LESSON PLAYS: PLANNING TEACHING versus TEACHING PLANNING

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Planning for instruction is an important and integral part of the complex activity of teaching. Learning how to plan for instruction continues to challenge teacher educators, who seek effective ways of supporting prospective teachers in this endeavor. Among different options available, creating “lesson plans” continues to be a popular one. In fact, almost everyone who has undergone a formal teacher education program has had to devise a lesson plan according to some prescribed format. We conjecture that almost no one, having become a teacher (even a very good one), plans lessons according to this same format. If so, why is there such discordance between what successful teachers do and what prospective teachers learn to do? How can teacher educators support and foster preparation for the practice of teaching a lesson, without turning that preparation into an activity of filling tables of rubrics? [1]

We examine the roots of the traditional lesson and address several studies of teachers’ planning. We then consider an example of a traditional plan and analyse its characteristic features and potential for success. With these features in mind, and somewhat as a counterpart to them, we introduce the notion of “lesson play,” in which part of a lesson is presented in dialogue format between a teacher and students. The “lesson play” is offered as a means to support the preparation for a lesson, which involves, as Lampert (2001) argues, both the work involved in being able “to teach a lesson, but also to learn from whatever happens in the lesson” (p. 119). However, in contrast with the lesson plan, our model of preparation is one that speaks to the possible, the contingent, and the imaginative. We provide an example of a lesson play and discuss the implementation of this strategy with a dual agenda— as a professional development tool for teachers and as a window for researchers to investigate “mathematical knowledge for teaching” (Ball & Bass, 2003; Davis & Simmt, 2006; Hill et al., 2007) and its various components.

Traditional lesson planning

The roots of the traditional instructional planning in general, and lesson planning in particular, can be traced to the work of Ralph Tyler (1949). His framework is based on four components: specifying objectives, selecting learning experiences for attaining objectives, organizing learning experiences, and evaluating effectiveness of learning experiences. Tyler considered the specification of objectives “the most critical criteria for guiding all the other activities of the curriculum-maker” (p. 62). Elaboration of Tyler’s ideas resulted in a variety of instructional design models, whose common components are the identification of: goals and objectives, a teacher’s and students’ activities (teaching and learning strategies), materials to be used in a lesson, feedback and guidance for students, and assessment/valuation procedures determining whether the identified objectives have been met (Freiberg & Driscoll, 2000).

The practical implementation of these models resulted in the creation of a variety of forms or templates, many of which do not explicitly embody the ideals and theories that justify their existence. As such, when a prospective teacher is handed a template, she is not receiving the full benefit of the work that went into creating it, but rather an empty shell that stands in the place of grounded theories of teaching practice. These templates have been criticised in the scholarly literature (see John, 2006; Maroney & Searcey, 1996) for oversimplifying what it means to teach, as well as for failing to consider how teachers actually plan. Of course, they are worth criticizing if and only if they are used as proxies for preparation, which often can be how they appear to future teachers. Future teachers can easily assume that the clear identification and organization of content outcomes will result in the acquisition of this same content by the students. We know now that the articulation of objectives, although necessary, is far from sufficient when planning for teaching.

Research from the 1970s and 1980s showed that specifying objectives is not a central part of teachers’ planning (Peterson et al., 1978; Zahorik, 1970). Yinger (1980) found that when using Tyler’s model “no provision was made for planning based on behavioral objectives or previously stated instructional goals” (p. 124). More recently, John (2006) conducted a comprehensive analysis and critique of the dominant Tylerian model and its extensions. He argued that the emphasis on “outcome-based education” has “led to teaching based on a restricted set of aims, which can in turn misrepresent the richer expectations that might emerge from constructive and creative curriculum documents” (p. 484), and that the approach does not acknowledge elements of teaching “that are not endorsed by the assessment structure” (p. 485). However, as Maroney and Searcy (1996) point out, the results of these studies have also had little influence on current practice: “teacher educators are not assisting teachers or their students by continuing to teach only traditional comprehensive lesson planning models, knowing that the majority of teachers will not use those models” (p. 200).

