cos\theta_W$ and $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$

$$\therefore \mathcal{L}_{eff}^{\nu e\to\nu e} = -\frac{G_F}{\sqrt{2}}\underbrace{\left[\bar{u}_\nu(k')\gamma^\mu(1-\gamma^5)u_\nu(k)\right]\left[\bar{u}_e(p')\gamma^\mu(g_V - g_A\gamma^5)u_e(p)\right]}_{\mathcal{M}\ \text{[after doing the }\int d^4x]}$$

⊛ Let's evaluate cross-section in the centre-of-mass

⊛ 

⊛ Let's neglect the electron mass

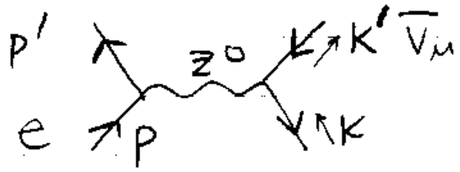
(Skipped over details)

$$\therefore \sum_{spins} |M|^2 = \mathrm{Tr}\left[\gamma^\mu(1-\gamma^5)\not{K}\gamma^\nu(1-\gamma^5)\not{K}'\right]$$

$$\times\ \mathrm{Tr}\left[\gamma_\mu(g_V-g_A\gamma^5)\not{p}\gamma_\nu(g_V-g_A\gamma^5)\not{p}'\right]$$

⊛ We will also want to do

$$\boxed{e^-\ \overline{\nu}_\mu \rightarrow e^-\ \overline{\nu}_\mu \qquad \therefore \text{Interchange } K \leftrightarrow K'}$$



⊛ Back to $\sum_{spins} = \mathrm{Tr}\left[\gamma^\mu\underbrace{(1-\gamma^5)^2}_{2(1-\gamma^5)}\not{K}\gamma^\nu\not{K}'\right]$

$$\times\ \mathrm{Tr}\left[\gamma_\mu(g_V-g_A\gamma^5)^2\not{p}\gamma_\nu\not{p}'\right]$$

⊛ Neutrino $\mathrm{Tr} = 2\times\mathrm{Tr}\left[(1+\gamma^5)\gamma^\mu\not{K}\gamma^\nu\not{K}'\right]$

$$= (2\times 4)\times\left[K^\mu K'^\nu - g^{\mu\nu}K\cdot K' + K'^\mu K^\nu \right. \qquad \mu\leftrightarrow\nu \text{ symmetric}$$

$$\left. -i\varepsilon^{\mu\alpha\nu\beta}K_\alpha K'_\beta\right] \qquad \mu\leftrightarrow\nu \text{ antisymm.}$$

⊛ For electron Tr : use $(g_V-g_A\gamma^5)^2 = g_V^2+g_A^2-2g_Vg_A\gamma^5$

$$\therefore = (4)\left[(g_V^2+g_A^2)(p_\mu p'_\nu - g_{\mu\nu}p\cdot p' + p'_\mu p_\nu)\right.$$

$$\left. -2g_Vg_A\, i\,\varepsilon_{\mu\alpha\nu\beta}p^\alpha p'^\beta\right]$$

⊛ [Symmetric] × [Symmetric]

$$= \left.\begin{aligned} &\overset{+1}{(K\cdot p)(K'\cdot p')} \overset{-1}{-(K\cdot K')(p\cdot p')} \overset{+1}{+(K\cdot p')(K'\cdot p)} \\ &-(K\cdot K')(p\cdot p')(1-4+1) \\ &= 2\left[(K\cdot p)(K'\cdot p') + (K\cdot p')(K'\cdot p)\right] \end{aligned}\right\} \times 32\,(g_V^2+g_A^2)$$

⊛ [Anti-Symm] × [Anti-Symm] $= \varepsilon^{\mu\alpha\nu\beta} k_\alpha k'_\beta \, \varepsilon_{\mu\rho\nu\sigma} P_\rho P'_\sigma$

(See proof on pg 41)

$$= -2\left(g^\alpha{}_\rho g^\beta{}_\sigma - g^\alpha{}_\sigma g^\beta{}_\rho\right) k_\alpha k'_\beta P_\rho P'_\sigma$$

$$= -2\left[(k\cdot p)(k'\cdot p') - (k\cdot p')(k'\cdot p)\right]$$

$\times\, 32(i)^2 \times 2 g_V g_A$

$$\therefore \sum_{spins} = 64\left[(g_V+g_A)^2 (k\cdot p)(k'\cdot p') + (g_V-g_A)^2 (k\cdot p')(k'\cdot p)\right]$$

⊛ At this point, $\boxed{\text{update our substitution rule:}}$

$$e\nu_\mu \to e\nu_\mu \implies e\bar{\nu}_\mu \to e\bar{\nu}_\mu$$

by: $k \Longleftrightarrow k'$

$\therefore$ by: $g_V \longrightarrow -g_V$

⊛ $\boxed{\text{Spin Averaged Scattering Probability}}$

⊛ Number of states / unit volume

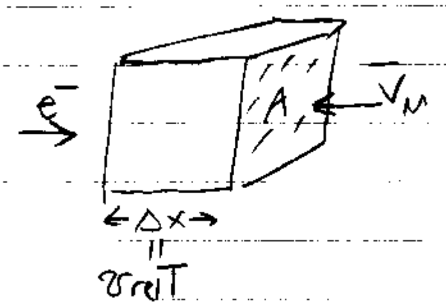
⊛ Recall our convention of 2E particles / unit volume

⊛ ∴ Divide by ~~2E to get~~ # states per particle

$$\bar{P} = \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \frac{d^3k'}{(2\pi)^3 2E_{k'}} \times \frac{1}{2}\sum_{spins} |M|^2 \times \frac{G_F^2}{2}$$

$\boxed{\text{C.O.M.}}$ $\quad * (2\pi)^4 \delta^4(P_f - P_i) \times V\, T$

⊛ neutrino only comes with $\underline{\underline{1}}$ polarization!



