Chapter 12 – Problem Solving

Definitions

• **Thinking**: formation of a new mental representation through the transformation of information by complex interaction of the mental attributes of judging, abstracting, reasoning, imagining, and problem solving.

• **Problem Solving**: thinking directed toward the handling of a particular situation involving both the formation of responses and the selection among possible responses.

What is a Problem?

• much of life is spent solving problems
• there is an initial state
• there is a goal state that differs from the initial state
• the process of going from the initial state to the goal state is not immediately obvious
Four Features of Problems

• a **goal**, or description of what constitutes a solution
• a description of **objects** relevant to achieving a solution
• a set of **operations** or allowable actions toward solution
• a set of **constraints** not to be violated

Parts of a Problem

• Problem solving involves attempting to move from the initial state to the goal state via the available operators while observing any constraints

Types of Problems

• **Knowledge-Lean Problems**: can be solved (though not always skillfully) by use of instructions for the task and general problem solving skills
  – e.g., finding a parking space in the mall
• **Knowledge-Rich Problems**: requires specific knowledge or skill to solve the problem
  – e.g., calculus, computer-programming problems
Two Kinds of Problems

• **Well-defined**: a problem having a clear-cut solution; can be solved by an algorithm
  – E.g., crossword puzzle or $3x = 2$ (solve for $x$)

• **Ill-defined**: a problem usually having multiple possible solutions; cannot be solved by an algorithm
  – E.g., writing a hit song or building a career

Herb Simon

• trained in political science; also worked in economics and (mainly) psychology
• studied protocols of subjects talking during problem solving
• won Nobel Prize in Economics (1978)

Problem Solving Process

- form initial problem representation
- try to plan potential solution
  - if succeed
  - try to reformulate problem
- execute plan and check results
  - if fail
  - if succeed
- take break and retry (STUCK)
  - if fail
  - if succeed
- DONE
Aspects of Problem Solving

- **Problem space**: the domain of the problem and the choices the solver evaluates during solution
- **Plan**: a hierarchical process that controls the order in which a sequence of operations is to be performed

Representation

- The price of a notebook is four times that of a pencil. The pencil costs 30 cents less than the notebook. What is the price of each?
- \( n = \) notebook; \( p = \) pencil
- **Initial**: \( n = 4p; p = n - 30 \)
- substitute: \( p = 4p - 30 \)
- subtract: \( p - 4p = 4p - 4p - 30 \)
- \( -3p = -30 \)
- **Solution**: \( p = 10; n = 40 \)

Mutilated Checkerboard Problem

- Wickelgren (1974)
  - 64 squares on chessboard
  - cut off two corners, leaving 62
  - 31 dominos
  - Each domino covers 2 squares
  - Can you cover the checkerboard with the dominos?
General Methods

• **generate-test** method (cf. “trial and error”)
  – would take too long
  – e.g. fruit with a vowel as its fourth letter
  – have to work through whole problem for each of the many candidates generated
• **means-end** analysis (a heuristic)
  – break problem into subgoals

3 Types of Problems (Greeno)
1. Arrangement Problems
   e.g., anagrams: KEROJ

3 Types of Problems (Greeno)
2. Inducing Structure Problems
   e.g., 1 2 8 3 4 6 5 6 _____
   A B M C D M _____
   or Washington is to 1
   as Lincoln is to 10 or 5?
What is the next item in the series?

What is the next item in the series?

3 Types of Problems (Greeno)

3. Transformation Problems
e.g., Tower of Hanoi puzzle or Rubik’s Cube

A          B          C

The Problem State Space

The Problem State Space

The Problem State Space
Reformulation

1 21 127 3 100
2 14 163 25 99
3 18 43 10 5
4 9 4 2 6 2 1
5 20 59 4 31
6 23 49 3 20
7 15 39 3 18
8 28 76 3 25

Mental Set (Luchins, 1942)

<table>
<thead>
<tr>
<th>Problem</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
<td>127</td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>163</td>
<td>25</td>
<td>99</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>43</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>42</td>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>59</td>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>49</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td>39</td>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>28</td>
<td>76</td>
<td>3</td>
<td>25</td>
</tr>
</tbody>
</table>

A Problem...