Why, despite the ongoing criticism and acknowledgement that “real teachers do not plan that way” has the traditional rational model sustained its popularity? John (2006) suggests several interrelated reasons. He believes that “much
of the attraction of this approach to planning lies in its elegant simplicity” (p. 485). Other identified reasons arise in the beliefs that prospective teachers need to know how to plan in a rational-traditional framework before they can attend to the complexities of particular curricular elements, national curriculum documents have prescribed the model for teachers to follow, the model creates unified agreement between school practice and teacher education institutions, and the use of the model reinforces a sense of control based on prediction and prescription.

In the mathematics education literature, teacher-researchers such as Lampert (2001) have shown how expert planning and preparation for teaching a lesson involves extensive work in connecting particular mathematics to particular students, moving back and forth between mathematics and the structure of tasks appropriate for particular learners. Thus, Lampert begins by designing mathematical tasks, but the implementation of these tasks shifts in accordance with students’ responses. Yinger might describe this type of by-the-seat-of-your-pants teaching as improvisation, for which the traditional lesson plan is ill-suited, to say the least, but which may well require even more preparation. [2] While the Yinger-style of improvisation may appear favourable, when handled by someone like Lampert, it may also quite easily draw on teachers’ own schooling experiences, which, as Lortie (1975) has argued, contribute to an “apprenticeship of observation” that cannot easily be changed, and may well align with outcome-based education.

In recent studies comparing Japanese and American teachers, Fernandez and Cannon (2005) examined what teachers think about when constructing mathematics lessons. Their results, which show differences in terms of responsiveness to students, can be summarized in terms of a content-versus-process interplay. That is, American teachers’ thinking emphasized students’ learning of specific content, while Japanese teachers attended to the process of students’ learning, focusing on the discovery of concepts. They also find that teachers differed in their attitudes towards planning – Japanese teachers considered planning as an important part of their work, reported spending on it a significant amount of time and “conveyed an attitude that good planning is difficult to achieve” (p. 494), while American teachers reported spending only a modest amount of time on preparation of lessons. Fernandez and Cannon conclude that “although it is important for teachers to emphasize the learning of content, this cannot be done without attending to how students learn content” (p. 494) and they see as “disconcerting” the fact that most American teachers did not describe the need to craft specific elements of their lessons to attend to students’ learning.

In this article we introduce the “lesson play,” which we propose might provide a novel juxtaposition to the traditional planning framework as a method of preparing to teach a lesson. These two exercises structure the act of preparation in two fundamentally different ways, and while the affordances of the lesson play may be layered on a modified lesson plan rubric, they are the defining principle and the unavoidable result of a lesson play. Before elaborating these affordances, we consider the specific values of a good “lesson plan.”

**Lesson plan: an example**

Figure 1 presents an example of a good “lesson plan.” We work from the premise that it is important to attend to one specific plan in order to illustrate its affordances before focusing on the components of preparation it necessarily ignores. Following a possible variation of the Tylerian model, this plan clearly identifies learning objectives, sets procedures for attaining these objectives, and specifies the procedures for evaluation. We note the following among other positive aspects of a lesson to be carried out according to this plan:

- Students are engaged in an activity of producing rectangular arrays. This occurs after the teacher has provided clear directions and illustrated using 6 as an example.
- Students are using manipulatives to construct the array.
- The teacher attempts to mediate between the students’ work with concrete objects and the mathematical ideas of prime and composite numbers.
- The teacher asks students to make observations based on a completed table. This represents a thoughtful attempt to build on students’ ideas rather than simply provide information.
- Students have an opportunity to share their ideas and observations regarding the patterns they see.
- The lesson is organized so that the main concept – prime numbers – can be built out of reflection on the activity.
- Evaluation procedures are set to check the degree to which the concepts of prime and composite numbers has been built.
- There is an opportunity for students who complete their work before their classmates to extend/challenge their understanding by exploring numbers greater than 100.

