Interest Rates, Moneyness, and the Fisher Equation

Lucas Herrenbrueck – Simon Fraser University

Zijian Wang – Wilfrid Laurier University

First version: February 2019
This version: December 2023
(Link to latest version)

ABSTRACT

The Euler equation of a representative consumer – or its long-run counterpart, the Fisher equation – is at the heart of modern macroeconomics. But it prices a bond – short-term, perfectly safe, yet perfectly illiquid – that does not exist in reality, where most safe assets can be easily traded or pledged as collateral to obtain money, or even for goods and services directly, and their prices reflect their *moneyness* as much as their dividends. In this paper, we deploy a New Monetarist framework to capture these facts and derive implications for monetary policy and asset pricing. Consistent with the model, we find that the return on a hypothetical illiquid bond, estimated via inflation and consumption growth, behaves very differently from the return on safe and liquid assets. This distinction helps resolve a great number of puzzles associated with the Euler/Fisher equation, and points to a better way of understanding how monetary policy affects the economy.

JEL Classification: E43, E44, E52

Keywords: Euler equation, Fisher rate, indirect liquidity, liquid assets, monetary policy

Email: herrenbrueck@sfu.ca, zijianwang@wlu.ca

We would like to thank David Andolfatto, Garth Baughman, Athanasios Geromichalos, Wenhao Li, Luba Petersen, Alberto Trejos, and many participants at seminar and conference presentations, for their very useful comments and suggestions. We also thank research assistants Marieh Azizirad and Hung Truong, and we acknowledge support from the Social Sciences and Humanities Research Council of Canada (435-2018-1263 and 435-2021-0725).

1 Money and interest rates

What is the opportunity cost of holding money? Economists give two answers to this question. First, money is a way to save, and the alternative to saving is to consume right away. Thus, the holding cost of money reflects patience and the changing value of consumption over time (summarized in a *stochastic discount factor*). Second, there are many ways to store wealth, so another alternative to holding money is to buy an asset – perhaps a Treasury Bill or a book entry in a deposit account – which may pay interest. If so, then this *interest rate* is another opportunity cost of holding money.

The usual inference is that the two answers coincide: interest rates must equal the inverse of the expected discount factor (including inflation, in case of nominal assets). If there is a representative consumer who consumes c, pays price p, and discounts the future by $\beta < 1$, this reasoning yields the famous Euler equation:

$$1 + i_t = \left(\mathbb{E}_t \left\{ \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \right\} \right)^{-1} \tag{1}$$

Although its interpretation differs across schools of thought (more on which in Section 5 below), this equation is at the heart of modern macroeconomics. This becomes a problem when empirical applications identify i_t with the monetary policy instrument, or a similar short-term rate. For instance, models estimated with U.S. data generally use the Federal Funds Rate [49], 3-month T-bill rate [47], or commercial paper rate [50, 38] as the empirical counterpart of i_t. Yet all of these rates price *highly liquid* assets that have as much in common with money as they have with long-term saving. Consider what the literature has identified as making for a liquid, "money-like", asset:

(a) Serving as a medium of exchange [ancient];
(b) Serving as collateral for a loan when money is needed [53];
(c) The ability to sell it on a secondary market when money is needed [22];
(d) The expectation that it turns into money soon, before that money is needed [25].

Any medium of exchange is money in its most direct sense (a). But a collateralizable or saleable asset is *indirect* money, because people who give up money to buy such an asset know that they have a way to get their money back if they need it badly enough (b,c). This channel is weaker if the collateral is subject to a large haircut or if selling the asset at a good price takes time, but it is always present, and surely so for U.S. Treasuries that can be pledged or sold within hours. Finally, if one needs money in two months, then a bond that matures in one month is as good as money (d). Indeed, accounting jargon makes all assets that mature in less than a year "cash equivalents". So which asset could in practice be perfectly safe, short-term, yet perfectly illiquid?

No, Equation (1) cannot possibly price the monetary policy instrument, which in most countries is an interest rate on debt as short-term and liquid as one can imagine. And there is a better way. Suppose we add to our models a bond which does not generally serve as a medium of exchange (hence it is distinct from "money", which does), but which is still "liquid" in ways (b)-(d).[1] Let us denote the nominal interest rate on this bond by i_t^P ("**policy rate**"). In contrast, consider a hypothetical illiquid bond whose payout must be used for buying consumption in the future; its nominal interest rate, i_t^F, satisfies Equation (1) by construction. Since that equation is the Fisher equation in the long run, Geromichalos and Herrenbrueck [23] proposed to call i_t^F the **Fisher interest rate**.

As people can always hold money (which pays no interest), the policy rate i_t^P cannot be less than zero. And since the liquid bond cannot be less valuable than the illiquid one, i_t^P cannot exceed i_t^F:

$$i_t^P \in [0, i_t^F]$$

Every fact about monetary policy makes more sense once we identify short-term monetary policy with i_t^P, rather than i_t^F, as the rest of this paper will demonstrate.

2 An informal model

In Section 4, we present a formal intertemporal model of money and liquid assets, but its essential insights can be explained informally. First, abstract from shocks, risk, and second-order terms, and suppose: (i) people expect consumption to grow at rate g and prices to grow at rate π; (ii) the marginal utility of consumption is proportional to $c^{-\theta}$, so that θ is the inverse elasticity of intertemporal substitution; (iii) people discount the future at rate $\rho \equiv 1/\beta - 1$; and (iv) the yield on the liquid bond (i^P) is a known function $\mathcal{G}$ of the opportunity cost of holding money (the Fisher rate i^F) and the relative supply of liquid bonds to money (B/M), increasing in both arguments.[2] Thus, the two rates i^F and i^P must satisfy:

$$i^P = \mathcal{G}\left(i^F, \frac{B}{M}\right) \tag{2}$$

$$i^F = \rho + \theta g + \pi \tag{3}$$

where the latter is simply the linearized right-hand side of the Euler equation (1).

[1] Empirically, U.S. T-Bills are quite liquid but still not perfect substitutes to U.S. money [35], and of course they can never be perfect substitutes to *non-U.S.* money. Safe assets do tend to be more liquid than risky ones, but this is an endogenous outcome and not a matter of definition, and the association is not perfect [36, 45, 27, 24, 26].

[2] A good number of formal models indeed yield something like this function $\mathcal{G}$ in reduced form; among them, [21, 54, 46, 5, 33, 23], and more. If money is not neutral (for example, in the short run), or if there are additional illiquid nominal assets in the economy, then $\mathcal{G}$ will depend separately on B and $1/M$.

In Section 3, we investigate Equation (3) empirically, and show that it fails to match the data for any values of θ and ρ. But the argument against treating i^F as the policy instrument goes beyond macrofinancial data, and touches the foundations of what we know about monetary policy. If i^F is the monetary policy instrument, how can a desired rate be implemented by a monetary authority? According to the equation, in one of three ways: through expected inflation, through expected consumption growth, or through shocks to ρ conveniently timed to coincide with monetary policy announcements. (The first of these is the path taken by much of monetary theory; the second, the path taken by New Keynesian models where prices are sticky, current consumption is endogenous, thus expected consumption growth 'clears' the equation.) In reality, however, measures of expected inflation and growth are either uncorrelated with policy rates or *negatively* correlated, at least in countries and times where these rates stay in single digits, and an explanation based on endogenous time preference does not amount to explaining anything.

But Equations (2)-(3) also present an opportunity to use the *return on the liquid bond*, i^P, as an alternative model of the policy rate.[3] Why does the policy rate correlate so weakly with g and π? Because the pass-through from i^P to i^F can be positive, zero, or even negative. How is a desired policy rate implemented? Through open-market operation that adjust the money supply in the background: a lower M/B ratio causes higher interest rates, moving up the "money demand curve" from the undergraduate textbook. Assuming $\mathcal{G}$ is invertible, we can write:

$$\begin{aligned} \frac{B}{M} &= \mathcal{H}\left(i^F, i^P\right) \\ i^F &= \rho + \theta g + \pi \end{aligned} \tag{4}$$

Now, i^F and i^P are logically independent instruments. They may still be correlated in the data depending on the nature of shocks (across big fluctuations in inflation the inequality $i^P \leq i^F$ will assert itself) or the rules governing policy (e.g., a Taylor rule). But the tight link between interest rates and inflation, implied by Equation (1), is broken. We call the spread between i^F and i^P the *aggregate liquidity premium*:

$$\ell \equiv i^F - i^P$$

Crucially, since i^P is an independent policy instrument and i^F is a macroeconomic equilibrium object, the spread ℓ acts like a residual. Mathematically, ℓ is a version of the *convenience yield* from the macrofinance literature [14, 44], but the role it plays in this economy is quite different. First, it is not exogenous, nor is it structural: the pass-through from i^P

[3] To our knowledge, the first microfounded model of money to do this is [3]; since then, the list has grown [5, 4, 18, 11, 23]. Others model monetary policy as implemented via open-market operations of a (partially or fully) liquid bond, but do not explicitly identify its yield with the *main* policy instrument [54, 32, 46].

to ℓ, from the policy rate to the aggregate liquidity premium, depends on the policy regime (see Section 3.2). Second, as the spread between the theoretically least liquid bond and the theoretically most liquid bond, it is of an order of magnitude larger than most estimates of convenience yields or liquidity premia, which are typically identified by comparing two assets that are fairly similar in the big picture (for example, on-the-run with off-the-run bonds [52], or T-notes with TIPS [6]).