⊛ Time to sweep out volume V:

$$(v_{rel} T)\, A = V$$

$$⊛\; T = \frac{V}{v_{rel} A}\;! = \frac{V}{2A}$$

⊛ Finally convert to

$P = \frac{V}{A}$ for 1 e^- $\left[* \frac{1}{2E_p} \frac{e^-}{V}\right]$

and 1 $\bar{\nu}_\mu$ $\left[* \frac{1}{2E_\mu} \frac{e^-}{V}\right]$

bottom p.99

⊛ Collect phase space integral pre-factors:

$$\underbrace{\frac{1}{2}}_{v_{rel}}\underbrace{\left(\frac{1}{2E}\right)^2}_{\text{incident particle densities}} \times \underbrace{\frac{(2\pi)^4}{[2(2\pi)^3]^2}}_{\text{Spin Average}} \times \underbrace{\frac{1}{2}}_{\text{Spin Average}} \times 64 \times \frac{G_F^2}{2}$$

$$\therefore \sigma^{C.O.M.}(e\nu_\mu \to e\nu_\mu) = \frac{G_F^2}{8\pi^2 E^2}$$

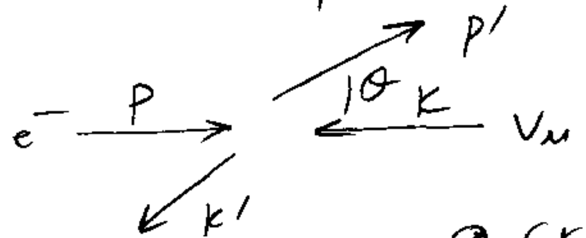
$$\times \int \frac{d^3p'}{\cancel{E_{p'}}}\frac{d^3k'}{\cancel{E_{k'}}}\,\delta^4(p+k-p'-k')\begin{bmatrix}(g_V+g_A)^2(k\cdot p)(k'\cdot p') \\ + \\ (g_V-g_A)^2(k\cdot p')(k'\cdot p)\end{bmatrix}$$

⊛ $\int d^3k'\,\delta^3(\ ) = 1$

⊛ $\int d^3p'\,\delta^0(2E-2E_{p'}) = (2\pi)\int d(\cos\theta)\,E'^2 dE' \times \frac{1}{2}\delta(E-E')$

$$= \pi\, \cancel{E^2} \int_{-1}^{1} d(\cos\theta)$$

Remember: $E = |\vec{p}|$ for all since we set $m_e = 0$



⊛ $(k\cdot p')(k'\cdot p)$
$= E^4(1+\cos\theta)^2$

⊛ $(k\cdot p)(k'\cdot p') = 4E^4$

⊛ $\int_{-1}^{1} d(\cos\theta)\left[4(g_V+g_A)^2 + (g_V-g_A)^2\underbrace{(1+\cos\theta)^2}_{1+\cancel{\cos\theta}+\cos^2\theta \,=\, 2+2/3 \,=\, 8/3}\right]$

$$= \frac{8}{3}\left[3(g_V+g_A)^2 + (g_V-g_A)^2\right] = \frac{8}{3}\times 4\left[g_V^2 + g_Vg_A + g_A^2\right]$$

$$\therefore \sigma(e\nu_\mu \to e\nu_\mu) = \frac{4G_F^2E^2}{3\pi}\left(g_V^2 + g_Vg_A + g_A^2\right)$$

$\boxed{\text{Recall: Cross-section is Lorentz Invariant!}}$ Use:

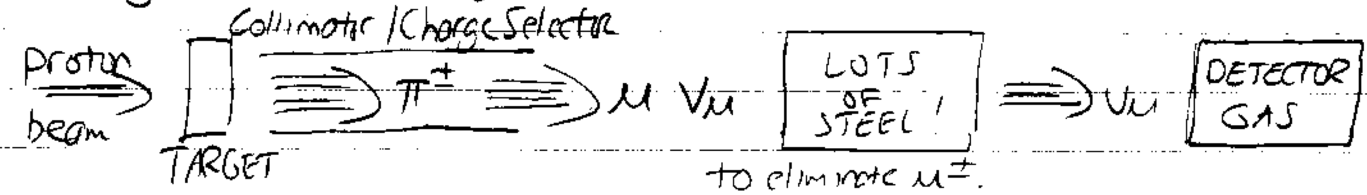
$$s = (p+k)^2 = 4E^2 \text{ in } e\nu_\mu \text{ C.O.M.}$$

$$\therefore\ \boxed{\begin{aligned}\sigma\begin{pmatrix} e\nu_\mu \to e\nu_\mu \\ e\bar{\nu}_\mu \to e\bar{\nu}_\mu \end{pmatrix} &= \frac{G_F^2 s}{3\pi}\left(g_V^2 \pm g_V g_A + g_A^2\right) \\ g_V = -\tfrac{1}{2} + 2\sin^2\theta_W &, \quad g_A = -\tfrac{1}{2}\end{aligned}}$$

Lorentz Invariant x-section

⊛ Cross-sections are extremely small $\therefore$ Need large neutrino fluxes, more important than large $\sqrt{s}$!

⊛ Best measurements from proton synchrotron at CERN, running in fixed-target mode



⊛ So in the lab frame, with target e^- essentially at rest:

$$S_{lab} = (p_\nu + p_e)^2 \simeq 2p_\nu \cdot p_e = 2m_e E_\nu \quad (\text{since } p_e^2 \ll 2m_e E_\nu)$$

⊛ Recent measurements (mid '90s by CHARM + CHARM II):

$$\sigma^{lab}_{expt}(e\nu_\mu) = (2.2 \pm 0.4 \pm 0.4)\times 10^{-42}\,\text{cm}^2\ E_\nu/\text{GeV}$$

$$\sigma^{lab}_{expt}(e\bar{\nu}_\mu) = (1.6 \pm 0.3 \pm 0.3)\times 10^{-42}\,\text{cm}^2\ E_\nu/\text{GeV}$$

⊛ Much better determination of $\sin^2\theta_W$ comes from the ratio (must be, among other things, that ν $\bar{\nu}$ absolute flux uncertainties cancel in ratio?):

$$R \equiv \frac{\sigma(e\nu_\mu)}{\sigma(e\bar{\nu}_\mu)} = \frac{3(1-4\sin^2\theta_W) + 16\sin^4\theta_W}{(1-4\sin^2\theta_W) + 16\sin^4\theta_W}$$

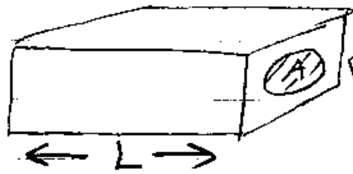
CHARM II
P. Vilain et al.
Phys. Lett. B335, 246 (1994)

$$\boxed{R_{expt} \Rightarrow \sin^2\theta_W = 0.224(9)}$$

⊛ $\boxed{\text{Side-Bar}}$ $\boxed{\text{Neutrinos interact \underline{extremely} weakly !}}$

⊛ $\sigma \approx 10^{-41}\,\text{cm}^2$ at $E_\nu \approx 10$ GeV

⊛ Probability of scattering $\underline{P} \equiv \sigma / A$



⊛ Let's take the number density of target particles (e^-/quarks!) to be : $n_T \approx 10^{25}/\text{cm}^3$ (e.g. Iron ?)

∴ The <u>total</u> probability for scattering of <u>one</u> incident neutrino :

$$P_{tot} = \frac{\sigma}{\cancel{A}} \times (\underbrace{n_T \cancel{A}\, L}_{\text{total \# scatterers}}) \equiv 1 \quad \text{for "definite" scattering}$$

∴ $\boxed{\text{Scattering Length } L = \frac{1}{\sigma n_T}} \approx 1/(10^{-41}\,\text{cm}^2 \times 10^{25}/\text{cm}^3)$

$\boxed{\approx 10^{16}\,\text{cm} \approx 0.01 \text{ Light-Years !!}}$

⊛ Well, either that, or a beam of $O(10^{14})$ ν's hitting an affordable target with $L = O(1\,\text{m})$!!