The plan appears to present a constructivist student-centred approach, in which concepts are built through reflection on an activity. Like an abstract, or a book review, it is descriptive, and thus summarises what a good lesson would look like. However, as John (2006) points out, “the model does not take into account contingencies of teaching” (p. 487). He further states that while the standard approach to lesson planning presents a “powerful generic idea, it tells us very little about the substance of the particular activity we apply it to” (ibid.). While economic, and perhaps even iconic, this particular lesson plan ignores:

- what definition for a prime number the teacher might use in relation to the manipulatives and the students’ prior experiences;
- what observations might emerge from considering the table;
LESSON PLAN

Objectives
SWAT
Model prime and composite numbers
Recognize prime and composite numbers
Define prime and composite numbers (explain which numbers are prime and which are not)

Materials
5–6 sets of 30 counters (pennies, cubes, chips)

<table>
<thead>
<tr>
<th>TEACHER’S ACTIVITY</th>
<th>STUDENTS’ ACTIVITY</th>
</tr>
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<tbody>
<tr>
<td>• Teacher provides instructions and exemplifies activity: <em>Our goal today is to make rectangular arrays from a given number of counters. We would like to make as many rectangular arrays as possible for any number. We will do this for every number from 2 to 30. For example, if we take 6 counters, they can be arranged in 1 row, in 1 column, in 2 rows and 3 columns or in 3 rows and 2 columns. So altogether we have 4 possible arrangements.</em></td>
<td>• Students working in groups of 3 build rectangular arrays. They record the information on the provided worksheet.</td>
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<tr>
<td>• Teacher asks students to consider the table they made and list what they notice.</td>
<td>• Students take notes.</td>
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<td>• Teacher asks students to share their notes.</td>
<td>• Students share observations about the table.</td>
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<td>• Teacher focuses on or explicitly provokes a specific observation: <em>which numbers can be built only in one row or in one column?</em></td>
<td>• Students list these numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29</td>
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<td>• Teacher asks why this is so.</td>
<td>• Students make suggestions.</td>
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<tr>
<td>• Teacher introduces the term “prime number” and describes what numbers are prime.</td>
<td>Students connect the notion of prime number to the table they created.</td>
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Evaluation:
Students are given a list of numbers between 5 and 100 and are asked to determine which of the numbers are prime.

Challenge:
Students are asked to find a prime number larger than 100 and explain why they think the number is prime.

Figure 1: Example of a ‘good’ lesson plan
• how students’ observations emerging from the table, which are not related to prime numbers, might be treated;
• what student difficulties are expected and how those might be addressed;
• what questions the teacher might use to assess or expand student understanding;
• what mathematical language might be introduced or supported.

These are not shortcomings of the specific lesson plan, but the artefacts of the planning structure, which is necessarily prescriptive and summative. The standard format for planning does not encourage, and at times does not leave room for, anticipation of faulty extensions, misconceptions, difficulties, possibilities for alternative explanations or examples, or consideration of interactions that take place in a lesson. Indeed, this is a lesson plan and not a teaching plan. But this only skirts the issue. In reality, the Tylerian planning framework, as well as variances of this framework, are explicitly designed to focus on predetermining outcomes; it is prescriptive.

Alternative structures
Given the limitation of the traditional model, several alternatives to traditional lesson planning have been suggested. For example, Egan (1988, 2005), who has also critiqued the Tylerian model, suggests creating frameworks that focus less on content delivery and more on the deployment of developmentally appropriate cognitive tools that foster the imaginative engagement of learners. However, while the role of the imagination in teaching and learning is masterfully outlined, the planning for instruction is reduced, yet again, to filling out templates of a pre-determined rubric.

As an alternative to traditional lesson planning, John advocates for a model that gradually adds layers to the Tylerian model. This model places the objective outcomes in the centre, and through a circular approach adds to this kernel additional consideration, or so called satellite components, without suggesting a fixed order. These components include, but are not limited to, key questions, students’ learning, professional values, resource availability, classroom control, and degree of difficulty of material. Though John’s model has the potential to capture many valuable aspects of teaching, it draws more on what experienced teachers do than on what novice teachers should learn to do. To elaborate, experienced teachers rely on their practice-based knowledge of students and of material in order to add layers to their plans, while novice teachers do not have sufficient resources to draw from. Moreover, while considering expert practice is important for novice teachers, this multi-layered model does not provide an instrument in which planning across the multiple layers can be captured and shared.

With particular attention to the teaching of mathematics, Japanese “lesson study” has more recently captured the attention of mathematics educators. Lesson study presents a unique approach to planning that involves a number of educators in a process of investigation, anticipation, implementation, reflection, and revision. The applicability of this process in preservice teacher education has limitations, however. The process is very time-intensive, requiring many hours of meetings spread over a long period. The process is heavily dependent on teachers’ experience to more effectively anticipate students’ reactions to specific activities. Researchers working within the context of lesson study have shown that anticipating student responses to questions and tasks stands out as one of the most challenging aspects of lesson study, especially for beginning teachers (Stigler & Hiebert, 1999).