To take this reasoning further, suppose we are pricing an asset X that can be liquidated whenever money is needed, in the same way as the liquid bond, but only with probability $\eta \in [0, 1]$. (Or, almost equivalently, one can use it as collateral but subject to a haircut $1 - \eta$.) Suppose this asset also depreciates at rate δ (which, again almost equivalently, could stand in for default risk or higher-order risk premia). Then, the linearized nominal rate of return on asset X will be:

$$\begin{aligned} & r = \delta + (1 - \eta) i^F + \eta i^P \\ \leftrightarrow \qquad & = \delta + i^F - \eta \ell \\ \leftrightarrow \qquad & = \delta + i^P + (1 - \eta) \ell \end{aligned} \tag{5}$$

This equation is a no-arbitrage condition among assets with varying real properties (δ, η). It illustrates how financial liquidity imbues an asset with *indirect moneyness*, lowers its return (relative to the fundamental) by an asset-specific liquidity premium $\eta\ell$, and integrates its return with the monetary policy rate i^P. The direct pass-through from i^P to r is less than one for all but the most liquid assets, but only if we hold all else (inflation expectations, risk premia, asset tradability) fixed, which may not be the case in real-world settings.

3 A look at the data

Trying to match up the right-hand side of Equation (1) with short-term interest rates results in a fit so famously poor that it has spawned entire branches of econometrics [28, 30]. Here, we make no claim to advance estimation in this way; however, we do want to provide a simple back-of-the envelope analysis of the data, since we think much can be learned from the *violations* of the Euler and Fisher equations. As it turns out, these are not mere noise, but highly systematic in ways that match with what else we know about how monetary policy is conducted, implemented, and intended.

We begin with quarterly postwar U.S. data on consumption growth, inflation, and interest rates.[4] To simplify matters, we assume that the elasticity of intertemporal substitution

[4] We take the 3-month T-bill secondary market rate as representative of a safe, short-term, highly liquid asset, but other choices are reasonable: federal funds, commercial paper, Libor, etc. Aside from a short window around 1980, these rates have never differed by more than a percentage point.

$1/\theta$ equals 1, corresponding to logarithmic utility of consumption.[5] Thus, the right-hand side variable in the Euler equation is simply the growth rate of nominal consumption, $p_t c_t$. We do not divide by the size of the population; most models of long-run growth (whether neoclassical or OLG) imply a positive effect from population growth to interest rates, usually with pass-through of either 1 or θ, but with logarithmic utility we have $\theta = 1$ anyways.

Postulating a simple log-linear trend for time preference, $\rho_t \equiv -\log(\beta_t) = \rho_0 + \rho_1 t$, and picking (ρ_0, ρ_1) to provide the best fit between ex-post consumption growth and the T-bill rate, we obtain the following result, illustrated in Figure 1:

$$\hat{\rho}_{1948} \approx -3\% \qquad \text{linearly increasing to} \qquad \hat{\rho}_{2022} \approx -1\% \qquad \text{(annually)}$$

Best fit it may be, but it flies in the face of received wisdom in many ways:

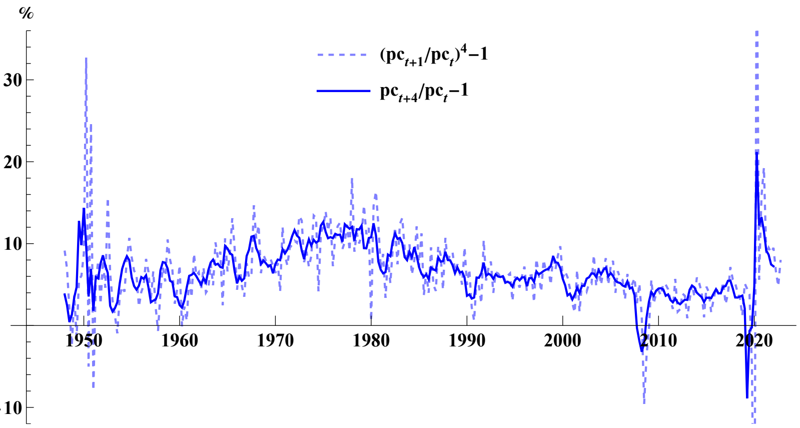
(a) Implied time preference is negative throughout (the risk-free rate puzzle);
(b) Implied time preference has *increased* over time (what about secular stagnation?);
(c) The fit is atrocious: the correlation is only 0.22 in levels, and -0.14 in first differences (0.35 and -0.14, respectively, if the sample ends in 2019).

Of course, these problems are well known [28, 30, 12, 41]. But they are usually understood as problems with "the Euler equation": short-term interest rates do not fit well with ex-post consumption growth, so it is the equation that must be misspecified, and solutions are to be found via a distinction between long-run and short-run risk (the Epstein-Zin literature), the particular interests of wealthy investors [40], or various other models of stochastic discount factors [41]. However, in the words of Canzoneri et al. [12], "the problem is fundamental: alternative specifications of preferences can eliminate the excessive volatility, but they yield an Euler equation rate that is strongly negatively correlated with the money market rate." The source of the problem may not be the equation as such, but the question of which interest rate it is estimated with: in the words of Rocheteau et al. [46], "to test the Fisher Equation one should not compare [inflation and the nominal rate on liquid bonds], but [inflation and] the nominal rate on an illiquid asset. That may be hard to implement empirically since most assets have some degree of liquidity."

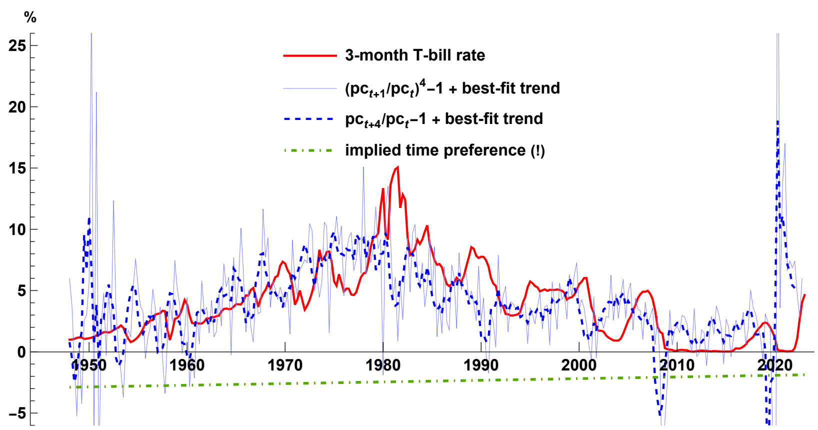
But we can sidestep this problem by estimating the right-hand side of the Fisher equation directly.[6]

[5] We also considered an infinite elasticity ($\theta = 0$) as a robustness check; in this case, only inflation is predicted to matter in the Fisher equation, not growth. All our results are robust to this alternative specification.

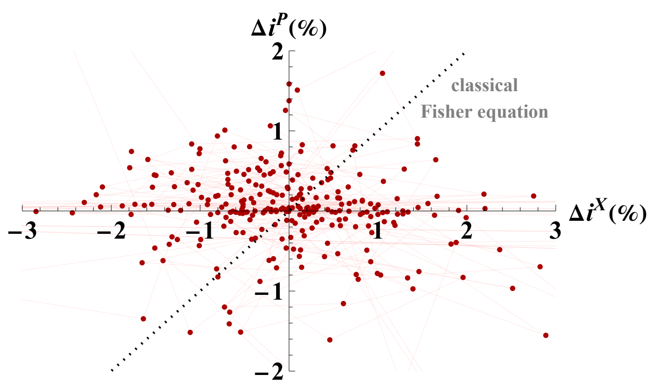
[6] Similar efforts have been made. [12] and [17] estimate the right-hand side of various real-asset Euler equations, and both suggest that the spread of the implied rate to the money market rate may be due to the "liquidity services" of the latter.



(a) U.S. nominal consumption growth, raw



(b) Best fit of nominal consumption growth on T-bill rate



(c) First differences thereof

Figure 1: Ex-post realized nominal consumption growth versus short-term interest rates (3-month T-bills, secondary market rate), from 1948-2022. Consumption growth is calculated as: $[(p_{t+1}c_{t+1})/(p_tc_t)]^4 - 1$ (quarterly, annualized) and $(p_{t+4}c_{t+4})/(p_tc_t) - 1$ (year-over-year), measured in percentage points. Best-fit estimates (throughout this paper) are computed after log points transformation $x \mapsto \log(1 + x)$.

3.1 Simple estimate of i^F

Abstracting away from issues of risk and Jensen's inequality, we use a loglinear representation of Equation (1) to define the ex-post counterpart of the Fisher rate (and multiply by 4 to convert to annual frequency):

$$x_{t+1} \equiv -\log(\beta) + 4\big[\log(p_{t+1}c_{t+1}) - \log(c_t p_t)\big]$$

and define the Fisher interest rate as a forecast of x_{t+1}: $\log(1+i_t^F) \equiv \mathbb{E}_t x_{t+1}$.

We compute three versions of the forecast: first, via a regression of x_t on one-quarter lags of itself, of the 3-month T-bill rate, and of the Moody's AAA corporate bonds rate; second, via a four-quarter moving average of the first estimate (where the estimates at time $t-3$, $t-2$, $t-1$, and t are averaged so as to provide a smoother forecast of x_{t+1}); third, via a regression of x_{t+1} on *one-year* lags of itself, of the 3-month T-bill rate, and of the Moody's AAA corporate bonds rate. This method yields the smoothest forecast of the three, but one that perhaps does not react enough to short-term movements in the variables.