!

$\bar{\nu}_\mu \rightarrow$

Figure 1.6 First example of weak neutral-current process $\bar{\nu}_\mu + e \rightarrow \bar{\nu}_\mu + e$ observed in heavy-liquid bubble chamber Gargamelle at CERN irradiated with a $\bar{\nu}_\mu$ beam (Hasert *et al.*, 1973). A single electron of energy 400 MeV is projected at a small angle ($1.5 \pm 1.5°$) to the beam, and is identified by bremsstrahlung and pair production along the track (see Chapter 2). <u>About 10^9 $\bar{\nu}_\mu$'s traverse the chamber in each pulse and three such events were observed in 1.4 million pictures.</u> (Courtesy CERN.)

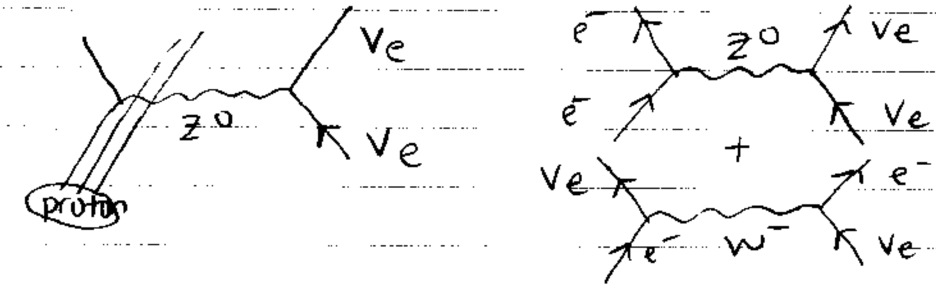
⊛ This picture from Perkins textbook

⊛ Hasert et al. Physics Letters B<u>46</u>, 138 (1973)
Nuclear Physics B<u>73</u>, 1 (1974)

Back to $\sin^2\theta_W$

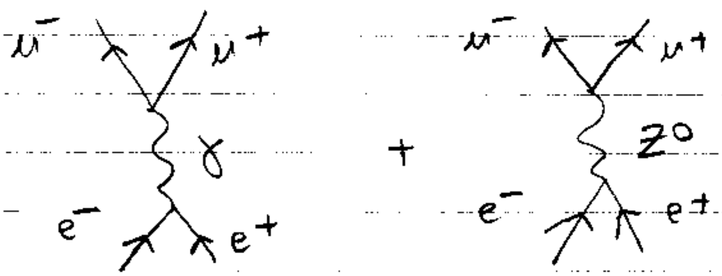
* Many different processes have been used to measure it (it's a universal, renormalizable, gauge-theory parameter)

* Exs:



Neutral Current + Charged Current

* Also, recall from last semester:



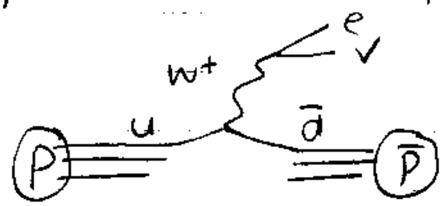
* Particle Data Group Best Fit:

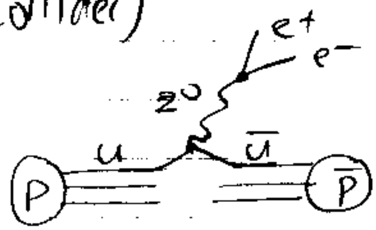
$$\boxed{\sin^2\theta_W = 0.22302(40)}$$

* So at last, predict:

	Expt
$M_W = \sqrt{\dfrac{\pi\,\alpha_{EM}}{\sqrt{2}\,G_F\sin^2\theta_W}} = 79.0\text{ GeV}$	$80.41(10)$ GeV
$M_Z = \dfrac{M_W}{\cos\theta_W} = 89.6\text{ GeV}$	$91.187(7)$ GeV

* Discovered in 1983 at CERN, re-tooled to run as a $p\bar{p}$ collider (currently e^-e^+ collider)

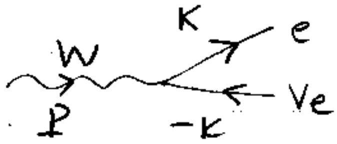




* $\boxed{\text{Assignment}}$ Calculate W and Z decay rates

(Peskin + Schroeder Problem 20.2 — But I'll give you some hints)

* Ex:



} Neglect fermion masses

invariant amplitude (unitary gauge)

$$\mathcal{M} = -i\frac{g}{\sqrt{2}}\, \varepsilon_W^{\mu}(p)\, \bar{u}(k)\, \gamma^{\mu}\, \tfrac{1}{2}(1-\gamma^5)\, v(k)$$

$$\sum_{\text{spins}} \varepsilon^{\mu}\varepsilon^{\nu} = -g^{\mu\nu} + \cancel{\frac{p^{\mu}p^{\nu}}{M_W^2}}$$

should $\sim (m_e/M_W)^2$ when contracted into lepton currents

* $$\boxed{\Gamma(W \to e\,\bar{\nu}_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \equiv \Gamma_W^0 = 0.23\text{ GeV}}$$

$W \not\to t\bar{b}$ since $m_t > m_W$!!

* $$\boxed{\Gamma^W_{tot} = \Gamma_W^0 * \Big(\underset{\text{families}}{3\text{ lepton}} + \underset{\text{families}}{2\text{ quark}} \times 3\text{ colors}\Big) = 9\,\Gamma_W^0 = 2.07\text{GeV}}$$

* $$\boxed{\Gamma^W_{expt} = 2.06(6)\text{ GeV}}\quad !$$

* You will do Γ_Z (formulas provided in assignment)

$$\boxed{\begin{aligned}\Gamma^Z_{theory} &\cong 2.55\ \text{GeV}\\ \Gamma^Z_{exp} &= 2.490(7)\ \text{GeV}\end{aligned}}$$

Radiative corrections give complete agreement: See PDG.