Based on Davis and Simmt’s (2006) distinction between planned (or prescribed) and emergent (or proscribed) events, we see the act of preparing to teach as one that is interpretive in nature, and that shifts focus “from what must or should happen toward what might or could happen” (p. 147). We have seen how traditional lesson planning does little to encourage interpretive planning, through which prospective teachers might consider the different possibilities occasioned by a question or task, the different responses a student might offer, or conceptions a student might build. One of our motivations for designing the lesson play has been to engage pre-service teachers in honing their ability to predict and reflection on students’ reactions through interpretive explorations of possibilities.

Lesson play
We suggest a structure that zooms in on one specific aspect of a lesson – interaction with students in general and with students’ emerging conceptions in particular. Our alternative attends to John’s suggestion that “the lesson plan should not be viewed as a blueprint for action, but should also be a record of interaction” (p. 495). As such, we propose the construct of a “lesson play,” which presents a record of imagined interaction related to a particular students’ difficulty.

The idea of lesson play grew out of our frustration with students’ writing ‘good’ lesson plans that did not attend, or had no place to attend, to what we consider important features in planning for instruction. Over the past four years it evolved from general instruction of “write a play as an imagined interaction” to an explicit request to attend to a presented problematic, the way it could have emerged and the way in could be resolved.

Figure 2 exemplifies a good lesson play that we constructed around the faulty conception that prime numbers are those numbers NOT located on the multiplication table. This is a misconception that we have encountered often in our own work with students, young and old, and that has been highlighted in the literature (Zazkis & Campbell, 1996).

Illustrating lesson play
Although the lesson plan makes quite clear the content in focus (identifying prime numbers), the lesson play and the dialogue between the teacher and the students draws much more attention to the process through which that content will be communicated in the classroom. At a mathematical level, the imagined verbal exchanges necessarily bring into focus both the actual use of mathematical language in communicating and the forms in which ideas are explained or justified. At the pedagogical level, the imagined exchange suggests something about the very nature of learning even
LESSON PLAY

SCENE 1
[Students were given a list of numbers and asked to determine which ones are prime and which ones are composite, and to explain their decisions. After about 5 minutes of silent individual work, some students are halfway through the task, while others are still working. The teacher decides to check some of the work to ensure students are on the right track.]

TEACHER: So, class, let’s check what we’ve came up with so far. Please pay attention, I know you haven’t finished, you can continue later.

Let’s start with the first number on our list – 23. Is it prime or composite? Yes, Susan.

Susan: Prime.

TEACHER: Okay, and why do you say this?

Susan: Because nothing goes into it.

TEACHER: Goes into?

Susan: I mean nothing divides it.

TEACHER: Nothing? Nothing at all?

Maria: She means no numbers other than 23 and 1. You can write it as 23 times 1, but no other options.

TEACHER: Good. So rather than “nothing,” we say 23 has exactly 2 divisors, 23 and 1. Susan: And also when we worked with chips we could only put them in one long line, and you couldn’t make another rectangle without leftovers.

TEACHER: Indeed, excellent. Let’s move on. How about 34, is it prime or composite? Yes, Jamie.

Jamie: Composite.

TEACHER: And you say this because …

Jamie: Because it is even.

TEACHER: So? Please explain.

Jamie: We know it is even, right, and if it is even it has 2 in it.

TEACHER: Has 2 in it? Hmm, I see 34, I see a 3 and a 4. Where is the 2?

Maria: What he means is 2 is a factor. Even numbers have 2 as a factor, so it cannot be prime.

TEACHER: So you are saying that an even number cannot be prime?

Maria: Sure. All even numbers are 2 times something, so they are not prime. Primes are odd.

TEACHER: And what about the number 2?

Jamie: 2 is prime, and 2 is even.

TEACHER: So I’m confused here. Can you help?

Maria: Sure. No need for confusion. What I mean to say is 2 is an exception. It is the only even prime because it is in the very beginning. The other primes are odd. 2 is the only exception.

TEACHER: Okay, good. We figured this out. Let us proceed – 68?

Marty: Composite of course. We just said that even numbers, not 2, but bigger even numbers cannot be prime. So no need to go over even numbers on the list, they are all composite.