These three measures of i_t^F, along with ex-post x_{t+1} and the 3-month T-bill rate as a measure of i_t^P, are shown in Figure 2. Clearly, while the forecasts differ at short horizons, they agree on the following big-picture facts: (1) i_t^F does not match up with i_t^P, neither in terms of slope nor intercept; (2) the i_t^F-series exceeds the i_t^P-series almost everywhere, which is consistent with the theoretical bounds $i_t^P \in [0, i_t^F]$. The reader will notice that this is not a sophisticated forecast. But since the raw ex-post outcome x_t is also shown, the reader may agree that a more sophisticated forecast is unlikely to change the story.

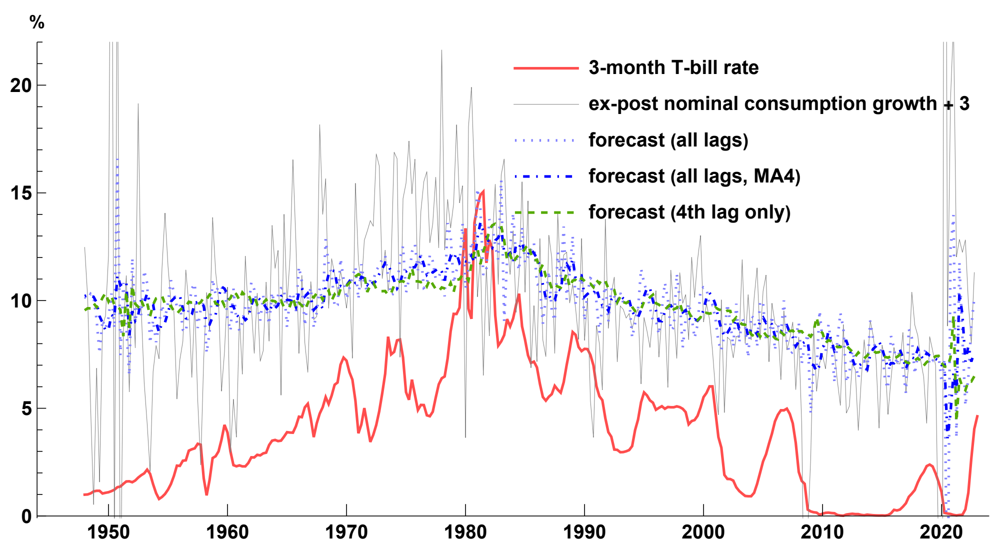


Figure 2: Policy rate i_t^P versus the ex-post Fisher rate x_{t+1} and various estimates of the ex-ante Fisher rate i_t^F. $\rho = 3\%$ is assumed.

One crucial parameter is not well identified: the time discount rate β, or equivalently the rate of time preference $\rho \equiv 1/\beta - 1$. Standard practice in macroeconomics has been to calibrate time preference to observed interest rates, but now an interest rate must *also* provide information about the liquidity of that particular asset. For now, we maintain $\rho = 3\%$ as a working hypothesis; it is the minimal ρ that is consistent with the constraint $i^P \leq i^F$ being satisfied throughout the dataset (save for a brief moment around 1980). Experimental and field studies that directly measure time preferences tend to find much larger values of $\rho \geq 10\%$ [2, 7]; if these numbers are correct, and representative of the population, then the implied Fisher rate – the opportunity cost of storing wealth as money – is much larger yet than shown in Figure 2.

3.2 Long-run relationship between i^F and i^P

As explained earlier, the correlation between first differences of x_t and i_t^P is actually negative. But x_t is measured ex-post – what about its forecastable component, i_t^F? Its correlation with i_t^P, in first differences, is still nearly zero. The long run is another matter, however: the Fisher equation (3) is a steady-state relationship and, as such, is only supposed to hold in the long run. In order to investigate this possibility, we split the dataset into 4-year bins and average i_t^P and i_t^F (now identified as x_t, without any attempt at a short-term forecast) in each bin. Figure 3 shows the result.

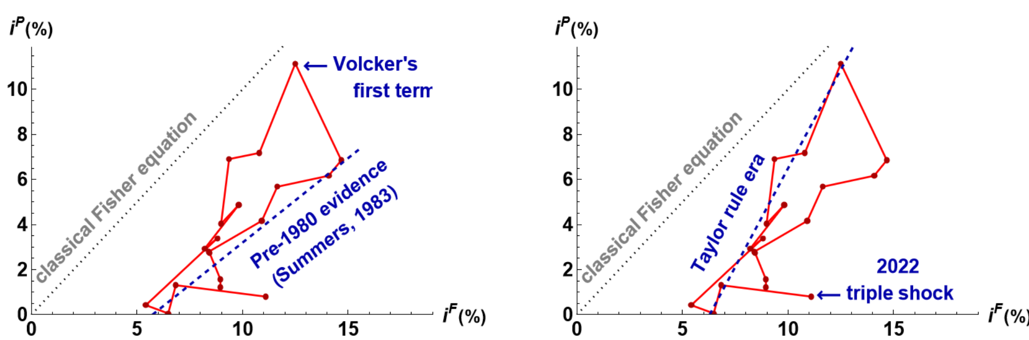


Figure 3: Four eras of U.S. monetary policy: i_t^P versus i_t^F from 1948-2022, averaged in 4-year bins (last bin is 2020-22). $\rho = 3\%$ is assumed. Trend lines are drawn with slope 0.75 (left) and 2 (right).

The classical hypothesis implies a slope of 1 and an intercept of 0 in these figures. With only 19 data points, a a slope coefficient of 1 cannot be rejected, but an intercept of 0 is clearly ruled out – liquid bonds carry a liquidity premium. But we are not trying to *estimate* the Fisher equation, we are looking for insight into *systematic violations* of it. In particular, monetary policy before 1980 was *not* conducted by setting interest rates; instead, a variety of money supply targeting schemes were employed (from the Gold Standard through the Monetarist Experiment), and interest rates were either a secondary target or left to adjust freely. Indeed, Summers [51] estimated a slope coefficient of inflation of 0.75, using data going back to the 1920s. This estimate is consistent with Equation (2) of the informal model: expected inflation (via i^F) and tightness of credit (via B/M) were varying autonomously and i^P was attenuated according to function $\mathcal{G}$.

Something changed, however, just as Summers' study was published: in his first term as Chair of the Federal Reserve, Paul Volcker raised policy interest rates ($i^P \uparrow$) via restrictive open-market operations ($B/M \uparrow$ in Equation 2), which brought down inflation and, in the short term, even growth ($i^F \downarrow$). Such a regime shift is also consistent with Equation (2), but it would not have been possible according to the classical Fisher equation (3).

In subsequent years, the U.S. Federal Reserve's policy has been consistent with a Taylor rule whereby it adjusts the policy rate strongly in response to inflation movements and modestly in response to GDP movements. We see the signature of this policy regime in the right panel of Figure 3: i^F reflects the sum of inflation and consumption growth, and indeed the 1980-2020 long-term pass-through of i^F to i^P was almost exactly 2. The two data points representing 2008-2015, encompassing the zero-lower-bound era, do not stand out, since nominal GDP growth was almost as stable as interest rates in that time.

Finally, the 2022 "triple shock" (pandemic-era fiscal expansion running into a supply-chain crunch and the Russia-Ukraine war) can be clearly seen in the most recent data point on the bottom right of the graph, even though this data point averages the years 2020-2022: a sudden run-up in inflation that had not yet been matched by higher interest rates as of 2022. But who doubts that the Fed's aggressive tightening cycle in 2023 will result in a counterclockwise turn on the graph, like Volcker's did a generation ago?

3.3 International comparisons

The postwar U.S. data can only take us so far in evaluating a truly long-term relationship. 75 years, when averaged into 4-year bins, only give us 19 data points, and 4-year windows might be too brief anyway. Therefore, in this section, we will test the Fisher equation against the *Macrohistory database* [34], a panel of annual data from 18 countries, stretching from 1870 to 2019. Following Jordà et al. [34], we identify the "long run" with a decade, and take geometric averages of nominal GDP growth (since consumption is not available for enough of the dataset) and short-term interest rates. (Unlike the arithmetic average, the geometric average represents the annualized *cumulative* rate of return from reinvesting wealth in short-term bills over a period of time, which is an important difference since this database spans some periods of extreme macroeconomic volatility.) After excluding the Germany-1920s pair (which includes the 1923 hyperinflation), we are left with 256 data points representing 18 countries over 15 decades. We graph the results in Figure 4, against Fisher equation benchmarks with time preference intercepts of 1%, 3%, and 5%.

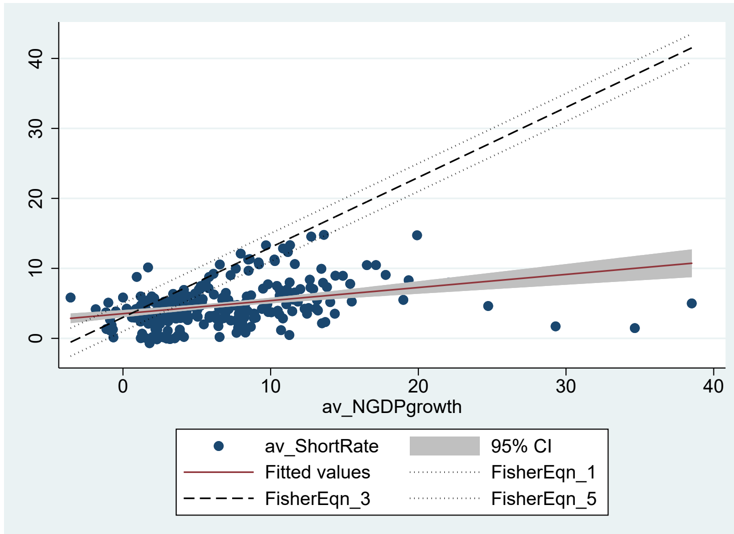


Figure 4: 256 country-decade averages of short-term interest rates and nominal GDP growth. "FisherEqn$X$" represents nominal GDP growth plus $X\%$ time preference, $X \in \{1, 3, 5\}$. The best-fit slope is 0.19, but the fit is poor ($R^2 = 0.12$).