Fermion Masses

First: Case of One Generation

* Recall the form of the mass term:

$$\mathcal{L}_{mass} = -m\,\overline{\psi}\psi = -m\left(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L\right)$$

* We cannot simply add this to our $SU(2)_L \times U(1)_Y$ model!

$$\text{e.g.}\quad \overline{e}_R e_L \xrightarrow{SU(2)_L} \underbrace{\overline{e}_R}_{\text{a singlet}}\left(e^{i\omega^a(x)T^a}\begin{pmatrix}0\\ e_L\end{pmatrix}\right) \neq \text{invariant}$$

$$\xrightarrow{U(1)_Y} \overline{e}_R e_L\, e^{i\beta(x)\underbrace{(Y_L - Y_R)}_{\neq 0}}\,! \neq \text{invariant}$$

* Well, we can generate gauge-invariant fermion masses "spontaneously", by the same Higgs mechanism that gave masses to the $W^\pm$, Z:

For a Hermitian $\mathcal{L}$

$$\boxed{\mathcal{L}^{(e)}_{ffh} = -\lambda_e\left[\overline{e}_R\underbrace{\left(\phi^{+} E_L\right)}_{SU(2)_L\text{ invariant}} + \left(\overline{E}_L\,\phi\right)e_R\right]}$$

$$q = +1 \quad -\tfrac{1}{2} \quad -\tfrac{1}{2} \quad \therefore\ U(1)_Y \text{ invariant}$$

* In the unitary gauge $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h(x)\end{pmatrix}$

* To identify the mass term, set $h(x)=0$ (including it gives the Higgs-electron interaction)

$$\therefore\ \phi^{+} E_L = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & v\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L = \frac{v}{\sqrt{2}}\,e_L$$

$$\overline{E}_L\,\phi = \begin{pmatrix}\overline{\nu}_e & \overline{e}\end{pmatrix}_L \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} = \frac{v}{\sqrt{2}}\,\overline{e_L}$$

$$\therefore\ \mathcal{L}^{e}_{ffh} = -\frac{\lambda_e v}{\sqrt{2}}\left[\overline{e}_R e_L + \overline{e}_L e_R\right] + \text{interactions}$$

$$= -m_e\,\overline{\psi}_e\psi_e$$

where $\boxed{\lambda_e = \frac{\sqrt{2}\,m_e}{v}} \cong \sqrt{2}\,\frac{\times 0.511\text{ MeV}}{250\text{ GeV}} \approx 10^{-6}$! Unnatural, unexplained hierarchy of Higgs couplings!

⊛ So in the doublet $E = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$, we've given a mass to the "down"-type member. $\boxed{\text{What about the up particle mass?}}$

$$\boxed{\mathcal{L}^{(up)}_{mass} = -\lambda_\nu \left[\bar{\nu}_R \left(\underbrace{\varepsilon^{ab} E_L^a \phi^b} \right) + \left(\varepsilon^{ab} \bar{E}_L^a \phi^{+b} \right) \nu_R \right]}$$

$Y = 0 \qquad -\frac{1}{2} + \frac{1}{2} = 0 \quad \therefore U(1)_Y$ singlet ✓

⊛ But is this an $SU(2)_L$ singlet? Yes, but first:

$$\varepsilon^{ab} E_L^a \phi^b \Big|_{h(x)=0} = \frac{1}{\sqrt{2}} \varepsilon^{ab} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L^a \begin{pmatrix} 0 \\ v \end{pmatrix}^b$$

$\leftarrow \therefore b = 2$ only

Hence $a = 1$ only

$$= +\frac{1}{\sqrt{2}} v \, \nu_{e,L}$$

$$\therefore \mathcal{L}^{up} = -\frac{\lambda_\nu v}{\sqrt{2}} \left(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R \right) \quad \text{and} \quad \left(m_\nu = \frac{\lambda_\nu v}{\sqrt{2}} \right)$$

⊛ To demonstrate the $SU(2)_L$ invariance, consider a G.T.:

$$\varepsilon^{ab} E^a \phi^b \longrightarrow \boxed{\varepsilon^{ab} U^{aa'} U^{bb'}} E^{a'} \phi^{b'}$$

⊛ This tensor is anti-symmetric under $a' \leftrightarrow b'$:

$$\varepsilon^{ab} \underbrace{U^{aa'} U^{bb'}}_{\text{just numbers}} = \varepsilon^{ab} U^{bb'} U^{aa'} = \underbrace{\varepsilon^{ba}}_{\text{minus}} U^{ab'} U^{ba'} \quad \text{relabel } b \rightleftarrows a$$

$$= -\varepsilon^{ab} U^{ab'} U^{ba'}$$

$\therefore \varepsilon^{ab} U^{aa'} U^{bb'}$ must $=$ constant $\times \, \varepsilon^{a'b'}$ $\qquad \therefore$ Proves Gauge Invariance of $\varepsilon^{ab} E_L^a \phi^b$

⊛ Now contract both sides with $\varepsilon^{a'b'}$

$$\therefore \underbrace{\varepsilon^{a'b'} \varepsilon^{ab} U^{aa'} U^{bb'}} = 2 \times \text{constant}$$

$$= 2! \det(U) = 2$$

$$\therefore \boxed{\varepsilon^{ab} U^{aa'} U^{bb'} = \varepsilon^{a'b'}}$$

⊛ $\boxed{\text{Can extend this to } SU(N)}$ Relevant to QCD - see pg. 120.

Higgs Couplings to Fermions

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$\therefore$ Same couplings λ for mass and interaction

$\therefore$  $\quad \lambda_f = \sqrt{2} \frac{m_f}{v} = O(1)$ for top

[Weak becomes strong!]

Multi-Generational Fermion Masses + Mixings

* $$\mathcal{L}_{mass} = -\lambda_d^{ij}\left(\bar{Q}_L^{\,i}\,\phi\right) d_R^{\,j} - \lambda_u^{ij}\,\varepsilon^{ab}\left(\bar{Q}_{La}^{\,i}\,\phi_b^{\dagger}\right) u_R^{\,j} + h.c.$$

Where now i, j labels the quark generation or family number

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \quad \cdots$$
$$i = \quad 1 \qquad\quad 2 \qquad\quad 3$$

* If we have a single generation, then
$$\frac{\lambda_{u,d}\, v}{\sqrt{2}} = m_{u,d}$$

But in a theory with several generations, $\lambda_{u,d}^{ij}$ does <u>not</u> have to be diagonal, or symmetric, or real

* $\mathcal{L}_{mass}$ = renormalizable for <u>arbitrary</u> $\lambda_{u,d}^{ij}$

* To find the <u>physical</u> quark (or lepton) fields, we must diagonalize the λ matrices

* An arbitrary complex matrix can be diagonalized by a "bi-unitary" transformation :

$$\lambda_u = U_u \, D_u \, W_u^{\dagger}$$

unitary matrices

* <u>diagonal</u> matrix with <u>positive</u> eigenvalues (the quark masses)

* Similarly

$$\lambda_d = U_d \, D_d \, W_d^{\dagger}$$

$\boxed{\text{Proof}}$ ① Since $\lambda_u \lambda_u^\dagger$ = Hermitian (with positive diagonal elements), it can be diagonalized by a unitary transformation:

$$\lambda_u \lambda_u^\dagger = U_u D_u^2 U_u^\dagger$$

Likewise: $\lambda_u^\dagger \lambda_u = W_u D_u^2 W_u^\dagger$

② Now we can show that:

$$h_u = U_u^\dagger \lambda_u W_u \quad \text{is hermitian}$$

* $h_u h_u^\dagger = U_u^\dagger \lambda_u W_u \cancel{W_u^\dagger} \cancel{\lambda_u^\dagger} U_u = D_u^2$