TEACHER: Does everyone agree? Great, so this makes our work easier, of course. Let’s go over odd numbers only. The next on our list is 49, Kevin?

Kevin: It is composite because … it almost looks like prime but then I remembered in my times tables it is 7 times 7. And the same is with the next one, 63, it is 7 times 9.

TEACHER: Very good. Your multiplication tables helped you decide. Okay. Now let us take a few more minutes and complete the work. If you have already decided whether each number is prime or composite, please turn to problem 7 on page 106.

SCENE 2

Students continue to work on their own. Some are just finishing up with the list of numbers provided while others have moved on to working on the problem in the text book.

TEACHER: Everyone finished? Good. Let’s check the rest of the numbers. How about 91?

Rita: 91 is prime.

TEACHER: And you say so because?

Rita: It is not anywhere on the times tables.

TEACHER: Interesting. So are you saying that only composite numbers are on our multiplication tables?

Rita: [hesitating] That’s what Kevin said and you said “Okay.”

TEACHER: What exactly did Kevin say?

Rita: That 49 is 7 times 7 and 63 is 7 times 9 on the times tables. And he is right, and you said “Okay,” and 91 is not there.

TEACHER: I see. When do we say that a number is prime?

Students: 2 factors only, no factors other than itself and 1.

TEACHER: So if 63 is 7 times 9, what do we know about its factors?

Tina: We know it has 7 and 9 as its factors.

TEACHER: Exactly, that is why it cannot be prime. But is it possible that 91 has factors that are not on our multiplication table?

Rita: [hesitating] No, I think, because it is smaller than 100.

TEACHER: Let’s look at 34. Can you find it on the
though it does not fall into any pre-fixed pedagogical “ism.”

In terms of the mathematical features, we elaborate on two main points. First, the lesson play deals explicitly with the use of mathematical language. Susan and the teacher negotiate meanings between “goes into” and “divides.” Later, Jamie and the teacher do the same for “has 2 in it” and “is even.” Both Jamie and Susan may see the teacher’s words as simple synonyms for their own, but in the lesson play, the teacher offers the more precise vocabulary that will be needed for effective communication about prime numbers, not just for Jamie and Susan, but for their classmates as well. The teacher’s responses not only offer alternative ways of talking about composite numbers, but also show how non-mathematical language such as “has 2 in it” can be communicatively misleading (since 34 clearly has no 2 in it). This close attention to language, and to the need for precision in communication cannot be separated from the content in question, but it is specific to the way in which the content is worked on in the classroom.

In addition to the language focus, the lesson play also makes explicit the various forms of mathematical reasoning that might emerge in the classroom. For instance, when Maria makes the argument that “all even numbers are 2 times something, so they are not prime,” the teacher evaluates the argument and proposes a counter-example. This occurs again with respect to Rita’s claim about composite numbers appearing on the times table. In both cases, the students have made quite a reasonable inference, perhaps even a necessary one given their current experiences, and the teacher must recognize them and then devise ways in which the students can come to more appropriate inferences. The actual counter-examples used by the teacher (2 for Maria and 39 for Rita) are highly specific in their responsiveness, and emerge directly from the dialogue.

In terms of the pedagogical features of the lesson play, we wish to draw attention to some aspects of its format. The structure of the lesson play – as a dialogue occurring over time with possibilities for different points of view – allows for the portrayal of the messy, sometimes repetitive interactions of a classroom. This structure stands in stark contrast to a necessarily ordered and simplified list of actions such as: take up homework, state definition, provide examples, give problems, and evaluate solutions. In this lesson play, we see the teacher revisiting definitions or “prime” and “composite” that were used in Scene 1 with the help of new ideas that emerge in Scene 2, such as the multiplication table. The lesson play communicates the fact that the meanings of definitions change for student as they encounter new examples or problems. It also probes the way in which student interpretations can lead to unexpected consequences.
For example, at the beginning of Scene 2, we see Rita defending her claim that 91 is prime because it’s not on the multiplication table: “That’s what Kevin said and you said ‘Okay.’” Here the teacher has the option of proposing a counter-example, returning to the definition of prime, or arguing about the context of her response to Kevin. The lesson play tests out these different options by ‘running’ them like a script and seeing how Rita (and other students) might respond. Being interpretations, these different options can now be critiqued, so that decisions can be evaluated. In contrast to a lesson plan, which may be “good” or “bad,” the lesson play, as an interpretation, invites questioning about the different ways in which teachers might respond to students, and the different conditions under which students might build understandings.

