Chapter 12 – Problem Solving

Definitions

- ***Thinking***: formation of a new mental representation through the transformation of information by complex interaction of the mental attributes of judging, abstracting, reasoning, imagining, and problem solving.
- ***Problem Solving***: thinking directed toward the handling of a particular situation involving both the formation of responses and the selection among possible responses.

What is a Problem?

- much of life is spent solving problems
- there is an initial state
- there is a goal state that differs from the initial state
- the process of going from the initial state to the goal state is not immediately obvious

Four Features of Problems

- a ***goal***, or description of what constitutes a solution
- a description of ***objects*** relevant to achieving a solution
- a set of ***operations*** or allowable actions toward solution
- a set of ***constraints*** not to be violated

Parts of a Problem

- Problem solving involves attempting to move from the initial state to the goal state via the available operators while observing any constraints

Types of Problems

- ***Knowledge-Lean Problems***: can be solved (though not always skillfully) by use of instructions for the task and general problem solving skills
  - e.g., finding a parking space in the mall
- ***Knowledge-Rich Problems***: requires specific knowledge or skill to solve the problem
  - e.g., calculus, computer-programming problems

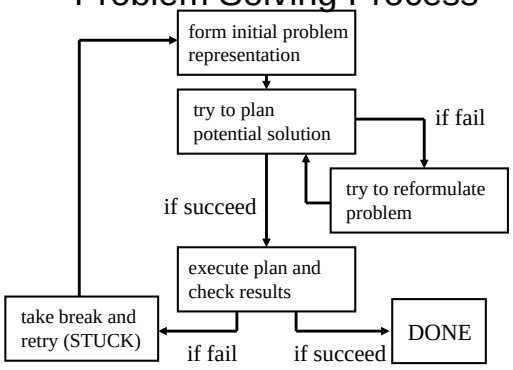
Two Kinds of Problems

- ***Well-defined***: a problem having a clear-cut solution; can be solved by an algorithm
  - E.g., crossword puzzle or 3x = 2 (solve for x)
- ***Ill-defined***: a problem usually having multiple possible solutions; cannot be solved by an algorithm
  - E.g., writing a hit song or building a career

Herb Simon

- trained in political science; also worked in economics and (mainly) psychology
- studied protocols of subjects talking during problem solving
- won Nobel Prize in Economics (1978)

Problem Solving Process



Aspects of Problem Solving

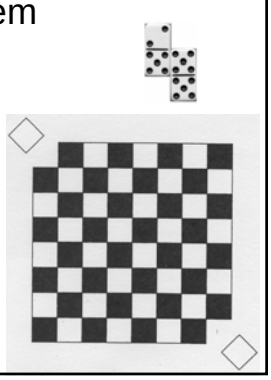
- ***Problem space***: the domain of the problem and the choices the solver evaluates during solution
- ***Plan***: a hierarchical process that controls the order in which a sequence of operations is to be performed

Representation

- The price of a notebook is four times that of a pencil. The pencil costs 30 cents less than the notebook. What is the price of each?
- n = notebook; p = pencil
- *Initial*: n = 4p; p = n - 30
- substitute: p = 4p - 30
- subtract: p - 4p = 4p - 4p - 30
- - 3p = -30
- *Solution*: p = 10; n = 40

Mutilated Checkerboard Problem

- Wickelgren (1974)
  - 64 squares on chessboard
  - cut off two corners, leaving 62
  - 31 dominos
  - Each domino covers 2 squares
  - Can you cover the checkerboard with the dominoes?



General Methods

- **generate-test** method (cf. “trial and error”)
  - would take too long
  - e.g. fruit with a vowel as its fourth letter
  - have to work through whole problem for each of the many candidates generated
- **means-end** analysis (a heuristic)
  - break problem into subgoals

3 Types of Problems (Greeno)

1. Arrangement Problems
   e.g., anagrams: KEROJ

3 Types of Problems (Greeno)

2. Inducing Structure Problems

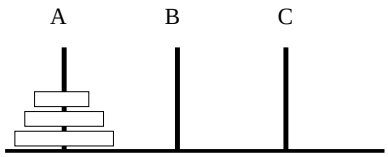
   e.g., 1 2 8 3 4 6 5 6 _____
   A B M C D M _____

   or Washington is to 1
   as Lincoln is to 10 or 5?