The figure also includes a line of best fit from a linear regression, with a slope coefficient of 0.19 (robust standard error 0.05). A Hausman test prefers the fixed-effects estimate of a slope of 0.17 (0.04), but that is beside the point: for a Fisher equation relationship that is supposed to be structurally stable and theoretically supreme in the long run, the fit is terrible.

But just as we saw in the previous section, the *deviations* from the postulated Fisher relationship turn out to be more instructive than any statistical trend. Four data points in the bottom right of the graph stand out as having exceptionally high inflation coexist with very low interest rates. It turns out that all four are in the 1940s, raising our suspicion that the low interest rates might have been the outcome of war-related financial repression rather than free capital markets. On the other hand, our theory postulates an inequality $i^P \leq i^F$, which is violated in several data points on the top left of the graph. But it turns out that most of *those* come from the 1920s, when high interest rates and low growth were the outcome of attempts to return currencies to the gold standard *at the pre-World-War-1 parity*, i.e., requiring massive deflation, a policy now considered one of the causes of the Great Depression [19].

Consequently, to give the Fisher equation its best shot at matching the macrohistory data, we repeat the analysis for a reduced dataset that excludes all of the 1920s and 1940s and show the results in Figure 5. Now, the line of best fit has a slope coefficient of 0.40 (0.04), with the Hausman test still preferring the fixed-effects estimate of a slope of 0.38 (0.03). The fit is much better than in Figure 4, but still very far from supporting a structural relationship with slope coefficient of one and intercept in the low single digits, as the Fisher equation postulates. Instead, Figure 5 fits well with our theoretical model: with notably rare exceptions on both

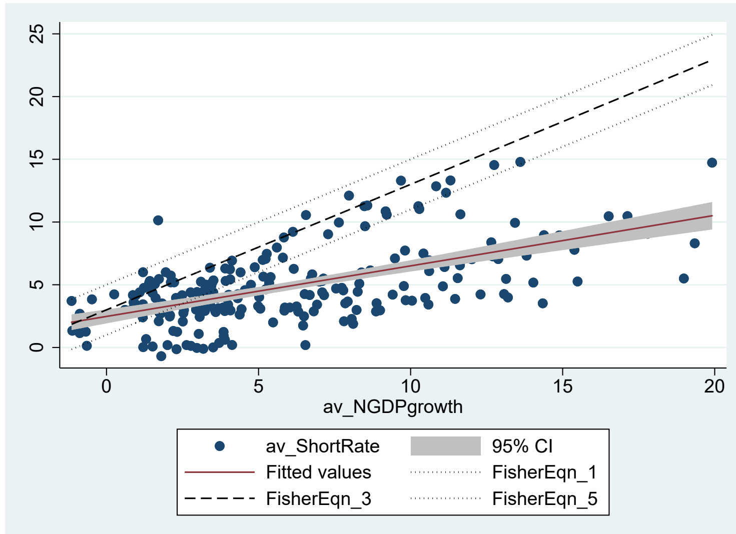


Figure 5: 239 country-decade averages of short-term interest rates and nominal GDP growth. “FisherEqn$X$” represents nominal GDP growth plus $X\%$ time preference, $X \in \{1, 3, 5\}$. The best-fit slope is 0.40, but the fit is still not great ($R^2 = 0.30$).

sides, interest rates fall in-between the zero lower bound and the Fisher equation upper bound $\rho + \theta g + \pi$, and a wide range of long-term combinations of interest rates, Fisher rates, and residual liquidity premia are observed in the data.

In analyzing the dataset from many different angles, we only found a single one that supports a nearly-Fisherian relationship: the Great Moderation, which we define as the three decades from 1980 to 2009. All country-decade pairs in this subsample averaged less than 15% inflation over a decade, except Portugal in the 1980s (which we therefore exclude). We show the results from our Great Moderation subsample in Figure 6; the line of best fit now has a slope of 1.14 (0.06) with a confidence interval that only just exceeds 1.

So, is this finally evidence in favor of the Fisher equation? Even here, the full story is more interesting. The slope from a fixed-effects estimate (which the Hausman test again favors) is 1.24 (0.07), now statistically significantly *larger* than one. What is going on? Just like we saw in the long-term U.S. data in the previous section, the era 1980-2009 is precisely when central banks started to actively set interest rates in order to achieve an inflation target, typically following the Taylor principle of reacting to inflation changes with a greater-than-one change in interest rates. The fact that the 1980-2009 era exhibited a historically strong relationship between nominal GDP growth and interest rates is precisely evidence that this relationship is *not* structural, but varies with the policy regime and the kinds of medium-run shocks affecting an economy.

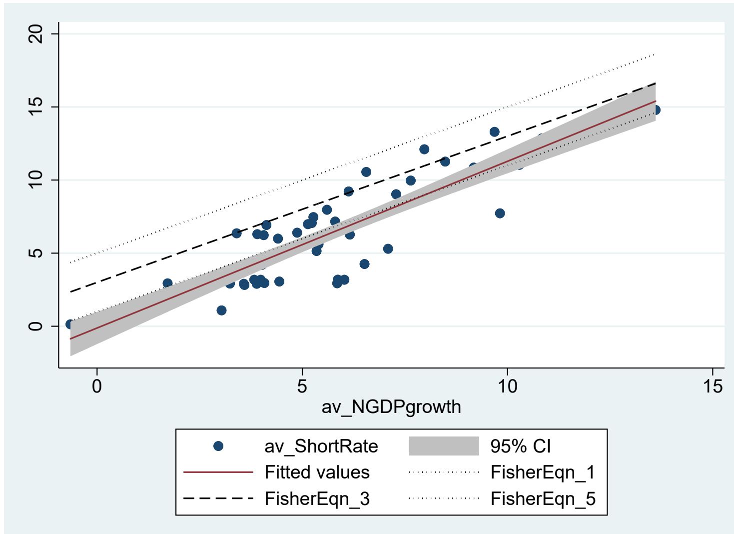


Figure 6: 53 country-decade averages of short-term interest rates and nominal GDP growth during the Great Moderation / Taylor Rule era. "FisherEqn$X$" represents nominal GDP growth plus $X\%$ time preference, $X \in \{1, 3, 5\}$. The best-fit slope is 1.14, with a good fit ($R^2 = 0.79$).

4 A formal model of monetary policy and asset pricing

In this section, we present a formal microfounded model which nests the informal model from Section 2 as a special case. Readers more interested in how our theory relates to (and differs from) canonical macroeconomic frameworks, and how it makes sense of monetary and asset pricing puzzles, can safely skip ahead to Sections 5 and 6.

4.1 Environment

We adapt the "Liquidity-Augmented Model" from Geromichalos and Herrenbrueck [23], with the following innovations. First, for simplicity we ignore capital and investment, and instead consider a Lucas tree as the generic asset which we seek to price in relation to money and bonds. Second, bonds are "short-term" in the sense that they mature (and turn into money) before all spending opportunities have materialized [25].

The economy consists of a unit measure of households and a consolidated government. Each household has two members: a worker and a shopper, who make decisions jointly to maximize the household's utility. All households are anonymous, therefore they cannot make long-term promises and trade must be quid-pro-quo.

Time $t = 0, 1, \dots$ is discrete and runs forever. Each period is divided into four subperiods: an asset market (AM), a production market (PM) further subdivided into an "early" and a "late" stage, and a centralized market (CM). There are three assets: fiat money (supply M_t) and one-period nominal discount bonds (B_t) issued by the government, and a Lucas tree in exogenous supply X_t. Each period, a fraction δ^X of trees is destroyed; new trees are created exogenously and given equally to all households. Changes in the stock of money and bonds can be implemented in one of two ways: either via open-market operations in the AM, or via lump-sum transfers in the CM. This environment is illustrated in Figure 7.

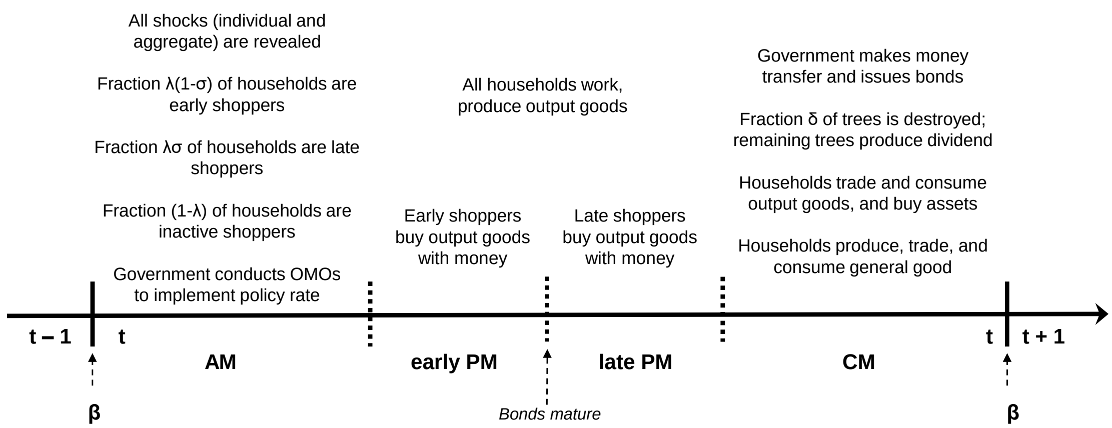


Figure 7: Timing of events in the formal model.