This leads to a final point about the lesson play that relates to its ‘playfulness’. By its very nature, the lesson play requires a focus on specific and particular imagined interactions. In a lesson plan, one can include directives such as “call on different students to answer questions.” In a lesson play, those students must be named, individually, and the lesson player has to decide quite explicitly whether, for example, Tina or Rita will answer a teacher’s question – the lesson player is forced to consider whether it is more important to make Tina follow through or to give Rita a chance to participate. This may, at one level, sound trivial, but we see it as part of the imaginative work that teachers must do to prepare and practice for the classroom – much the same way children practice routines of communication in their self-talk. By being forced to make a choice, one must follow through with the consequences of each option, and one might even find it necessary to evaluate the outcomes of different choices. Further, the lesson player must do this imaginative work not only for the teacher (the role she will eventually play), but also for the students – the lesson player must try to think or talk like a student. We conjecture that this type of role-playing might help teachers develop better models of students’ conceptual schemes (see Steffe & Thompson, 2000). While crafting lesson plays cannot replace real experiences of teaching or of listening to student ideas, it can help teachers develop a larger repertoire of possible actions and reactions.

Conclusion

Lesson planning, as has been mentioned, is limited in its ability to allow teachers to prepare for teaching. Its very structure is built around generalities and well laid plans in the absence of students’ questions and alternate conceptions of the topic being taught.

Having realized these limitations, attempts are made by teacher educators to introduce prospective teachers to students’ thinking by other means. Analysis of video-clips – which has gained popularity with the advances of video technology – is one way to draw attention to the detail of communication and is considered to be an effective tool in teacher education (Maher, 2008). This may include the study of effective teaching and the revisiting of one’s own teaching. Analysis of video-clips helps prospective teachers examine the relationship between teacher’s actions and students’ learning, study subtle details of classroom interactions and in such become more aware of their practice and inform their future planning.

However, not diminishing the importance of discussion and reflection provided by the examination of video-clips, we feel the lesson play requires prospective teachers to practice and play in the particulars of their own. Centrally, the lesson play provides an opportunity to imagine the future, being informed by the past, rather than reexamine the past. Its structure is built around the specific conceptions of a particular student, or group of students, learning the details of a mathematical concept, with the preciseness of mathematical language, through the relationship of teaching. It is not a description of how things will occur in the classroom, but an imagined account of how things might occur. We see this kind of interpretive exercise consonant with Maxine Greene’s (and others’) vision of aesthetic engagement, in which one experiencing the world as if, as a hypothetical world of “what is not yet, or what might, unpredictably, still be experienced” (1995, p. 62). For Greene, such engagement – in which we bring together prior experiences, reconfigure and reconstruct them, finding in them familiarity and strangeness – opens the doors to transformative teaching and learning.

We further hypothesise that through several instances of detailed planning for such detailed encounters a prospective teacher can build up general strategies that allow for improvisation in other contexts. In this, the instantiation of lesson plays that we are introducing and exemplifying here resembles the Italian theatre tradition of commedia dell’arte wherein actors rehearse particular characters and roles as a way to build up a repertoire of personas that they can flexibly apply to a large variety of scenes. This is the preparation required for effective improvisation.

While we have focused on the lesson play as a pedagogical tool, it can also be used as a research tool. Our forthcoming research investigates the prospect that lesson play serves both as a lens and a window – a lens through which prospective teachers can examine teaching and a window through which we, as teacher educators, can examine prospective teachers’ conceptions of teaching and student learning.

Notes

[1] Although the word “rubric” is often used in the context of assessment, we draw on its etymology roots: form the Latin rubrica (red ochre), rubric referred to directions given (in red) in prayer books, which were meant to help the congregation follow along. The sections of the standard lesson plan play a similar guiding role.


References


Show that the sum of the semi-circumferences of the two small (white) circles is equal to the semi-circumference of the large (light grey) circle, and find the centres of the two small (dark grey) circles.

(from Archimedes; selected by Leo Rogers of the Oxford Problem Café)