⊙ $h_u^\dagger h_u = W_u^\dagger \lambda_u^\dagger \cancel{U_u U_u^\dagger} \lambda W_u = D_u^2$

so eigenvalues of h_u are real

③ Hence h_u can be diagonalized by a unitary transformation

$$h_u \rightarrow (Z_u U_u^\dagger) \lambda_u (W_u \; Z_u^\dagger) = D_u$$

$$\text{or} \quad \tilde{U}_u \equiv U_u Z_u^\dagger \quad \text{and} \quad \tilde{W}_u \equiv W_u Z_u^\dagger$$

④ We can always force the eigenvalues of h_u to be positive choosing $\tilde{\tilde{W}}_u = W_u \times (\text{Diagonal Matrix})$

$\boxed{\text{QED}}$

$\boxed{\text{Now to diagonalize } \mathcal{L}_{mass}}$

$\boxed{\text{First:}}$ Make unitary transformation on $\boxed{\text{Right}}$ Handed Fields

$$u_R^i \rightarrow \cancel{W_u^{ij}} u_R^j$$

$$d_R^i \rightarrow \cancel{W_d^{ij}} d_R^j$$

⊛ up-type and ⊛ down-type fields

⊙ This cancels off the W matrices in both of:

$$\lambda_u = U_u D_u \cancel{W_u^\dagger}$$

$$\lambda_d = U_d D_d \cancel{W_d^\dagger}$$

⊛ The Right-Handed fields are coupled directly to one another in the Z^0_μ and A^{EM}_μ neutral currents

e.g. $A^{EM}_\mu \Big(Q_u \underbrace{\sum_i \bar{u}_R^i \gamma^\mu u_R^i} + Q_d \sum_i \bar{d}_R^i \gamma^\mu d_R^i \Big)$

All the e.g. Right-Handed up type quarks have the same charge $\therefore \sum_i \bar{u}_R^i u_R^i \rightarrow \bar{u}_R \cancel{W_u^\dagger} \cancel{W_u} u_R$

⊛ Ditto for the J_Z current, since the coupling is $Q \sin^2\theta_W$ again the same for all up or down-type quark,

$\therefore$ W_u and W_d disappear from the action

$\boxed{\text{Now}}$ Make similar transformation on

$$u_L \rightarrow U_u u_L$$
$$d_L \rightarrow U_d d_L$$

⊛ The mass terms have been successfully diagonalized:

$$\mathcal{L}^{(down)}_{mass} = - D_d^{ij} \left(\bar{Q}_L^i \phi \right) d_R^j + h.c.$$
$$\hookrightarrow = (\bar{u} \ \bar{d}) \tfrac{1}{\sqrt{2}} \binom{0}{v+h}$$

⊛ Diagonal

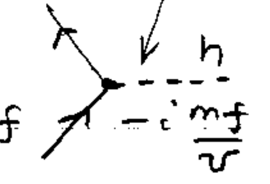
$$= - m_d^i \bar{d}_L^i d_R^i \left(1 + \frac{h(x)}{v} \right) + h.c.$$

where $m_d^i = \frac{1}{\sqrt{2}} D_d^{ii} v$

⊛ $\boxed{\text{Also notice the Higgs couplings}}$

The Higgs field couples in proportion to the fermion mass.

⊛ Strong couplings $\simeq 170/250$ to the top quark!!



$\boxed{\text{However, the } U \text{ matrices do } \underline{\text{not}} \text{ disappear}}$ ⊛ As before, they $\underline{\text{do}}$ drop out of J_Z and J_{EM},

⊛ $J_W^\mu = \frac{1}{\sqrt{2}} \bar{u}_L^i \gamma^\mu d_L^i$ $\underline{\text{before}}$ mass diagonalization

in terms of "weak eigenstates"

$\rightarrow \frac{1}{\sqrt{2}} \bar{u}_L^i \gamma^\mu (U_u^\dagger U_d)^{ij} d_L^j$ $\underline{\text{after}}$ mass diagonalization

Now physical quark states

⊛ So we recover the concept of quark mixing that we saw in $K^\pm \rightarrow \pi^\pm e \nu$ decay

$$V^{ij} = (U_u^\dagger U_d)^{ij} = \text{quark mixing matrix}$$

⊛ For two generations :

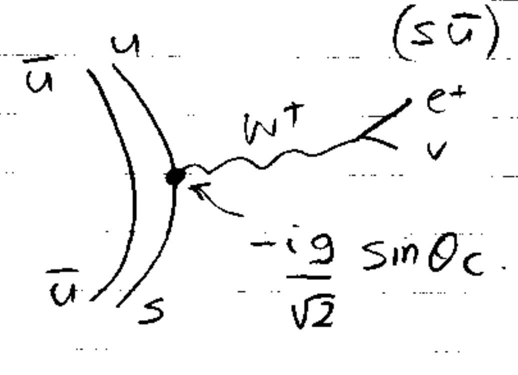
$$\begin{pmatrix} u \\ d \end{pmatrix}_i \equiv \underset{i=1}{\begin{pmatrix} u \\ d \end{pmatrix}}, \underset{i=2}{\begin{pmatrix} c \\ s \end{pmatrix}}, \underset{i=3}{\begin{pmatrix} t \\ b \end{pmatrix}}$$

and it is possible to reduce the mixing matrix to :

$$V_{ij} = \begin{pmatrix} \cos\theta_c & +\sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} = \begin{pmatrix} V_{ud} & \boxed{V_{us}} \\ V_{cd} & V_{cs} \end{pmatrix}$$

⊛ What we determined in $K^- \rightarrow \pi^- e^+ \bar{\nu}$

$(s\bar{u}) \rightarrow (u\bar{u})$



Glashow - Iliopoulos - Maiani Mechanism

⊛ Prediction of charm quark on the basis of loop effects

⊛ An important feature of the weak interactions is the dramatic suppression of flavour-changing neutral currents

Ex: $$\frac{\Gamma(K_L^0 \xrightarrow{?} \mu^+\mu^-)}{\Gamma(K^- \to \mu^-\bar{\nu}_\mu)} \approx 3\times 10^{-9}$$

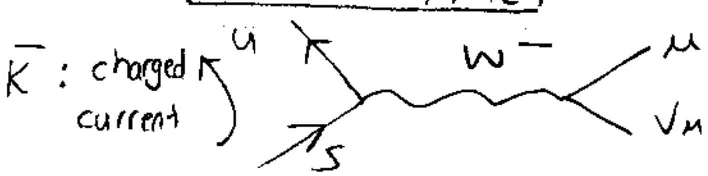
⊛ These are "flavour changing" processes, since

$$K^0 \equiv s\bar{d} \quad \text{and} \quad K^- = s\bar{u}$$

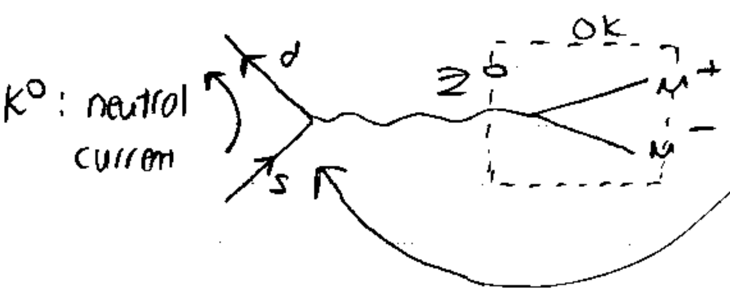
have "naked" or "open" flavour, unlike the $\pi^0 = u\bar{u}$.