At the very beginning of a period, all shocks are revealed. A randomly selected fraction $\lambda_t(1-\sigma_t)$ of shoppers learn that they will enter the "early" PM where they can buy goods from workers; a fraction $\lambda_t\sigma_t$ learns that they will enter the "late" PM; the other shoppers $(1-\lambda_t)$ are "inactive" and will not participate in the PM in that period. This information induces differing liquidity needs among the shoppers, motivating trade in the AM. In the equilibrium we are interested in, early shoppers will seek to hold money (and sell bonds and trees, which are illiquid to them), late shoppers will prefer to buy bonds (since they mature and become money in time to be spent on consumption, hence bonds are liquid to them), and inactive shoppers will seek to earn a return on their idle money by spending it on bonds and trees. The government can also trade money and bonds in the AM, and we interpret the market price of bonds in the AM as the main policy instrument.

Next, the economy proceeds to the PM stage. Each worker produces y_t units of consumption goods at zero cost, and workers can choose freely whether to sell these in the "early" or "late" PM. Due to anonymity, credit is not feasible so shoppers must pay for the goods with a suitable medium of exchange. We assume that money is the only asset that can fulfill this role – an assumption here, but see [23] for a discussion of the microfoundations behind it – however, since bonds mature in time to take advantage of "late" consumption opportunities, shoppers faced with these opportunities will treat bonds effectively as money within that period. Thus, we can interpret the parameter σ, the fraction of PM trade that is "late", as the short-term-ness of the bonds: if $\sigma \to 1$, the bonds are truly cash equivalents.

The final subperiod, the CM, is where consumption takes place, and consumption can come from two sources: produced goods y_t or the fruit of Lucas trees, d_t per tree. (Thus, the aggregate supply of consumption goods satisfies $c_t = y_t + d_t X_t$, which we treat as exogenous for the purposes of this paper.) In the CM, all households can trade consumption goods among themselves, as well as all assets.

In summary, the three assets differ in their "liquidity" as follows. Money is the most liquid: it can be traded for any other commodity, in any market. Bonds are the next most liquid: a fraction η^B of them can be traded in the AM (or used as collateral for a monetary loan, in which case $1 - \eta^B$ is the haircut), and because they mature partway through the PM, a fraction σ of agents will effectively treat them as money there. Trees are the least liquid: only a fraction η^X of each tree can be traded (or used as collateral) in the AM, and while the math is general, we think of the case $\eta^X < \eta^B \leq 1$ as best representing reality.

Finally, there exists a "general" consumption good, $g \in \mathbb{R}$, which households can produce, consume, and trade during the CM (negative values mean production and positive ones mean consumption). This good is in zero aggregate supply; its only function in the model is to induce linear preferences for an individual household, collapse the portfolio problem into something tractable, and create a representative household. Households discount the future at rate $\beta < 1$ and have the following per-period utility function:

$$U_t(c_t, g_t) = u(c_t) + g_t,$$

where u is a twice continuously differentiable function that satisfies $u' > 0$ and $u'' < 0$.

4.2 Equilibrium

We start with the household's problem in the CM. Consider a household with portfolio (m_t, b_t, x_t, y_t) of money, bonds, the Lucas tree, and output goods. The household chooses its consumption (c_t and g_t), as well as the asset portfolio $(m_{t+1}, b_{t+1}, x_{t+1})$ to be carried into the next period. The prices of general goods (p_t^G), output goods (p_t), bonds (p_t^B), and Lucas trees (p_t^X; all in terms of money) are taken as given, and the transfer of money from the government (T_t) is also taken as given. Let V^{CM} and V^{AM} denote the value functions in the CM and AM subperiods, respectively. We can describe the household's choice as follows:

$$\begin{aligned} V^{CM}(m_t, b_t, x_t, y_t) &= \max_{c_t, g_t, m_{t+1}, b_{t+1}, x_{t+1}} \left\{ u(c_t) + g_t + \beta \mathbb{E}\{V^{AM}(m_{t+1}, b_{t+1}, x_{t+1})\} \right\} \\ \text{s.t. } & p_t c_t + p_t^X x_{t+1} + p_t^G g_t + m_{t+1} + p_t^B b_{t+1} \\ &\qquad = p_t y_t + p_t(1 - \delta^X) d_t x_t + p_t^X [(1 - \delta^X) x_t + \tilde{x}_t] + m_t + b_t + T_t, \end{aligned}$$

where $\tilde{x}_t$ represents newly created Lucas trees that are endowed to the household.

Next, in the PM, active shoppers decide how much output good (y_t) to purchase, and workers decide which market to sell the output good. Define the prices of the the output good in the early and late goods markets as p_t^E and p_t^L, respectively. Then, it must be that $p_t^E = p_t^L$.[7] Now, define $q = p_t^E/p_t = p_t^L/p_t$. Let $\Lambda_{t+1} \in \{0, 1\}$ denote whether an individual shopper is an active shopper ($\Lambda_t = 1$) or inactive shopper ($\Lambda_t = 0$). Let $S_t \in \{0, 1\}$ denote whether an active shopper is a late shopper ($S_t = 1$) or an early shopper ($S_t = 0$). Finally, let V^{PM} denote the value function in the PM. We can describe the household's choice as follows:

$$V^{PM}(m_t, b_t, x_t, \Lambda_t, S_t) = \max_{y_t} \left\{ V^{CM}(m_t + S_t b_t - p_t q_t y_t, (1 - S_t) b_t, x_t, y_t) \right\},$$
$$\text{s.t. } y_t \leq \Lambda_t \frac{m_t + S_t b_t}{p_t q_t}.$$

Finally, consider the AM. After the shocks Λ_t and S_t have been realized, early shoppers will want to sell assets other than money, while late shoppers will seek to sell Lucas trees but buy bonds, since their consumption opportunities arrive after bonds mature. Inactive shoppers will want to buy bonds and Lucas trees. Let (χ_t, ξ_t) denote the amounts of bonds and Lucas trees sold by an active shopper with $S_t = 0$. Let (ζ_t^B, ξ_t) denote the amount of money spent on buying bonds and the amount of Lucas trees sold by an active shopper with $S_t = 1$. Finally, let (ζ_t^B, ζ_t^X) denote the amounts of money spent on bonds and Lucas trees bought by an inactive shopper. Households take the prices of bonds and capital as given; we denote them by s_t^B and s_t^X in terms of money. Hence, we can describe the households' choices as follows:

$$\begin{aligned}
V^{AM}_{\Lambda_t=0}(m_t, b_t, x_t) &= \max_{\zeta_t^B, \zeta_t^X} \left\{ V^{CM}(m_t - \zeta_t^B - \zeta_t^X, b_t + \zeta_t^B/s_t^B, x_t + \zeta_t^X/s_t^X) \right\} \\
&\text{s.t. } m_t - \zeta_t^B - \zeta_t^X \geq 0; \\
V^{AM}_{\Lambda_t=1, S_t=0}(m_t, b_t, x_t) &= \max_{\chi_t, \xi_t} \left\{ V^{PM}(m_t + s_t^B \chi_t + s_t^X \xi_t, b_t - \chi_t, x_t - \xi_t, 1, 0) \right\} \\
&\text{s.t. } \chi_t \leq \eta_t^B b_t, \text{ and } \xi_t \leq \eta_t^X x_t; \\
V^{AM}_{\Lambda_t=1, S_t=1}(m_t, b_t, x_t) &= \max_{\chi_t, \xi_t} \left\{ V^{PM}(m_t - \zeta_t^B + s_t^X \xi_t, b_t + \zeta_t^B/s_t^B, x_t - \xi_t, 1, 1) \right\} \\
&\text{s.t. } m_t + s_t^X \xi_t \geq \zeta_t^B, \text{ and } \xi_t \leq \eta_t^X x_t.
\end{aligned}$$

We leave the detailed solutions to the above problems to the Appendix and highlight only the results here. The Euler equations for money, bonds, and Lucas trees are:

[7] *Proof*: suppose that $p_t^E < p_t^L$. First, note that $p_t^L \leq p_t$, since the output good is also traded in the following CM. Next, if $p_t^E < p_t^L$, no workers will sell their output good in the early market. However, early shoppers are willing to pay up to $p_t^E = p_t$ to buy the output good, which means that $p_t^E < p_t^L$ cannot be part of a competitive equilibrium. Similarly, $p_t^E > p_t^L$ cannot be part of a competitive equilibrium either.

$$\frac{u'(c_t)}{p_t} = \beta \mathbb{E}\left\{\frac{u'(c_{t+1})}{p_{t+1}}\left[\frac{\lambda_{t+1}\sigma_{t+1}}{s^B_{t+1}q_{t+1}} + \frac{\lambda_{t+1}(1-\sigma_{t+1})}{q_{t+1}} + \frac{1-\lambda_{t+1}}{s^B_{t+1}}\right]\right\} \tag{6}$$

$$\frac{u'(c_t)p^B_t}{p_t} = \beta \mathbb{E}\left\{\frac{u'(c_{t+1})}{p_{t+1}}\left[\frac{\lambda_{t+1}\sigma_{t+1}}{q_{t+1}} + \lambda_{t+1}(1-\sigma_{t+1})\left(1-\eta^B_{t+1} + \frac{\eta^B_{t+1}s^B_{t+1}}{q_{t+1}}\right) + 1 - \lambda_{t+1}\right]\right\}$$

$$\frac{u'(c_t)p^X_t}{p_t} = \beta \mathbb{E}\left\{(1-\delta^X)(p_{t+1}d_{t+1} + p^X_{t+1})\frac{u'(c_{t+1})}{p_{t+1}}\right.$$
$$\left.\ldots\left[1 - \lambda_{t+1}\eta^X_{t+1} + \frac{\lambda_{t+1}\eta^X_{t+1}[(1-\sigma_{t+1})s^B_{t+1} + \sigma_{t+1}]}{q_{t+1}}\right]\right\} \tag{7}$$

which share a common structure: the cost of obtaining the asset is matched on the right-hand side by a 'fundamental return' multiplied by a liquidity premium term that accounts for using the assets in a trade in the AM or PM.