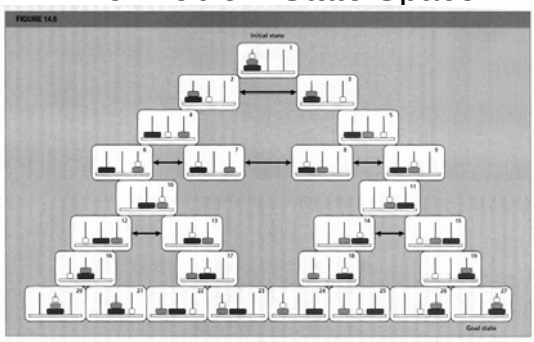
What is the next item in the series?

3 Types of Problems (Greeno)

3. Transformation Problems
   e.g., Tower of Hanoi puzzle or Rubik's Cube

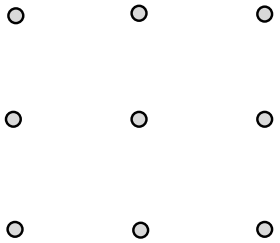


The Problem State Space



The problem space of legal moves in the Tower of Hanoi problem. If boxes touch each other, or are joined by arrows, this indicates that one can move from one state to the other using a legal operator.

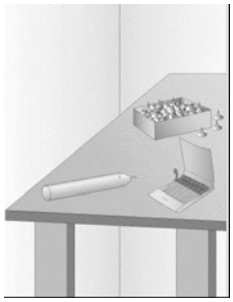
Reformulation



Mental Set (Luchins, 1942)

Problem	A	B	C	Goal
• 1	21	127	3	100
• 2	14	163	25	99
• 3	18	43	10	5
• 4	9	42	6	21
• 5	20	59	4	31
• 6	23	49	3	20
• 7	15	39	3	18
• 8	28	76	3	25

A Problem…



- Duncker (1945)
- instructions: using only the objects shown in the picture, mount the candle to the wall
- people do not think to use the box (the origin of “thinking outside the box”?)

Terminology

- ***Functional Fixedness***: treating an object as having only one function; not thinking creatively
- How do we deal with problems? (Wertheimer, 1959)
  - ***Reproductive thinking***: learning by rote
  - ***Productive thinking***: learning in a way that makes generalization to new situations easier and more flexible

Gick & Holyoak (1980)

- subjects first read a story about a general attacking a fortress. The roads around the fortress were mined
  - general split up his forces and had them converge on the fortress from many directions
- later, they were given a radiation problem
  - patient has an inoperable tumor
  - rays strong enough to kill the tumor would damage the healthy tissue

Their Results

- Control (no story): 8% correct
- General story: 76% correct
- Story + hint: 92% correct

Using Analogies

Gick & Holyoak (1983):

- we tend not to use analogies when the relation between problems is not obvious
- that is, when the problems differ markedly in surface features
- therefore, the difficult part of applying past knowledge is finding the relevant knowledge

Analogy

- puppy : kitten :: dog :
- puppy : dog :: kitten :

- A : B :: C :
- C : L :: X :

Allan Newell & Herbert Simon

- Carnegie Mellon University
- together, developed General Problem Solver
- earliest computer simulations of methods of problem solving
- earliest chess programs

Protocol Analysis

- Newell & Simon (1972)
- thinking aloud during problem solving
- Hint R = 6

```
  SEND
+ MORE
------
 MONEY
```

Solving a Problem

```
  CROSS
+ ROADS
-------
 DANGER
```

A Protocol

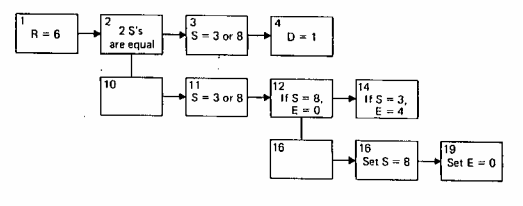
TABLE 12.5 Beginning of Protocol for Subject on CROSS + ROADS = DANGER Problem