⊛ In principle, one might have expected a neutral current Z^0 boson to mediate K^0 decay:

$\longrightarrow$ Time



Yes, present in the standard model.



$\boxed{\text{NO}}$ not present,

$J_Z = \bar{s}_L\gamma^\mu s_L + \bar{d}_L\gamma^\mu d_L$

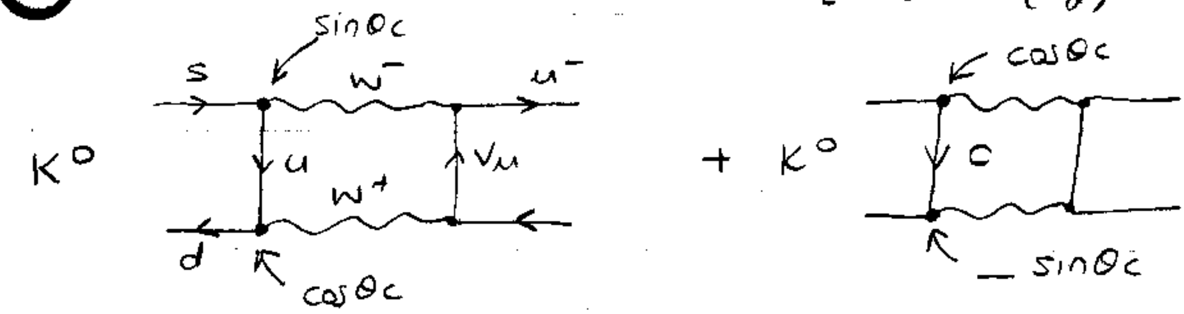
but not $\bar{s}_L\gamma^\mu d_L$

⊛ In fact, the absence of this neutral current arose in a natural way, in order to get V−A charged current but pure V EM current

⊛ We ended up with flavour neutral J_{EM} and J_{Z^0}.

⊛ $K^0 \to \mu^+\mu^-$ process does arise in the standard model, not as a F.C.N.C., but as a loop process:

Time ⟶ [Here pictured in a 2 generation world: $\binom{u}{d}$ and $\binom{c}{s}$.]



⊛ Let's first live in a world with only u, d, s quarks

⊛ Although this process is suppressed because it first occurs at loop-level (not at tree-level), the decay rate would be much too large with only u, d, s:

$$\text{Amplitude} \sim g^4 \int d^4k \frac{1}{(k^2 - m_W^2)^2}$$

[Dropping m_u and, $k^\mu k^\nu / m_W^2$ parts of the W boson propagator — How can that be? GIM will do that too!]

⊛ But also gone in R_ξ gauge, how to see agreement with unitary gauge without GIM??

$$\therefore \text{Amp} \sim \frac{g^4}{m_W^2} \sim G_F g^2 \sim G_F \frac{\alpha_{EM}}{\sin^2\theta_W}$$

(The integral receives maximum contribution from $k \sim m_W$!)

$$\therefore \frac{\Gamma(K^0 \to \mu^+\mu^-)}{\Gamma(K^- \to \mu^-\bar{\nu}_\mu)} \approx \frac{\alpha_{EM}^2}{\sin^4\theta_W} \sim 10^{-3} \quad \left[\text{instead of } 10^{-9}!!\right]$$

⊛ With the charm quark, the unitarity of the quark mixing matrix introduces a relative sign between the two diagrams:

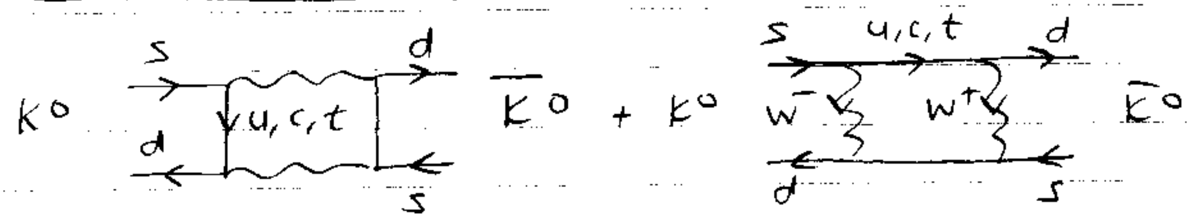
$$\therefore \text{Amp} \sim \frac{g^4}{m_W^2} \times \left(\frac{m_c^2 - m_u^2}{m_W^2}\right)$$

⟵ ⊛ Taylor expand the loop integral in quark mass, $O(m^0)$ term subtracts out!

$\therefore \Gamma$ suppressed by additional ~~factor~~ of $(m_c/m_W)^4 \approx \left(\frac{1.5\,\text{GeV}}{80\,\text{GeV}}\right)^4 = 10^{-7}$!

* G.I.M. first proposed the existence of the charm quark (1970) in order to get this cancellation mechanism as a "natural" explanation for the very small size of all flavour-changing neutral current processes

* $\boxed{\text{A similar mechanism occurs in } K^0 - \bar{K}^0 \text{ mixing}}$ Gaillard + Lee (1974)



* So the weak interactions mix the flavour eigenstates $K^0(s\bar{d})$ and $\bar{K}^0(\bar{s}d)$ to produce mixed-flavour mass eigenstates

* Eigenstates of $H_{total} = a|K^0\rangle + b|\bar{K}^0\rangle$

* Recall that in charged weak interactions:

Left-Handed Particles only $\xrightarrow[\text{Conjugation}]{\text{Charge}}$ ~~Left-Handed Anti-Particles~~ $\xrightarrow{\text{Parity}}$ Right-Handed Anti-Particles only

∴ CP combined operation was thought to be a good quantum number.