- Duncker (1945)
- Instructions: using only the objects shown in the picture, mount the candle to the wall
- People do not think to use the box (the origin of “thinking outside the box”?)
Terminology

- **Functional Fixedness**: treating an object as having only one function; not thinking creatively
- How do we deal with problems? (Wertheimer, 1959)
  - Reproductive thinking: learning by rote
  - Productive thinking: learning in a way that makes generalization to new situations easier and more flexible

Gick & Holyoak (1980)

- subjects first read a story about a general attacking a fortress. The roads around the fortress were mined
  - general split up his forces and had them converge on the fortress from many directions
- later, they were given a radiation problem
  - patient has an inoperable tumor
  - rays strong enough to kill the tumor would damage the healthy tissue

Their Results

- Control (no story): 8% correct
- General story: 76% correct
- Story + hint: 92% correct
Using Analogies
Gick & Holyoak (1983):
• we tend not to use analogies when the relation between problems is not obvious
• that is, when the problems differ markedly in surface features
• therefore, the difficult part of applying past knowledge is finding the relevant knowledge

Analogy
• puppy : kitten :: dog :
• puppy : dog :: kitten :
• A : B :: C :
• C : L :: X :

Allan Newell & Herbert Simon
• Carnegie Mellon University
• together, developed General Problem Solver
• earliest computer simulations of methods of problem solving
• earliest chess programs
Protocol Analysis

• Newell & Simon (1972)
• thinking aloud during problem solving
• Hint R = 6

SEND
+ MORE
MONEY

Solving a Problem

\[
\text{CROSS} + \text{ROADS} = \text{DANGER}
\]

A Protocol

<table>
<thead>
<tr>
<th>1. Experimenter: (S = 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Since 6 is 4 and the two 2's are equal …</td>
</tr>
<tr>
<td>3. (S) must be equal to 5, or 6.</td>
</tr>
<tr>
<td>4. And 6 must be equal to 1 …</td>
</tr>
<tr>
<td>5. Therefore 1 and 6 must be greater than 19 …</td>
</tr>
<tr>
<td>6. So S must be even.</td>
</tr>
<tr>
<td>7. Would you make it 19?</td>
</tr>
<tr>
<td>8. Experimenter: (S = 19)</td>
</tr>
<tr>
<td>9. And noting that two 2’s are equal …</td>
</tr>
<tr>
<td>10. They must be either 3 or 9.</td>
</tr>
<tr>
<td>11. If this you …</td>
</tr>
<tr>
<td>12. Then (T) would be 0.</td>
</tr>
<tr>
<td>13. If they add …</td>
</tr>
<tr>
<td>14. And so on …</td>
</tr>
<tr>
<td>15. Would you make it (T = 4)</td>
</tr>
<tr>
<td>16. Experimenter: (T = 4)</td>
</tr>
<tr>
<td>17. And noting that (S) and (T) are even …</td>
</tr>
</tbody>
</table>
Problem Behaviour Graph

Answer...

\[
\begin{array}{c}
96233 \\
+ 62513 \\
\hline
158746
\end{array}
\]

General Problem Solver

• computer program that solves a variety of problems
• an explicit, testable theory
• uses means-end analysis; subgoals
• solves well-defined problems; no reformulation
Expert Systems

• solves specific problem (not general)
  – e.g., medical diagnosis

• sophisticated but lack generality & flexibility

Definitions

• Creativity: a new solution to an old problem, when it has never been solved before, and when we recognize it as such

• Insight: a deep, useful understanding of the nature of something, especially a difficult problem

Creativity

• 4 steps to the creative process (Wallas, 1926):
  – preparation = formulating and beginning
  – incubation = setting aside
  – illumination = achieving insight
  – verification = checking solution
Metcalfe & Wiebe (1987)

“Warmth Ratings” for correct and incorrect answers as a function of time of rating prior to answering.

High

Warmth Rating

10

Incorrect

Correct

Low

0

3rd Last 2nd Last Last Solution Interval Interval Interval

Incubation & Insight

• Why should an interruption help?

• Posner (1973) suggested three reasons:
  – recovery from fatigue
  – forget inappropriate approaches
  – reorganization

Hobbits & Orcs

Hob

move everyone to the other side of the river; boat can hold only two at a time

never leave more orcs than hobbits on either side, or the orcs will eat the hobbits

Orc
Insight #1

• A man needing a haircut goes into a small town where everyone in the town gets their haircut by one of two barbers. The man visits the first—a very clean shop where the barber has a terrific haircut. Then he visits the second—a very dirty shop where the barber has a terrible haircut.
• Where does the man get his hair cut?

Insight #2

• A boat is floating at the dock with a rope ladder hanging off the side. The rungs of the ladder are one foot apart, and there are three rungs under water. The tide is rising at one foot per hour.
• How many rungs will be under water after 3 hours?

Insight #3

• An archaeologist claims to have discovered the oldest coin ever unearthed, dated 542 BC. A psychologist claims that the archaeologist is a fraud. Who is right?