1. Experimenter: R is 6.

```
  C6OSS
+ 6OADS
-------
 DANGE6
```

2. Since R is 6 and the two S's are equal...
3. S must be equal to 3, or 8.
4. And D must be equal to 1...
5. Because C plus R can't be greater than 19...
6. Or greater than 16 in this case.
7. So D must be a 1.
8. Would you make D a 1?
9. Experimenter: D is 1.
10. And seeing that two S's are equal...
11. They must be either 3 or 8.
12. If they are 8...
13. Then E would be 0.
14. If they are 3...
15. Then E would be 4.
16. So let's try the S's as 8.
17. Could you make the S an 8?
18. Experimenter: S is 8.
19. That would make E a 0.
20. Would you make E a 0?
21. Experimenter: E is a 0.

Problem Behaviour Graph



Answer…

96233

+ 62513

158746

General Problem Solver

- computer program that solves a variety of problems
- an explicit, testable theory
- uses means-end analysis; subgoals
- solves well-defined problems; no reformulation

Expert Systems

- solves specific problem (not general)
  - e.g., medical diagnosis
- sophisticated but lack generality & flexibility

Definitions

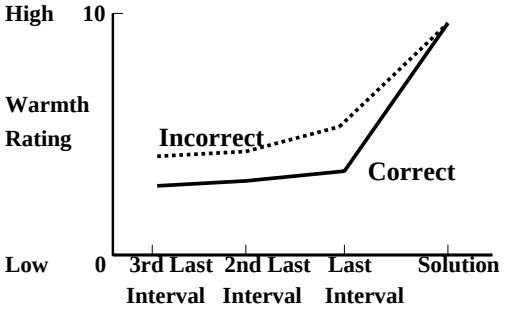
- ***Creativity***: a new solution to an old problem, when it has never been solved before, and when we recognize it as such
- ***Insight***: a deep, useful understanding of the nature of something, especially a difficult problem

Creativity

- 4 steps to the creative process (Wallas, 1926):
  - ***preparation*** = formulating and beginning
  - ***incubation*** = setting aside
  - ***illumination*** = achieving **insight**
  - ***verification*** = checking solution

Metcalfe & Wiebe (1987)

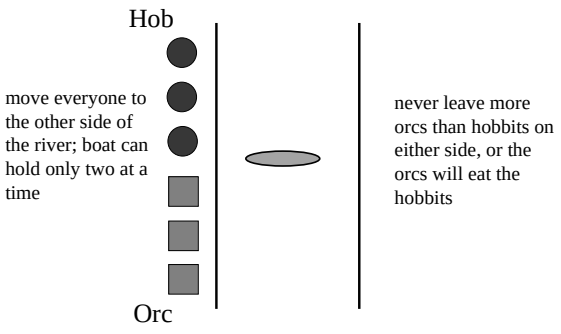
"Warmth Ratings" for correct and incorrect answers as a function of time of rating prior to answering.



Incubation & Insight

- Why should an interruption help?
- Posner (1973) suggested three reasons:
  - recovery from fatigue
  - forget inappropriate approaches
  - reorganization

Hobbits & Orcs



Insight #1

- A man needing a haircut goes into a small town where everyone in the town gets their haircut by one of two barbers. The man visits the first—a very clean shop where the barber has a terrific haircut. Then he visits the second—a very dirty shop where the barber has a terrible haircut.
- Where does the man get his hair cut?

Insight #2

- A boat is floating at the dock with a rope ladder hanging off the side. The rungs of the ladder are one foot apart, and there are three rungs under water. The tide is rising at one foot per hour.
- How many rungs will be under water after 3 hours?

Insight #3

- An archaeologist claims to have discovered the oldest coin ever unearthed, dated 542 BC. A psychologist claims that the archaeologist is a fraud. Who is right?