NAME: Student Number:

STAT 450
Final Examination:

Instructions: This is an open book exam. You may use notes, books and a calculator. I am permitting people access to their computers for the purpose of looking at their notes or the text or other material on line. I am permitting use of Wolfram alpha or maple or mathematica or R. I am NOT permitting communication with other people. The exam is out of **60**. You should have **15** pages including this one and two pages for extra work. **DON'T PANIC.**

GRADE SHEET

1a		5	1b		3						
2a		2	2b		3						
3a		5	3b		5						
4a		5	4b		2	4c		5	4d		3
5a		5	5b		6	5c		1	5d		2
6a		3	6b		5						

Total		60

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1. Suppose $U_1, \ldots, U_n$ are independent and identically distributed with density

$$f(u;\lambda) = \begin{cases} \lambda^2 u e^{-\lambda u} & u > 0 \\ 0 & \text{otherwise} \end{cases}$$

for some unknown parameter $\lambda > 0$.

(a) Find the likelihood, log-likelihood, and score functions. [5 marks]

(b) Find the maximum likelihood estimator, $\hat{\lambda}$, of λ. [3 marks]

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2. Suppose $X_1, \ldots, X_n$ are independent and identically distribution Normal(μ, σ^2) random variables.

   (a) Use the central limit theorem (and the definition of the χ^2 distribution on $n-1$ degrees of freedom) to prove that

   $$\sqrt{n}(s^2 - \sigma^2) \Rightarrow N(0, \psi^2)$$

   and give a formula for ψ^2. Here s is the sample standard deviation. [2 marks]

   (b) Now prove that

   $$\sqrt{n}\log(s/\sigma) \Rightarrow N(0, \tau^2)$$

   and give a formula for τ. [3 marks]

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3. Suppose X has a $N(0, \sigma^2)$ distribution. Consider the null hypothesis that $\sigma = 1$ and the alternative that $\sigma = 2$.

   (a) Find the critical region which minimizes $\alpha + \beta$, the sum of the probability of a Type I error and the probability of a Type II error. Give the simplest formula you can. [5 marks]

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(b) For the region you found in the previous part what are the values of α and β? Again, give the simplest formula possible; you don't need to get a calculator out to produce numerical values. [5 marks]

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4. Suppose that $x_1, \ldots, x_n$ are n constants (values of a non-random covariate). Suppose that $Y_1, \ldots, Y_n$ are independent. The variable

$$U_i = x_i Y_i$$

has an Exponential distribution with rate λ.

(a) What is the Cramér-Rao lower bound for the variance of an unbiased estimate of λ? [5 marks]

(b) Is there an estimate which achieves that lower bound? [2 marks]

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(c) Consider estimating $\mu = 1/\lambda$ using

$$\tilde{\mu} = aT$$

where

$$T = \sum x_i Y_i.$$

For what value of a is the MSE of $\tilde{\mu}$ minimized. [5 marks]

(d) Give an explicit formula for the approximate standard error of the maximum likelihood estimator of λ. [3 marks]

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5. Consider an experiment to compare two methods. We have two groups of subjects. In the first group Method 1 gives X_1 successes in n_1 trials. In the second group Method 2 gives X_2 successes in n_2 trials. Use the notation p_i for $i = 1, 2$ for the probability of success for each of the two methods. Define

$$\phi = \log\left(\frac{p_1/(1-p_1)}{p_2/(1-p_2)}\right).$$

(a) Find the maximum likelihood estimates, $\hat{p}_1$, $\hat{p}_2$ and $\hat{\phi}$ of p_1, p_2 and ϕ. [5 marks]

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(b) Find approximate standard errors and estimated standard errors for $\log\{\hat{p}_1/(1-\hat{p}_1)\}$ and $\hat{\phi}$. [6 marks]

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(c) Explain how to use the answers to the previous part to find confidence intervals for ϕ. [1 mark]

(d) Is there an unbiased estimate of ϕ? Explain. [2 marks]

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6. Suppose $X_1, \ldots, X_n$ are independent Bernoulli(α) variables. Let $X = \sum_i X_i$. Assume $n \geq 3$.

   (a) Show that
   $$P(X_1 = 1|X = k) = \frac{k}{n}$$
   and
   $$P(X_1 = 1, X_2 = 1|X = k) = \frac{k(k-1)}{n(n-1)}$$
   and
   $$P(X_1 = 1, X_2 = 1, X_3 = 1|X = k) = \frac{k(k-1)(k-2)}{n(n-1)(n-2)}$$
   for $n \geq 3$. [3 marks]

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(b) Find the UMVUE of $\phi = \alpha(1-\alpha)(2\alpha-1)$. [5 marks]

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