But how does the monetary authority implement a particular policy rate? We define the *policy rate* as the implied yield on liquid bonds in the AM ($1 + i^P_t = 1/s^B_t$). Market clearing in the AM then implies that:

$$1 + i^P_t = \frac{1}{s^B_t} = \frac{\lambda_t(1-\sigma_t)}{1-\lambda_t(1-\sigma_t)}\left[\frac{\eta^B_t B_t}{M_t} + \frac{\eta^X_t(1-\delta^X)[p_t d_t + p^X_t]X_t}{M_t}\right] \tag{8}$$

This is the short-term counterpart to Equation (2) of the informal model, defining the function $\mathcal{G}$. Seen this way, the secondary market yield on liquid bonds depends on the demand for money in that period, $\lambda_t/[1-\lambda_t(1-\sigma_t)]$, on the supply of liquid bonds relative to money, B_t/M_t, as well as the saleable market value of other liquid assets relative to the money supply. We can invert Equation (8) to obtain the bond quantity corresponding to a particular level of the policy rate:

$$B_t = \frac{1-\lambda_t(1-\sigma_t)}{\eta^B_t\lambda_t(1-\sigma_t)}(1+i^P_t)M_t - \frac{\eta^X_t}{\eta^B_t}(1-\delta^X)(p_t d_t + p^X_t)X_t \tag{9}$$

In the short run, the Fisher rate does not appear directly in either formula, but expectations of inflation and growth still affect these equations via the equilibrium price of trees and the level of real balances.

Define the *ex-post Fisher rate* as:

$$1 + i^F_{t+1} = \frac{u'(c_t)}{\beta u'(c_{t+1})} \cdot \frac{p_{t+1}}{p_t}$$

and impose a balanced growth path for consumption at rate g, assuming the elasticity of $u'(c)$ equals $-\theta$, and a balanced growth path for the money supply and price level at rate π.

Then, the long-run Fisher rate satisfies $1 + i^F = (1 + g)^\theta (1 + \pi)/\beta$. After linearizing (and defining $\rho = 1/\beta - 1$), we obtain Equation (3) from the informal model.

On this balanced growth path, the rest of our equilibrium variables satisfy:

$$
\begin{aligned}
q(i^F, i^P) &= \frac{(1 + i^P)\sigma\lambda + (1 - \sigma)\lambda}{1 + i^F - (1 - \lambda)(1 + i^P)} \\
p(i^F, i^P) &= \frac{\left[1 + \frac{\sigma}{1-\sigma}(1 + i^P)\right] M + \lambda\sigma(1 - \eta^B)B}{q(i^F, i^P)y} \\
p^X(i^F, i^P) &= \frac{(1 - \delta^X)[1 + \eta^X(i^F - i^P)/(1 + i^P)]d}{i^F - \eta^X(i^F - i^P)/(1 + i^P)} p(i^F, i^P)
\end{aligned} \tag{10}
$$

where linearizing the latter equation yields Equation (5) from the informal model, thereby demonstrating how the policy rate and the Fisher rate separately affect the pricing of a generic asset X. Note that any direct dependence on the policy rate disappears when $\eta^X = 0$, i.e., asset X is entirely illiquid.

And finally, we have the long-run version of Equation (8):

$$
\frac{B}{M} = \frac{1 - \lambda(1 - \sigma)}{\eta^B \lambda(1 - \sigma)}(1 + i^P) - \frac{(1 - \delta^X)[p(i^F, i^P)d + p^X(i^F, i^P)]\eta^X X}{\eta^B M}
$$

which is the formal counterpart of Equation (4), defining the 'money demand curve' $\mathcal{H}$.

5 Comparison with other frameworks

5.1 The New-Keynesian / Neo-Wicksellian framework

In the New Keynesian model, Equation (1) is called the "expectations-augmented IS curve" and interpreted as follows. i_t is set by monetary policy while expectations of future consumption and inflation are (to a first approximation) given; thus it is c_t, consumption today, that adjusts to make the equation true. This is the main way monetary policy affects the economy: higher interest rates reduce output via consumption demand.

However, even apart from questions of empirical fit of the IS curve itself, the idea that monetary policy can "set" i_t is problematic. In simple New Keynesian models, this is left as a black box. In models that open the box, such as [13] where i_t is set via fiscal interventions in households' budgets, the fiscal implications run counter to the facts.

Empirical estimation of a New Keynesian DSGE model sometimes allows for a "convenience yield" on Treasuries [14], observationally equivalent to the liquidity premium postulated by our model. However, this convenience yield is estimated as an independently varying noise term; the pass-through from the monetary policy rate to the rate in the consumer's Euler equation is still 1 after the noise is averaged out. Here, by contrast, we argue that the monetary policy rate can be set independently of the Fisher rate (which is determined in general equilibrium), and that the liquidity premium is best understood as the residual $i^F - i^P$.

5.2 Monetarism, old and new

In most of monetary theory, from money-in-the-utility-function beginnings [48] to the microfounded cutting edge [37], the main instrument of monetary policy is money growth which then determines prices and inflation, and ultimately interest rates via the Fisher equation (3). Inflation affects the economy through a real balance effect; generally, this hurts any kind of economic activity where cash is used. As in the New Keynesian model, higher interest rates therefore correlate with reduced output, but the mechanism is very different. Changing interest rates may be how monetary policy is *communicated*, but the true causal variable is expected inflation. This approach is problematic since expected inflation does not vary much over the business cycle in developed countries [29], and larger shifts over longer horizons do not correspond well with movements in interest rates (see Section 3).

Certainly, monetary economists have known for a long time that some bonds are liquid, and these have a lower yield because of a liquidity premium [21]. However, the full implications of this fact have gone underappreciated even in monetary theory. Part of the reason is that in some models (e.g., [53]), the bond liquidity premium appears as a constant markdown $\bar{\mu}$ such that:

$$i_t^P = (1 - \bar{\mu}) i_t^F$$

So even though i^F and i^P are now distinct rates, as may be accounted for in a calibration, it is still the case that policy changes to i_t^P must correspond one-to-one to changes in i_t^F. This does not help much. The risk-free rate puzzle is resolved (liquid bond rates are low because of the liquidity premium), but other puzzles remain (e.g., the structural shifts in the link between i_t^P and inflation). The solution lies in models where i_t^P also depends on the quantity of liquid bonds relative to money, so it can be implemented via open-market operations *independently* from the evolution of i_t^F. This is quite easy to accomplish within New Monetarist microfoundations, as we demonstrate with our own model in Section 4.

5.3 The natural rate of interest

The Fisher rate i^F and the "natural rate" are related concepts which share a mathematical expression (Equation 3) and reflect the same economic forces: time preference, expected growth, and expected inflation. But there are important conceptual differences in the interpretation of these rates and how we think of their representation in the data. First, the natural rate arises in sticky-price models as the interest rate that would prevail if prices were flexible;

here, prices are already flexible, and the reason observed rates differ from the Fisher rate is due to the moneyness of bonds. Second, the Fisher rate is the theoretical *upper bound* on the policy rate (and on all short-term safe rates), whereas the natural rate is supposed to be the *average* of actual rates over time. Third, more than just being the average, the natural rate is supposed to be an *attractor* of policy rates, at least in a determinate model (given 'active' policy, e.g. the Taylor principle in a New Keynesian model). Not so here: through altering the money supply (relative to bonds and relative to demand), the monetary authority can pick any $i^P \in [0, i^F]$, hence there is no theoretical reason to expect an attractor to exist.

5.4 Barnett's user cost of money

Barnett [10] observed that many assets are valued for their liquidity services, and proposed to account for them as follows: construct a broad monetary aggregate where assets other than narrow money are weighted inverse to their *user cost*, which Barnett proposed to estimate as the interest rate on a fairly illiquid benchmark asset – possibly adding a linear offset of 3%. For an example, if the benchmark rate was 6%, and the yield on T-bills was 3%, then T-bills should enter the monetary aggregate with a weight of $(6\% - 3\%)/6\% = 0.5$.

Clearly, Barnett's formula for the user cost of money is closely related to our concept of the Fisher rate, and the formula for the weights corresponds to our concept of the liquidity premium on a specific asset divided by the Fisher rate. However, despite resembling this aspect of Barnett's theory, our model does not support its construction of a monetary aggregate.[8] The deep reason is our concept of *indirect liquidity*: many assets can be easily turned into money, but once this is done it is still money that is exchanged for goods and services. Other assets have no role in the quantity equation even as they are priced for their anticipated-but-indirect liquidity services.

6 Questions answered and puzzles resolved

A long list of questions and puzzles is resolved when we properly distinguish between the yields on liquid and illiquid bonds: how monetary policy can set interest rates in the first place, the lack of a Fisher effect in the short run, the post-2008 persistence of low inflation despite low interest rates, the risk-free rate puzzle, the equity premium puzzle, the UIP puzzle, and more. Each of these puzzles has stimulated its own theoretical and empirical literature, most with plausible improvements on the standard model, and we make no claim that any of these do not contribute to explaining the respective puzzle – or that our theory explains

[8]See Equation (A.5) in the Appendix: while the money supply relevant for PM trade can be expressed as a weighted sum of money and other assets, the weights are mainly composed of exogenous variables and cannot be expressed in terms of the assets' rates of return.

all of it. But there are vast disconnects between these literatures, as what explains one puzzle often fails find applications – or even empirical support – when applied to other areas of macroeconomics. Azariadis [9] puts it better than anyone: "Attempts to explain [basic asset pricing facts] involve narrowly specialized structural assumptions that would limit the applicability of any model in other areas of macroeconomics."