∴ these would be eigenstates

$$K_S = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$K_L = \frac{1}{\sqrt{2}}\left(\quad - \quad\right)$$

$\boxed{\text{Gaillard + Lee}}$

* Experimental $\frac{m_L - m_S}{m} \approx 10^{-14}$ explained if $m_c = 1.5$ GeV

(Just a convention without physical consequences)

* Defining $CP|K^0\rangle \equiv (+)|\bar{K}^0\rangle$, $CP\, K_{S,L} = \pm K_{S,L}$

∴ $K_S \xrightarrow{\text{"shorter" lifetime}} 2\pi$ would be allowed
$K_L \xrightarrow{\text{"longer" lifetime}} 3\pi$ only would be allowed

see pg. 37

* Actually, weak interactions also "slightly" break CP!!
$Br(K_L \to 2\pi) \approx 10^{-3}$ See Holzen + Martin, Perkins

Particle Data Group, Physical Review D45, Part II (1992)

Before top discovery at FNAL 1995

(115)

Additional small effects are included in the numbers. Vertex corrections in $Z \to b\bar{b}$ can be approximated by $A^b \to A^b + \rho_t/3$, $V^b \to V^b + \rho_t/3$ [28].

For 3-fermion families the total widths are

$$\Gamma_Z \approx 2.478 \pm 0.002 \; (2.504) \text{ GeV}$$

$$\Gamma_W \approx 2.08 \pm 0.02 \text{ GeV} \quad (26)$$

for $m_t = 100$ (200) GeV. QCD introduces an additional uncertainty of ≈ 5 MeV in Γ_Z. (Fermion masses have been included in Γ_Z). This is to be compared with the experimental results [20–27]: $\Gamma_Z = 2.487 \pm 0.010$ GeV and $\Gamma_W = 2.12 \pm 0.11$ GeV.

Experimental results: Fits to the Z-line shape yield M_Z, Γ_Z, and the peak (QED-corrected) cross sections

$$\sigma_p^f = \frac{12\pi}{M_Z^2} \frac{\Gamma_{e\bar{e}}\Gamma_{f\bar{f}}}{\Gamma_Z^2} \quad (27)$$

for $e^+e^- \to f\bar{f}$ [20–25]. The values of the principle Z-pole observables are listed in Table 2, along with the Standard Model predictions for $M_Z = 91.173 \pm 0.020$, $m_t = 150^{+23}_{-26}$ GeV (for $M_H = 250$ GeV), and 50 GeV $< M_H <$ 1 TeV. The values and predictions of M_W [26], M_W/M_Z [27], and the Q_W for cesium [16,17] are also listed. The agreement is remarkable. The only hints of a discrepancy are in $A_{FB}(b)$ and Q_W, but even these agree at $\sim 1\frac{1}{2}\sigma$. The observables in Table 2 (including correlations on the LEP observables), as well as all low-energy neutral-current data [5], are used in the global fits described below. The parameter $\sin^2\theta_W$ can be determined from the Z-pole observables and M_W, and from a variety of neutral-current processes spanning a very wide Q^2 range. The results [5], shown in Table 3, are in impressive agreement with each other, indicating the quantitative success of the Standard Model.

The best fit to all data yields $\sin^2\hat{\theta}_W(M_Z) = 0.2337 \pm 0.0003$ for the weak angle in the $\overline{\text{MS}}$ scheme for $m_t = 100$ GeV and yields 0.2310 ± 0.0003 for $m_t = 200$ GeV, both for $M_H = 250$ GeV. In all fits the errors include full statistical, systematic, and theoretical uncertainties. The result is dominated by M_Z, with the error reflecting both ΔM_Z (± 0.0002) and the low-energy uncertainty of ± 0.0009 in Δr (± 0.0003). In the on-shell scheme $\sin^2\theta_W$ is more sensitive to m_t [29]. One obtains $\sin^2\theta_W = 0.2315 \pm 0.0003$ (0.2191 ± 0.0003) for $m_t = 100$ (200) GeV.

The derived $\sin^2\theta_W$ is sensitive to the isospin breaking [5] associated with a large m_t, as can be seen in Fig. 1. Consistency of the

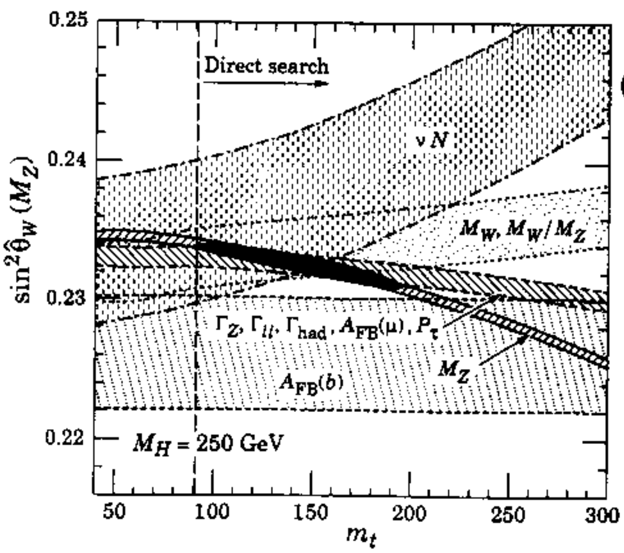


Fig. 1. One standard deviation uncertainties in $\sin^2\hat{\theta}_W$ as a function of m_t, the direct constraint $m_t > 91$ GeV [30], and the 90% CL region in $\sin^2\hat{\theta}_W - m_t$ allowed by all data, assuming $M_H = 250$ GeV.

Table 2. Principal LEP and other recent observables, compared with the Standard Model predictions for $M_Z = 91.173 \pm 0.020$ GeV, 50 GeV $< M_H <$ 1 TeV, and the global best fit value $m_t = 150^{+23}_{-26}$ GeV (for $M_H = 250$ GeV). The LEP averages of the ALEPH [21], DELPHI [22], L3 [23], and OPAL [24] results include common systematic errors [25]. $\Gamma_{\ell\ell}$ is the average of Γ_{ee}, $\Gamma_{\mu\mu}$, and $\Gamma_{\tau\tau}$; Γ_{had} is the width into hadrons. The invisible width Γ_{inv} corresponds to $N_\nu = 3.00 \pm 0.05$ light neutrino flavors. $\bar{g}_A$ and $\bar{g}_V$ are effective leptonic couplings determined from $A_{FB}(\mu)$ and $\Gamma_{\ell\ell}$ ($\bar{g}_A$ is not independent). At tree level, $\bar{g}_A = A^e$, $\bar{g}_V = V^e$. $A_{FB}(b)$ is corrected for $B\overline{B}$ oscillations. The second error in Q_W (for cesium) is theoretical [17]. In the Standard Model predictions, the first uncertainty is from M_Z and Δr, while the second is from m_t and M_H. There is an additional QCD error of ~ 5 MeV in Γ_Z and Γ_{had}.

Quantity	Value	Standard Model
M_Z (GeV)	91.173 ± 0.020	input
Γ_Z (GeV)	2.487 ± 0.010	$2.488 \pm 0.002 \pm 0.006$
$\Gamma_{\ell\ell}$ (MeV)	83.0 ± 0.6	$83.7 \pm 0.1 \pm 0.2$
Γ_{had} (MeV)	1736 ± 11	$1737 \pm 2 \pm 4$
Γ_{inv} (MeV)	502 ± 9	$501 \pm 0.3 \pm 1$
$\bar{g}_A^2$	0.2492 ± 0.0012	$0.2513 \pm 0.0002 \pm 0.0004$
$\bar{g}_V^2$	0.0012 ± 0.0003	$0.0011 \pm 0 \pm 0.0001$
P_τ	0.134 ± 0.035	$0.136 \pm 0.003 \pm 0.006$
$A_{FB}(b)$	0.126 ± 0.022	$0.091 \pm 0.002 \pm 0.004$
M_W (GeV)	80.22 ± 0.26	$80.21 \pm 0.03 \pm 0.16$
M_W/M_Z	0.8798 ± 0.0028	$0.8798 \pm 0.0002 \pm 0.0017$
Q_W [16,17]	$-71.04 \pm 1.58 \pm 0.88$	$-73.21 \pm 0.08 \pm 0.03$

$\sin^2\theta_W$ values derived from the various reactions requires [5] $m_t < 194$ GeV at 90% CL ($m_t < 201$ GeV at 95% CL) for $M_H \leq 1000$ GeV. (Similar limits hold for the mass splittings between fourth-generation quarks or leptons.)

When m_t is left as a free parameter one obtains $\sin^2\hat{\theta}_W(M_Z) = 0.2325 \pm 0.0008$ ($\overline{\text{MS}}$), or $\sin^2\theta_W = 0.2259 \pm 0.0029$ (on-shell), and (*) $m_t = 150^{+23}_{-26} \pm 16$ GeV. The $\sin^2\theta_W$ errors include m_t and M_H (assuming 50 GeV $< M_H <$ 1 TeV). The central value and first error in m_t is for $M_H = 250$ GeV, while the second error is from M_H. The fits cannot significantly constrain M_H until m_t is known independently. The $\sin^2\hat{\theta}_W(M_Z)$ value is in striking agreement with the prediction 0.233 ± 0.003 of grand unified theories based on the minimal supersymmetric extension of the Standard Model, but disagree with the prediction 0.211 ± 0.002 of nonsupersymmetric unified theories.

One can also determine the radiative correction parameters Δr [Eq. (5)]: one obtains $\Delta r = 0.049 \pm 0.009$ and $\Delta\hat{r}_W = 0.063 \pm 0.007$, where the error includes m_t and M_H. The data also yield $\alpha_s(M_Z) = 0.127 \pm 0.015$ (mainly from $\Gamma_{\text{had}}/\Gamma_{\ell\ell}$), in excellent agreement with the value 0.115 ± 0.008 obtained from event shapes and jet studies [31].

Deviations from the Standard Model: The Z-pole, W mass, and neutral-current data can be used to search for and set limits on deviations from the Standard Model. For example, the relation in Eq. (5) between M_W and M_Z is modified if there are Higgs multiplets with weak isospin $> 1/2$ with significant vacuum expectation values. In order to calculate to higher orders in such theories one must define a set of four fundamental renormalized parameters. It is convenient to take these as α, G_F, M_Z, and M_W, since M_W and M_Z are directly measurable. Then $\sin^2\theta_W$ and ρ_0 can be considered dependent

(*) Subsequent FNAL discovery and measurement $m_t = 173.8 \pm 5.2$ GeV

Particle Data Group, European Physical Journal C3, 1 (1998)

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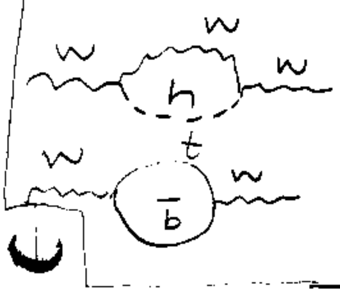
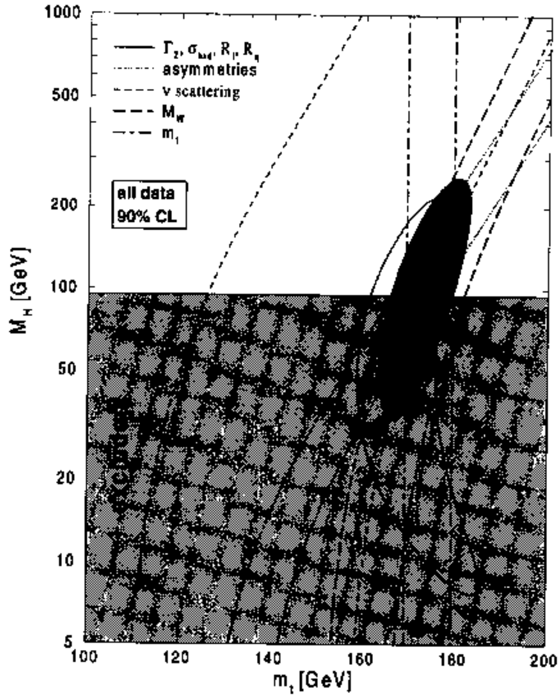
One can also carry out a fit to the indirect data alone, *i.e.*, without including the value, $m_t = 174.3 \pm 5.1$ GeV, observed directly by CDF and DØ. (The indirect prediction is for the $\overline{\rm MS}$ mass, $\widehat{m}_t(\widehat{m}_t) = 158.7^{+9.1}_{-7.0}$ GeV, which is in the end converted to the pole mass using a BLM optimized [113] version of the two-loop perturbative QCD formula [114]; this should correspond approximately to the kinematic mass extracted from the collider events.) One obtains $m_t = 168.2^{+9.6}_{-7.4}$ GeV, with little change in the $\sin^2\theta_W$ and α_s values, in remarkable agreement with the direct CDF/DØ value. The central M_H value of this fit (see the third line of Table 10.5) is below the direct lower bound; keeping $M_H = 100$ GeV fixed results in $m_t = 172.2 \pm 4.0$ GeV in even better agreement. The relations between M_H and m_t for various observables are shown in Fig. 10.1.

e.g. mw influenced by mt and mH





⊗ http://pdg.lbl.gov - Electroweak Model and Constraints on New Physics

Figure 10.1: One-standard-deviation (39.35%) uncertainties in M_H as a function of m_t for various inputs, and the 90% CL region ($\Delta\chi^2 = 4.605$) allowed by all data. $\alpha_s(M_Z) = 0.120$ is assumed except for the fit to all data. The 95% direct lower limit from LEP 2 is also shown.

Using $\alpha(M_Z)$ and $\widehat{s}^{\,2}_Z$ as inputs, one can predict $\alpha_s(M_Z)$ assuming grand unification. One predicts [115] $\alpha_s(M_Z) = 0.130 \pm 0.001 \pm 0.01$ for the simplest theories based on the minimal supersymmetric extension of the Standard Model, where the first (second) uncertainty is from the inputs (thresholds). This is slightly larger, but consistent with the experimental $\alpha_s(M_Z) = 0.1192 \pm 0.0028$ from the Z lineshape, and with the world