This is precisely the motivation of this section. We argue that a simple principle resolves, or at least improves in the correct direction, a vast array of puzzles spanning macroeconomics and finance: money is not a 'special' asset deserving to be studied in a "boutique subfield" (as Wright [55] bemoans), but central to macrofinancial analysis. Without taking substitution with money into account, all asset pricing theory is incomplete.

1. **How exactly is the monetary authority able to set interest rates?**

   As in Macro 101: via open-market operations in secondary asset markets. This works because the monetary authority controls the money stock (at the margin; the point is unchanged if private agents can create some money but cannot outcompete the monetary authority). Thus, they can achieve any $i^P \in [0, i^F]$.

2. **The awkward coexistence of the Fisher effect and a short-run liquidity effect**

   The Fisher effect applies to i^F and the liquidity effect applies to i^P. Specifically:

   Fisher effect: higher money supply $\Rightarrow$ higher inflation $\Rightarrow$ higher i^F

   Liquidity effect: higher money supply $\Rightarrow$ bonds are scarce $\Rightarrow$ lower i^P

   In the long run, the two rates may show comovement as the inequality $i^P \leq i^F$ asserts itself, but there is no structural correlation, especially across regime shifts (see Figure 3). And in the short run, any correlation is possible, depending on which shocks are hitting the economy. There is no Neo-Fisherian puzzle of policy rates affecting inflation causally, since policy rates do not obey the Fisher equation.

3. **The "lowflation" puzzle**

   The coexistence of low interest rates and low but positive inflation is a puzzle in the standard New Keynesian model, which predicts either accelerating inflation (in the standard equilibrium) or deflation (in the liquidity trap equilibrium), unless the "natural" real interest rate has cratered (the secular stagnation hypothesis) [16]. Not so here: since Equations (2)-(3) hold even in a monetary steady state, coexistence between a low policy rate and a high Fisher rate is no problem (see also [1]).

4. **Did the U.S. run the Friedman rule in 2009-2014 and 2020-21?**

   Certainly not. See Figure 2: the policy rate was zero in that time but the Fisher rate was far from it. We cannot be sure about ρ, but even eliminating time preference altogether implies $i^F_{2014} \approx 4\%$ and $i^F_{2020} \approx 3\%$. Our preferred value of $\rho = 3\%$ (which corresponds to $i^F \approx i^P$ during their Volcker-era peaks) implies $i^F_{2014} \approx 7\%$ and $i^F_{2020} \approx 6\%$; historical lows, but still far from the Friedman rule.

5. **The risk-free rate puzzle**

   Resolved almost by construction, since a liquidity premium reduces the yields on liquid bonds below their fundamental levels.

6. **The equity premium puzzle**

   Explained by Lagos [36]: even if equity is *almost* as liquid as bonds, a small difference is enough to account for the equity premium. See also Equations (5) and (10).

7. **The positive term premium**

   Explained by Geromichalos et al. [25]: short-term assets are inherently more liquid because they turn into money when they mature. Long-term assets must be liquidated instead, which can be subject to delays and/or transaction costs.

8. **The prominence of a "liquidity factor" in empirical asset pricing**

   Observed by Liu [43], and explained by Equation (5).

9. **The uncovered interest parity puzzle**

   Violations of UIP – interest rate changes that do not correspond to matching exchange rate movements – can be explained by differential liquidity of the bonds involved [42, 39], and exchange-rate movements more broadly also support the liquidity story [20].

10. **The forward guidance puzzle**

    Iterating Equation (1) forward, consumption today should be affected by all future rates equally, without discounting of far-future rates. This is implausible and not supported by experience [15]. Common explanations include excess discounting via bounded rationality or a lack of common knowledge [8]. We have no reason to discount these explanations, but the Euler equation for money from our formal model (Equation 6) offers an additional one: future interest rates on the right-hand side of the Euler equation are discounted by σ and $(1 - \lambda)$, both less than one.

11. **The fact that interest rates do not forecast consumption growth well**

    Noted by Hall [28], and many others since [31]. Equation (6) suggests two explanations:

    (i) The fact that $\lambda_t > 0$ on average (liquidity is valued);

    (ii) The possibility that λ_t is positively correlated with interest rates and/or consumption growth (it may even be driving the cycle), which soaks up explanatory power.

12. **The long-run volatility of the risk-free rate of return**

    Jordà et al. [34] noted that the risk-free rate of return is more volatile than the risky one on a decade-by-decade basis, and more variable between countries. This is a puzzle for standard asset pricing theory where the risky rate equals the safe one plus a risk premium. It is no problem for the Liquidity-Augmented Model, however, as Equation (5) shows: the return on safe-and-liquid assets (i^P) is governed by monetary policy, whereas the return on risky-and-illiquid assets (r^X with $\eta^X \to 0$) is governed by fundamentals (the Fisher rate plus the risk premium). So no matter how volatile the risk premium and growth expectations are in the short term, in the long term such volatility may be averaged out, unlike changes in the stance of monetary policy that can be slow-moving and persistent.

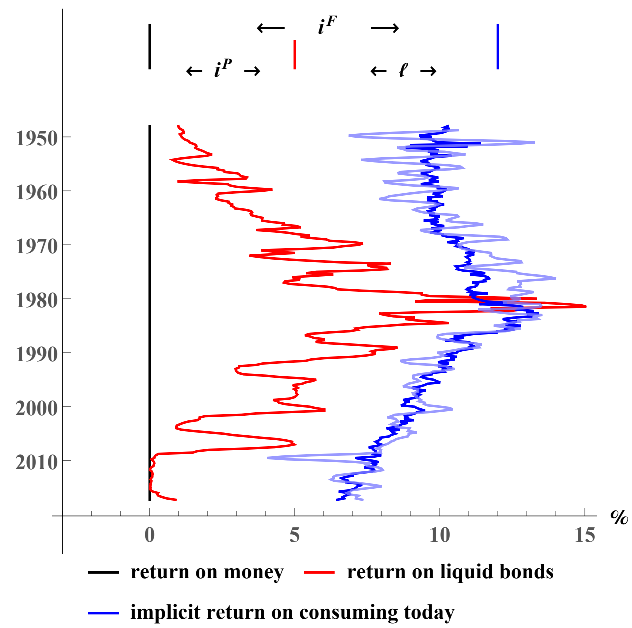


Figure 8: Black line: nominal return on the medium of exchange, which has traditionally been zero but may become something else if the means of payment of the future is an interest-paying CBDC. Red line: 3-month U.S. T-bill rate. Blue lines: estimates of i_t^F as forecast of U.S. nominal consumption growth (see Section 3).

7 Summary

We close by returning to the question that opened this paper: what is the opportunity cost of holding zero-interest cash? The answer depends on what we compare it to. If the alternative is spending the cash on consumption right now, then the opportunity cost equals the return i^F on a virtual, illiquid bond, which must be estimated rather than just observed. This return is composed of two spreads (Figure 8) which affect the economy in different ways. First, the opportunity cost of holding money versus a liquid bond, which is the interest rate i^P. Second, the opportunity cost of holding a liquid bond versus buying consumption right away, which is the aggregate liquidity premium ℓ. Both of these spreads – the spread from 0 to i^P and the spread from i^P to i^F – combine to define the stance of monetary policy.

Appendix

From the CM problem, we have

$$\partial_m V^{CM} = \partial_b V^{CM} = \frac{\partial_x V^{CM}}{(1-\delta^X)[p_t d_t + p_t^X]} = \frac{\partial_y V^{CM}}{p_t} = \frac{u'(c_t)}{p_t}.$$

The PM problem implies that so long as $q_t \leq 1$, we have $y_t = \Lambda_t \frac{m_t + S_t b_t}{p_t q_t}$. First order conditions of the AM problem imply the following:

$$\begin{aligned}
\partial_m V^{CM} + \theta_t^M &= \frac{\partial_b V^{CM}}{s_t^B}, & \partial_m V^{CM} + \theta_t^M &= \frac{\partial_x V^{CM}}{s_t^X}, \\
\partial_b V_{10}^{PM} + \theta_t^B &= s_t^B \partial_m V_{10}^{PM}, & \partial_x V_{10}^{PM} + \theta_t^X &= s_t^X \partial_m V_{10}^{PM} \\
\partial_m V_{11}^{PM} + \theta_t^M &= \frac{\partial_b V_{11}^{PM}}{s_t^B}, & \partial_x V_{11}^{PM} + \theta_t^X &= s_t^X \frac{\partial_b V_{11}^{PM}}{s_t^B},
\end{aligned}$$

where, for the ease of exposition, we define $V_0^{AM} = V_{\Lambda_t=0}^{AM}$, $V_{10}^{AM} = V_{\Lambda_t=1,S_t=0}^{AM}$, and $V_{11}^{AM} = V_{\Lambda_t=1,S_t=1}^{AM}$. Furthermore, we have

$$\begin{aligned}
\frac{\partial_b V^{CM}}{s_t^B} = \frac{\partial_x V^{CM}}{s_t^X} \Leftrightarrow & \frac{\partial_m V^{CM}}{s_t^B} = \frac{(1-\delta^X)[p_t d_t + p_t^X]\partial_m V^{CM}}{s_t^X} \\
\Leftrightarrow & s_t^X = (1-\delta^X)[p_t d_t + p_t^X] s_t^B,
\end{aligned}$$

and

$$\begin{aligned}
\partial_m V_0^{AM} &= \frac{\partial_b V^{CM}}{s_t^B} = \frac{u'(c_t)}{s_t^B p_t} \\
\partial_m V_{10}^{AM} &= \partial_m V_{10}^{PM} = \frac{\partial_y V^{CM}}{p_t q_t} = \frac{u'(c_t)}{p_t q_t} \\
\partial_m V_{11}^{AM} &= \frac{\partial_b V_{11}^{PM}}{s_t^B} = \frac{\partial_y V^{CM}}{s_t^B p_t q_t} = \frac{u'(c_t)}{s_t^B p_t q_t} \\
\partial_b V_0^{AM} &= \partial_b V^{CM} = \frac{u'(c_t)}{p_t} \\
\partial_b V_{10}^{AM} &= \partial_b V_{10}^{PM} + \eta_t^B \theta_t^B
\end{aligned} \tag{A.1}$$

$$
\begin{aligned}
&= \eta_t^B s_t^B \partial_m V_{10}^{PM} + (1-\eta_t^B)\partial_b V_{10}^{PM} \\
&= \left(\eta_t^B \frac{s_t^B}{q_t} + 1 - \eta_t^B\right) \frac{u'(c_t)}{p_t} \\
\partial_b V_{11}^{AM} &= \partial_b V_{11}^{PM} = \frac{\partial_y V^{CM}}{p_t q_t} = \frac{u'(c_t)}{p_t q_t} \\
\partial_x V_0^{AM} &= \partial_x V^{CM} = (1-\delta^X)(d_t + p_t^X/p_t)u'(c_t) \\
\partial_x V_{10}^{AM} &= \partial_x V_{10}^{PM} + \eta_t^X \theta_t^X \\
&= \eta_t^X s_t^X \partial_m V_{10}^{PM} + (1-\eta_t^X)\partial_x V_{10}^{PM} \\
&= \left[1 - \eta_t^X + \frac{\eta_t^X s_t^B}{q_t}\right](1-\delta^X)(d_t + p_t^X/p_t)u'(c_t) \\
\partial_x V_{11}^{AM} &= \partial_x V_{11}^{PM} + \eta_t^X \theta_t^X \\
&= \eta_t^X s_t^X \frac{\partial_b V_{11}^{PM}}{s_t^B} + (1-\eta_t^X)\partial_x V_{11}^{PM} \\
&= \left[1 - \eta_t^X + \frac{\eta_t^X}{q_t}\right](1-\delta^X)(d_t + p_t^X/p_t)u'(c_t). \qquad \text{(A.2)}
\end{aligned}
$$

The first-order conditions of the CM problem imply the following:

$$
\begin{aligned}
\frac{u'(c_t)}{p_t} &= \beta \mathbb{E}\{\lambda_{t+1}\sigma_{t+1}\partial_m V_{11}^{AM} + \lambda_{t+1}(1-\sigma_{t+1})\partial_m V_{10}^{AM} + (1-\lambda_{t+1})\partial_m V_0^{AM}\}, \quad \text{(A.3)} \\
\frac{u'(c_t)p_t^B}{p_t} &= \beta \mathbb{E}\{\lambda_{t+1}\sigma_{t+1}\partial_b V_{11}^{AM} + \lambda_{t+1}(1-\sigma_{t+1})\partial_b V_{10}^{AM} + (1-\lambda_{t+1})\partial_b V_0^{AM}\}, \\
u'(c_t) &= \beta \mathbb{E}\{\lambda_{t+1}\sigma_{t+1}\partial_x V_{11}^{AM} + \lambda_{t+1}(1-\sigma_{t+1})\partial_x V_{10}^{AM} + (1-\lambda_{t+1})\partial_x V_0^{AM}\}. \qquad \text{(A.4)}
\end{aligned}
$$

Substitute (A.1)-(A.2) into (A.3)-(A.4) to get (6)-(7). On the balanced growth path, we have:

$$
\begin{aligned}
1 + i^F &= \frac{(1+i^P)\sigma\lambda}{q} + \frac{(1-\sigma)\lambda}{q} + (1-\lambda)(1+i^P), \\
(1+i^F)\frac{p^X}{p} &= (1-\delta^X)(d + p^X/p)\left[\lambda\left[1 - \eta^X + \frac{\eta^X[1-\sigma+\sigma(1+i^P)]}{(1+i^P)q}\right] + 1 - \lambda\right] \\
&= (1-\delta^X)(d + p^X/p)\left[\lambda\left[1 - \eta^X + \eta^X\frac{1+i^F-(1-\lambda)(1+i^P)}{(1+i^P)\lambda}\right] + 1 - \lambda\right] \\
&= (1-\delta^X)(d + p^X/p)\left[1 - \lambda + \frac{\eta^X(1+i^F) + (\lambda-\eta^X)(1+i^P)}{1+i^P}\right] \\
&= (1-\delta^X)(d + p^X/p)[\eta^X(1+i^F)/(1+i^P) + 1 - \eta^X]
\end{aligned}
$$

So:

$$
q = \frac{(1+i^P)\sigma\lambda + (1-\sigma)\lambda}{1+i^F-(1-\lambda)(1+i^P)} \qquad \text{and} \qquad \frac{p^X}{p} = \frac{(1-\delta^X)[1+\eta^X(i^F-i^P)/(1+i^P)]d}{i^F - \eta^X(i^F-i^P)/(1+i^P)}
$$

Finally, in the PM, late shoppers obtains $1/s^B$ units of bonds for each unit of money they have. Hence, the total money spent in the PM is equal to

$$
\begin{aligned}
M_t^{PM} = & \underbrace{\lambda_t(1-\sigma_t)M_t}_{\text{Money carried by early shoppers from the CM}} \\
& + \underbrace{s_t^B \lambda_t(1-\sigma_t)[\eta_t^B B_t + (1-\delta^X)[p_t d_t + p_t^X]\eta_t^X X_t]}_{\text{Money obtained by early shoppers from the AM}} \\
& + \underbrace{\frac{\lambda_t \sigma_t M_t + s_t^B \lambda_t \sigma_t (1-\delta^X)[p_t d_t + p_t^X]\eta_t^X X_t}{s_t^B}}_{\text{Bonds obtained by late shoppers from the AM}} \\
& + \underbrace{\lambda_t \sigma_t B_t}_{\text{Bonds carried by late shoppers from the CM}}
\end{aligned}
$$

Substitute in the AM market clearing condition (8) to get

$$
\begin{aligned}
M_t^{PM} =& \lambda_t(1-\sigma_t)M_t + [1-\lambda_t(1-\sigma_t)]M_t \\
& + (1+i_t^P)\lambda_t\sigma_t M_t + \lambda_t\sigma_t[(1-\delta^X)[p_t d_t + p_t^X]\eta_t^X X_t + \eta_t^B B_t] + \lambda_t\sigma_t(1-\eta_t^B)B_t \\
=& M_t + (1+i_t^P)\lambda_t\sigma_t M_t + (1+i_t^P)\frac{\sigma_t[1-\lambda_t(1-\sigma_t)]}{1-\sigma_t}M_t + \lambda_t\sigma_t(1-\eta_t^B)B_t \\
=& \left[1 + \frac{\sigma_t}{1-\sigma_t}(1+i_t^P)\right] M_t + \lambda_t\sigma_t(1-\eta_t^B)B_t
\end{aligned} \tag{A.5}
$$

Hence, on the balanced growth path, we have

$$
p = \frac{M^{PM}}{qy} = \frac{\left[1 + \frac{\sigma}{1-\sigma}(1+i^P)\right] M + \lambda\sigma(1-\eta^B)B}{\frac{(1+i^P)\sigma\lambda + (1-\sigma)\lambda}{1+i^F-(1-\lambda)(1+i^P)} y}
$$

Substitute this into (9) to get

$$
\begin{aligned}
\frac{B}{M} =& \frac{1-\lambda(1-\sigma)}{\eta^B \lambda(1-\sigma)}(1+i^P)\ldots \\
& -(1-\delta^X)d\frac{1+\frac{\sigma}{1-\sigma}(1+i^P) + \lambda\sigma(1-\eta^B)B/M}{\frac{(1+i^P)\sigma\lambda+(1-\sigma)\lambda}{1+i^F-(1-\lambda)(1+i^P)}y} \frac{1+i^F-\delta^X[1+\eta^X(i^F-i^P)/(1+i^P)]}{i^F-\eta^X(i^F-i^P)/(1+i^P)} \frac{\eta^X X}{\eta^B}
\end{aligned}
$$

$$
\Rightarrow \frac{B}{M} = \frac{\frac{1-\lambda(1-\sigma)}{\eta^B\lambda(1-\sigma)}(1+i^P) - (1-\delta^X)d\frac{1+\frac{\sigma}{1-\sigma}(1+i^P)}{\frac{(1+i^P)\sigma\lambda+(1-\sigma)\lambda}{1+i^F-(1-\lambda)(1+i^P)}y}\frac{1+i^F-\delta^X[1+\eta^X(i^F-i^P)/(1+i^P)]}{i^F-\eta^X(i^F-i^P)/(1+i^P)}\frac{\eta^X X}{\eta^B}}{1+\frac{(1-\delta^X)d\lambda\sigma(1-\eta^B)}{\frac{(1+i^P)\sigma\lambda+(1-\sigma)\lambda}{1+i^F-(1-\lambda)(1+i^P)}y}}
$$

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